T. 1 - PARTÍCULAS E INTERACCIONES

Quarks   Leptones $s=1/2$ (fermiones), $Q=\pm 1/3, 2/3$

u d ν_e e^-

c s ν_μ μ^- → Modelo quark

t b ν_τ τ^-

MQ + REL → ∃ antimateria

qqq → Barión   & $q\bar{q}$ → Mesón

$\bar{q}\bar{q}\bar{q}$ → Anti "

Interacción → Bosones $Q=0,\pm 1$, $J=0,1$

$J=1$ $J=0$, $Q=0$

γ, G^a Higgs H

Z^0, $W^\pm$

Simetría gauge → $M=0$

3 réplicas $Q=2/3$ $Q=-1/3$

1ª fila $\boxed{u \quad d \quad \nu_e \quad e^-}$

Relativid especial

$c=1$

$ds^2 = (x_1 - x_2)^2$ → Invariante relativista

$ds^2 = ds'^2 = dt^2 - d\vec{x}^2$ → $\begin{cases} >0 & \text{temporal} \rightarrow t_2 \neq t_1, \exists\, d\vec{x}=0 \\ <0 & \text{espacial} \rightarrow \text{desconectados}, \exists\, dt=0 \\ =0 & \text{luz} \end{cases}$

g... x^μ → contravariante

x_μ → covariante

$ds^2 = c^2 d\tau^2 = c^2 dt^2\left(1-\frac{v^2}{c^2}\right)$ → $\tau = t\sqrt{1-\frac{v^2}{c^2}}$ = tiempo propio

$u^\mu = \frac{dx^\mu}{ds} = \gamma(1,\vec{v})$ → $u_\mu u^\mu = 1$

$P^\mu = mu^\mu$ / $P^2 = m^2$ → $E^2 = p^2 + m^2$

$= (E, \vec{p})$

$x'^\mu = \Lambda^\mu{}_\nu x^\nu$ (filas, columnas) / $x\cdot y = x'\cdot y'$ ⇒ $\boxed{\Lambda^T g \Lambda = g}$

- $(\det\Lambda)^2 = 1$ $\{ +, -$ sig$(\det\Lambda)$
- $(\Lambda^0{}_0)^2 \geq 1$ $\{ \uparrow, \downarrow$ sig$(\Lambda^0{}_0)$

→ $\mathcal{L} = \mathcal{L}_+^\uparrow + \mathcal{L}_+^\downarrow + \mathcal{L}_-^\uparrow + \mathcal{L}_-^\downarrow$ ← Inv. temp.

~~Propios (+)~~

$\mathcal{L}_+^\uparrow$: Propio ortocrono (rotas, boosts) $\mathcal{L}_-^\uparrow$: Parid

$\Lambda^\mu{}_\nu = g^\mu{}_\nu + \Delta\omega^\mu{}_\nu$ / $\Delta\omega_{\alpha\beta} = -\Delta\omega_{\beta\alpha}$

$T'^{\mu\nu} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta T^{\alpha\beta}$

$\Lambda_\mu{}^\nu = (\Lambda^{-1})^\nu{}_\mu$, $(\Lambda^{-1})^\beta{}_\nu$

$T'^\mu{}_\nu{}^\sigma = \Lambda^\mu{}_\alpha \Lambda_\nu{}^\beta \Lambda^\sigma{}_\gamma T^\alpha{}_\beta{}^\gamma$

$g'^{\mu\nu} = g^{\mu\nu}$ (tensor invariante)

$\epsilon^{\mu\nu\rho\sigma}$, $\epsilon^{0123} = +1$

$\Lambda = \begin{pmatrix} ch\omega & 0 & sh\omega \\ & 1 & \\ sh\omega & 0 & ch\omega \end{pmatrix}$ $\gamma = ch\omega$, $\beta = \frac{v}{c} = th\omega$

Sistema natural de unidades

$\hbar = c = 1$ $\quad (\hbar c = 197{,}33 \text{ MeV fm})$

↳ velocid = 1, acción = 1

$[E] = [m] = [\frac{1}{T}] = [\frac{1}{L}]$

Cinemática relativista

$p^2 = m^2$

$p_a^\mu + p_b^\mu \rightarrow \sum_{i=1}^{N} p_i^\mu \rightarrow 3N-4$

$s_{ij} = (p_i + p_j)^2 \rightarrow$ Masas invariantes $\geq (m_i + m_j)^2$

$t_{ai} = (p_a - p_i)^2 \rightarrow$ Transfer E de mom $\leq (m_a - m_i)^2$

$S \equiv S_{ab} \geq (\sum m_i)^2 \equiv S_{th}$

$\rightarrow p_i p_j \underset{= m_i m_j}{\text{máx}}$ si $\vec{v}_i = \vec{v}_j \rightarrow \frac{\vec{p}_i}{E_i} = \frac{\vec{p}_j}{E_j}$

→ Formulario SCM + SLAB

Campo EM

$\vec{\nabla} \cdot \vec{B} = 0 \quad ; \quad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

$\vec{\nabla} \cdot \vec{E} = \rho \quad ; \quad \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J}$

$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$

$\vec{B} = \vec{\nabla} \times \vec{A}$

$\downarrow J^\mu = (\rho, \vec{J}) \quad ; \quad A^\mu = (V, \vec{A})$

$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$ tensor campo EM $; \quad \partial_\mu = \frac{\partial}{\partial x^\mu}$

$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\sigma\rho} F_{\sigma\rho}$

$\downarrow$

$\partial_\mu \tilde{F}^{\mu\nu} = 0$

$\partial_\mu F^{\mu\nu} = J^\nu$

↳ $\partial_\nu J^\nu = 0 \rightarrow Q = \int d^3x \, \rho$ se conserva

$\rightarrow \Box A^\nu - \partial^\nu (\partial_\mu A^\mu) = J^\nu$

$A'^\mu = A^\mu + \partial^\mu \Lambda$

Gauge Lorentz : $\partial_\mu A^\mu = 0 \quad \rightarrow \Box A^\nu = J^\nu$

Part. libre $\Box A^\mu = 0 \rightarrow m = 0 \rightarrow \nexists$ sist. en reposo fotón, va a c.

$p^\mu = i\hbar \partial^\mu \quad ; \quad E = i\hbar \partial^0$

Helicid $\quad \lambda = \frac{\vec{J} \cdot \vec{p}}{|\vec{p}|} \qquad 2J+1$ polarizacs

$\lambda \ (m \neq 0) \rightarrow$ invariante relativista

Parid cambia $\lambda \rightarrow \exists$ 2 polarizacs γ (simetría EM)

FORMULARIO CINEMÁTICA RELATIVISTA

- $a + b \longrightarrow 1 + 2 + \ldots + N$

$s = (p_a + p_b)^2$

$\boxed{\text{S. C.M.}} \rightarrow \vec{p}_a^{\,*} + \vec{p}_b^{\,*} = \vec{0}$

$$E_a^* = \frac{s + m_a^2 - m_b^2}{2\sqrt{s}}$$

$$E_b^* = \frac{s - m_a^2 + m_b^2}{2\sqrt{s}}$$

$$|\vec{p}_a^{\,*}| = |\vec{p}_b^{\,*}| \equiv |\vec{p}| = \frac{\lambda^{1/2}(s, m_a^2, m_b^2)}{2\sqrt{s}}$$

$\boxed{\text{S. LAB}} \rightarrow \vec{p}_b^{\,L} = \vec{0}$

$$E_a^L = \frac{s - m_a^2 - m_b^2}{2m_b}$$

$$E_b^L = m_b$$

$\vec{p}_b^{\,L} = \vec{0}$

$$|\vec{p}_a^{\,L}| = \frac{\lambda^{1/2}(s, m_a^2, m_b^2)}{2m_b}$$

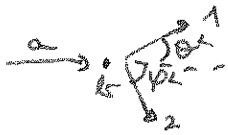
$N = 2$:

$$s \equiv (p_a + p_b)^2 = (p_1 + p_2)^2 \geq \max\left[(m_a + m_b)^2, (m_1 + m_2)^2\right]$$

$$t \equiv (p_a - p_1)^2 = (p_2 - p_b)^2 \leq \min\left[(m_a - m_1)^2, (m_2 - m_b)^2\right]$$

$$u \equiv (p_a - p_2)^2 = (p_1 - p_b)^2 \leq \min\left[(m_a - m_2)^2, (m_1 - m_b)^2\right]$$

$$s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2$$



$$E_1^* = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}$$

$$E_2^* = \frac{s - m_1^2 + m_2^2}{2\sqrt{s}}$$

$$|\vec{p}_1| = |\vec{p}_2| \equiv |\vec{p}\,'| = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}}$$

$$\cos\theta^* = \frac{s(t-u) + (m_a^2 - m_b^2)(m_1^2 - m_2^2)}{\lambda^{1/2}(s, m_a^2, m_b^2)\,\lambda^{1/2}(s, m_1^2, m_2^2)}$$

$$E_1^L = \frac{m_1^2 + m_b^2 - u}{2m_b}$$

$$E_2^L = \frac{m_2^2 + m_b^2 - t}{2m_b}$$

$$|\vec{p}_1^{\,L}| = \lambda^{1/2}(u, m_b^2, m_1^2)/2m_b$$

$$|\vec{p}_2^{\,L}| = \lambda^{1/2}(t, m_b^2, m_2^2)/2m_b$$

$$\cos\theta^L = \frac{t + 2E_a^L E_1^L - m_a^2 - m_1^2}{2\,|\vec{p}_a^{\,L}|\,|\vec{p}_1^{\,L}|}$$

$$t_\pm = \frac{1}{4s}\left\{(m_a^2 - m_b^2 - m_1^2 + m_2^2)^2 - \left[\lambda^{1/2}(s, m_a^2, m_b^2) \pm \lambda^{1/2}(s, m_1^2, m_2^2)\right]^2\right\}$$

$\rightarrow t_+ < t < t_-$

$$\tan\theta^L = \frac{\sin\theta^*}{\frac{|\vec{p}_b|^*}{m_b}\cdot\frac{E_1^*}{|\vec{p}_1^{\,*}|} + \frac{E_b^*}{m_b}\cos\theta^*} \underset{\sqrt{s} \gg m_i}{\approx} \frac{2m_b}{\sqrt{s}}\tan\left(\frac{\theta^*}{2}\right)$$

(exacta si $m_i = m_j$)

- $A \longrightarrow 1+2+3$ (A en reposo)

$t_i \equiv (P - p_i)^2$; $s_{jk} \equiv (p_j + p_k)^2$; $\sum_{i=1}^{3} t_i = m_A^2 + m_1^2 + m_2^2 + m_3^2$

$$E_1 = \frac{m_A^2 + m_1^2 - s_{23}}{2m_A} \qquad |\vec{p}_1| = \lambda^{1/2}(m_A^2, m_1^2, s_{23})/2m_A$$

$$E_2 = \frac{m_A^2 + m_2^2 - s_{13}}{2m_A} \qquad |\vec{p}_2| = \lambda^{1/2}(m_A^2, m_2^2, s_{13})/2m_A$$

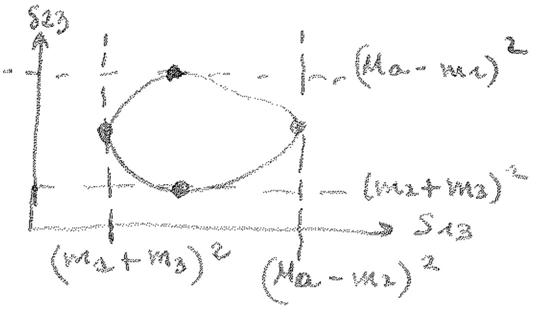
$$E_3 = \frac{m_A^2 + m_3^2 - s_{12}}{2m_A} \qquad |\vec{p}_3| = \lambda^{1/2}(m_A^2, m_3^2, s_{12})/2m_A$$

$\to s_{12}(s_{13}, s_{23}, m_i)$

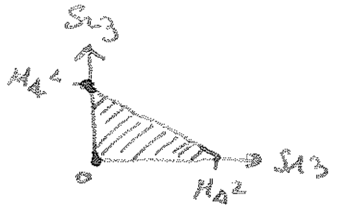
$(m_1 + m_3)^2 \leq s_{13} \leq (m_A - m_2)^2$ (idem intercambiando $i \leftrightarrow j \leftrightarrow k$)

$$s_{23\pm} = \frac{1}{4s_{13}}\left\{(m_A^2 - m_1^2 - m_2^2 + m_3^2)^2 - \left[\lambda^{1/2}(m_A^2, s_{13}, m_2^2) \pm \lambda^{1/2}(s_{13}, m_1^2, m_3^2)\right]^2\right\}$$

$s_{23+} < s_{23} < s_{23-}$

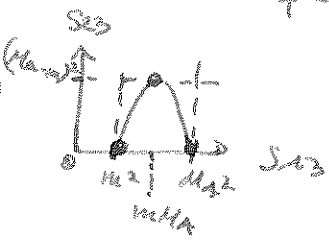


- $m_1 = m_2 = m_3 = 0 \longrightarrow s_{23\pm} = \begin{cases} 0 \\ M_A^2 - s_{13} \end{cases}$



(rompe esquina izqda)

- $m_1 = m,\ m_2 = m_3 = 0 \longrightarrow s_{23\pm} = \begin{cases} 0 \\ (M_A^2 - s_{13}) + m^2\left(1 - \frac{M_A^2}{s_{13}}\right) \end{cases}$



T. II - MECÁNICA CUÁNTICA RELATIVISTA

$[x^i, p^j] = i\hbar\,\delta^{ij}$

$[t, E] = -i\hbar$

$E = i\hbar\frac{\partial}{\partial t}$; $\vec{p} = -i\hbar\vec{\nabla}$ $\rightarrow E = \frac{p^2}{2m} + V(x)$

$\ast\; i\frac{\partial}{\partial t}(\psi^*\psi) = i\frac{\partial \rho}{\partial t} = -\vec{\nabla}\vec{j}$ $\longrightarrow \partial_\mu j^\mu = 0$

$Q = \int d^3x\, j^0(x)$

$\delta[f(x)] = \sum_{x_0} \frac{\delta(x-x_0)}{f'(x_0)}$ con $f(x_0) = 0$

$\delta(k^2 - m^2) = \frac{1}{2|k^0|}\{\delta(k^0+) + \delta(k^0 - \ldots$

$\left.\begin{array}{l}\longrightarrow E = p^2 + m^2 \\ p^\mu = i\hbar\partial^\mu\end{array}\right\}$ $(\Box + m^2)\,\psi(x) = 0$! Ec. Klein-Gordon

$P^\mu e^{\pm ipx} = \pm p^\mu e^{\pm ipx}$

Ondas planas $\rightarrow$ sol. ecuac K-G

$\hookrightarrow \psi(x) = \sum_k a(\vec{k})\cdot e^{-ikx} + b^+(\vec{k})e^{ikx}$; $\sum_k \equiv \int \frac{d^4k\,\delta(k^2-m^2)}{(2\pi)^3}\,\theta(k^0) = \frac{1}{(2\pi)^3}\int\frac{d^3k}{2k^0}$

$E \gtrless 0$

$\ast\; \psi^*(\Box + m^2)\psi - ((\Box + m^2)\psi^*)\psi$

$\partial_\mu j^\mu = 0$; $j^\mu = \frac{i}{2m}(\psi^* \overleftrightarrow{\partial_\mu} \psi)$; $Q = \frac{1}{2m}\sum_k \{|a(\vec{k})|^2 - |b(\vec{k})|^2\}$

$(i\gamma^\mu\partial_\mu - m)\,\psi(x) = 0$ Ec. de Dirac

$\hookrightarrow \hat{=}$ K-G, $\frac{1}{2}[\,] + \frac{1}{2}\{\,\}$

$\hookrightarrow \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$(i\gamma^0\partial_0 - m)\psi \longrightarrow i\frac{\partial}{\partial t}\psi = H\psi \rightarrow \begin{array}{l}H = H^+\\ p = p^+\end{array} \rightarrow \gamma^{\mu +} = \gamma^0\gamma^\mu\gamma^0$

$Tr(\gamma^\mu) = 0$; $\lambda(\gamma^0) = \pm 1$; $(\gamma^0)^2 = -(\gamma^i)^2 = 1$

$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\cdot\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \gamma_5^+$; $\epsilon^{0123} = +1$; $\gamma_5^2 = I$

$\epsilon \rightarrow$ impar bajo paridad

$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $\sigma^{\mu\nu +} = \gamma^0\sigma^{\mu\nu}\gamma^0$

$\{\gamma_5, \gamma^\mu\} = 0$; $[\gamma_5, \sigma^{\mu\nu}] = 0$

$P_{R,L} = \frac{1 \pm \gamma_5}{2}$; $P_R + P_L = 1$, $P_{R,L}^2 = 1$, $P_R\cdot P_L = 0$

$\gamma_5 \rightarrow$ autovalores $\pm 1 \rightarrow$ quiralidad

$\bar{\psi}$ Ec. D ψ + Ec. D.$(\bar{\psi})\cdot\psi \longrightarrow j^\mu = \bar{\psi}(x)\gamma^\mu\psi(x)$

$\psi'(x') = S(\Lambda)\psi(x)$

$S^{-1}\gamma^\mu S = \Lambda^\mu{}_\nu\gamma^\nu$ $\begin{array}{l}[\gamma_5, S] = 0 \text{ si propia}\\ \{\gamma_5, S\} = 0 \text{ '' improp}\end{array} \rightarrow \det(\Lambda)$

$\Lambda^\mu{}_\nu = g^\mu{}_\nu + \Delta\omega^\mu{}_\nu \longrightarrow S(\Lambda) = I_4 - \frac{i}{4}\sigma_{\mu\nu}\Delta\omega^{\mu\nu}$

$\bar{\psi}\psi$ es invariante Lorentz

$S(\Lambda)^{-1} = \gamma^0 S(\Lambda)^+\gamma^0$

$\not{k} = k_\mu\gamma^\mu$; $\not{k}^2 = k^2$

Para ondas planas $\Rightarrow (i\not\partial - m)\psi = (\not{k} - m)\psi \rightarrow (k^2 - m^2)\psi = 0$

$\left(\frac{1 \pm \gamma^0}{2}\right) \rightarrow$ proyectores ψ, χ

$\hookrightarrow$ masa m, espín $\frac{1}{2}$, $E > 0$

$\hookrightarrow$ '' '' '' , $E < 0$

$$\psi(x) = \sum_{K,r} \{ a_r(\vec{K}) u_r(\vec{K}) e^{-iKx} + b_r^+(\vec{K}) v_r(\vec{K}) e^{iKx} \}$$

$$\sum_K = \frac{1}{(2\pi)^3} \int \frac{d^3K}{2K^0} \;;\quad K^0 = + \sqrt{K^2 + m^2}$$

$$\overline{u_r}(\vec{K})\, u_s(\vec{K}) = 2m\delta_{rs}$$

$$\overline{V_r}(\vec{K}) V_s(\vec{K}) = -2m\delta_{rs}$$

$$\overline{U_r}(\vec{K}) V_s(\vec{K}) = \overline{V_r}(\vec{K}) U_s(\vec{K}) = u_r^+(\vec{K}) V_s(-\vec{K}) = 0$$

$$(\not{K} - m) U_r(\vec{K}) = 0 = \overline{V_r}(\vec{K})(\not{K} - m) = (\not{K} + m) V_r(\vec{K}) = \overline{V_r}(\vec{K})(\not{K} + m)$$

$$\Lambda_\pm = \frac{m \pm \not{K}}{2m} \quad \text{(proyector)}$$

$$\sum u_r \overline{u_r} = \not{K} + m$$

$$\sum V \overline{V} = \not{K} - m$$

$E = \pm K^0$ · $|E| \geq m$ → gap, mar de Dirac → falla $E<0$ → tiene $E>0$ (signo todo excepto K^0 y m.

Masa nula

$$H = \gamma^0 \vec{\gamma} \cdot \vec{p}$$

quiralidad ± 1 → helicidad $\pm\frac{1}{2}$

signo quiralidad y helicidad = si $m = 0$
$\downarrow$
γ_5

$H = p^2/2m + \frac{m\omega^2 x^2}{2} = \omega(N + \frac{1}{2})$

$N = a^+ a$

$a = (2m\omega)^{-1/2}[m\omega x + ip]$

$[x, p] = i$

$[a, a^+] = 1$, $[a, a] = [a^+, a^+] = 0$, $[N, a] = -a$, $[N, a^+] = a^+$ → operadores escalera

$\langle\psi|N|\psi\rangle \geq 0$

$N|n\rangle = n|n\rangle$, $n \geq 0$, $E_n = (n + \frac{1}{2})\omega$

Heisenberg

$|\psi\rangle_H = e^{iHt}|\psi\rangle_S$

$O^H = e^{iHt} O^S e^{-iHt} \rightarrow \frac{dO^H}{dt} = i[H, O^H(t)]$

$\frac{da}{dt} = -i\omega a \rightarrow a(t) = a(t_0)\cdot e^{-i\omega(t - t_0)}$

$S = \int d^4x \, \mathcal{L}(\phi_i(x), \partial_\mu \phi_i(x))$

$\delta S = 0 \rightarrow$ Euler - Lagrange $\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\right) = 0$

$L(t) = \int d^3x \, \mathcal{L} \rightarrow H = p_i \dot{q}_i - L(q_i, \dot{q}_i)$

$p_i = \frac{\partial L}{\partial \dot{q}_i}$, $\dot{q}_i = \frac{dq_i}{dt}$ $\mathcal{L}$ invariante

$H = \int d^3x \{\underbrace{\pi_i(x)\dot{\phi}_i(x) - \mathcal{L}}_{\mathcal{H}}\}$, $\pi_i(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_i(x)}$

$\dot{\phi}_i = \frac{\partial \mathcal{H}}{\partial \pi_i}$, $\dot{\pi}_i = -\frac{\delta \mathcal{H}}{\delta \phi_i} = -\left[\frac{\partial \mathcal{H}}{\partial \phi_i} - \partial^k\left(\frac{\partial \mathcal{H}}{\partial(\partial^k \phi_i)}\right)\right]$

$[\phi_i(\vec{x}), \pi_j(\vec{y})]_{x^0 = y^0} = i\delta_{ij}\,\delta^{(3)}(\vec{x} - \vec{y}) = [\phi_i^+(\vec{x}), \pi_j^+(\vec{y})]$; resto $= 0$

$i\frac{d}{dt}O^H = [O^H, H] \rightarrow O(t) =$ cte de movim. si $[O, H] = 0$

$U = e^{i\alpha^i T^i}$ unitario $(T^i = T^{i+})$, $[T^i, T^{j+}] = i f^{ijk} T^k$

$\delta H = 0 \rightarrow [T^i, H] = 0$ } generadores grupo simetría

$\exists \; j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\delta\phi_i - B^\mu$ / $\partial_\mu j^\mu = 0$ } $\mathcal{L}$ invariante salvo derivada total; Tma. de Noether

$Q = \int d^3x \, j^0(x)$ conservada

$\mathcal{L}(\phi, \phi^+)$

$\rightarrow j^\mu = i Q_i \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i^+)}\phi_i^+ - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\phi_i\right)$

$Q = i \sum_i Q_i \int d^3x \{\pi_i^+ \phi_i^+ - \pi_i \phi_i\}$

$[Q(y), \phi_i(x)] = -Q_i \phi_i(x)$ → ϕ_i destruye carga Q_i

$[Q, \phi_i^+(x)] = Q_i \phi_i^+(x)$ → ϕ_i^+ crea " "

$Q|q\rangle = q|q\rangle$

$\phi_i'(x) = U\phi_i U^+ = e^{-iQ_i\theta}\phi_i(x)$, $U = e^{i\theta Q}$

$\mathcal{L}$ → escalar bajo traslads

$j^\mu = a_\nu T^{\mu\nu} \rightarrow \partial_\mu T^{\mu\nu} = 0$

$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \partial^\nu \phi_i - g^{\mu\nu}\mathcal{L}$

$P^\nu \equiv \int d^3x \left[\pi_i(x)\partial^\nu \phi_i(x) - g^{0\nu}\mathcal{L}\right] = \int d^3x\, T^{0\nu} \equiv \int d^3x \left(\mathcal{H}(x), \pi_i \partial^k \phi_i\right)$

$= \left(H(x), -\int d^3x\, \pi_i \vec{\nabla}\phi_i\right)$

$[\phi_i, P^\mu] = i\partial^\mu \phi_i$

$[\pi_i, P^\mu] = i\partial^\mu \pi_i$

$P^\mu |p^\mu\rangle = p^\mu |p^\mu\rangle$

$P^\mu (\phi(x)|p^\mu\rangle) = (-i\partial_\mu \phi + p^\mu \phi)|p^\mu\rangle$

$k \to a^* = (p^\mu \underset{+}{-} k^\mu)\,\phi\,|p^\mu\rangle$ campos destruyen / crean cuadrimo = partícula
$\quad a^+$

tensor mo angular

$J^{\mu,\alpha\beta} = x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \Sigma^{\alpha\beta}\phi_i$

$J^{\alpha\beta} = \int d^3x\, J^{0,\alpha\beta}$

T. IV – PARTÍCULAS SIN ESPÍN

$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2$ K-G real

$\mathcal{H} = \pi\dot\phi - \mathcal{L} \geq 0 = \frac{1}{2}(\pi^2 + (\vec\nabla\phi)^2 + m^2\phi^2) \geq 0$

$\pi = \frac{\partial\mathcal{L}}{\partial\dot\phi} = \dot\phi$

$\phi(x) = \sum_k \left(a(\vec k)e^{-ikx} + a^+(\vec k)e^{ikx}\right)$; $\sum_k = \frac{1}{(2\pi)^3}\int \frac{d^3k}{2k^0} = \frac{1}{(2\pi)^3}\int d^4k\, \delta(k^2-m^2)\,\theta(k^0)$

$k^0 = \sqrt{m^2+\vec k^2}$

$\dot\phi(x) = -i\sum_k k^0 (\ - \ )$

$a(\vec k) = i\int d^3x \left[e^{ikx}\overleftrightarrow{\partial_0}\phi(x)\right]$ $\quad [f\overleftrightarrow{\partial^\mu}g] = f(\partial^\mu g) - (\partial^\mu f)g$

$\int d^3x\, e^{\mp ikx}\overleftrightarrow{\partial_0} e^{\pm ik'x} = \pm i \underbrace{2k^0(2\pi)^3\delta^{(3)}(\vec k - \vec k')}_{\Delta \vec k \vec k'}$

$\sum_{k'} \Delta\vec k\vec k' f(\vec k') = f(\vec k)$

$\int d^3x\, e^{\pm ikx}\overleftrightarrow{\partial_0} e^{\pm ik'x} = 0$

$\int dx\, e^{ikx} = 2\pi\delta(k)$

$a^+(\vec k) = \int d^3x\, e^{-ikx}[k^0\phi - i\pi]$

$[\phi(x), \pi(y)]_{x^0=y^0} = i\delta^{(3)}(\vec x - \vec y)$

$[\phi(x), \phi(y)]_{x^0=y^0} = [\pi(x),\pi(y)]_{x^0=y^0} = 0 = [a(\vec k), a(\vec k')] = [a^+(\vec k), a^+(\vec k')]$

$[a(\vec k), a^+(\vec k')] = \Delta\vec k\vec k'$

$N(\vec k) = a^+(\vec k)a(\vec k)$

$[N(\vec k), a(\vec k')] = -\Delta\vec k\vec k' a(\vec k)$; $[N(\vec k), a^+(\vec k')] = \Delta\vec k\vec k'\, a^+(\vec k')$

$H = \sum_k k^0\left[N(\vec k) + \frac{1}{2}[a(\vec k), a^+(\vec k)]\right]$

$P^\mu = \sum_k k^\mu\left(N(\vec k) + \frac{1}{2}[a(\vec k), a^+(\vec k)]\right)$

Lo que mides es $:H: = H - \langle 0|H|0\rangle$

$\langle 0|:H:|0\rangle$; $:H: = \sum_k k^0 N(\vec k)$ $\quad :P: = \sum k^\mu N(\vec k)$

$a^+(\vec k)|0\rangle = |\vec k\rangle$

$\langle \vec k|\vec k'\rangle = \Delta\vec k\vec k'$

$|\vec k_1, \vec k_2\rangle = |\vec k_2, \vec k_1\rangle$ → estado 2 partículas, simétrico → Sistema de Bosones idénticos

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Campo K-G complejo

$\mathcal{L} = \partial_\mu\phi^+\partial^\mu\phi - m^2\phi^+\phi$

$\phi(x) = \sum_k\left[a(\vec k)e^{-ikx} + b^+(\vec k)e^{ikx}\right]$

$\pi = \dot\phi^+$; $\pi^+ = \dot\phi = \frac{\partial\mathcal{L}}{\partial\dot\phi^+}$

$\mathcal{H} = \pi\dot\phi + \pi^+\dot\phi^+ - \mathcal{L} = \pi^+\pi + \vec\nabla\phi^+\vec\nabla\phi + m^2\phi^+\phi$

$[\phi, \pi] = i\delta^{(3)}(\vec x - \vec y) = [\phi^+(x), \pi^+(y)]$

$[a, a^+(\vec k')] = [b(\vec k), b^+(\vec k')] = \Delta\vec k\vec k'$

$\phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$

$\mathcal{L}_{\phi_i} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi_i - \frac{1}{2}m^2\phi_i^2$

$\mathcal{L}_\phi = \mathcal{L}_{\phi_1} + \mathcal{L}_{\phi_2}$; $[a_i, a_j^+] = \delta_{ij}\Delta_{kk'}$

$a(\vec{k}) = \frac{1}{\sqrt{2}}[a_1(\vec{k}) + i a_2(\vec{k})]$

$b^+(\vec{k}) = \frac{1}{\sqrt{2}}[a_1^+(\vec{k}) + i a_2(\vec{k})]$

$:p^\mu: = \sum_k K^\mu (N(\vec{k}) + \bar{N}(\vec{k}))$ → $N = a^+a$, $\bar{N} = b^+b$

$p^\mu = :p^\mu: + \frac{1}{2}\sum_K K^\mu ([a(\vec{k}), a^+(\vec{k})] + [b(\vec{k}), b^+(\vec{k})])$

$a^+(\vec{k})|0\rangle = |\vec{k}\rangle_a$ $\quad a|0\rangle = 0$ → = masa, bosones ...
$b^+ \quad = |\vec{k}\rangle_b$ $\quad b|0\rangle = 0$

$\phi' = e^{-iQ\alpha}\phi(x)$ → invariancia fase → $\phi\phi^+ = \phi'^+\phi'$

$Q = iQ\int d^3x [\pi^+\phi^+ - \pi\phi] = iQ\int d^3x (\phi^+ \overleftrightarrow{\partial_0} \phi)$

$Q = :Q: = Q\sum_K [N(\vec{k}) - \bar{N}(\vec{k})]$

$[N, a^+] = \Delta_{kk'} a^+(\vec{k})$
$[N, a] = -\Delta_{kk'} a(\vec{k})$
$[Q, a] = -Q a(\vec{k})$; $[Q, a^+] = +Q a^+$; $[Q, b] = Qb$; $[Q, b^+] = -Qb^+$

$:Q: (a^+(\vec{k})|0\rangle) = \underline{+} Q\, a^+(\vec{k})|0\rangle$
$\quad b^+$

$|k_a\rangle = |\vec{k}, +Q\rangle$; $|k_b\rangle = |\vec{k}, -Q\rangle$ → Partículas – Antipartículas

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$[\phi(x), \phi(y)] = i\Delta(x-y)$; $\Delta(x-y) = i\sum_K [e^{ik(x-y)} - e^{-ik(x-y)}]$ K-G real
$= \Delta^+(x) + \Delta^-(x)$

Complejo:
$[\phi(x), \phi(y)] = 0$; $[\phi(x), \phi^+(y)] = i\Delta(x-y)$

- $(\Box + m^2)\Delta = 0$
- $\Delta(x) = -\Delta(-x)$
- $\Delta(x^0 = 0) = 0$ (tb si $(x-y)^2 < 0$ tb $[O(x), O'(y)]$ Microcausalid
- $\partial^0\Delta(x)|_{x^0=0} = -\delta^{(3)}(\vec{x})$

$\phi(x)|0\rangle = \sum_K e^{ikx}|\vec{k}\rangle$

$\langle 0|\phi(x)\phi(y)|0\rangle = i\Delta^+(x-y)$ → $x^0 > y^0$ $y \to x$; $\Delta^+(x) = e^{-ikx}$

$T[\phi(x)\cdot\phi(y)] \equiv \theta(x^0-y^0)\phi(x)\phi(y) + \theta(y^0-x^0)\phi(y)\phi(x)$

$\langle 0|T(\phi(x)\phi(y))|0\rangle = i[\theta(x^0-y^0)\Delta^+(x-y) - \theta(y^0-x^0)\Delta^-(x-y)] = i\Delta_F(x-y)$

Δ_F: propagador de Feynman → respeta causalid

Complejo:
$\phi(x)|0\rangle = \sum_K e^{ikx}|\vec{k}, -Q\rangle$; $\phi^+(x)|0\rangle = \sum_K e^{ikx}|\vec{k}, +Q\rangle$

$\langle 0|T[\phi(x), \phi^+(y)]|0\rangle = i\Delta_F(x-y) = i[\theta(x^0-y^0)\underset{\text{part}}{\Delta^+} - \theta(y^0-x^0)\underset{\text{Antip}}{\Delta^-}] = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ikx}}{k^2-m^2+i\epsilon}$

$\int_a^b f(x)dx = 2\pi i\, \text{Res}\{f(z), z_k\}$

$\text{Res} = \lim_{z\to z_0}\frac{1}{(p-1)!}\cdot\frac{d^{p-1}}{dz^{p-1}}[(z-z_0)^p f(z)]$

$(\Box + m^2)\Delta_F = -\delta^{(4)}(x)$

- Si $(x-y)^2 < 0$ → $T(\phi(x)\phi(y)) = \phi(x)\phi(y)$
- Si $(x-y)^2 > 0$ → $\mathcal{L}^\uparrow$ no cambia orden temporal

T.V - PARTÍCULAS DE ESPÍN 1/2

$\{a_i, a_j^+\} = \delta_{ij}$ $\rightarrow$ c. signo bajo intercambio $1 \leftrightarrow 2$ con $j \neq i$

$\{a_i, a_j\} = \{a_i^+, a_j^+\} = 0$

$[N_i, a_j] = -\delta_{ij} a_j$ $\rightarrow$ $N_i^2 = N_i$ $\rightarrow$ $n_i = 0, 1$

$[N_i, a_j^+] = +\delta_{ij} a_j^+$

↳ no puedes tener 2 partículas iguales
stma. fermiónico

$\mathcal{L}(x) = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x)$; $\bar{\psi} = \psi^+\gamma^0$;

$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\bar{\psi}\gamma^0 = i\psi^+$

$\mathcal{H} = \pi_\alpha \dot{\psi}_\alpha - \mathcal{L} = i\psi^+\dot{\psi}$

$\psi(x) = \sum_k \sum_{r=1}^{2} [a_r(\vec{k}) u_r(\vec{k}) e^{-ikx} + b_r^+(\vec{k}) v_r(\vec{k}) e^{ikx}]$

$a_r(\vec{k}) = \int d^3x\, e^{ikx} u_r^+(\vec{k}) \psi(x)$ $\quad b_r(\vec{k}) = \int d^3x\, e^{ikx} \psi^+(x) v_r(\vec{k})$

$a_r^+(\vec{k}) = \int d^3x\, e^{-ikx} \psi^+(\cancel{x}) u_r(\vec{k})$ $\quad b_r^+(\vec{k}) = \int d^3x\, e^{-ikx} v_r^+(\vec{k}) \psi(x)$

⇓ Impone

$\{\psi_\alpha(x), \pi_\beta(y)\}_{x^0=y^0} = i\delta^{(3)}(\vec{x}-\vec{y})\delta_{\alpha\beta} = i\{\psi_\alpha(x), \psi_\beta^+(y)\}_{x^0=y^0}$

$\{\psi_\alpha(x), \psi_\beta(y)\}_{x^0=y^0} = \{\psi_\alpha^+(x), \psi_\beta^+(y)\}_{x^0=y^0} = 0$

$\{a_r(\vec{k}), a_s^+(\vec{k}')\} = \{b_r(\vec{k}), b_s^+(\vec{k}')\} = \delta_{rs}\Delta_{\vec{k}\vec{k}'}$, resto $= 0$

$P^\mu = \int d^3x\, \psi^+ i\partial^\mu \psi = \sum_{k,r} k^\mu [a^+a - bb^+]$

inv. fase $\rightarrow j^\mu = \bar{\psi}\gamma^\mu\psi$

↳ $Q = Q\int d^3x\, \psi^+\psi = Q\sum_{k,r}[a^+a + bb^+]$

$:P^\mu: = \sum_{k,r} k^\mu [N + \bar{N}]$

$:Q: = Q\sum_{k,r}[N - \bar{N}]$

$[P^\mu, a_r(\vec{k})] = -k^\mu a_r(\vec{k})$ $\quad [P^\mu, b_r] = -k^\mu b_r$

$[P^\mu, a_r^+(\vec{k})] = k^\mu a_r^+(\vec{k})$ $\quad [P^\mu, b_r^+] = k^\mu b_r^+$

$a_r^+|0\rangle \equiv |\vec{k}\rangle$

$[Q, a_r(\vec{k})] = -Q a_r(\vec{k})$ $\quad [Q, b_r] = Q b_r$

$[Q, a_r^+(\vec{k})] = Q a_r^+(\vec{k})$ $\quad [Q, b_r^+] = -Q b_r^+$

$a_r^+(\vec{k})|0\rangle \equiv |m, r, \vec{k}, Q\rangle$ $\quad b_r^+(\vec{k})|0\rangle \equiv |m, r, \vec{k}, -Q\rangle$

Partícula $\quad$ Antipartícula

Fermiones
⇒ $\mathcal{H} \geq 0$
Interp. part-antip.

campos fermiónicos $\rightarrow \{\ \}$
bosónicos $\rightarrow [\ ]$

$\{\psi_\alpha(x), \bar{\psi}_\beta(y)\} = (i\not{\partial}_x + m)_{\alpha\beta} \sum_k [e^{-ik(x-y)} - e^{ik(x-y)}] = i(i\not{\partial}_x + m)\Delta(x-y)$

$[O_1(x), O_2(y)] = 0$ si $(x-y)^2 < 0$ $\rightarrow$ Microcausalid

	$\boxed{K-G}$	$\boxed{Dirac}$
Estadística	Bose	Fermi
H	≥ 0	≥ 0
Q	$\pm$	$\pm$
	$\Delta(x-y)$	

Teorema espín-estadística (Pauli)

RAB • Micro causalid $[O_1(x), O_2(y)]$
• $\mathcal{H} \geq 0$, $\exists |0\rangle$ $\Big\}$ $\Big\{ S = n \to$ bosones; $S = \frac{n}{2} \to$ fermiones

$O \sim \bar{\psi} \Gamma \psi$

$[AB, CD] = A\{B,C\}D - AC\{B,D\} + \{A,C\}DB - C\{A,D\}B$

Propagador fermiónico

$$S_F(x) = (i\cancel{\partial} + m)\Delta_F(x) = \int \frac{d^4k}{(2\pi)^4} \cdot \frac{\cancel{k}+m}{k^2 - m^2 + i\epsilon} e^{-ikx}$$

$$T[\psi_\alpha(x)\bar{\psi}_\beta(y)] = \theta(x^0 - y^0)\,\psi_\alpha(x)\bar{\psi}_\beta(y) - \theta(y^0 - x^0)\,\bar{\psi}_\beta\,\psi_\alpha(x)$$

T. VI - CAMPOS EN INTERACCIÓN

$$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle \rightarrow |\psi(t)\rangle_s = e^{-iHt}|\psi(t=0)\rangle_s$$

Imagen de Heisenberg → quitar (t) de ψ → pasarla a O

$|\psi\rangle_H = U|\psi\rangle$; $U = e^{iHt}$ → $O_H = U\, O_s U^+$

$i\frac{d}{dt} O^H(t) = [O^H, H]$ Ev. temp. operador ; $H_H = H_S = H$

Imagen de interacción → op. y func. dependen t

$U = e^{iH_0 t}$; $H = H_0 + H_{int}$

$O^I = U O_s U^+$; $i\frac{d}{dt}O^I = [O^I, H_0]$; $H_{int}^I = U H_{int} U^+$

$\lim_{\substack{t_i \to -\infty \\ t_f \to \infty}} H_I = 0$ → $\langle \phi | S | \psi \rangle$

$i\frac{dU^I}{dt} = -iH_I^I U^I$

$S = \lim_{\substack{t_i \to -\infty \\ t_f \to +\infty}} U^I(t_f, t_i) = \bar{H}$

↳ $= U e^{-iH(t_f - t_i)} U^+$

$$U^I(t_f - t_i) = 1 - \int_{t_i}^{t_f} dt_1\, H_I^I(t_1)\,(U^I(t_1 - t_i))$$

$$\text{Dyson} \rightarrow U^I(t_f - t_i) = \sum_{n=0}^{\infty} (-i)^n \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_1} dt_2 \ldots \int_{t_i}^{t_{n-1}} dt_n\, H_I^I(t_1)\ldots H_I^I(t_n)$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{+\infty} dt_1 \ldots dt_n \cdot T(H_I^I(t_1)\ldots H_I^I(t_n))$$

$$= T\left\{\exp\left\{-i\int_{-\infty}^{+\infty} dt\, H_I^I(t)\right\}\right\} \rightarrow = T\left\{\exp\left\{-i\int d^4x\, \mathcal{H}_I^I(x)\right\}\right\}$$

$\downarrow \alpha(\varphi, \cancel{\partial})$ $\quad \mathcal{L}_I = -\mathcal{H}_I$

$$S = T\left(\exp\left\{i\int d^4x\, \mathcal{L}_I^I(x)\right\}\right) = T\cdot e^{i\tilde{S}\ldots\text{acción}}$$

$\downarrow \mathcal{L}_I = -\frac{\lambda}{4!}\phi^4$

$\simeq 1 - \frac{i\lambda}{4!}\int d^4x\, \phi_I^4 + O(\lambda^2)$

$\langle f|\, S - I\, |i\rangle$ → lo q medimos

↳ $= -i(2\pi)^4\, \delta^{(4)}\left(\sum_i p_i^\mu - \sum_f p_f^\mu\right)\cdot M_{i\to f}$

$\overbrace{a(\vec{k})\, a^+(\vec{p})} = \Delta\vec{k}\,\vec{p}$

$\phi_1\phi_2 = \phi_1^+\phi_2^+ + \phi_1^+\phi_2^- + \phi_1^-\phi_2^+ + \phi_1^-\phi_2^-$ $\qquad \phi_1^+ \propto a$

$:\phi_1\phi_2: = \;$ " $\;\overset{+\text{bos}}{\underset{-\text{ferm}}{\pm}}\phi_2^-\phi_1^+ \;+\;$ " $\;$ "

$\phi_1\phi_2 = :\phi_1\phi_2: + [\phi_1^{(+)}, \phi_2^{(-)}]$

$:\phi_1\phi_2: = \pm :\phi_2\phi_1:$

$T(\phi_1\phi_2) = :\phi_1\phi_2: + \overbrace{\phi_1\,\phi_2} = \phi_1\phi_2\,\Theta^> \pm \phi_2\phi_1\,\Theta^<$

$\langle 0|T(\phi_1\phi_2)|0\rangle = \overbrace{\phi_1\phi_2}$ ≡ propagador de Feynman

K-G real ⇒ $\langle 0|T(\phi(x)\phi(y))|0\rangle = i\Delta_F(x-y)$

" comp ⇒ $\langle 0|T(\phi(x)\phi^+(y))|0\rangle = i\Delta_F(x-y)$; $\langle 0|T(\phi\phi)|0\rangle = 0$

Dirac ⟹ $\overbrace{\psi(x)\,\bar{\psi}(y)} = iS_F(x-y)$

$$\Delta_F(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2 - m^2 + i\epsilon}$$

$$S_F(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2 - m^2 + i\epsilon}(\cancel{k} + m)$$

$T(\phi_1\phi_2\phi_3) = :\phi_1\phi_2\phi_3: + \overbrace{\phi_1\phi_2}\phi_3 + \phi_1\overbrace{\phi_2\phi_3} + \overbrace{\phi_1\phi_2\phi_3}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \pm \phi_2\overbrace{\phi_1\phi_3}$

$\langle 0 | T \downarrow | 0 \rangle = 0$

$T(\phi_1 \ldots \phi_{2N+1}) = 0$

<u>Tma. de Wick</u>

$T(\phi_1\phi_2\cdots\phi_N) = :\phi_1\phi_2\ldots\phi_N: + \sum_{i\neq j} :\phi_1\phi_2\ldots\overbrace{\phi_i\ldots\phi_j}\ldots\phi_N:$

$+ \sum_{i\neq j\neq k\neq l} :\ldots\overbrace{\phi_i\ldots\phi_j}\ldots\overbrace{\phi_k\ldots\phi_l}\ldots: + \ldots$ TODO CONTRAÍDO (nº par) (últ. término)

$\langle 0 | T \ldots | 0 \rangle =$ últ. térm.

<u>Tma</u> útil entre Hs, dentro del mismo, conmutan

Cálculo de amplitudes

$\langle f | S | i \rangle = -i(2\pi)^4 \delta^{(4)}(\ldots)\cdot \mathcal{M}$

<u>Reglas de Feynman</u> (bosones) $\qquad \delta^{(4)}(\qquad)$

1) Líneas externas ... de la antip.

$1\cdot\delta^4(\,)(2\pi)^4 \quad \ldots e^{-ipx}|0\rangle \quad \longrightarrow \quad 1$

$1\cdot\delta(\,) \quad \langle 0| e^{ipx}$

2) Líneas internas (propagadores)

y ---- x $\quad i\int \frac{d^4k\, e^{-ik(x-y)}}{(2\pi)^4(k^2-m^2+i\epsilon)} \longrightarrow \frac{1}{k^2-m^2+i\epsilon}$

3) Vértices de interacción

$-i\frac{\lambda}{3!}\int d^4x \longrightarrow \lambda$

$-i\frac{\lambda}{4!}\int d^4x \longrightarrow \lambda$

4) Factor de simetría, canales incompletos $\longrightarrow 1/n_s$
factores $1/n_s$ $\quad n_s$: factor simetría diagrama

5) $\frac{1}{n!}\cdot n! = 1$ $\qquad (i)^L = i$

$\langle f | S - I | i \rangle = -i(2\pi)^4 \delta^{(4)}(\sum p_{in}^\mu - \sum p_{out}^\mu)\cdot \mathcal{M}_{i\to f}$

$V=6$, $I=7$ (líneas internas), 6) loops $L = I - V + 1$ (nº loops) $\quad \left(\frac{1}{2\pi}\right)^{4L} \cdot i^L \longrightarrow i\int \frac{d^4k}{(2\pi)^4}$

Cada vértices $(-i)$

Q se conserva en cada vértice $\longrightarrow$ restringe topologías permitidas (canales)

T. VII — OBSERVABLES

Probabilid de transición

$\langle f | S - I | i \rangle = -i (2\pi)^4 \delta^{(4)} (p_f^\mu - p_i^\mu) \cdot M_{i\to f}$ (amplitud de probabilid)

$|\delta^4(p_f^\mu - p_i^\mu)|^2 = \delta^{(4)}() \cdot \int \frac{d^4x}{(2\pi)^4} = \lim \frac{V\cdot T}{(2\pi)^4} \cdot \delta^{(4)}()$

$W(i \to all) = \sum_f W(i\to f) = (2\pi)^4 \sum_f \delta^{(4)}() \cdot |M_{i\to f}|^2$, $f = 1+2+\dots+N$

$\sum_f \to \sum_{\lambda_f} \sum_{p_f}$

$\sum_{p_f} (2\pi)^4 \delta^4 = \int dQ_N \quad \to W(i \to 1+2+\dots+N) = \sum_{\lambda_f} \int dQ_N \, |M_{i\to f}|^2$

$\Gamma(A \to 1+\dots+N) = \frac{1}{2M_A} \sum_{\lambda} \int dQ_N \, |M_{A\to 1+\dots+N}|^2$

$\overline{\sum_{\lambda_i \lambda_f}} = \frac{1}{2S_A+1} \cdot \sum_{\lambda_f}$ $= \frac{1}{(2S_a+1)(2S_b+1)} \cdot \sum_{\lambda_f}$

$\sigma(a+b \to 1+\dots+N) = \frac{1}{2\lambda^{1/2}(s, m_a^2, m_b^2)} \overline{\sum_{\lambda_i\lambda_f}} \int dQ_N \, |M_{i\to f}|^2$ $\quad a \to \square$

$\int dQ_2 = \frac{1}{(2\pi)^2} \frac{p_{cm}}{4\sqrt{s}} \int d\Omega_{cm}$

$dQ_3 = \frac{\Theta(1-\cos^2\theta_{12})\, ds_{13}\, ds_{23}}{128\pi^3 s}$ → Dalitz, restricciones

$dQ_N = \frac{1}{(2\pi)^{3N-4}} \prod_{l=1}^{N} \frac{d^3 p_l}{2E_l} \, \delta^{(4)}$

Análisis dimensional

$[c] = 1$, $[\hbar] = 1$ $\to [E] = [T^{-1}] = [L^{-1}]$

$[\Gamma] = [T^{-1}] = [E]$, $[\sigma] = [L^2] = [E^{-2}]$

$S = \int d^4x \mathcal{L} \to S$ adim $\to [\mathcal{L}] = [E^4]$

$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 \to [\varphi] = [E]$ ... espín 0, bosónicos, W, Z, dim E entera

$\mathcal{L} = \bar\psi(i\gamma^\mu\partial_\mu - m)\psi \to [\psi] = [E^{3/2}]$ ... " fracc, fermiónico ; dim E fracc.

nunca habrá ψ suelto x dimensiones!

si aparece f, aparece $\bar f$ (nº fermiónico conservado)

$\mathcal{L}_{\phi^3} = -\frac{\lambda}{3!}\phi^3 \to [\lambda] = [E]$; $\mathcal{L}_{\psi^3}$ $[\lambda] = [E^{-1/2}]$

$\mathcal{L}_{\phi^4} = -\frac{\lambda}{4!}\phi^4 \to [\lambda] = 1$; $\mathcal{L}_{\psi^4}$ $[\lambda] = [E^{-2}]$

ϕ^3: $M \sim \lambda^2 + \lambda^4$, $\sigma \sim M^2 \sim \lambda^4$. $\overset{E^2?}{\Rightarrow} \sigma \sim \frac{\lambda^4}{s^3}$ ... baja enseguida $\to \sigma \sim \frac{\lambda^4}{s^3}(1 + \theta(\frac{\lambda^2}{s}))$

ϕ^4: $M \sim \lambda$, $\sigma \sim \frac{\lambda^2}{s}\{1 + \theta(\lambda)\}$ ... baja + suave! $\Leftrightarrow$ ψ^4: $\sigma \sim \lambda^2 \cdot s$

$\mathcal{L}_I = -\frac{g}{2}\Phi\varphi^2$; $[g] = [E] \to \sigma \underset{s \gg m_i^2}{\sim} \frac{\lambda^4}{s^3}(1 + \theta(\frac{\lambda^2}{s}))$

Pero si $m^2 \ll s \ll M^2$

↳ $M \sim g^2 \frac{1}{s^2 - M^2} \sim -\frac{g^2}{M^2} = \underset{\lambda}{\underset{\shortparallel}{cte}} \Rightarrow$ cte efectiva ϕ^4

$\sim \left(\frac{g^2}{M^2}\right)^2$

↳ teoría efectiva a bajas E, ctes acoplo escondem info sobre objetos + pesados

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$\mu^- \to G_F \sim E^{-2} \to \Gamma \sim G^2 \sim E^{-4} \to \times m_\mu^5$ $\quad \Gamma \sim \frac{G_F^2 m_\mu^5}{128\pi^3}$

T. VIII QED

Física en gotée

$m, s, \psi \rightarrow \mathcal{L}_0 \; \bar{\psi}(i\not\partial - m)\psi$ (fermiones)

↳ invariante bajo cambio de fase → $e^{iQ\theta}$ → $\exists\, j^\mu, Q$

Principio de Gauge

→ Impones invariancia $\mathcal{L}$ bajo tr. fase locales $\theta(x)$

⇒ Derivada covariante / $D_\mu \psi'(x) = e^{iQ\theta(x)} D_\mu \psi(x)$

$D_\mu = \partial_\mu + i X_\mu \longrightarrow X'_\mu = X_\mu - Q\partial_\mu\theta$

$\mathcal{L} = \bar{\psi}(iD_\mu\gamma^\mu - m)\psi$ ↳ campo vectorial con ley de transf.

$D_\mu = \partial_\mu + i e Q A_\mu$

$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\theta$ Acoplo mínimo

$\mathcal{L} = \mathcal{L}_0 - eQA_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x) \to \mathcal{L}_I = -eA_\mu(x)J^\mu(x)$ ....$(\rho, \vec{J})$

$J^\mu(x) = Q\bar{\psi}\gamma^\mu\psi$ → Corriente fermiónica conservada } Único vértice de interacción

A^μ → campo EM ; e → cte acoplo A_μ, intensid. interacción

Q → carga conservada, no fijada

↳ $(\Box + M^2)A^\mu(x) = 0$ → "Conexión"

↳ $\mathcal{L}_K = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (Lagrang. ED) → término M no sería invariante → $M_\gamma = 0$

↳ $\partial_\mu F^{\mu\nu} = eJ^\nu = e\sum_i Q_i \bar{\psi}_i \gamma^\nu \psi_i$ (Ec. Maxw) ⇐ falta cuantizar

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; $\partial_\mu F^{\mu\nu} = (g^{\mu\nu}\Box - \partial^\mu\partial^\nu)A_\mu = 0$ (libre)

$A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\theta$

Gauge Lorentz → $\partial_\mu A^\mu = 0$; $\Box\theta = 0$

$\mathcal{L}_\xi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2$

↓ No válido, imponer sobre $\langle\ \rangle_{\psi\text{ físico}}$

Condición de Gupta – Bleuler : $\boxed{\partial_\mu A^\mu(x)\big|_{\text{destrucción}}|\psi\rangle = 0 \quad \forall\, |\psi\rangle_{\text{física}}}$

↳ los que no cumplan: espúreos

$A_\mu(x) = \sum_{r=0}^{3}\sum_k a_r(k)\epsilon_r^\mu(k)e^{-ikx} + a_r^+(\vec{k})\epsilon_r^{\mu *}(\vec{k})e^{ikx}$ → $\epsilon_{1,2}{}^\mu k_\mu = 0$

↳ $(a_0 - a_3)|\psi\rangle = 0$ → $\langle\phi|N_0 - N_3|\psi\rangle = 0$

1) $|\vec{r}, \vec{k}\rangle, k^2 = 0$

2) Físicos $N_0 = N_3$; $:P^\mu: = \sum_k k^\mu(N_1 + N_2)$ sólo contribuyen 1-2, 0-3: espúreos, no observables

3) $N_0 = 0$

$\sum_r(-g_{rr})\epsilon_r^\mu(\vec{k})\epsilon_r^{\nu *}(\vec{k}) = g^{\mu\nu}$ ↳ relac. de completitud

Propagador de Feynman :

$\langle 0|T(A^\mu(x)A^\nu(y))|0\rangle = -ig^{\mu\nu}\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2+i\epsilon}$

T. adicional → $i\delta^{\mu 0}\delta^{\nu 0}\frac{1}{4\pi|\vec{x}-\vec{y}|}\cdot\delta(x^0 - y^0)$ potee de Coulomb interacción estática, local, nada se propaga

→ 2 polarizaciones físicas propagándose + término residual Coulomb

$\overline{\psi}\, a_{r1}^{+}(\vec{p_1})|0\rangle = \{\psi, a_{r1}^{+}\}|0\rangle = u_{r1}(\vec{p_1})\, e^{-ip_1 x}|0\rangle$

$a_{r2}(\vec{p_2})\,\overline{\psi} = \bar{u}_{r2}(\vec{p_2})\, e^{ip_2 x}$

$\mathcal{L}_I = -e A_\mu \sum_f Q_f \overline{\psi}_f \gamma^\mu \psi_f \quad ; \quad S = T\exp\{i\int d^4x\, \mathcal{L}_I(x)\}$

- Fermión in $u_r^i(\vec{p})$ out $\bar{u}_r^i(\vec{p})$
- Anti " " $\bar{v}_r(\vec{p})$ out $v_r^i(\vec{p})$
- Fotón in $\epsilon_r^\mu(\vec{p})$ out $\epsilon_r^\mu(\vec{p})^*$
- Prop. fermiónico $\dfrac{\not{k} + m_i}{k^2 - m^2 + i\epsilon}$
- " fotónico $\dfrac{-g^{\mu\nu}}{k^2 + i\epsilon}$
- Vértice $e Q_i \gamma^\mu$
- Bucles cerrados $i\int \dfrac{d^4k}{(2\pi)^4}$
- Signo - y Traza por cada loop fermiónico
- " " por cada cruce de 2 líneas fermiónicas