UIMP Universidad Internacional Menéndez Pelayo

1) $d\vec{r} = \frac{\partial \vec{r}(t)}{ds} \cdot ds = \vec{\tau}\, ds \qquad \frac{d\vec{r}}{dt} = v \cdot \vec{\tau}$

Frenet

$\vec{a} = \dot{v}\vec{\tau} + v\dot{\vec{\tau}} = a_t \cdot \vec{\tau} + \underbrace{v \cdot \dot{\theta}}_{a_n = \frac{v^2}{\rho}} \cdot \vec{n} \qquad \vec{a} - a_t \cdot \vec{\tau} = a_n \vec{n}$

$\dot{\vec{\tau}} = \frac{v}{\rho} \cdot \vec{n} = \frac{ds}{dt} \cdot \frac{\vec{n}}{\rho} \qquad a^2 - a_t^2 = a_n^2$

↳ Rebote

Curvilíneas:

$\vec{u_i} = \frac{1}{h_i} \frac{\partial \vec{r}}{\partial q_i} \qquad h_i = \left| \frac{\partial \vec{r}}{\partial q_i} \right|$

$T_{uuv} = \begin{pmatrix} u_1 \\ v_2 \\ v_3 \end{pmatrix} \qquad d\vec{r} = h_1 \cdot \vec{u_1}\, dq_1 + \ldots$

②

$$\left.\begin{aligned}\vec{L} &= \vec{r} \times \vec{p} \\ \vec{M} &= \vec{r} \times \vec{F}\end{aligned}\right\} \quad \vec{M} = \frac{d\vec{L}}{dt}$$

$$W_{vit} = \int_A^B = -\Delta U = U_A - U_B \qquad dU = \vec{\nabla} U \cdot d\vec{r}$$

$$F_{conserv} \begin{cases} \vec{F} = -\vec{\nabla} U \overset{\Delta}{\Leftrightarrow} \vec{\nabla} \times \vec{F} = 0 \\ \frac{\partial U}{\partial t} = 0 \;;\; F \neq f(t) \end{cases}$$

$$U(x) \simeq U(x_0) + U'(x_0) \cdot (x - x_0) + \frac{1}{2} \underbrace{U''(x_0)}_{k} \cdot (x - x_0)^2$$

$-\vec{\nabla} U =$

OSCILACIONES

OAS

$$\ddot{\psi} + \omega_0^2 \psi = 0$$

$\leadsto \psi \propto e^{rt}$ $\longrightarrow$ $\psi = A\cos\omega_0 t + B\sin\omega_0 t$

$r = \pm i\omega_0 \leadsto$ c.l.

Superposición

$\leadsto x_1 + x_2 = \underbrace{(A_1 e^{i\theta} + A_2 e^{i\phi})}_{x(0)} e^{i\omega t}$ $\longrightarrow$ $A_{cte} = |x(0)|$

$\delta = \frac{\mathrm{Im}\, x}{\mathrm{Re}\, x}$

$\phi - \theta = 0 \longrightarrow$ Cons

$= \pi/2 \longrightarrow$ Cuad

$= \pi \longrightarrow$ Destr.

$\neq \nu \leadsto A(t) = |x|^2$ $2A_1A_2\cos(\omega_1-\omega_2)t$

si $A_1 = A_2$

$\hookrightarrow x = 2A\cos\underbrace{\frac{\omega_1-\omega_2}{2}}_{\omega_{mod}}t \,\cos\underbrace{\frac{\omega_1+\omega_2}{2}}_{\omega_{pul}}t$ $\longrightarrow$ $x = \underbrace{A(t)}_{2A\cos\omega_{mod}}\cdot\cos\omega_p t$

$E_{osc} = T + U = \frac{1}{2}kA^2$

Curvas de Lissajous

$\frac{\omega_x}{\omega_y}$ irracional $\leadsto$ trayectoria abierta

racional $\leadsto$ cerrada

OAA

$$\ddot{\psi} + 2\beta\dot{\psi} + \omega_0^2\psi = 0$$

β: factor amortiguado

$\tau = \frac{1}{2\beta}$ $2\beta = \frac{b}{m}$

$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$

$\rightarrow \beta > \omega_0 \longrightarrow$ Aperiódico sobreamortiguado

$\rightarrow \beta = \omega_0 \longrightarrow$ " crítico

$\rightarrow \beta < \omega_0 \longrightarrow$ Periódico infraamortiguado

$\psi(t) = \underbrace{A\cdot e^{-\beta t}}_{A(t)}\cdot\cos(\omega' t + \varphi)$ $\omega'^2 = (\omega_0^2 - \beta^2)$

$\frac{A(t+T)}{A(t)} = e^{-\delta}$ $\leadsto$ $\delta = \beta\cdot\frac{2\pi}{\omega'}$

δ: decremento logarítmico

$\langle E\rangle = E_0\cdot e^{-2\beta t}$

$\tau \leadsto 1/e$

$Q = 2\pi\frac{\langle E\rangle}{\langle E_{dis}\rangle} = \omega'\tau = \frac{\omega'}{2\beta} = \frac{\pi}{\delta}$

○ OAAF

$$\ddot{\psi} + 2\beta\dot{\psi} + \omega_0^2\psi = A_{ext}(t)$$

$A_{ext} = \frac{F_0}{m} \sin \omega_F t$

$\psi = \psi_h + \psi_p$ → soluc. forzada → $t \gg \tau$

ψ_h → solución transitoria (si $\beta \neq 0$)

$$\psi_p = D(\omega_F)\cdot \sin(\omega_F t + \delta)$$

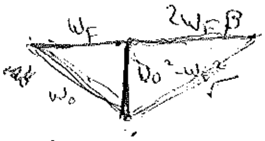
$$\boxed{D = \frac{A_{ext}}{\sqrt{(\omega_0^2-\omega_F^2)^2 + (2\omega_F\beta)^2}}}$$

$$\delta = \arctg \frac{-2\omega_F\beta}{(\omega_0^2-\omega_F^2)}$$

→ $\omega_F < \omega_0$ → $\delta < |\frac{\pi}{2}|$

→ $\omega_F = \omega_0$ → $\delta = |\pi/2|$

→ $\omega_F > \omega_0$ → $\delta > |\pi/2|$



$$\omega_{res}^2 = \omega_0^2 - 2\beta^2$$

$\beta \ll \omega_0$ → $D_{max} \approx \frac{A}{2\omega_0\beta}$

$$\Delta\omega = 2\beta$$

$Q \propto \frac{\omega_R}{\Delta\omega}$

- δ_{max} si $\omega_0 = \omega_F$
- A_{max} si $\omega_0^2 = \omega_F^2 - \beta^2$

F, v fase → resonancia

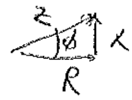
F, x $\pi/2$

β si F adelantada vs

20 v adelantada

F = desfasados si $\omega_F = \omega_0$

$$\langle P_{abs}(t)\rangle = \frac{1}{2}\frac{F_0^2}{|Z|^2}R$$



$\frac{P_{max}}{2}$ → $\Delta\omega$

× Principio superposición

$\psi_i \to F_i$

$\psi_p = \sum c_i \psi_{p_i}$ (A)

$$L(\psi) = \left(\frac{d^2}{dt} + a\frac{d}{dt} + b\right)\cdot\psi$$

× Teorema de Fourier

$f(t)$ = suma de senos / cosenos $k\cdot\omega_{fund.}$

periódica

- osciladores no lineales o anarmónicos
- " paramétricos

SISTEMAS DE PARTÍCULAS

$$\vec{R}_{CM} = \frac{\sum_i m_i \cdot r_i}{\sum m_i}$$

$$\vec{V}_{CM} = \dot{R}_{CM}$$

$\vec{r_i}' = \vec{r_i} - \vec{R}_{CM}$ → coordenada relativa

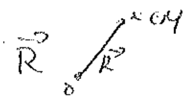
$$\vec{F_i} = \vec{F}_{i\,ext} + \vec{f_i} \rightarrow \vec{f_i} = \sum_j \vec{f_{ij}} \qquad \vec{f_{ij}} = -\vec{f_{ji}}$$

$$\vec{P} = \sum_i \vec{p_i} = M \cdot \vec{V}_{CM} \rightarrow \dot{\vec{P}} = \vec{F}_{ext} \rightarrow \vec{F}_{ext} = 0 \rightarrow \vec{P}_{total} = cte$$

$$\vec{L} = \sum_i \vec{L_i} = \sum_i \vec{r_i} \times \vec{p_i}$$

1er Tma König

$$\vec{L} = \vec{R} \times \vec{P} + \sum_i \vec{r_i}' \times \vec{p_i}' = \vec{L}_{CM} + \vec{L}_{rel}$$



$$\rightarrow \dot{\vec{L}} = \vec{M}_{ext} \rightarrow \vec{M}_{ext} = 0 \rightarrow \vec{L} = cte$$

2º Tma König

$$T = \frac{1}{2} M |V_{CM}|^2 + \sum_i \frac{1}{2} m_i |\vec{v_i}'|^2 = T_{CM} + T_{rel}$$

- 2 cuerpos

$$\vec{r_1} = \vec{R} + \underbrace{\frac{m_2 \vec{r}}{m_1 + m_2}}_{r_1'} \qquad \vec{r_2} = \vec{R}_{CM} - \underbrace{\frac{m_1 \vec{r}}{m_1 + m_2}}_{r_2'} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$\vec{L}_{rel} = r \times (\mu \vec{v})$ ← vel. relativa

$T = \frac{1}{2} M V_{CM}^2 + \frac{1}{2} \mu \vec{v}^2$ ← v. relativa

- Conserva energía mecánica

$$dW = \sum_i F_i \, dr_i$$

$$W = W_{ext} + W_{int}$$

$$\left.\begin{aligned} W_{int} &= -\Delta U_{int} \\ W_{ext} &= -\Delta U_{ext} \end{aligned}\right\} \text{(conservativas)} \rightarrow W = -(\Delta U^{int} + \Delta U^{ext}) = \Delta T$$

$$\hookrightarrow E_{total} = \underbrace{T + U^{int}}_{E_{propia} = T_{CM} + E_{interna}} + U^{ext} = cte$$

$\hookrightarrow = U_{int} + T_{rel}$

- SIMETRÍAS

Noether: cada regla de simetría → ley de conservación

- Homogeneidad del espacio → $\vec{P} = cte$ $\quad \sum \vec{F_i} = 0$ $\quad$ invariancia frente a traslaciones espaciales
- " " tiempo → $E = cte$ $\quad \frac{dT}{1} + \frac{dU}{1} = \frac{dE}{dt} = 0$ $\quad$ " " " " temporales
- Isotropía del espacio → $\vec{L} = cte$ $\quad \sum_i \vec{M_i} = 0$ $\quad$ " " " giros

- Teorema del Virial

$$S(t) = \sum_i \vec{p_i} \cdot \vec{r_i} \quad \text{acotada}$$

Promedio temporal: $\langle A \rangle = \frac{1}{\tau} \int_0^\tau A\,dt$

$$\langle \dot{S}(t) \rangle = \frac{1}{\tau} \int_0^\tau \dot{S}\,dt = \frac{S(\tau) - S(0)}{\tau}$$

mov. periódico → $S(\tau) = S(0)$
" no " → $S(t)$ acotada $\tau \gg$ $\Big\} \langle \dot{S}(t) \rangle = 0$

$$\langle \sum_i \vec{p_i} \dot{\vec{r_i}} \rangle = -\langle \sum_i \dot{\vec{p_i}}\, \vec{r_i} \rangle$$

$$\langle T \rangle = -\frac{1}{2} \langle \sum_i \vec{F_i} \cdot \vec{r_i} \rangle$$

cons $= \frac{1}{2} \langle \sum_i \vec{r_i} \cdot \vec{\nabla}_i U_i \rangle$

int. $= -\frac{1}{2} \langle \left(\sum \vec{F_i}^{ext} \vec{r_i} + \sum_{ij} \vec{f_{ij}} \vec{r_{ij}} \right) \rangle$

central $\propto r^n$, $n \in \mathbb{Z}$ $= \frac{n+1}{2} \langle U \rangle$

TEMA 5

FORMULACIÓN LAGRANGIANA Y HAMILTONIANA

- $\dot{x}_i = \sum_j \frac{\partial x_i}{\partial q_j}\dot{q}_j + \frac{\partial x_i}{\partial t}$
- $\frac{\partial \dot{x}_i}{\partial q_j} = \frac{d}{dt}\frac{\partial x_i}{\partial q_j}$
- $\frac{\partial \dot{x}_i}{\partial \dot{q}_j} = \frac{\partial x_i}{\partial q_j}$

① Coordenadas:

$x_i \rightarrow q_j \quad x_i = x_i(q_j; t)$

3N ecuacs transf.

→ se prescinde de las fuerzas activas (ligaduras) → sólo las q originan movi∅ → Fuerzas activas

→ ≡ principios, formulac sofisticada

→ α escalar → ED del movi∅ tantas como variables libres

→ ecuacs formal∅ ≡s

② Mo∅s

$p_i = \frac{\partial T(\dot{x}_i)}{\partial \dot{x}_i} \quad \pi_j = \sum_{i=1}^{3N} p_i \frac{\partial x_i}{\partial q_j} \Rightarrow \pi_j = \frac{\partial T(q_j, \dot{q}_j, t)}{\partial \dot{q}_j}$

③ Fuerzas

$F_i \quad Q_j = \sum_{i=1}^{3N} F_i \frac{\partial x_i}{\partial q_j} = \sum_{k=1}^{N} \vec{F}_i \frac{\partial \vec{r}_k}{\partial q_k} \Rightarrow Q_j$

proyecciones s/ unitarios, no amplificadas + otras magnitudes

d → F

θ → M

④ Ec. dinámica (general)

$\dot{p}_i = F_i \quad \rightarrow \quad \dot{\pi}_j - \frac{\partial T}{\partial q_j} = Q_j$ (Ec. generalizadas del Newton)

⑤ " (C + NC)

$\dot{p}_i = F_i^{nc} - \frac{\partial V}{\partial x_i} \quad \rightarrow \quad \mathcal{L} = T - V$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j^{NC}$$

- q_c es cíclica si $\frac{\partial \mathcal{L}}{\partial q_c} = 0$

↳ Si q_c es cíclica y F^{cons} ⇒ Π_c se conserva (no cambia con t)

Ligaduras: constricciones al movi∅ del sistema

$f(q) = 0 \rightarrow -m$ coord. lig → $\mathcal{L}'$ Lagr. libre

holónoma $\neq f(\dot{q}_i) \rightarrow f_i(q_j, t)_{i=1...3N,\ j=1...s}$ → escleronómas $\neq f(t)$; reónomas $\Rightarrow f(t)$

no holónoma $\neq f(q_i) = f(\dot{q}_i)$

Multiplicadores de Lagrange

$$\mathcal{L}_\lambda(q_j, \dot{q}_j, \lambda_\alpha; t) = \mathcal{L}(q_j, \dot{q}_j; t) + \sum_\alpha \lambda_\alpha \cdot f_\alpha(q_j, t)$$

EL $\partial \alpha \rightarrow f_\alpha$

$$\text{EL } \partial q_j \rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = \sum_\alpha \lambda_\alpha \frac{\partial f_\alpha}{\partial q_j} = Q_j^{lig}$$

Fuerza / Mo∅

Función Hamiltoniana

$H = \sum \pi_j \dot{q}_j - \mathcal{L}$

$\frac{dH}{dt} = -\frac{\partial \mathcal{L}}{\partial t}$ (Fac)

→ Si el tiempo no explíc. en la Lagrangiana ⇒ $H = cte$ (se conserva)

→ Si las ligaduras son esclerónomas ⇒ $H = T + V = E = cte$

⇒ Si " " " reónomas ⇒ $H \neq E$ $(q_j(t))$

↳ $\mathcal{L}(t)$ → H no se conserva

↳ $\mathcal{L} \neq \mathcal{L}(t)$ → H se conserva

Ecuaciones Canónicas o de Hamilton

$\dot{q}_j = \frac{\partial H}{\partial \pi_j}$ $\dot{\pi}_j = -\frac{\partial H}{\partial q_j}$

2s ecuas 1er orden simétricas salvo signo ⟷ $\mathcal{L}$ s ecuad 2º orden

$\pi_j = \frac{\partial \mathcal{L}}{\partial \dot{q}_j}$ (Fac)

→ si q_c cíclica → π_c se conserva

$H = H(q_j, \pi_j; t)$

$\mathcal{L} = \mathcal{L}(q_j, \dot{q}_j, t)$

Principio variacional / de Hamilton

$A = \int_{t_1}^{t_2} \mathcal{L}(q_j, \dot{q}_j; t)\, dt$ → Acción

→ Trayectoria física es aquella que minimiza la acción. (principio de mínimo)

↳ $V_{pot\,min}$ + T_{max}

$\delta A = 0 \Leftrightarrow \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_j} - \frac{\partial \mathcal{L}}{\partial q_j} = 0$

↓ sent. coord y no sólo qu ↳ t

- - → postulado básico

- invariante respecto coordenadas concretas elegidas $\mathcal{L}$
- ... teoría campos

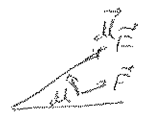
TEMA 6

CAMPOS Y MOVIMIENTO EN CAMPOS CENTRALES

Campos centrales:

Fuerza central: $\vec{F} = F(r)\,\vec{u_r}$

- conservativas $\left(< \begin{matrix} \text{no dep. } t \\ \vec{\nabla} \times \vec{F} = 0 \end{matrix} \right)$
- asociado una energía potencial central $U - U_0 = -\int F(r)\,dr \Rightarrow U(r)$



- energía potencial central origina f. central $\vec{F} = -\vec{\nabla} U = -\frac{dU}{dr} \cdot \vec{u_r}$

Newtoniano		Coulombiano
$\alpha = -GMm$	**Fuerzas** $\vec{F} = \frac{\alpha}{R^2}\vec{u_r}$	$\alpha = \frac{Qq}{4\pi\varepsilon_0}$
	Principio superposición $\vec{F} = \sum_i \vec{F_i} = \sum_i \frac{\alpha_i}{R_i^2}\vec{u_{R_i}}$	
$U = -G\frac{Mm}{R}$	Origen energía potencial en el ∞ $U(r) = \frac{\alpha}{R}$	$U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R}$
$\vec{g} = \frac{\vec{F}}{m}$	**Campos** $\vec{g} = \frac{\vec{F}}{mag}$ ... normalizar x unid magnitud activa	$\vec{E} = \frac{\vec{F}}{q}$
	Principio superposición $\vec{g} = \sum_i \vec{g_i}$	
$V = \frac{U}{m} = -G\frac{M}{R}$	**Potencial** $V = \frac{U}{mag}$	$V = \frac{U}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$
	Principio superposición $V = \sum_i V_i = \sum \frac{\alpha_i}{R\,mag}$	
$\vec{g} = -\vec{\nabla} V$	$\vec{g} = -\vec{\nabla} V$	$\vec{E} = -\vec{\nabla} V$

Características vectoriales

$\oint \vec{g} \cdot \vec{dl} = 0$	× Rotac: $\vec{\nabla} \times \vec{F} = 0$ (f. dif.) — Circul: $\oint \vec{g} \cdot \vec{dl} = 0$	$\oint \vec{E} \cdot \vec{dl} = 0$
$\oint \vec{g} \cdot \vec{ds} = -4\pi GM$	× Div: Flujo: $\oint \vec{g} \cdot \vec{ds} = \frac{4\pi\alpha}{mag}$ (f. int.) T. Gauss	$\oint \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}$
$\vec{\nabla} \cdot \vec{g} = -4\pi G \rho_M$	$\vec{\nabla} \cdot \vec{g} = 4\pi \frac{d}{dV}\left(\frac{\alpha}{mag}\right)$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho_q}{\varepsilon_0}$

× Ecuac de Poisson - Laplaciano

$\Delta V = \vec{\nabla} \cdot (\vec{\nabla} V)$

$\Delta V = 4\pi G \rho_M$	$\Delta V = -4\pi \frac{d}{dV}\left(\frac{\alpha}{mag}\right) = -\vec{\nabla} \cdot \vec{g}$	$\Delta V = -\frac{\rho_q}{\varepsilon_0}$

Movi∅ en un potencial central.

- Conservación del momento angular

$\vec{M} = \vec{r} \times F(r)\cdot\vec{u}_r = 0 \Rightarrow \vec{L} = cte$

→ Movi∅ se desarrolla en un plano $\vec{r} \perp \vec{L}$

→ Ley de las áreas → v_a es cte, $v_a = \frac{1}{2}r^2\dot\phi = \frac{L}{2m}$ $\quad L = mr^2\dot\phi$

- Ecuaciones del movimiento

$\mathcal{L} = \frac{1}{2}m(\dot r^2 + r^2\dot\phi^2) - U(r)$ → ϕ cíclica → p_ϕ se conserva $\dot\phi = \frac{l}{mr^2}$ 1D!

La coordenada radial

$$m\ddot r - \frac{l^2}{mr^3} + \frac{dU}{dr} = 0$$

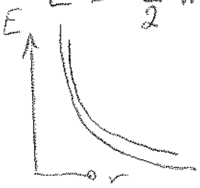
- En. potencial 1D!

$$E = \frac{1}{2}m(\dot r^2 + r^2\dot\phi^2) + U(r) = \frac{1}{2}m\dot r^2 + \underbrace{\frac{1}{2}\frac{l^2}{mr^2} + U(r)}_{U'(r)}$$

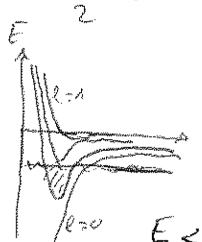
$U'(r)$ → potencial efectivo (como si) xa cada cond. inicial (del dentro bidim.)

... $E(l)$ → F ficticia asociada

→ $\nabla E_2 = mr\dot\phi^2$ → centrífuga



$l >$ → hacia dcha
- profundo



$E<0$
↳ movi∅ acotado entre pericentro r_p (cerca) y apocentro r_a

Órbita circular $\dot r = 0$ $\quad E = U'(r_c) = E_{min} \to \frac{dU'}{dr} = 0$ mínimo $\quad r_c$ pozo

función $\in \phi$

$|E \geq 0$
↳ movi∅ no acotado r_p, ~~r_a~~

$r(t)$, $\phi(t)$ → $T_r = k\,T_\phi$ → cerrada
$\neq$ → abierta

- Problema de los dos cuerpos → 2 partículas potencial coulomb, sin f. ext.

$U = \frac{\alpha}{|\vec{r}_1 - \vec{r}_2|}$

$\mathcal{L} = \frac{1}{2}m\dot{\vec{r}}_1^{\,2} + \frac{1}{2}m\dot{\vec{r}}_2^{\,2} - \frac{\alpha}{\vec{r}_1 - \vec{r}_2}$ (6 grados libertad) → CM + Rel$_{CM}$

$= \frac{1}{2}M\dot{\vec{R}}_{CM}^{\,2} + \frac{1}{2}\mu\dot{\vec{r}}^{\,2} - \frac{\alpha}{r} \equiv \mathcal{L}_{CM}(\vec R, \dot{\vec R}) + \mathcal{L}_{rel}(\vec r, \dot{\vec r})$

↳ $M\ddot{\vec{R}}_{CM} = 0$ → CM partícula libre

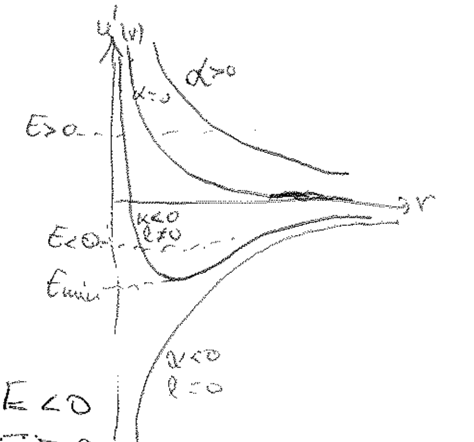
↳ $\mu\ddot{\vec{r}} = -\frac{\alpha}{r^2}\vec{u}_r$ → Mov. relativo → F central. L_{relCM} se conserva

↳ Movi∅ es un plano → $\mu\ddot r = \frac{l^2}{\mu r^3} + \frac{\alpha}{r^2}$ (1 coord. de interés)

- Análisis del potencial

$$E = \frac{1}{2}\mu\dot r^2 + \underbrace{\frac{1}{2}\frac{l^2}{\mu r^2} + \frac{\alpha}{r}}_{U'}$$

1. Órbita circular → $r_c = \frac{l^2}{\mu\alpha}$
   → v
   → E_{min}
2. Pericentro y apocentro
   r_p $\quad r_a$
3. Tipos de movi∅
   → $\alpha < 0$ (atractivo) → órbita acotada si $E<0$
   → $\alpha < 0$ → " no " si $E \geq 0$
   → $\alpha > 0$ (repulsivo → " " " → $E>0$ necesari∅



- Ecuas del moviϕ

8

- Trayectoria

$$\frac{1}{r} = -\frac{\alpha\mu}{\ell^2} + A\cos(\phi - \phi_0)$$

$\rightarrow \phi_0 = 0 \qquad A = \ldots \qquad \mathcal{E}$

$$\frac{p}{r} = 1 + \mathcal{E}\cos\phi \qquad \text{(ecua de una cónica)}$$

?

$\alpha < 0 \rightarrow E < 0 \rightarrow 0 < \mathcal{E} < 1 \rightarrow$ Elipse $\qquad a =$

$\rightarrow E = 0 \rightarrow \mathcal{E} = 1 \rightarrow$ Parábola

$\rightarrow E > 0 \rightarrow \mathcal{E} > 1 \rightarrow$ Hipérbola rama +

$\alpha > 0 \rightarrow E > 0 \rightarrow \mathcal{E} > 1 \rightarrow$ " " $-$

- Leyes de Kepler

1. Los planetas se mueven describiendo elipses con el sol en uno de los focos
2. Radio vector barre áreas =s en tiempos =s
3. $T^2 \propto a^3$

$$T^2 = \frac{4\pi^2}{G(m_1+m_2)}\,a^3 \approx \frac{4\pi^2}{G M_{sol}}\cdot a^3 \qquad \text{(a semieje mayor)}$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad < \begin{matrix} \text{Leyes} = s \\ c = cte \ \forall s \end{matrix}$$

$\rightarrow (ct)^2 - (x^2 + y^2 + z^2)$ $\quad (ct, \vec{r})$

$$\begin{pmatrix} ct \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct' \\ z' \end{pmatrix} \rightarrow \begin{matrix} ct = \gamma(ct' + \beta z') \\ z = \gamma(\beta ct' + z') \end{matrix}$$

$$ct' = \gamma(ct - \beta z)$$
$$z' = \gamma(z - \beta ct)$$

$\angle \rightarrow$ simultáneas 2 posic. en reposo

Δs^2 = intervalo
- $\rightarrow 0$ tipo luz $\rightarrow \left|\frac{\Delta z}{\Delta t}\right| = c$ — 2 sucesos separados sólo cubrirse luz
- $\rightarrow > 0$ temporal $\rightarrow$
- < 0 espacial

define universo cuando
$< c \rightarrow$ Causal. físicamente puedes interactuar conectados
$>$ causal. desconectados
fuera cono

Todos compartimos cono luz determina de dónde información

$\times \rightarrow$ ~~...~~ puede ser fut/pasado(s)
$\rightarrow$ en conectado $\forall$ a todos fut/pasado

$(c\Delta t,$
$(c\Delta t)^2 - \ldots$

$$d\tau = \frac{dt}{\gamma(u)} = \frac{dt'}{\gamma(u')} = \frac{ds}{c}$$

Suceso causal $S \rightarrow$ causal S', sig Δt = sig $\Delta t'$
S no causal $\rightarrow$ no conserva signo necesariamente

Pasado $\Delta t < 0$, $\Delta s^2 > 0$
Futuro $\Delta t > 0$

$$X^\mu = (ct, x, y, z)$$

$$\frac{dx^\mu}{d\tau} = (\gamma c, \gamma(u)\vec{u}) = U^\mu$$

$$\begin{pmatrix} \gamma c \\ \gamma(u) u_z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \gamma(u')c \\ \gamma(u) v_z' \end{pmatrix} ?$$

$$U^\mu U_\mu = c^2$$

$$u_x = \frac{u_x'}{\gamma\left(1 + \frac{v v_z}{c^2}\right)} \qquad u_z = \frac{v_z' + v}{\left(1 + \frac{v u_z}{c^2}\right)} \qquad \boxed{u_z' = \frac{u_z - v_z}{1 - \frac{u_z v_z}{c^2}}}$$

$$u_x' = \frac{u_y}{\gamma(u)\left(1 - \frac{v_z v_z}{c^2}\right)}$$

$p^\mu = m u^\mu = (m\gamma c, m\gamma u) = (p_0, \underset{m\gamma\vec{u}}{\underset{\parallel}{\vec{p}}})$ trio relativo y p^0 se conservan (4 cantid.) ?

$\frac{E}{c} = p_0 \Rightarrow E_0 = mc^2$

$p^\mu p_\mu = p^2 = m^2c^2$ → vector $(E, \vec{p})$

$E = c p_0 = m\gamma(u) c^2$

$x^\mu x_\mu = s^2$

$u^\mu u_\mu = c^2$

$p^\mu p_\mu = m^2$

$\hookrightarrow p^\mu = \left(\frac{E}{c}, \vec{p}\right)$

$\| \| \to E^2 = m^2c^4 + p^2c^2$

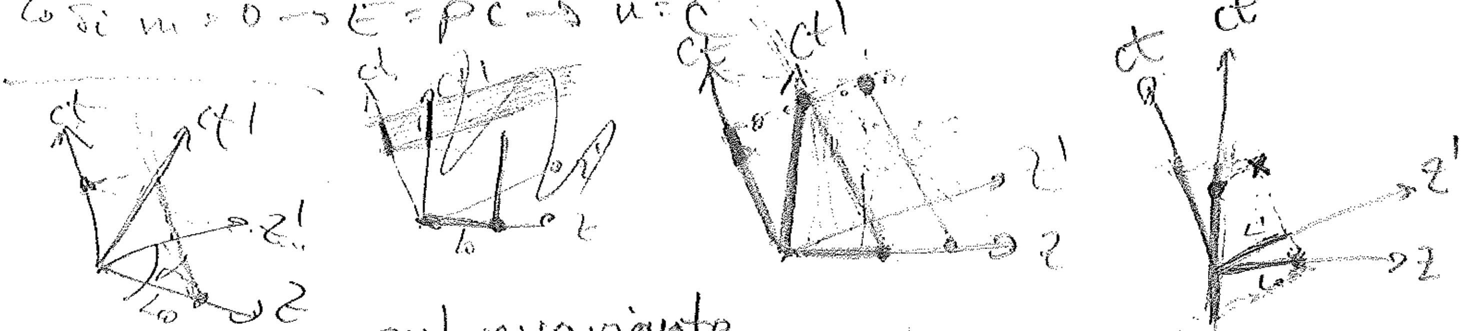
$|\vec{p}| = m\gamma |\vec{u}|$ ... no acotado ($\vec{p}$ y E)

$u \ll c \to \gamma = 1 + \frac{1}{2}\frac{u^2}{c^2} + \dots$

$\vec{p} = m\vec{u}$ → $E = \underset{E_0}{mc^2} + \underset{T_{NR}}{\frac{mu^2}{2}}$ → $E - E_0 = \underline{T_{NR}}$

$\frac{p}{E} = \frac{u}{c^2} \to \frac{pc}{E} = \beta < 1$

Lo si $m = 0 \to E = pc \to u = c$



contravariante

$a^\mu = (a_0, +\vec{a}) \longrightarrow a_\mu = (a_0, -\vec{a})$ covariante

$a'_\mu = \Lambda^\mu{}_\nu a^\nu$ $\quad \Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$

$a \cdot b = a^\mu b^\nu g_{\mu\nu} = a^\mu b_\mu$ $\quad g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$a \cdot b = a^\mu b_\mu = (a_0 b_0 - \vec{a} \cdot \vec{b})$

$c = 1$

$E^2 = m^2 + p^2$

$\frac{d\vec{p}}{dt} = \vec{F}$ $\quad \frac{dW}{dt} = m u' \frac{d}{dt}(\gamma u') = \frac{d}{dt} T$

Fuerza Minkowsky $F^\mu = \frac{dp^\mu}{d\tau} = m\frac{du^\mu}{d\tau}$

$a + b \to 1 + 2$

$p^\mu{}_{in} = p^\mu{}_{fin} \Rightarrow$ $E_a + E_b = E_1 + E_2$, $\vec{p}_a + \vec{p}_B = \vec{p}_1 + \vec{p}_2$ $\qquad E^2 = \vec{p}^2 + m^2 \quad (c=1)$

Ecin se compensa con masas al cuadrado

$m_a^2 + m_b^2 - (m_1^2 + m_2^2) = \dots$

CM

$\vec{p}_a = -\vec{p}_B = \vec{p} \qquad \vec{P} = 0$

$|\vec{p}| \to$ Conservac $\to E_{part}$, m conocidas $(\gamma, \gamma\beta)$ $(\gamma\beta, \gamma)$

$|p^1| = f(\Theta)$

LAB

$\vec{p}_B = 0$

$|\vec{p}_a|$ + dato final $(\Theta / |\vec{p}_1|)$

$\hookrightarrow E, \vec{p}, \Theta$

$|\vec{p}| = m\gamma(u)\,u$

$E = m\gamma(u)\,c^2$

$\to v = \frac{|\vec{p}|}{E_b^{CM}}$ $\qquad \frac{|p|}{E} = \beta$

$\to$ invariantes

Invariantes

$s = (p_a + p_b)^2 = (p_1 + p_2)^2 \qquad p$ (cuadri)

$t = (p_a - p_1)^2 = (p_2 - p_b)^2 \quad \to E, \vec{p} \quad$ CM, LAB

$p^2 = E^2 - \vec{p}^2 = m^2$

$s + t + u = m_a^2 + m_b^2 + m_1^2 + m_2^2$

s

$\boxed{CM}$

$(E_a^{CM} + E_b^{CM}, \vec{0})^2 = (E_a^{CM} + E_b^{CM})^2$

$\sqrt{s} = E_a^{CM} + E_b^{CM}$

$m_a^2 - m_b^2 = (E_a^{CM})^2 - (E_b^{CM})^2 \quad (a+b)(a-b)$

$= \sqrt{s}\,(E_a^{CM} - E_b^{CM})$

$E_a^{CM} = \frac{1}{2\sqrt{s}}(s + m_a^2 - m_b^2) \to |\vec{p}|$

$E_b^{CM} = \frac{1}{2\sqrt{s}}(s - m_a^2 + m_b^2)$ $\qquad |\vec{p}_a| = \sqrt{E_a^2 - m_a^2}$

$E_{1CM}, E_{2CM}, |\vec{p}^1|$

$\boxed{LAB}$

$s = p_a^2 + p_b^2 + 2p_a \cdot p_b$

$= m_a^2 + m_b^2 + 2E_a^{LAB} m_b$

$E_a^{LAB} = \frac{1}{2m_b}(s - m_a^2 - m_b^2)$

$\hookrightarrow |\vec{p}_a^{LAB}| = \sqrt{E_a^2 - m_a^2}$

t

$\boxed{CM}$

$t = p_a^2 + p_b^2 - 2p_a \cdot p_B = m_a^2 + m_b^2 - 2E_a^{CM} E_b^{CM} + 2|\vec{p}||\vec{p}^1| \cos\Theta$

$\hookrightarrow \Theta$

$\boxed{LAB}$

$t = p_2^2 + p_b^2 - 2p_a p_b$

$= m_2^2 + m_b^2 - 2E_2^{LAB} m_b$

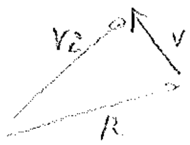
$E_2^{LAB} = \frac{1}{2m_b}(-t + m_2^2 + m_b^2)$

$\downarrow$

$|\vec{p}_2^{LAB}|$

(si $|\vec{p}_A^{LAB}|$, $|\vec{p}_2^{LAB}|$ $\Rightarrow |\vec{p}_1^{LAB}|, \theta_1, \theta_2$

$$\frac{d\vec{A}}{dt} = \frac{\partial \vec{A}}{\partial t} + \vec{\omega}\times\vec{A}$$



$$\vec{v}_I = \frac{d\vec{R}}{dt} + \vec{v}_{NI} + \vec{\omega}\times\vec{r} = \frac{\partial \vec{R}}{\partial t} + \vec{v}_{NI} + \vec{\omega}\times\vec{r}$$

$$\vec{a}_I = \underbrace{\frac{d^2\vec{R}}{dt}}_{-\text{Arrastre}} + \vec{a}_{NI} + \underbrace{2\vec{\omega}\times\vec{v}_{NI}}_{-\text{Coriolis}} + \underbrace{\vec{\omega}\times(\vec{\omega}\times\vec{r})}_{-\text{Centríf.}} + \underbrace{\dot{\vec{\omega}}\times\vec{r}}_{-\text{Azim.}}$$

Tierra

$$\vec{a}_{NI} = \vec{F}' + m\vec{g}_{ef} - 2m\vec{\omega}\times\vec{v}$$

Centrífuga

$$-\omega^2 R\,\mathrm{sen}\,\theta\,(0, \cos\theta, -\mathrm{sen}\,\theta)$$

Coriolis

$$\vec{\omega} = \omega(0, \mathrm{sen}\,\theta, \cos\theta)$$

$\vec{i}$ ESTE
$\vec{j}$ NORTE
$\vec{k}$ ARRIBA

$$\vec{F}_{coriol} = 2m\omega\left(v_y\cos\theta - v_z\,\mathrm{sen}\,\theta, -v_x\cos\theta, +v_x\,\mathrm{sen}\,\theta\right)$$

Aproximaciones sucesivas

$\theta \neq \theta(t)$, $x_0(0) = y_0(0) = 0, z(0) = h$

1. $\omega = 0 \to x, y, z$
2. $\omega \neq 0$
3. $\vdots$

Foucalt

$$\vec{T} = T\left(-\frac{x}{L}, -\frac{y}{L}, \frac{\theta}{L}\right) \quad 2\text{ ec} \quad \to \ddot{x}, \ddot{y}$$

$$\ddot{\xi} + 2i\omega\cos\theta\,\dot{\xi} + \frac{g}{L}\xi = 0$$

$$\hookrightarrow \xi(t) = c\cdot e^{i\omega\cos\theta t}\cos\left(\sqrt{\frac{g}{L}}\,t + \phi\right) \quad \to \tau = \frac{2\pi}{\omega|\cos\theta|} \in [24h, +\infty]$$

$\vec{r}(t) = \vec{R}(t) + \mathbb{R}(t)\cdot\vec{r'}(0)$

$\mathbb{W} = \dot{\mathbb{R}}\cdot\mathbb{R}^T$ → antisimétrico

$\vec{\omega}_{0'} - \vec{\omega}_{0''}$

Angulos Euler

$\mathbb{R} = R_\psi R_\theta R_\phi$

$i \to K$

$i_1 \ j_1 \ K$

$i_2 \ j_2 \ K_2$

$i_3 \ j_3 \ K_2$

$\dot\psi\,(\text{sen}\theta\,\text{sen}\psi, -\text{sen}\theta\cos\psi, \cos\ldots)$

$\dot\theta\,(\cos\phi, \text{sen}\phi; 0)$

$\vec{\dot\psi} = \dot\psi\,(0, 0, 1)$

$\vec{\dot\theta} = \dot\theta\,(\cos\psi, -\text{sen}\psi, 0)$

$\vec{\dot\phi} = \dot\phi\,(\text{sen}\theta\,\text{sen}\psi, \text{sen}\theta\cos\psi, \cos\theta) \neq \dot\phi\,(0, 0, 1).$

$\mathbb{I}_{ij} = \sum_\alpha m_\alpha \left((\vec{r'}_\alpha)^2\delta_{ij} - r'_{\alpha i} r'_{\alpha j}\right) = \mathbb{I}_{ij\,CM} + M(a^2\delta_{ij} - a_i a_j)$

• $T = \frac{1}{2} M \dot{\vec{R}}(\dot{\vec{R}}_{CM} + \dot{\vec{R}}'_{CM}) + \frac{1}{2}\vec{\omega}^T\,\mathbb{I}\,\vec{\omega}$

• $\vec{P} = M\dot{\vec{R}}_{CM} = M(\dot{\vec{R}}_{CM}' + \dot{\vec{R}})$

• $\vec{L} = M\vec{R}\times\dot{\vec{R}}_{CM} + M\vec{R}'_{CM}\times\dot{\vec{R}} + \mathbb{I}\cdot\vec{\omega}$

$O'_{CM} \to = \mathbb{I}\vec{\omega} + \vec{R}_{CM}\times\vec{P}$

$$\frac{d\vec{L}}{dt} = \vec{N} = \frac{\partial\vec{L}}{\partial t} + \vec{\omega}\times\vec{L} = \begin{pmatrix} I_1\dot\omega_1 + (I_3 - I_2)\,\omega_3\omega_2 \\ 2 \quad 2 \quad (1 - 3) \quad 1 \quad 3 \\ 3 \quad 3 \quad (2 - 1) \quad 1 \quad 2 \end{pmatrix}$$

PEQUEÑAS OSCILACIONES ACOPLADAS

$X_\alpha = X_\alpha(q_1 \dots q_N)$

$V = \frac{1}{2} \underbrace{\frac{\partial^2 V}{\partial q_i q_j}\Big|_{q_i^0 q_j^0}}_{V_{ij}} \eta_i \eta_j \qquad T = \frac{1}{2} \underbrace{m_{ij}(q_1^0 \dots q_N^0)}_{T_{ij}} \dot{\eta}_i \dot{\eta}_j$

$\mathcal{L} = \frac{1}{2} T_{ij} \dot{\eta}_i \dot{\eta}_j - \frac{1}{2} V_{ij} \eta_i \eta_j$

$\Rightarrow \mathbb{T} \ddot{\vec{\eta}} + \mathbb{V} \vec{\eta} = 0 \sim\!\!\rightarrow \ddot{\vec{Q}} + \mathbb{W}_D \vec{Q} = 0$

$\vec{Q} = \mathbb{A}^{-1} \vec{\eta}$

$\det(\mathbb{T}^{-1}\mathbb{V} - \omega^2 \mathbb{I}) = 0 = \det(\mathbb{V} - \omega^2 \mathbb{T})$

$\rightarrow \mathbb{A}^T T \mathbb{A} = \mathbb{1}$

$(\ddot{\vec{\eta}} + M \vec{\eta} = 0)$

tensor masas

$W_0(1 \pm e)$

saber

triad ?

1 1 1 - 3

1 1 1 - 3, ...

14d

152 ...

$\vec{Q}\ \frac{1}{\sqrt{m}}$?

156

$\cos a + \cos b = 2 \cos \frac{+}{2} \cos \frac{-}{2}$

$\cos a - \cos b = -2 \sin \frac{+}{2} \sin \frac{-}{2}$

$\sin \alpha + \sin \beta = 2 \sin \frac{+}{2} \cos \frac{-}{2}$

Ampl. cond. 14

140 143

156

158

habrá problema cuántico

th puede haber cuestión

emisión social

Fourier → ω

$\omega = k v$? $\omega(k)$?

$v(k)$! v_{group} ?

115, 116

(Hall ??)

ONDAS

$$\frac{\partial^2 \psi}{\partial t^2} - v^2 \frac{\partial^2 \psi}{\partial x^2} = 0 \iff \psi(x,t) = \psi_+(x-vt) + \psi_-(x+vt)$$

Estac: $k = w/v$ $\qquad v^2 = \frac{E}{\rho} = \frac{Y_p}{\rho}$

$$\psi(x,t) = 4|A||B|\cos(wt+\beta)\cos\left(\frac{w}{v}x+\alpha\right) = 2|A||B|\left(\cos\left(\frac{wx}{v}+wt+\alpha+\beta\right) + \cos\left(\frac{w}{v}x - wt + \alpha - \beta\right)\right)$$

ψ_-, ψ_+

V. fase

$v_+ = \frac{dx}{dt} \qquad v_- = -\frac{dx}{dt}$ $\qquad T = \frac{2\pi}{w}$; $k = \frac{2\pi}{\lambda}$; $v = \lambda/T$

nº ondas

Relac dispersion

$w(k) = k\,v(k)$ $\qquad$ si $v(k) = cte \to$ no dispersivo

$v_f = \frac{w}{k}$

$v_g = \frac{\partial w}{\partial k}$ $\to v_g = v_f - \lambda \frac{\partial v_f}{\partial \lambda}$ (no disp $\to$ =)

medio normal: > 0 $\quad v_g < v_f$

anómalo: < 0 $\quad >$

$$\boxed{\begin{aligned}\Delta k \Delta x &= 2\pi \\ \Delta t \cdot \Delta w &= 2\pi\end{aligned}}$$

$f = 2a\cos(wt-kx)\cos(\Delta w t - \Delta k x)$

$C e^{i(wt-k_2x)}$ $\quad A e^{i(wt-k_1x)} + B e^{i(wt+k_1x)}$

$k_2 \neq k_1$, w cte en ambas ondas $\to v_2 \neq v_1$

$C = \frac{2k_1}{k_1+k_2} A$

$B = \frac{k_1-k_2}{k_1+k_2} A$

$R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} = \frac{|B|^2}{|A|^2}$

$T = 1 - R = \frac{4k_1k_2}{(k_1+k_2)^2} = \left|\frac{k_2}{k_1}\right| \cdot \frac{|C|^2}{|A|^2}$

$T = 2\pi$

$f(t) = \sum a_n \cos\frac{n\pi t}{T} + b_n \sin\frac{n\pi t}{T}$

$a_n = \frac{1}{T}\int_{-T}^{T} f(x)\cos\left(\frac{n\pi t}{T}\right)dt$

$b_n = \frac{1}{T}\int_{-T}^{T} f(x)\sin\left(\frac{n\pi t}{T}\right)dt$