POSTULADOS MQ

1) • Estados stma. físico $\rightarrow |\alpha\rangle \in \mathcal{H}$ ... compuesto $\mathcal{H}_i \times$ ...
   • $|\alpha\rangle \simeq c|\alpha\rangle$

2) • observable A $\rightarrow$ operador hermítico sobre esp. estados
   • resultado medida $\rightarrow$ autovalores
   • $A|a'\rangle = a'|a'\rangle$ .. autovector

3) • $\langle\alpha|A|\alpha\rangle$: valor promedio medida A sobre $|\alpha\rangle$

4) • $[X_i, X_j] = [P_i, P_j] = 0$; $[X_i, P_j] = i\hbar\delta_{ij}$

5) • $|\alpha\rangle \xrightarrow[A]{} a' \rightarrow$ colapso a $|a'\rangle$
   • $\rho \rightarrow$ medida filtrante $\Delta \rightarrow \rho_{A,\Delta} = \dfrac{\sum_{\cancel{a\in\Delta}} \Lambda_a \rho \Lambda_a}{\sum_{a\in\Delta} Tr(\rho\Lambda_a)}$

6) • $i\hbar \frac{\partial}{\partial t} |\alpha, t_0; t\rangle = H(t) |\alpha, t_0; t\rangle$

7) • $\left\{ \begin{array}{l} \text{Bosones: } \psi \text{ simétrica} \\ \text{Fermiones: } \psi \text{ antisimétrica} \end{array} \right\}$ bajo intercambio de dos partículas

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- $(XY\ldots Z)^\dagger = Z^\dagger \ldots Y^\dagger X^\dagger$
- $Tr\{XY\ldots Z\} = Tr\{ZXY\ldots\}$
- $\langle X, Y\rangle \equiv Tr\{X^\dagger Y\}$

$$\left(|X\psi\rangle\right)^\dagger = \langle X\psi| = \langle\psi|X^\dagger$$

↳ Diagonalizar $(H + xA) = c(x) \rightarrow \lambda$s dtos

T.1 - Conceptos fundamentales

- Stern - Gerlach, Ag, $\vec{\mu} \sim \vec{S}$, $l=0$ (5s), $\vec{\nabla}\cdot\vec{B}$, $F \propto \mu_z \propto S_z$ → S cuantizado
- indeterminismo, átomo a átomo
- $SGz+$ → $Sx+$ → $Sz <$ → lenguaje estados cuánticos, c. lin.
- Espacio de estados de stma físico
  - $\{|\alpha\rangle\} \equiv \mathcal{H}$, $\lambda|\alpha\rangle + \mu|\beta\rangle \in \mathcal{H}$ (ppo. superposición) → $\mathcal{H}$ esp. vectorial
  - $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^* \in \mathbb{C}$ → producto escalar def. positivo
  - $\langle\alpha|\alpha\rangle \geq 0$, $\in \mathbb{R}^+$; $\langle\alpha|\alpha\rangle = 0 \iff |\alpha\rangle = 0$
  - $\langle\gamma|\lambda\alpha + \mu\beta\rangle = \lambda\langle\gamma|\alpha\rangle + \mu\langle\gamma|\beta\rangle$ (linealidad
  - Dual ($\mathcal{H}$) = $\mathcal{H}^*$ → bra
    ↓ket
- Operador $X : \mathcal{H} \to \mathcal{H}$ → $X|\alpha\rangle = |X\alpha\rangle$
  - $(X+Y)|\alpha\rangle = X|\alpha\rangle + Y|\alpha\rangle$
  - Asociativa, Conmutativa ; Lineales
  - adjunto $(X) = X^+$; $X = X^+$ → hermítico
  - $X \cdot Y$ asociativa, elemento neutro, no conmuta, $\nexists$ simétrico (semigrupo)
  - $|\alpha\rangle\langle\beta|$ → operador (proyector)
- Observables
  - Valores propios operador hermítico → reales ← medida
  - Vectores " con distintos λ son ortogonales
  - $|\alpha\rangle = \frac{1}{\langle\alpha'|\alpha'\rangle}|\alpha'\rangle$ → Normalizar
  - Asociado a λ → subespacio vectorial $V_a \subset \mathcal{H}$ / $\forall |\alpha\rangle \in V_a \to A|\alpha\rangle = a|\alpha\rangle$
  - $V_{a'} \perp V_{a''}$ si $a' \neq a''$ (subespacios ortogonales)
  - $\exists \langle\alpha_i|\alpha_j\rangle = \delta_{ij}$ (base O.N.)
  - $\exists$ b.o.n. de vectores propios $|a_i\rangle$
  - $I = \sum_a |a\rangle\langle a|$ $\forall$ b.o.n
  - $\sum |c_a|^2 = 1$
  - $\Lambda_a = |a\rangle\langle a|$ proyector → $\sum \lambda_a = I$
  - Espectro < continuo / discreto < ∞ / finito → repres. matricial
  - $X_{ij} = \langle i|X|j\rangle$; $X = \langle i|X|j\rangle |i\rangle\langle j|$
  - $\langle\alpha|\beta\rangle = \sum_i \langle i|\alpha\rangle^* \langle i|\beta\rangle = (\ldots)\begin{pmatrix}\cdot\\\cdot\\\cdot\end{pmatrix}$
- Teorema espectral
  - A hermítico → $A = \sum_a \Lambda_a \cdot a$
- Cambio de base
  - $U|j\rangle = |j'\rangle$ → $U_{ij} = \langle i|j'\rangle$
  - $\langle i|\alpha'\rangle = U^+_{ij}\langle j|\alpha\rangle$; $|v'\rangle = U^+|v\rangle$; $X' = U^+ X U$
- Traza $tr(X) = \sum x_{ii}$
  1) Propiedad cíclica
  2) $tr(|\beta\rangle\langle\alpha|) = \langle\alpha|\beta\rangle$
  3) Invariante bajo cambio de base

T. 2 - Observables y medidas

- medida no es transf. unitaria → stma. colapsa a $|a\rangle$ al medir a.

$|\alpha\rangle \longrightarrow \boxed{A} \xrightarrow[a]{} c_a|a\rangle$ ó $\sum_i c_{a,i}|a,i\rangle$ (falta normalizar)

$c_a|a\rangle = \lambda_a|\alpha\rangle$

Λ_a no es unitario → $\Lambda_a^n = \Lambda_a$

- $\langle A\rangle_\alpha = \sum_a a\cdot p_\alpha(a) = \langle\alpha|A|\alpha\rangle = \sum_a a\cdot\overbrace{|\langle a|\alpha\rangle|^2}^{p_\alpha}$ ó $p_\alpha = \langle\alpha|\Lambda_a|\alpha\rangle$

- $e^{i\delta}|\alpha\rangle / \delta\in\mathbb{R}$ mý estado → rayo
  ↳ fases globales sin consecuenc observables

matrices de Pauli

$\boxed{|S_x\pm\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1\\ \pm1\end{pmatrix}}$; $\boxed{|S_y\pm\rangle = \frac{1}{\sqrt2}\begin{pmatrix}1\\ \pm i\end{pmatrix}}$; $\boxed{\vec\sigma = \left(\begin{pmatrix}0&1\\1&0\end{pmatrix};\begin{pmatrix}0&-i\\i&0\end{pmatrix};\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right)}$

$[S_x, S_y] = i\hbar S_z$; $[S_i, S_j] = i\hbar\,\epsilon_{ijk} S_k$ → operadores de mom angular

$\boxed{[\sigma_i,\sigma_j] = 2i\,\epsilon_{ijk}\sigma_k}$; $\boxed{\sigma_i\sigma_j = \delta_{ij}I + i\,\epsilon_{ijk}\sigma_k}$

$\boxed{\{\sigma_i,\sigma_j\} = 2\delta_{ij}\mathbb{I}}$

$[\sigma^2, \sigma_i] = 0$; $S^2 = \frac34\hbar^2 I$

- Funciones de un operador

$\boxed{F(A) = \sum_a F(a)|a\rangle\langle a|}$ → base de vectores propios

- $f(z) = \sum_n \alpha_n z^n \quad |z|<R$
  ↳ $f(A) = \sum_a\left(\sum_n \alpha_n a^n\right)|a\rangle\langle a| \rightarrow A^n = \sum_a a^n|a\rangle\langle a| \rightarrow f(A) = \sum \alpha_n A^n$

- Propieds exponencial
  - $e^A = \exp(A)$
  - $\exp(0) = I$
  - $\exp(A+B) = \exp\left(\frac12[A,B]\right)\cdot\exp(B)\cdot\exp(A)$

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Observables$_{AB}$ compatibles $\iff [A,B]=0$ → mismos $|v\rangle$, $\neq\lambda$

I) a no degenerado → $|a\rangle$ es propio de B; $\dim(V_a)=1$

II) a es " → $\dim(V_a)\geq 2$; ∃ b.o.n de $|v\rangle_B$ en V_a → $\{|a,b\rangle\}$

...$|a,b,c,...\rangle$ → etiquetas hasta tener un CCOC → conmutan to2 ; no degenera2

- medidas sucesivas ≠ simultáneas
  ↳ precisa + estado sin destruir lo anterior, proyecta +

× Observables incompatibles $\iff [A,B]\neq0$ → ∄ $\{|a,b\rangle\}$ base común
↳ puede haber un $|v\rangle$ propio de ambos xo no toda la base

Relaciones de incertidumbre

Observables $\to A \begin{cases} \langle A \rangle & \text{valor esp.} \\ \Delta A_\psi & \text{Desviación cuadrática media} \end{cases}$

$$\boxed{(\Delta A)_\psi = \sqrt{\sum_a p_\psi(a)\,(a - \langle A\rangle_\psi)^2} = \sqrt{\langle A^2\rangle_\psi - \langle A\rangle_\psi^2}}$$

A, B

$\hookrightarrow \Delta A_\psi \, \Delta B_\psi \geq \frac{1}{2} \left| \langle [A,B] \rangle_\psi \right|$ → restricción si A y B son incompatibles

$\hookrightarrow [X,P] = i\hbar$

$$\Delta x_\psi \, \Delta p_\psi \geq \hbar/2$$

- histograma → valores discretos
- X, P → continuo ⇝ ESPECTRO CONTINUO

$X|x_n\rangle = x_n|x_n\rangle \;/\; \langle x_n|x_{n'}\rangle = \delta_{nn'}$ $\qquad \lim_{a\to 0}$

$$\begin{cases} \delta_{nn'} \to a\,\delta(x-x') \\ \sum \to \frac{1}{a}\int dx \\ |x_n\rangle \to \sqrt{a}\,|x\rangle \end{cases} \qquad \begin{aligned} &\langle x|x'\rangle = \delta(x'-x) \\ &I = \int dx\, |x\rangle\langle x| \\ &\hookrightarrow |\psi\rangle = \int dx\,|x\rangle \langle x|\psi\rangle \end{aligned}$$

$\psi(x)$ → función de onda

$$P(x\in\Delta) = \sum P(x_i) \longrightarrow P_\Delta = \int_{x-\varepsilon/2}^{x+\varepsilon/2} dx'\,|\psi(x')|^2 \underset{\varepsilon\to 0}{\simeq} \varepsilon\,|\psi(x)|^2$$

$\langle\phi|\psi\rangle = \int dx\,\phi^*\psi$; $\langle\phi|\phi\rangle = \int dx\,|\phi(x)|^2$ ... Densidad de probabilidad x unidad de longitud

$\langle\phi|A|\psi\rangle = \int dx\int dx'\,\phi^*\,\langle x'|A|x\rangle\,\psi(x)$ → $A = f(x)$

$\hookrightarrow \langle\phi|f(x)|\psi\rangle = \int_{-\infty}^{+\infty} dx\,\phi^*\, f(x)\,\psi$

$\phi_a(x)$ ... func. onda operador H, osc. armón.

$A \to \{|a\rangle\} \to |\psi\rangle = \sum_a c_a|a\rangle \to \langle x|\psi\rangle = \sum c_a \langle x|a\rangle$

$\sum \phi_a^*\phi_a = \delta(x-x')$

... 3D → $\vec{X}|\vec{x}'\rangle = \vec{x}'|\vec{x}'\rangle \to \int d^3\vec{x}'\,|\vec{x}'\rangle\langle\vec{x}'| = I$; $\langle\vec{x}'|\vec{x}''\rangle = \delta^3(\vec{x}'-\vec{x}'')$

$\hookrightarrow \vec{P} \to \exists$ b.o.n. $\{|\vec{p}'\rangle\} / \vec{P}|\vec{p}'\rangle = \vec{p}'|\vec{p}'\rangle \to \int d^3\vec{p}'\,|\vec{p}'\rangle\langle\vec{p}'| = I$; $\underbrace{\langle\vec{p}'|\psi\rangle}_{\text{f. onda en el espacio de mom.}} \equiv \psi(\vec{p}')$

Operador de traslaciones: $\vec{l}\in\mathbb{R}^3$; $\boxed{U(\vec{l}) = e^{-i\frac{\vec{P}\cdot\vec{l}}{\hbar}}}$; $U^+ = e^{+i\ldots}$; $\vec{P}\cdot\vec{l}$ hermítico conmuta

- $U^+U = I$
- $U(\vec{l})\,U(\vec{l}') = U(\vec{l}+\vec{l}')$

$$\{U(\vec{l}) \,/\, \vec{l}\in\mathbb{R}^3\} \longrightarrow \begin{cases} 1) \text{ ley comp. interna} \\ 2)\ U(0) = I \quad \text{(e.n.)} \\ 3)\ U(\vec{l}) = U^+ \quad \text{(unit.)} \\ \quad U^{-1} = U^+ = U(-\vec{l}) \quad \text{(simetr.)} \\ 4)\ U(\vec{l}')U(\vec{l}) = U(\vec{l})\cdot U(\vec{l}') \quad \text{(conm)} \end{cases} \Longrightarrow \text{grupo abeliano}$$

Tma. de Hadamard

$$e^{A}\cdot B\cdot e^{-A} \to B(\lambda) = B + \lambda[A,B(\lambda)] + \frac{\lambda^2}{2!}[A,[A,B(\lambda)]] + \ldots = e^{\lambda A}\,B\cdot e^{-\lambda A}$$

$|\psi\rangle \to \langle\vec{x}|\psi\rangle = \int d^3\vec{p}\, \langle\vec{p}\,|\psi\rangle\langle\vec{x}\,|\vec{p}\rangle$

$\langle \mathcal{U}^+(\vec{\ell})x^\circ|\psi\rangle = \langle\vec{x}|U|\psi\rangle = \psi(\vec{x}) - I\vec{\ell}\,\vec{\nabla}\psi(\vec{x}) + \ldots = \psi(\vec{x}) - I\frac{\vec{\ell}}{\hbar}\langle\vec{x}|\vec{p}\rangle\psi$

$\hookrightarrow \langle\vec{x}|\vec{p}|\psi\rangle = -i\hbar\vec{\nabla}\psi(\vec{x}) = -i\hbar\vec{\nabla}\langle x|\psi\rangle$

$\phi_{\vec{p}\,'}(\vec{x}) = N_{p'}\cdot e^{i\frac{\vec{p}\,'\vec{x}}{\hbar}}$; $N_{\vec{p}} = \frac{1}{(2\pi\hbar)^{3/2}}$

Oscilador armónico

$H = p^2/2m + \frac{1}{2}m\omega^2 X^2$ $\qquad X = \sqrt{\frac{\hbar}{2m\omega}}(a + a^+)$

$a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{iP}{m\omega}\right)$; a^+ $\qquad P = \sqrt{\frac{\hbar m\omega}{2}}\, i\,(a^+ - a)$

$[a, a^+] = I$; $N = a^+a$ (operador número) $\qquad N|n\rangle = n|n\rangle$; $n = \langle n|N|n\rangle$
$N = N^+$

$[N, a] = -a$; $[N, a^+] = a^+$

$\begin{cases} a|n\rangle = \sqrt{n}\,|n-1\rangle \\ a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle \end{cases}$; $\exists |0\rangle$, $n \in \mathbb{N}_0^+$ $\longrightarrow$ $|n\rangle = \frac{(a^+)^n}{\sqrt{n!}}|0\rangle$

$H = \hbar\omega\left(N + \frac{I}{2}\right)$; $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ $\qquad \langle x|0\rangle = \frac{1}{\sqrt{x_0\sqrt{\pi}}}e^{-\frac{1}{2}\left(\frac{x}{x_0}\right)^2}$; $x_0 = \sqrt{\frac{\hbar}{m\omega}}$

Mom angular:

1) Orbital $\quad L_i = \epsilon_{ijk}x_j p_k$; $[L^2, L_i] = 0$ $\quad$ (CCOC) $\{L^2, L_z\}$ $\qquad [L_i, L_j] = i\hbar\epsilon_{ijk}L_k$

$\vec{L} = \vec{x}\times\vec{p}$

$L^2|\ell m\rangle = \ell(\ell+1)\hbar^2|\ell, m\rangle$ $\qquad Y_{\ell m}(\theta,\varphi) = \langle\vec{x}, \ell, m\rangle \to$ armónicos esféricos

$L_z|\ell, m\rangle = m\hbar|\ell, m\rangle$ $\qquad \hookrightarrow$ continua cada $2\pi \longrightarrow m = \underbrace{-\ell, \ldots \ell}_{2\ell+1}$, ℓ entero

2) General

$\{J_x, J_y, J_z\}$ son oper. mom ang. $\iff [J_i, J_j] = i\hbar\epsilon_{ijk}J_k$

$\to [J^2, J_i] = 0$ $\qquad [J_+, J_-] = 2\hbar J_z$

$\{J^2, J_z\}$; $J_\pm = J_x \pm iJ_y$ $\qquad [J_z, J_\pm] = \pm\hbar J_\pm$

$[J^2, J_\pm] = 0$

- $j = n/2$, $n \in \mathbb{N}_0^+$
- $m = -j, \ldots +j \quad \to 2j+1$

$J^2|jm\rangle = j(j+1)\hbar^2|jm\rangle$

$J_z|jm\rangle = m\hbar|jm\rangle$

$J_\pm|jm\rangle = \hbar\sqrt{(j\mp m)(j\pm m+1)}\cdot|j, m\pm1\rangle$

T. 3 - OPERADOR DENSIDAD

Puro: todas las partículas en un mismo estado cuántico

Mezcla: $\begin{cases} \{|\psi_n\rangle\} & \text{no necesariamente ortogonales} \\ \{p_n\} & \text{proporciones } 0 \le p_n \le 1 \,/\, \sum_n p_n = 1 \end{cases}$

$$\langle A\rangle = \sum_n p_n \langle\psi_n|A|\psi_n\rangle = \sum_n p_n \, Tr(|\psi_n\rangle\langle\psi_n|A) = Tr\{\underbrace{\sum p_n|\psi_n\rangle\langle\psi_n|}_{\rho} A\}$$

$$\boxed{\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|}$$

$\boxed{p(a) = Tr(\rho\Lambda_a)}$; $\langle A\rangle = \sum a\, p(a)$

Base $\{|i\rangle\}$ o.n $\rightarrow \rho = \langle i|\rho|j\rangle |i\rangle\langle j|$ $\begin{cases} \langle i|\rho|i\rangle & \text{poblaciones} \\ \langle i|\rho|j\rangle & \text{coherencias} \end{cases}$

Propieds.:

1) $\rho^+ = \rho$
2) $Tr(\rho) = 1$
3) $\langle\psi|\rho|\psi\rangle \ge 0$
4) b.o.n $\{|\omega\rangle\} \rightarrow \rho = \sum \omega|\omega\rangle\langle\omega| \rightarrow \sum\omega = 1$; $\sum\omega^2 \le 1$

c. Puro $\Longleftrightarrow Tr(\rho^2) = 1$; $\rho_{puro} = \sum |\omega_0\rangle\langle\omega_0|$

Medidas

$$\boxed{\rho' = \frac{\sum_{a\in\Delta}\Lambda_a\rho\Lambda_a}{\sum_{a\in\Delta} Tr\{\rho\Lambda_a\}} = \frac{\sum_{a\in\Delta}\Lambda_a\rho\Lambda_a}{\sum_{a\in\Delta} p(a)}} \quad ; \; Tr(\rho') = 1 \checkmark$$

- $a\in\Delta$ no degenerados $\rightarrow \rho' = \sum w_a|a\rangle\langle a|$; $w_a = \dfrac{\langle a|\rho|a\rangle}{\sum_{a\in\Delta}\langle a|\rho|a\rangle} = \dfrac{p_a}{\sum_{a\in\Delta} p_a}$
- $\Delta = a \rightarrow \rho' = \dfrac{\Lambda_a\rho\Lambda}{Tr(\Lambda_a\rho)} = |a\rangle\langle a| \rightarrow$ C. Puro
- Δ = todos $\rightarrow \Lambda_\Delta = I \rightarrow \rho' = \sum_a \Lambda_a\rho\Lambda_a = \sum_a p(a)|a\rangle\langle a|$ (sin deg.)

Vector de polarización (spin 1/2)

2x2 $\rightarrow A = a_0 I + \vec a\cdot\vec\sigma \rightarrow a_0 = \frac12 Tr(A)$; $a_i = \frac12 (Tr\,\sigma_i A)$

$\rho = \frac12(I + \vec p\cdot\vec\sigma)$; $\boxed{\vec p = 2\vec a = Tr(\vec\sigma\rho) = \frac{\langle\vec S\rangle}{\hbar/2}}$ $\rightarrow$ Vector de polarización

$\rho|\vec S\hat p, \pm\rangle = \frac12(1\pm|\vec p|)|\vec S\hat p,\pm\rangle \rightarrow \rho = \frac12(1+|\vec p|)|\vec S\hat p,+\rangle\langle\vec S\hat p,+| + \frac12(1-|\vec p|)|\vec S\hat p,-\rangle\langle\vec S\hat p,-|$

- $|\vec p| = 1 \rightarrow$ C.P $\rightarrow$ mezcla totalm polarizada
- $0 < |\vec p| < 1 \rightarrow$ " parcialm "
- $|\vec p| = 0 \rightarrow$ " no polarizada $\rightarrow$ isótropo
- Esfera de Block $|\psi\rangle = \cos\frac\theta2|+\rangle + e^{i\varphi}\,\text{sen}\frac\theta2|-\rangle$

- Medidas no filtrantes de spin (1/2)

$\vec p_f = (\vec p_i\cdot\hat n)\,\hat n$

T.4 - Evolución de sistemas y observables

$|\psi, t\rangle = U(t,t_0)\,|\psi, t_0\rangle$

U : operador de evolución temporal

1) unitario

2) $U(t_2,t_1)\,U(t_1,t_0) = U(t_2,t_0)$

3) $U(t_0,t_0) = I$

• $U(t+dt,t) = I - i\Omega(t)\,dt + O(dt^2) \rightarrow \Omega = \Omega^+ \rightarrow \hbar\Omega = H(t)$

↳ $\boxed{i\hbar \frac{\partial}{\partial t} U(t,t_0) = H(t)\cdot U(t,t_0)}$ → separas evol. temporal ∀ estado

↳ $\boxed{i\hbar \frac{\partial}{\partial t}|\psi,t\rangle = H(t)|\psi,t\rangle}$ Ecuac de Schrödinger

I) Si $H \neq H(t) \rightarrow \boxed{U(t,t_0) = e^{-\frac{i}{\hbar}(t-t_0)H}}$

II) $[H(t),H(t')] = 0 \;\; \forall t,t' \rightarrow \boxed{U(t,t_0) = e^{-\frac{i}{\hbar}\int_{t_0}^{t'} dt\, H(t)}}$

$H = p^2/2m + V$

↳ $-i\hbar \frac{\partial}{\partial t}\psi(\vec{x},t) = -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{x},t) + V(\vec{x},t)\,\psi(\vec{x},t)$ Ecuac de Schrödinger para ψ

$\boxed{i\hbar \frac{d}{dt}\langle A\rangle = \langle [A,H] + i\hbar \frac{dA}{dt}\rangle}$

$A(t)$ es constante de movimiento $\Longleftrightarrow [A,H] + i\hbar\frac{dA}{dt} = 0$

$\Longleftrightarrow A' = U^+AU$ no depende de t / $A'^+ = A'$

$A' = \sum a|a\rangle\langle a|$, $A = \sum a\, |a,t\rangle\langle a,t|$ $\quad$ a ctes $\quad |a,t\rangle = U(t,t_0)|a\rangle$

1 ↳ a ctes (resultado medida)

2 ↳ $\Lambda_a(t) = U(t,t_0)|a\rangle\langle a|U^+(t,t_0)$ ctes movi$\varnothing$

3 ↳ $P_\psi(a) = \langle \Lambda_a(t)\rangle$ independiente de t

4 ↳ H cte movi$\varnothing$ $\Longleftrightarrow H \neq H(t)$

5 ↳ Si $A \neq A(t) \rightarrow A$ cte mov. si $[A,H(t)] = 0$

Stma. conservativo → H cte, $U = e^{-\frac{i}{\hbar}(t-t_0)H} \rightarrow H|\psi_E\rangle = E|\psi_E\rangle$

$|\psi_E,t\rangle = e^{-\frac{i}{\hbar}(t-t_0)E}|\psi_E\rangle$ → estados estacionarios

$B \neq B(t) \rightarrow \langle B\rangle_{\psi_E,t} = $ cte

• degenerad en H → $\{H,A\}$ ccoc → $U(t,t_0) = \sum_a |a\rangle\langle a|\, e^{-\frac{i}{\hbar}(t-t_0)E_a}$

$\langle B\rangle = \sum_{a,a'} e^{-\frac{i}{\hbar}(t-t_0)(E_a - E_{a'})}\langle\psi,t_0|a'\rangle\langle a'|B|a\rangle\langle a|\psi,t_0\rangle$

Tma. Ehrenfest

$i\hbar\frac{d}{dt}\langle\vec{x}\rangle_\psi = \langle[\vec{x},H]\rangle_\psi \Longrightarrow \frac{d}{dt}\langle\vec{x}\rangle_\psi = \frac{\langle\vec{p}\rangle_\psi}{m}$

$i\hbar\frac{d}{dt}\langle\vec{p}\rangle_\psi = \langle[\vec{p},H]\rangle_\psi \Longrightarrow \frac{d}{dt}\langle\vec{p}\rangle_\psi = -\vec{\nabla}\langle V(\vec{x})\rangle \neq -\vec{\nabla}V(\langle\vec{x}\rangle)$

Imágenes de evolución

$V(t)\,V^+(t) = I$ (V unitario)

$\langle\psi|A|\psi\rangle = \langle\psi|V^+VAV^+V|\psi\rangle = \langle\psi'|A'|\psi'\rangle$

☆ Imagen de Heisenberg $\boxed{V(t) = U^+}$

$|\psi\rangle_H = V\psi = VU\psi_0 = \psi_0 \rightarrow |\psi,t\rangle_H = |\psi,t_0\rangle_S$

$A_H(t) = U^+ A_S U$; $A_H(t_0) = A_S$

$V\ A_S\ V^+$ ✓

$\Downarrow$

$i\hbar \frac{d}{dt} A_H = [A_H, H_H] + i\hbar \frac{\partial}{\partial t} A_H(t)$

$[A(t), B(t)] = c(t) \rightarrow [A_H(t), B_H(t)] = V c(t) V^+ = C_H(t)$

$[X, P] = [X_H, P_H]$; $c \propto I$

$A_H(t)$ cte de movim si $\frac{d}{dt}A_H = 0$

Stma. conservativo $H \neq H(t) \longrightarrow H_H = H_S$

Tma. de Ehrenfest : $\frac{d}{dt}\vec{X}_H(t) = \frac{\vec{p}_H(t)}{m}$; $\frac{d}{dt}\vec{p}_H(t) = -\vec{\nabla} V_H(\vec{X}_H, t)$

Evolución del oscilador armónico: (imagen de Heisenberg)

$i\hbar \frac{da}{dt} = \hbar\omega\, a(t) \rightarrow \boxed{i\frac{da}{dt} = \omega\, a(t)}$ → para operadores

$a(t) = e^{-i\omega t} a(0)$; $a^+(t) = e^{i\omega t} a^+(0)$

$X(t) = X(0)\cos\omega t + \frac{p(0)}{m\omega}\sin\omega t$

$p(t) = p(0)\cos\omega t - m\omega X(0)\sin\omega t$

$\begin{pmatrix} X \\ \frac{p}{m\omega} \end{pmatrix} = \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix} \begin{pmatrix} X \\ p/m\omega \end{pmatrix}$

Evolución de estados mezcla

$\rho = U\rho_0 U^+ \longrightarrow i\hbar \frac{d\rho}{dt} = [H, \rho]$ ... cambio signo

Ev. del vector de polarización

$\frac{d\vec{P}}{dt} = \omega(\vec{n} \times \vec{P})$ Precesión $\rightarrow \vec{p}(t) = R_{\vec{n}}(\omega t)\,\vec{p}(0)$

Oscilaciones de neutrinos

proyectas Λ_a → evolución $H \rightarrow \Lambda_a \rightarrow$ my? N^o $[H, A] \neq 0$

$p_{a'} = |\sum_v \langle v|a\rangle e^{-\frac{iE_v t}{\hbar}} \langle a'|v\rangle|^2$; $U(t,0)|a\rangle = \sum \langle v|a\rangle U|v\rangle$

$|\nu_1\rangle, |\nu_2\rangle$ masa definida $\quad |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$

$|\nu_e\rangle, |\nu_\mu\rangle$ familia " $\quad |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$ } $\cos\theta = \langle\nu_1|\nu_e\rangle$

$|\nu_e, t\rangle = e^{-\frac{iE_1 t}{\hbar}}\cos\theta|\nu_1\rangle + e^{-i\frac{E_2 t}{\hbar}}\sin\theta|\nu_2\rangle$

$p(\nu_e \rightarrow \nu_\mu, t) = \sin^2(2\theta)\sin^2((E_2 - E_1)\,t/2\hbar)$ $\qquad\theta$: ángulo de mezcla

$p(\nu_\mu \rightarrow \nu_e, t) = 1 - p(\nu_e, \nu_\mu, t)$

$\Delta m_{21}^2 = m_2^2 - m_1^2 \rightarrow p(\nu_e \rightarrow \nu_\mu, t) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 c^3 L}{4\hbar E}\right)$

T. 5 POTENCIALES DEPENDIENTES DEL TIEMPO.

$H = H_0 + V(t)$

$|\psi, V=0\rangle = \sum_n C_{n\psi}|n\rangle \rightarrow |\psi, t\rangle = \sum C_{n\psi}|n\rangle e^{-iE_nt/\hbar}$

↳ $V \neq 0$ $\quad |\psi, t\rangle = \sum_n C_{n\psi}(t) \cdot e^{-i\frac{E_nt}{\hbar}}|n\rangle$

$t=0 \rightarrow |\psi\rangle = |i\rangle \longrightarrow |i, t\rangle = \sum_n C_{ni}(t)\, e^{-i\frac{E_nt}{\hbar}}|n\rangle$

$|\psi, t\rangle = e^{-\frac{itH_0}{\hbar}} \sum C_{n\psi}(t)|n\rangle \longrightarrow |\psi, t\rangle_I = e^{+\frac{itH_0}{\hbar}}|\psi, t\rangle$ → imagen de interacción

$= \sum C_{n\psi}(t)|n\rangle$ → estados evolucionan × V

si $A = H_0 \rightarrow H_{0I} = H_0 \longleftarrow\longrightarrow A_I = e^{\frac{itH_0}{\hbar}} A\, e^{-itH_0/\hbar}$ → operadores " × H_0

$V_I = e^{+} V e^{-}$

$i\hbar \frac{d}{dt}|\psi, t\rangle_I = V_I(t)\,|\psi, t\rangle_I \rightarrow i\hbar\frac{d}{dt}C_n(t) = \sum_m e^{i\omega_{nm}t} V_{nm} C_{mi}(t)$

↳ acoplado

$i\hbar \frac{d}{dt} A_I = [A_I, H_0] \quad$ si $A \neq A(t)$

2 niveles con perturbₙ armónica

$|c_2(t)|^2 = \frac{\gamma^2}{\hbar^2\Omega^2}\, \text{sen}^2(\Omega t)$; $\Omega^2 = \left(\frac{\gamma}{\hbar}\right)^2 + \left(\frac{\omega - \omega_{21}}{2}\right)^2$, Ω = frecuₑ de Rabi

$|c_1(t)|^2 = \text{"}\ \cos^2(\Omega t)$

$|c_2|^2_{max} = \frac{\gamma^2/\hbar^2}{\left(\frac{\gamma}{\hbar}\right)^2 + \left(\frac{\omega-\omega_{21}}{2}\right)^2} \longrightarrow$ "Breit-Wigner"

$\Delta\omega = 4\gamma/\hbar$

$\omega_{res} = \omega_{21}\ \forall \omega$

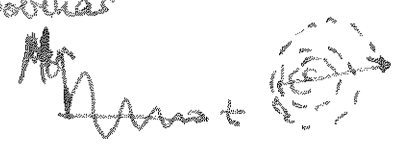
RMN

E definida ↑↓ precesa → meter B_0 ; $H = -\frac{\hbar}{2}\omega_0 \sigma_z$; $\omega_0 = \frac{2\mu B_0}{\hbar}$; $\vec{\mu} = g\cdot\vec{S}$

$\frac{n_2}{n_1} = e^{-(E_2-E_1)/K_BT}$; $\nu_0 = \frac{\omega_0}{2\pi}\ (B = 1T) = 42{,}6$ MHz (frecuₑ de Larmor)

↳ campo ortogonal (t) → precesión → inducción bobinas

$H = -\frac{\hbar\omega_0}{2}\sigma_z + \hbar q\,(\sigma_x \cos\omega t + \sigma_y \,\text{sen}(\omega t))$



resonancia ω_0, dtos. picos (I, ω) → n° y campos.

$\omega(B)$, $\vec{\nabla}\cdot\vec{B}$

Teoría de perturbₙs

$|\psi, t\rangle_I = \underbrace{e^{+}\, U\, e^{-}}_{U_I}|\psi, t_0\rangle_I \qquad U_I(t_0, t_0) = I$

U_I : operador de evolución temporal en la im. de interacción

$i\hbar\frac{d}{dt}U_I = V_I U_I \longrightarrow U_I = I - \frac{i}{\hbar}\int_{t_0}^{t} dt'\, V_I(t')\, U_I(t', t_0)$

1º $\quad U_I = I - \frac{i}{\hbar}\int_{t_0}^{t} dt'\, V_I(t')$

2º $\quad U_I = I - \frac{i}{\hbar}\int_{t_0}^{t} dt'\, V_I(t') + \left(\frac{-i}{\hbar}\right)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt''\, V_I(t')\, V_I(t'')$

n: $\quad U_I = I + \sum_{n=1}^{\infty}\left(-\frac{i}{\hbar}\right)^n \int_{t_0}^{t} dt_1\, V_I(t_1) \int_{t_0}^{t_1} dt_2\, V_I(t_2) \cdots \int_{t_0}^{t_{n-1}} dt_n\, V_I(t_n) \rightarrow$ SERIE DE DYSON

Probabilid transición

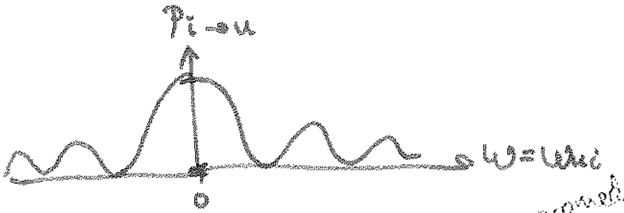
$$c_{ni}^{(0)}(t) = \delta_{ni}$$

$$c_{ni}^{(1)}(t) = -\frac{i}{\hbar}\int_{t_0}^{t} dt_1 \langle n|V_I(t_1)|i\rangle = -\frac{i}{\hbar}\int_{t_0}^{t} dt_1 V_{ni}(t_1)\, e^{i\omega_{ni}t_1}$$ ... im. Schröd!

$$c_{ni}^{(2)}(t) = \left(-\frac{i}{\hbar}\right)^2 \sum_m \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, V_{nm}(t_1)\cdot e^{i\omega_{nm}t_1}\, V_{mi}(t_2)\cdot e^{i\omega_{mi}t_2}$$

$$V(t) = \begin{cases} 0 & t<0 \\ V & t>0 \end{cases}$$

$$P_{i\to n} = \left[\frac{|V_{ni}|}{\hbar}\, \frac{\operatorname{sen}\left(\frac{\omega_{ni}t}{2}\right)}{\omega_{ni}/2}\right]^2$$ ~ 2 niveles, Rabi $\frac{2|V_{ni}|}{\hbar} \ll \omega_{ni}$



$$\left(t\, \operatorname{sinc}\left(\frac{\omega_{ni}t}{2}\right)\right)^2$$

promedio a $\lim_{t\to\infty} t\left(\frac{\operatorname{sen} tx}{tx}\right)^2 = \pi\cdot\delta(x)$

Prob. transic/t

$$W_{i\to n} = \frac{P_{i\to n}}{t} = \frac{2\pi}{\hbar}|V_{ni}|^2 \rho(E_i)$$ Regla de oro de Fermi

Probabilid desintegración ... a 2º orden

$$|c_{ii}(t)| = 1 - \frac{4}{\hbar^2}\sum_{n\neq i}\frac{|V_{ni}|^2}{\omega_{ni}^2}\operatorname{sen}^2\left(\frac{\omega_{ni}t}{2}\right) = 1 - \sum_{n\neq i} P_{i\to n}(t)$$

$$\Gamma_i = \sum 2\pi |V_{ni}|^2 \rho(E_i)$$

$$|c_{ii}|^2 \sim 1 - \frac{\Gamma_i}{\hbar}t \longrightarrow |c_{ii}(t)|^2 = e^{-\frac{\Gamma_i t}{\hbar}}$$ → Ley de desintegrac exp.

$$c_{ii}(t) = e^{-\frac{i}{\hbar}\left(\delta E_i - i\frac{\Gamma_i}{2}\right)t} \quad ; \ \delta E_i \in \mathbb{R}$$ → modifica niveles δ + los hace inestable

$$|f(E)|^2 = \frac{\hbar^2}{(E-\tilde{E}_i)^2 + \frac{\Gamma_i^2}{4}}$$ ... B-W

Energía nivel i no bien definida → anchura $\Gamma_i \propto \tau$ vida media

Relac de indeterminac energía - tiempo (de evolución)

$\to \tau_i \Gamma_i = \hbar$