T. 5 – ÍNDICE DE REFRACCIÓN

Ec. Maxwell $\Leftarrow$ $+2 < \vec{D} = \epsilon\vec{E} = \epsilon_0\vec{E} + \vec{P}$, $\vec{B} = \mu\vec{H}$

$\vec{P} = \epsilon_0 \chi_e^{(1)} \vec{E}$

$\vec{p} = q\cdot\vec{r} \propto \vec{E} \to \chi(p)$ → dipolo oscilante, carga acelerada, oscilador forzado amort. por radiac.

Modelo de Lorentz $U \approx \frac{1}{2}K|\vec{r}|^2 \to \vec{F} = -\vec{\nabla}U = -m\omega_0^2 r = -e\vec{E}$

$$\frac{d^2\vec{r}}{dt^2} + 2\gamma\frac{d\vec{r}}{dt} + \omega_0^2\vec{r} = -\hat{e}\,e\frac{E_\omega}{m}e^{-i\omega t} + c.c. \qquad 2\gamma \approx \frac{1}{4\pi\epsilon_0}2e^2$$

$$\vec{r}_h = 2\vec{r}_0\,e^{-\gamma t}\cos\left[\left(\omega_0 - \frac{\gamma^2}{2\omega}\right)t + \phi_0\right] \quad \text{transitorio}$$

$$\vec{r}_p = -\hat{e}\,\frac{e}{m}\left[\frac{E_\omega}{\omega_0^2 - \omega^2 - 2i\gamma\omega}e^{-i\omega t} + c.c.\right] \to \vec{p} = -e\vec{r}_p, \quad \vec{P} = N\cdot\vec{p}$$

$$\vec{p} = \frac{e^2}{m}\hat{e}\ldots \to \vec{P} \approx \frac{Ne^2}{m}\hat{e}\ldots \quad \left(\begin{matrix}\lambda \gg L\\ N \to N\Delta\end{matrix}\right) \quad P_\omega = \ldots e^{-i\omega t}E_\omega = \epsilon_0\chi^{(1)}(\omega)E_\omega$$

$$\chi^{(1)}(\omega) = \frac{Ne^2}{m}\frac{1}{\omega_0^2 - \omega^2 - 2i\gamma\omega} = \chi'(\omega) + i\chi''(\omega)$$

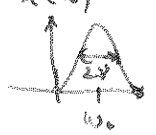
$n^2 = 1 + \chi \to Re(n) \approx 1 + \frac{1}{2}\chi'(\omega)$, $Im(n) = \frac{1}{2}\chi''(\omega)$

$n \approx 1 + \frac{1}{2}\chi$

$$\vec{E} = \hat{e}\,E_\omega\,e^{-i(\omega t - k_0 Re(n) z)}\,e^{-k_0 Im(n) z} \to \text{atenuación}$$

$n(\omega)$ → dispersión cromática

gas diluido



Luz difundida

Potencia $W \propto \omega^4 |\vec{p}(\omega)|^2$ Sección eficaz $\sigma = \frac{W}{I}$

$\omega \gg \omega_0 \to \sigma_{Th} \sim \frac{8\pi}{3}r_0^2$ D. Thomson

$\omega \ll \omega_0 \to \sigma_R \sim \frac{8\pi}{3}r_0^2\frac{\omega^4}{\omega_0^4}$ D. Rayleigh

× Polarización

$\hat{e}$ incidente

$\hat{u}_R(\hat{u}_R\cdot\hat{e}) - \hat{e}$

↓ de donde miro; ↳ pol. incidente

Metales

$\omega_0 = 0$

$$n^2 = 1 - \left(\frac{\omega_p}{\omega}\right)^2 + 2i\gamma\frac{\omega_p^2}{\omega^3}, \quad \omega_p = \sqrt{\frac{Ne^2}{\epsilon_0 m}}, \quad R = \left|\frac{n(\omega) - 1}{n(\omega) + 1}\right|^2$$

filtros

Óptica no lineal

$U(r) = \frac{1}{2!}K_0 r^2 + \frac{1}{3!}K_2 r^3 + \frac{1}{4!}K_3 r^4$ < centrosimétricos $K_2 = 0$, $U(r) = U(-r)$; no "

$\vec{F} = -\vec{\nabla}U$

$L\cdot r + br^3 = -\frac{e}{m}E_\omega e^{i\omega t} + c.c.$ → solución perturbativa

↳ frecuencia triple

Efecto Kerr, $\chi = 1 - f(|E_\omega|^2)$

T. 6 - Interferencias

Ppo. de superposición $\nabla^2 E - \frac{1}{v^2}\frac{\partial}{\partial t^2}\vec{E} \equiv L\vec{E}$

$L\vec{E} = L\vec{E_1} + L\vec{E_2} = 0$ → operador lineal

$I = v\langle W\rangle = \langle|\vec{E_r}\times\vec{H_r}|\rangle = \frac{\varepsilon c}{n}\langle|\vec{E_r}|^2\rangle = v\langle W\rangle$ ↳ densid E $= \varepsilon\langle|\vec{E_r}|^2\rangle$

$\langle\vec{E_r}\cdot\vec{E_r}\rangle = [\omega\tau \gg 1] = \frac{1}{2}\vec{E}\cdot\vec{E}^*$

↳ armónicos

$\vec{E} = E_{r1} + E_{r2}$

$I = I_1 + I_2 + I_{12}$ → $I_1 = \frac{\varepsilon c}{2n}\vec{E_1}\cdot\vec{E_1}^*$

I_{12} → $I_2 = \frac{\varepsilon c}{2n}\vec{E_2}\cdot\vec{E_2}^*$

↳ ω_i = s (+ coherε pulso) → $I_{12} = \frac{\varepsilon c}{2n}\begin{cases}\vec{E_1}\vec{E_2}^* + \vec{E_1}^*\vec{E_2} & \text{si } \omega_1 = \omega_2 \\ 0 & \omega_1 \neq \omega_2 \ (\Delta\omega\tau \gg 1)\end{cases}$

↳ polarizad paralela

1) Ondas planas

$I_{12} = E_{01x}E_{02x}\cos\delta_x + \ldots$ $\quad \delta_x = \phi_{2x} - \phi_{1x}$, $E_{1i} = E_{01i}(\vec{r})\cdot e^{-i\phi_{1i}(\vec{r})}$

$\phi_{1i} = -\vec{k_1}\vec{r} + \varphi_{1i}$

si $= \varphi_i$

⬇ $I_{12} = \vec{E_{01}}\cdot\vec{E_{02}}\cdot\cos\delta$, $\quad \delta = (\vec{k_2} - \vec{k_1})\cdot\vec{r} - \Delta\varphi$

2) Ondas esféricas

$E \propto \frac{1}{r}$ $\quad \vec{E_i}(\vec{r_i}) = \vec{E_{0i}}(\vec{r_i})\, e^{i(\omega t - kr_i + \varphi_i)}$ $\quad \delta(\vec{r}) = \phi_2 - \phi_1 = k(r_2 - r_1) + \varphi_2 - \varphi_1$

$I \propto \frac{1}{r^2}$ $\quad = k_0\Delta L - \Delta\varphi$

3) O. L. P. $\delta(\vec{r}) = k_0\Delta L - \Delta\varphi$

< Young esféricas puntual

$\vec{E_1} = \hat{e_1}\frac{E}{2r_1}e^{i(\omega t - kr_1 + \varphi_1)}$ → $I(y) = 2I_0\left[1 + \cos\left(\frac{4\pi a y}{\lambda D}\right)\right]$

$i = \frac{\lambda D}{2a}$

real $r_2 - r_1 = cte$ → hiperboloide

× $\hat{=}$ fuente desplazada

$I(y) = 2I_0\left[1 + \cos\left(\frac{4\pi a}{\lambda D}\left(y + y_s\frac{D}{d}\right)\right)\right]$

× fuente lineal incoherente

$\frac{y_s}{d} = \frac{00'}{D}$

→ simetría → ↳ muy + luz

× rendijas lineales

↳ simetría en y a muy z → + luz, muy patrón

× abertura

$\vec{E} = \frac{A}{\sqrt{N}}\hat{e}\left(\sum_{n=1}^{N} e^{i(\omega t - k(FMS_1 + S_1P) + \phi_n)} + e^{i(\omega t - k(FMS_2 + S_2P) + \phi_n)}\right)$

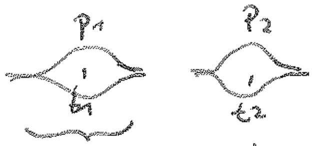
→ $I = 2I_0\left(1 + \text{sinc}\,\Omega\cos\left(\frac{4\pi a y}{\lambda D}\right)\right)$ $\quad \Omega = \frac{kah}{d}$, $V = \text{sinc}\,\Omega$

$I = \int\frac{2I_0}{h}\left(1 + \cos\left(\frac{4\pi a}{\lambda D}\left(y + y_s\cdot\frac{D}{d}\right)\right)\right)dy_s$ → T. Fourier

↳ interferómetro de Michelson

Coherencia

pulso, limitado tiempo



τ_c: tiempo de coherencia

$t_2 - t_1 > \tau_c \Rightarrow \nexists$ interferencia

$t_2 - t_1 < c\tau_c \rightarrow$ hay interf.

$c\tau_c = \ell_c$ = long. de coherencia

tiempos ⟶ frecuencias, T. Fourier

monocr ⟶ $\tau_c = \infty$

cuasimonocr ⟶ $\Delta\omega$ ⟶ $\tau_c < \infty$

$$E^{r}(t) = |a(t)| \cos(\omega_0 t + \phi(t))$$

$a(t)$ ⟶ es calcular

$$\tilde{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dt\, E^{(r)}(t)\, e^{-i\omega t}$$

$$\tilde{E}(\nu) = \frac{\tau_c}{\pi} \left(\operatorname{sinc} \pi(\nu - \nu_0)\tau_c + \operatorname{sinc} \pi(\nu + \nu_0)\tau_c \right)$$

$$\delta(x) = \lim_{p \to \infty} \operatorname{sinc} px$$

$\Delta\nu = \frac{1}{\tau_c}$ (sinc = 0)

$$E^{(r)}(t) = \int_0^{\infty} d\nu\, b(\nu) \cos(2\pi\nu t + \phi_0) = 2\int_0^{\infty} d\nu\, \mathrm{Re}\left[e^{i2\pi\nu t}\, \tilde{E}(\nu)\right] \quad (\nu \geq 0)$$

espectro $S(\nu) = |b(\nu)|^2$

$$S(\nu) = \frac{\tau^2}{\nu\pi^2} \cdot \operatorname{sinc}^2(\pi(\nu - \nu_0)\tau_c)$$

Young

↳ $I_1 = \langle |a(t)|^2 \rangle$

$I_2 = K^2 \langle |a(t)|^2 \rangle$

Autocorrelación:

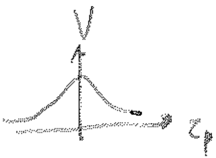
$$G(\tau_p) = \langle a(t) \cdot a^{*}(t + \tau_p) \rangle$$

$$I_{12} \doteq 2K\, |G(\tau_p)| \cos(2\pi\nu\tau_p + \phi_{cp})$$

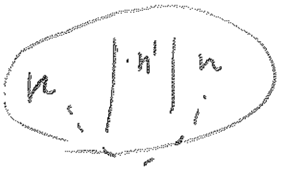
$$= 2\sqrt{I_1 I_2} \left| \frac{G(\tau_p)}{G(0)} \right| \cdot \cos \delta_{Young}$$

$\left|\frac{G(\tau_p)}{G(0)}\right| = g(\tau_p)$ ⟶ grado de coherencia

$V = |g(\tau_p)|$



T. 7 - INTERFERE X DIV. DE AMPLITUD



$\Delta L = 2n'd\cos\varepsilon'$

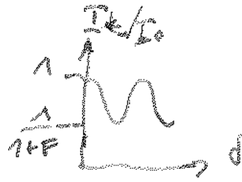
$\delta = \frac{4\pi}{\lambda_0} d n' \cos\varepsilon'$

$F = \frac{4R}{(1-R)^2}$

Reflexión $A'' = \frac{(1-e^{-i\delta})\sqrt{R}\,A}{1-Re^{-i\delta}} \Rightarrow \frac{I''}{I} = \frac{F\sin^2\delta/2}{1+F\sin^2\delta/2} = R_{global}$

} Fórmulas de Airy

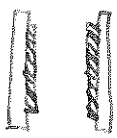
$A' = \frac{T}{1-Re^{-i\delta}} A$ $\qquad \frac{I'}{I} = \frac{1}{1+F\sin^2\delta/2} = T_{global}$



$\to \uparrow R, F \to$ + picada $\qquad \varepsilon \simeq \frac{4}{\sqrt{F}}$ (1/2)

$\to$ filtro interferee

Interferómetro Fabry - Perot



prisma.... interfere en interior
d fijo $\to$ Etalón

$r \to |r| e^{i\phi}$

$\boxed{\delta = \delta_0 - 2\phi}$ $\qquad 2\phi \ll \delta_0$ (10^3)

↳ anillos de = inclina $\qquad D_p \simeq \frac{2f'}{1}\left(\frac{n'\lambda_0}{d}\right)^{1/2}\sqrt{p+e-1}$

$m_0 = m_1 + e$ ↳ decimal

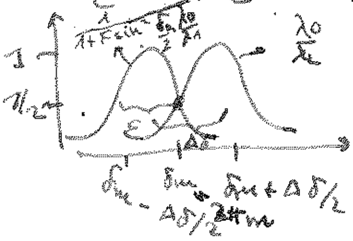
→ láser; atd & prop fibra óptica
⇒ líneas espectrales estrellas
⇒ eleqs químicos λs pegadas
⇒ efecto Zeemann, niveles atómicos
⇒ método de Tolansky
⇒ filtro interferee

Ancho

Fineza interferómetro $\mathcal{F} = \frac{\Delta\delta}{\varepsilon} = \frac{2\pi}{\varepsilon} = \frac{\pi\sqrt{F}}{2}$ $\qquad \delta = \frac{4\pi}{\lambda_0} d n' \cos\varepsilon'$

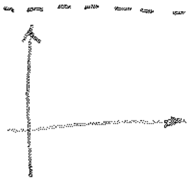
$\lambda_1, \lambda_2 \to \delta_1 = \delta_m - \frac{\Delta\delta}{2}$

$\delta_2 = \delta_m - \frac{\Delta\delta}{2}$



$\Delta\delta_{min} \geq \varepsilon$

Análisis de espectros



$I_{tot} = I\left(\delta - \frac{\Delta\delta}{2}\right) + I\left(\delta + \frac{\Delta\delta}{2}\right)$

$|d\delta| = \frac{4\pi}{\lambda^2} n d\cos\varepsilon' |d\lambda|$

$\to \left|\frac{\lambda_0}{\Delta\lambda}\right| = \frac{2\pi|m|}{\varepsilon} = \mathcal{F}|m| \sim 10^6 \Rightarrow$ Poder resolvente del interferómetro

Multicapas dieléctricas

$1 = n_0 \mid n_1 \mid \dots \mid n_N \mid n_{N+1} = n'$

$$\begin{pmatrix} e_y^{(i)}(z) \\ h_x^{(i)}(z) \end{pmatrix} = \begin{pmatrix} \cos(\beta_i z) & -\frac{1}{p_i}\sin(\beta_i z) \\ -i p_i \sin(\beta_i z) & \cos(\beta_i z) \end{pmatrix} \begin{pmatrix} e_y^{(i)}(0) \\ h_x^{(i)}(0) \end{pmatrix}$$

$$T(\varepsilon) = \prod_{i=1}^{N} T_i(\varepsilon, n_i, d_i)$$

$$\begin{pmatrix} e_y^{(N)} \\ h_x^{(N)} \end{pmatrix} = T(\varepsilon) \begin{pmatrix} e_y(0) \\ h_x(0) \end{pmatrix} \to r, t \dots$$

1 lámina

$$r = \frac{r_{01} + r_{12} e^{-2i d \beta_1}}{1 + r_{01} r_{12} e^{-2i d \beta_1}} \dots = \frac{r_{01} + r_{12} e^{-i\delta}}{1 + r_{01} r_{12} e^{-i\delta}}$$

$n - n' - n \to$ F. Airy

Lámina antirreflejante

$$R = |r|^2 = \frac{r_{01}^2 + r_{12}^2 + 2|r_{01}||r_{12}| \cos\delta_t}{1 + r_{01}^2 r_{12}^2 + 2|r_{01}||r_{12}| \cos\delta_t}$$

$\delta_t = \delta \pm \pi$ si tipo sombrero

$$d_m = \frac{\lambda_0}{2 n_1 \cos\varepsilon_1} \cdot \begin{cases} m + 1/2 \\ m + 1 \end{cases}$$

$d_{min}, m_{min} \to R_{min} = \dfrac{(n n' - n_1^2)^2}{(n n' + n_1^2)^2}$ $\quad \to n_1 = \sqrt{n n'} \to R = 0$

se cumple 1 λ_0, n_{ext}, $\varepsilon = 0$, 1 capa; + capas

+ capas → cristal fotónico unidimensional ... ≠ bandas prohibidas

Cuña

$$\delta(x) = \frac{4\pi}{\lambda_0} n' d(x) \cos\varepsilon'_1 \pm \pi \overset{\varepsilon = 0}{\simeq} \frac{4\pi n' d(x)}{\lambda_0} \pm \pi = 2\pi m$$

$d(x) \simeq \alpha x$

ó $d_m(x) = \dfrac{1}{2n'}(m \mp 1/2)\lambda_0$

$$x_m = \left(\frac{m + 1/2}{2\alpha}\right)\lambda \qquad m = 0, 1, 2, \dots$$

Franjas de Fizeau equiespaciadas

→ Mapa topográfico de franjas de = espesor

perfilometría de superficies (precisión λ)

T. 8 – DIFRACCIÓN

Luz coherente → V siempre máxima

$\lambda_0 \sim d$

$(\nabla^2 + k_0^2 n^2)\vec{E} = 0$ Ec. Helmholtz } Aproximac escalar $\vec{\mathcal{E}}(\vec{x},t) = \vec{E}(\vec{x}) e^{i\omega t}$

$\Downarrow$ polariz fija → $\vec{E}(x) = \hat{u}\, u(\vec{x})$

$(\nabla^2 + k_0^2 n^2)\, u(\vec{x}) = 0$

$u(\vec{x}) = u(\vec{x}_t, z) = A(\vec{x}_t, z)\, e^{-ik_0 n z}$

↓ conjugados $\beta \leftrightarrow k$

Helmholtz

$\frac{\partial^2 A}{\partial z^2} - 2ik_0 n \frac{\partial A}{\partial z} + \nabla_t^2 A = 0$ → $\left|\frac{\partial^2 A}{\partial z^2}\right| \ll 2k_0 n \left|\frac{\partial A}{\partial z}\right|$

Aproximac paraxial

$$\boxed{\nabla_t^2 A - 2i\underbrace{k_0 n}_{\beta_0} \frac{\partial A}{\partial z} = 0}$$

$x \to k$

$\hat{A}(\vec{k}_t, z) = \frac{1}{2\pi} \int d^2 x_t\, A(\vec{x}_t, z)\, e^{i\vec{k}_t \cdot \vec{x}_t}$

$A(\vec{x}_t, z) = \int d^2 k_t\, \hat{A}(k_t, z)\, e^{-i\vec{k}_t \cdot \vec{x}_t}$ ← $\hat{A}(\vec{k}_t, z) = \hat{A}(k_t, 0) \cdot e^{\frac{i}{2\beta_0}|k_t|^2 z}$

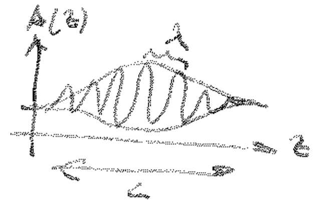
↓ $d^2 x_t\, A(\vec{x}_t, 0)\, e^{\cdots}$

→ $\int \ldots$, $G(x - x', z) = \frac{i\beta_0}{2\pi z} e^{-\frac{i\beta_0}{2z}(\vec{x} - \vec{x}')^2}$

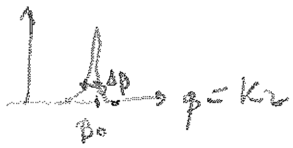
$$u(\vec{x}, z) = e^{-i\beta_0 z} \frac{i}{\lambda z} \int d^2 x'\, e^{-i\frac{\beta_0}{2z}(\vec{x} - \vec{x}')^2}\, u(\vec{x}', 0)$$ INT. DIFRACCIÓN DE FRESNEL

Aprox. paraxial

$2^{\underline{a}}$ a $1^{\underline{er}}$ orden $\left|\frac{\partial^2 A}{\partial z^2}\right| \ll 2\beta_0 \left|\frac{\partial A}{\partial z}\right| \; \forall z$



envolvente suave respecto a f. armónica dentro

$\hat{A}(\beta) =$ 

$\Delta\beta \sim \frac{1}{L}$

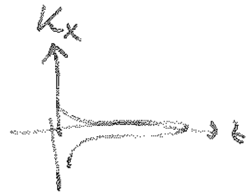
$\frac{\partial A}{\partial z} \sim \Delta\beta \cdot A$

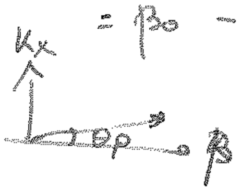
$\frac{\partial^2 A}{\partial z^2} \sim \Delta\beta^2 A$ → $\Delta\beta \ll \beta_0$ ó k_t

$\beta = \sqrt{\beta_0^2 - |k_t|^2} = \beta_0 \sqrt{1 - \frac{k_t^2}{\beta_0^2}}$ → $|\vec{k}_t| \ll 1$

$= \beta_0 - \frac{|k_t|^2}{2\beta_0}$

$z \equiv t$, $\beta \to \beta$ } $\Delta E \cdot \Delta t \geq$ cte





$\sin\theta_p = \left|\frac{k_x}{\beta_0}\right| \ll 1$ θp pequeños → paraxiales

$\Delta\beta$ grande → ya no válida aproximac

Difracción de Fraunhofer (campo lejano)

$$u(\vec{x}, z) = \frac{i e^{-i\beta_0 z}}{\lambda z} e^{-i\frac{\beta_0 |\vec{x}|^2}{2z}} \int_\Omega d^2x' \, e^{i\frac{\beta_0}{z}\vec{x}\vec{x}'} \cdot e^{-i\frac{\beta_0}{2z}|\vec{x}'|^2} \cdot u(\vec{x}', 0)$$

$$\hookrightarrow \frac{\beta_0 |\vec{x}'|^2}{2z} \leq \frac{\beta_0 a^2}{2z} = \frac{\pi a^2}{\lambda z} = \pi F$$

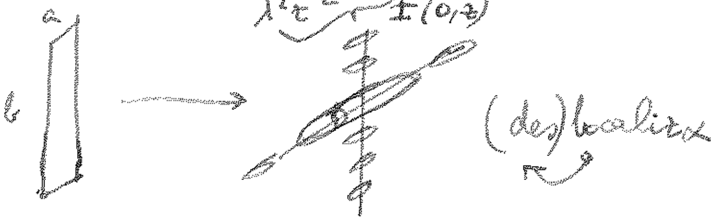
F (nº de Fresnel) $= \frac{a^2}{\lambda z}$ → $F \gtrsim 1$ d. Fresnel; → $F \ll 1$ d. Fraunhofer

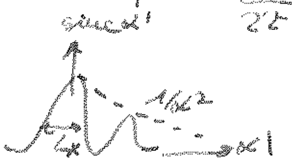
Fraunh.:

$$u(\vec{x}, z) \simeq \frac{i e^{-i\beta_0 z}}{\lambda z} e^{-i\frac{\beta_0}{2z}|\vec{x}|^2} \int_\Omega d^2x' \, e^{i\frac{\beta_0}{z}\vec{x}\cdot\vec{x}'} u(\vec{x}', 0)$$ → doble transformada de Fourier

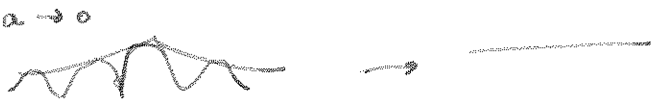
$a \times b$ → $I = \underbrace{\frac{|u_0|^2 A^2}{\lambda^2 z^2}}_{I(0,z)} \mathrm{sinc}^2 \alpha' \, \mathrm{sinc}^2 \beta'$ $\alpha' = \frac{\beta_0 a x}{2z}$

$\beta' = \frac{\beta_0 b}{2z} y$





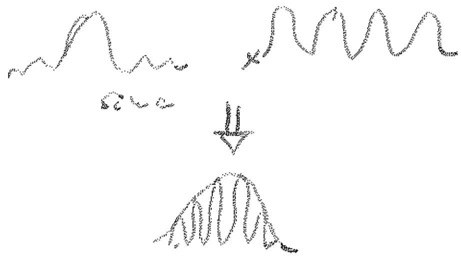
$\Delta x \sim \frac{1}{a}$



$$u_a(\vec{x}, z) = e^{i\frac{\beta_0}{z}\vec{x}\cdot\vec{a}} u_0(\vec{x}, z)$$

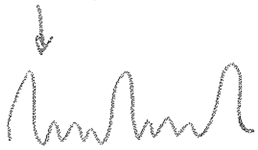
$\vdots \; (\vec{x} - \vec{a})$

Young $\hookrightarrow 2 \underbrace{I_0(\vec{x}, z)}_{\text{sinc}} \left(1 + \underbrace{\cos}_{\text{Young}} \frac{2\pi c}{\lambda z} x\right)$



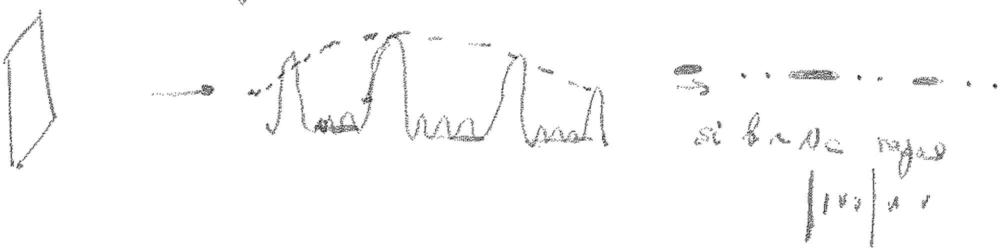
N aberturas equidistantes (red de difracción)

$$I = \frac{\sin^2 N\delta/2}{\sin^2 \delta/2} \cdot \underbrace{I_0(\vec{x}, z)}_{\text{difracción x 1 abertura}}$$



→ $\delta = 2\pi m$ → máximos abs.
$\delta N = 2\pi m$ → mínimos relat.

red lupida (?) $b \gg N_c$



si $b \sim N_c$ rojo (?)