$|z_1 + z_2| \leq |z_1| + |z_2|$

$d(z_1, z_1) \geq 0 \iff z_1 = z_2$

$d(z_2, z_1) = d(z_1, z_2)$

$d(z_1, z_3) \leq d(z_1, z_2) + d(z_2, z_3)$

Disco abierto

$D_r(z_0) = \{ z \in \mathbb{C} / |z - z_0| < r \}$

perforado

$D_r^P(z_0) =$ $0 < \ldots$

Curva de Jordan → sin puntos múltiples

Circuito ≡ curva Jordan cerrada

$\gamma : z(t) = x(t) + i y(t)$ (+)

$|z| = \sqrt{z \cdot z^*}$

$x = \dfrac{z + z^*}{2}$

$y = \dfrac{z - z^*}{2i}$

$z^n = r^n (\cos n\theta + i \sin n\theta$

$\lim_{z \to z_0} f(z) = w_0$ $\quad \forall z \in |z-z_0|<\delta \rightarrow |f(z)-w_0|<\epsilon$

$\forall \epsilon \ \exists \delta(\epsilon); \forall z \in D_\delta(z_0) \rightarrow f(z) \in D_\epsilon(w_0)$

i) $\exists$ lim $\longleftrightarrow$ $\exists$ u, v

ii) Si $\exists$ " $\rightarrow$ único + no dep. camino $\rightarrow$ cualquier γ en el plano

iii) $\lim_{z \to z_0} f(z)^* = w_0^*$

CONTINUIDAD

$f(z)$ en z_0, definida en $D \subset \mathbb{C}$

1) $\exists f(z_0)$

2) $\exists \lim_{z \to z_0} f(z) = w_0$ $\Big\}$ $\Rightarrow \lim_{z \to z_0} f(z) = f(z_0) \Rightarrow f(z)$ continua en z_0

$\Updownarrow$

u, v lo son en $z_0(x, y)$

3) $f(z_0) = w_0$

DIFERENCIABILIDAD (pto)

anal. = dif $\Rightarrow$ cont z_0
regul.
holom. $\Rightarrow$ E. C. R

dif = regular, holomorfa, analítica (D)

$f(z)$ dif. en z_0 si $\exists$

$$\lim_{\Delta z \to 0} \frac{f(z_0+\Delta z) - f(z_0)}{\Delta z} = f'(z_0) = \frac{df}{dz} \in \mathbb{C}$$

dif. $\Longleftarrow$ E. C. R (?)

CONDICIONES CAUCHY-RIEMANN

dif. en un pto. $\Rightarrow$ $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$; $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

$\Longleftarrow$ E. C-R + ∂ continuas en (x_0, y_0)

?

FUNCIÓN REGULAR, ANALÍTICA U HOLOMORFA EN UN PUNTO

$f(z)$ diferenciable en entorno abierto del punto + z_0 $D_r(z_0)$

↳ punto regular $\rightarrow f$ regular en dicho punto

singular $\rightarrow f$ no " " " "

$f(z)$ analítica si es desarrollable en $\sum_{n=0}^{\infty}$ potencias (+) (Taylor no singular) $|z-z_0|<R$

FUNCIONES MULTIVALUADAS / MULTIFORMES

$\sqrt{\ }$

$\theta \in [0, 2\pi[$ 1ª hoja Riemann

$\theta \in [2\pi, 4\pi[$ 2ª hoja "

ó $[-\pi, \pi[$ $[\pi, 2\pi[$

CORTE en $0, 2\pi$

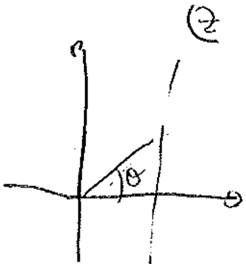
$0 \rightarrow$ pto de ramificac (branching point) origen corte

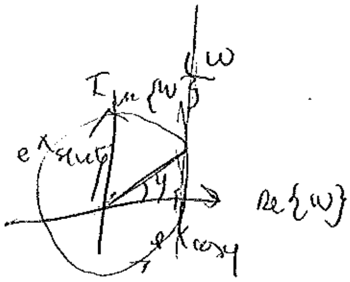
log $\rightarrow \infty$s hojas

④

F. EXPONENCIAL

$w = e^{z}$



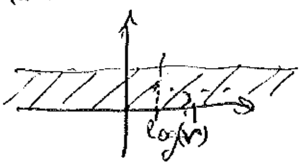


$e^{z_0} = e^{x} e^{i(y+2\pi K)}$

$W = \log z = \log r + i\theta$ $\theta_p \in [0, 2\pi[$

↳ corte cada 2π, $\theta = \theta_p + 2\pi K i$

$= \underbrace{\log r + i\theta_p}_{\text{Log} z} + 2\pi K i$



$\log(z_1 z_2) = \log z_1 + z_2$

$\text{Log}(z_1 \cdot z_2) = \text{Log} z_1 + \text{Log} z_2 + 2\pi i K$

$\text{sen} z = \dfrac{e^{iz} - e^{-iz}}{2i}$ no acotada $|\text{sen} z| \leq 1$

$\text{sen}(z_1 + z_2) = \text{sen} z_1 \cos z_2 + \text{sen} z_2 \cos z_1$

$\cos z = \dfrac{e^{iz} + e^{-iz}}{2}$ $\cos iz = \text{ch} z$

$\text{senh} z = \dfrac{e^{z} - e^{-z}}{2}$ ~~sen~~ $\cosh' z = \text{senh} z = \text{senh}''' z = \text{senh}'' z$

$\cosh z = \dfrac{e^{z} + e^{-z}}{2}$

F. Potencial

$w = z^{a}$ $a \in \mathbb{C}$

Multivaluada si $a \notin \mathbb{Z}$

↳ $w(K)$
↳ hoja Riemann

$w = e^{\log z^{a}} = \left(e^{a \log r} e^{a i \theta}\right) e^{a 2K\pi i} = w(K)$

$I = \int_{z_i, c}^{z_f} f(z) \cdot dz$ $c: z(t)$ ó $y(x), x(y)$

$= \int_a^b f(t) \cdot z'(t)\, dt = \int_{x_i,y_i}^{x_f,y_f} u(x,y)dx - v(x,y)dy + i \int_{x_i,y_i}^{x_f,y_f} v(x,y)dx + u(x,y)dy$

$\int_\gamma f(z)dz \leq M \cdot L(\gamma)$

Teorema Cauchy – Goursat

$f(z)$ analítica en dominio abierto + simplemente conexo en $\mathbb{C}$ ($\forall \gamma$ puntos internos)

$\hookrightarrow \oint_\gamma f(z)\, dz = 0 \quad \gamma \subset D$

T. de Green

$\oint P(x,y)dx + Q(x,y)dy = \oint (P,Q)(dx,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$

Integral no depende del camino

$\int_{z_i, \gamma}^{z_f} f(z)\, dz = \int_{z_i, c}^{z_f} f(z) dz$

T. de Morera

$\oint_c f(z)\, dz = 0 \Longrightarrow f(z)$ es analítica

+ $f(z)$ continua en D

inverso Cauchy
+ restrictivo

analít $\rightarrow I = 0$
analít. $\leftarrow I = 0 \ \forall \gamma$ + cont

Funciones multivaluadas

$\oint \sqrt{z}\, dz = \int_0^{2\pi}$ (1ª hoja) $\neq 0$ → circuito completo: $\int_0^{4\pi}$

corte, no cierres

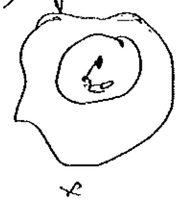
log ∞s vueltas, no circuito

$\sqrt[n]{z} \rightarrow n$ vueltas $(2\pi n)$

Índice camino (winding number)

$$\oint \frac{1}{z} dz = n \cdot 2\pi i$$

Deformación circuito integración



→ Mientras no englobes otra singularid
zona ampliación todo analítico

Fórmula integral Cauchy

$$\oint_\gamma \frac{f(z)}{z - z_0} dz = 2\pi i \, f(z_0)$$

si $f(z)$ analítica en interior γ y
$z_0 \in$ interior (no en γ)

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + re^{i\theta}) \equiv \text{valor promedio}$$

$f(z_0)$ depende entorno

$\downarrow \frac{d}{dz_0}$

$$\oint \frac{f(z)}{(z - z_0)^{K+1}} dz = \frac{2\pi i}{K!} f^{(K)}(z_0)$$

Sucesiones numéricas: 'sequence'

$z_i \in \mathbb{C}$

$\{z_n\}_{n=1}^{\infty}$

$\lim_{n\to\infty} z_n = w$: $\forall \epsilon \; \exists N \in \mathbb{N} \;/\; \forall n > N \Rightarrow |z_n - w| < \epsilon$

$z_n = \frac{1}{n} + \frac{n-1}{n} i \rightarrow |z_n - w| = \frac{|1-i|}{n} < \epsilon \rightarrow N = E\left(\frac{\sqrt{2}}{\epsilon}\right)$

$\{i^n\}$ no convergente

$z_{n+1} = z_n^2 + c \rightarrow$ Mandelbrot $\quad c=0: \{0\}$, $c=1: \{0, 1, 2, 5, \dots\}$, $c=-1: \{0, -1, 0, -1, \dots\}$, $c=i: \{0, i, (-1+i), -i, \dots\}$

Series numéricas

$z_1, z_1+z_2, \dots, S_n = \sum_{k=1}^{n} z_k$

$\downarrow S_1 \quad \downarrow S_2$

$S = \lim_{n\to\infty} \sum_{k=1}^{n} z_k = \sum_{n=1}^{\infty} z_n$; $z_n = \sum a_n + i\sum b_n = a + ib$ $\quad C \Leftrightarrow \sum a_n, \sum b_n \; C$

C necesaria $\lim_{n\to\infty} z_n = 0 \begin{cases} a_n \\ b_n \end{cases}$ (no sufic, ritmo...) $\hookrightarrow \sum \frac{1}{n}$ D

Convergencia absoluta

$\sum |z_n|$ C

Criterios

- D'Alembert

$$\lim \left| \frac{z_{n+1}}{z_n} \right| = L \quad \begin{cases} L > 1 & D \\ L < 1 & CA \\ L = 1 & ? \end{cases}$$

- Raíz o de Cauchy

$$\lim \sqrt[n]{|z_n|} = L$$

Sucesiones y series de funciones

$f_1(z), f_2(z), \dots \quad \{f_n(z)\}_{n=1}^{\infty} \quad z \in D \subset \mathbb{C} \qquad f(n,z)$

$f(z) = \lim_{n\to\infty} f_n(z) \quad \forall \epsilon, \forall z \in D \; \exists N(\epsilon, z) \;/\; \forall n > N \Rightarrow |f_n(z) - f(z)| < \epsilon$

$\hookrightarrow$ Convergencia puntual

- Convergencia uniforme

Si N no depende de $z \rightarrow N(\epsilon)$ (coges el N > de todos los zs del intervalo $\rightarrow \alpha$)

$D: |z| \in [\alpha, \infty[$ C.U. $\quad N > \frac{1}{\alpha\epsilon} \quad N(\epsilon)$

$D: |z| \in]0, \alpha]$ ~~C~~ $\quad N > \frac{1}{|z|\epsilon} \quad N(\epsilon, z)$

Series

$f_1(z), \; f_1+f_2, \dots, \underbrace{f_1+\dots+f_n}_{S_n}, \dots$

$\underbrace{f_1(z)}_{S_1}$

$$S(z) = \lim_{n\to\infty} S_n(z) = \lim_{n\to\infty} \sum_{k=1}^{n} f_k(z) = \sum_{k=1}^{\infty} f_k(z) \qquad \sum_{k=0}$$

↳ Criterio Weierstrass

$$\forall \varepsilon, \forall z \in D, \exists N(\varepsilon, z) \,/\, \forall n > N \Rightarrow \left|\sum_{k=1}^{n} f_k(z) - S(z)\right| < \varepsilon$$

Criterio M de Weierstrass

Si $\exists$ serie $\{M_n\}$ C / $|f_n(z)| \le M_n \quad \forall n, \forall z \in D$

↳ $\sum_{n=1}^{\infty} f_n(z)$ converge absoluta y uniforme$\not{}$

Teorema Weierstrass

Si $\sum$ C.U. $\longrightarrow \int_\Gamma \underbrace{\left(\sum_{n=0}^{\infty} f_n(z)\right)}_{S(z) \text{ analítica}} dz = \sum_{n=0}^{\infty} \int_\Gamma f_n(z)\, dz$

$$S'(z) = \left(\sum_{n=0}^{\infty} f_n(z)\right)' = \sum_{n=0}^{\infty} f'_n(z)$$

Sucesiones + series de potencias

$f_n(z) = a_n(z-z_0)^n$

$\sum_{n=0}^{\infty} a_n z^n \to$ Qué region C?

D'Alembert: $\lim \left|\frac{f_{n+1}}{f_n}\right| < 1 \longrightarrow \left|\frac{a_{n+1}}{a_n}\right| |z| < 1 \Leftrightarrow |z| < \underbrace{\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|}_{R \,\div\, \text{radio converge}}$

Serie de Taylor

$$f(z) = \sum a_n (z-z_0)^n = \sum_{n=0}^{\infty} \underbrace{\frac{f^{(n)}(z_0)}{n!}}_{a_n} (z-z_0)^n = \frac{1}{2\pi i} \oint \frac{f(\omega)}{\omega - z}\, d\omega$$

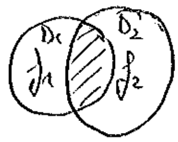
$|z-z_0| < R$

$$R_n = \frac{1}{2\pi i} \oint \left(\frac{z}{\omega}\right)^{n+1} \frac{f(\omega)}{\omega - z}\, d\omega \qquad R \to \text{simetría!}$$

analítica $\equiv$ expansionable en serie de Taylor en z_0

Prolonga / continuación analítica

$f_1(z) = f_2(z) \;\forall z \in D_1 \cap D_2, \; D_1 \ne D_2$



$w = \operatorname{sen} x$
$\Downarrow$
$w = \operatorname{sen} z$
recta real $\cap$

1 prolonga xa $\Rightarrow$ intersecciones

si incontras $\longrightarrow$ no func. analítica

Serie de Laurent

alrededor singularid

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-z_0)^n = \text{P. Principal}\Big|_{-\infty}^{-1} + \text{P. regular (analítica)}$$

$R_1 < |z-z_0| < R_2$

↳ Anillo

Singularids

a) Evitable $\to$ No tiene potencias negativas $\to f(z) \begin{cases} f(z) & z \ne z_0 \\ \lim_{z\to z_0} f(z) & z = z_0 \end{cases}$

b) Polo de orden p $\to f(z) = \frac{a_{-p}}{(z-z_0)^p} + \frac{a_{-p+1}}{(\;)^{p-1}} + \dots + \frac{a_{-1}}{z-z_0} + a_0 + a_1(z-z_0) + \dots$

$0 < |z-z_0| < R$ \\ aislada

c) Singularid esencial

∞s términos negativos, $p \to \infty$

Teorema del residuo

Supong z_0 singularid aislada $f(z)$

$$f(z) = \sum_{n=-\infty}^{+\infty} a_n (z-z_0)^n$$

$$\oint f(z)\, dz = 2\pi i \cdot a_{-1} \rightarrow \text{Res} = a_{-1} \qquad 0 < |z - z_0| < R$$

Teorema de los residuos

(deformar circuito) → varias singularids aisladas

$$\int_\gamma f(z)\, dz = 2\pi i \sum_{k=1}^{n} w_k \,\text{Res}\{f(z), z_k\}$$

(w_k ↳ winding number)

- Evitables
  $a_{-1} = 0$
- Polo orden p → ceros orden n f. regular son aislados
  $$\text{Res}\{f(z), z_0\} = \lim_{z \to z_0} \frac{1}{(p-1)!} \frac{d^{p-1}}{dz^{p-1}} \left[(z-z_0)^p \cdot f(z)\right]$$
  ↳ Polo simple $\text{Res} = \dfrac{F(z_0)}{G'(z_0)}$

$$\frac{1}{1-s} \approx 1+s \quad s \to 0$$

- Esencial
  $p \to \infty$
  desarrollar y buscar a_{-1}

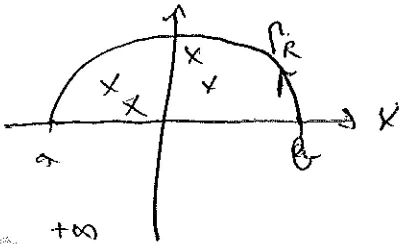
Integrales impropias (reales)

a) $\mathcal{P}\int_{-\infty}^{+\infty} f(x)\,dx = \lim_{a\to\infty}\lim_{b\to\infty}\int_a^b f(x)dx$; $\mathcal{P}\int_0^{\infty} f(x)dx = \lim_{b\to\infty}\int_0^b f(x)dx$

↳ valor principal de Cauchy

b) $\mathcal{P}\int_a^b f(x)\,dx = \lim_{\epsilon\to0}\int_a^{c-\epsilon} f(x)dx + \lim_{\epsilon\to0}\int_{c+\epsilon}^b f(x)dx$ partes divergentes se cancelan?

↳ singularid $c\in\,]a,b[$



$f(x)\to f(z)$

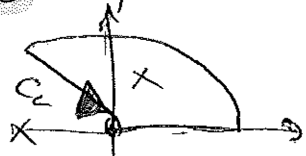
$\int_a^b f(x)\,dx \underset{\uparrow}{=} 2\pi i\sum \mathrm{Res}\{f(z), z_k\}$

condiciones

Comprobad: nº real, dimensiones, par-impar, Re, Im sin, cos

① $\int_{-\infty}^{+\infty} R(x)dx$ ⟶ i) $R(x)$ sin polos en eje real
ii) $\lim_{x\to\infty}|x\,R(x)|=0$ (num 2 grados ≤ denom)

② Diferentes circuitos



I ~~…~~ $+\int_{C_\epsilon} = 2\pi i\sum \mathrm{Res}$

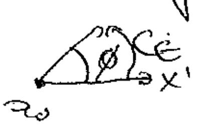
③ $\int_{-\infty}^{+\infty} R(x)\begin{Bmatrix}\cos\lambda x\\ \sin\lambda x\end{Bmatrix}dx = \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\left\{\int_{-\infty}^{+\infty}R(x)e^{i\lambda x}dx\right\}$ → i) $R(x)$ sin polos
ii) $\lim_{x\to\infty}|R(x)| = 0$

$\mathrm{Im}\{z_k>0\}$ si $\lambda>0$

$= \begin{matrix}\mathrm{Re}\\ \mathrm{Im}\end{matrix}\left\{2\pi i\sum\mathrm{Res}\{R(z)e^{i\lambda z}, z_k\}\right\}$ – conjugate

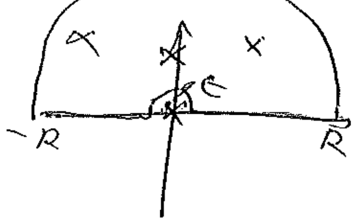
④ Con polos en camino → simples

Lema integración



$\lim_{\epsilon\to0}\int_{\phi:\,|z-z_0|=\epsilon} f(z)dz = i\phi\,\mathrm{Res}\{f(z), z_0\}$

C_ϵ

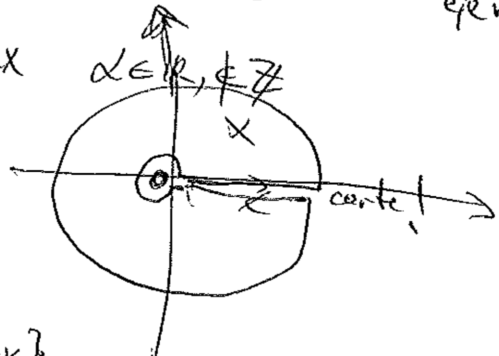


⟹ = s condiciones ↑
– polos → ∞

$\mathcal{P}\int_{-\infty}^{+\infty} f(z)dz = 2\pi i\left(\sum_{\mathrm{Im}\{z_k\}>0}\mathrm{Res}\{f(z), z_k\} + \frac{1}{2}\sum \mathrm{Res}\right)$ (eje real)

⑤ Funciones multiformes $\int_0^{\infty} x^\alpha R(x)dx$ $\alpha\in\mathbb{R}, \notin\mathbb{Z}$

i) $R(x)$ sin polos eje real
ii) $\lim_{x\to\infty}|x^{\alpha+1}R(x)|=0$
iii) $\lim_{x\to0}|x^{\alpha+1}R(x)|=0$



$\mathcal{P}\int_0^{\infty} x^\alpha R(x)dx = \dfrac{2\pi i\sum_k \mathrm{Res}\{z^\alpha R(z), z_k\}}{1-e^{2\pi\alpha i}}$

↳ nº real

Función Gamma

$$n! = \int_0^{\infty} t^n e^{-t}\, dt$$

$$\Gamma(n) = (n-1)! = \int_0^{\infty} t^{n-1} e^{-t}\, dt \quad , \quad \Gamma(x+1) = \int_0^{\infty} t^x e^{-t}\, dt$$

$x \in \mathbb{R}$ $\quad x > -1$

$$\Gamma(n+1) = n\Gamma(n)$$

$$\Gamma(x+1) = x\Gamma(x) \quad x>0$$

$$\underset{\substack{x>0\\ x\in\mathbb{R}}}{\Gamma(x)} = \int_0^{\infty} t^{x-1} e^{-t}\, dt$$

Función Gamma de Euler

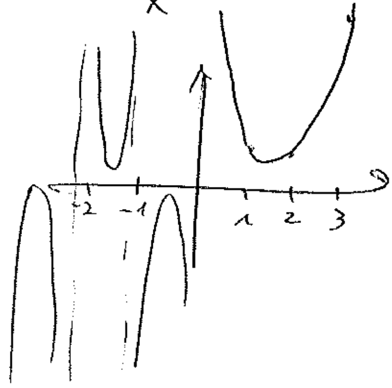
$$\Gamma(1/2) = \sqrt{\pi}$$

$$\Gamma(3/2) = \frac{\sqrt{\pi}}{2}$$

$$\Gamma(-1/2) = -2\sqrt{\pi}$$

$$n! \approx \sqrt{2\pi}\, n^{n+1/2} e^{-n}$$

$$\Gamma(x) = \begin{cases} \int & x>0 \\ \dfrac{\Gamma(x+1)}{x} & x<0 \end{cases}$$



Integral de Euler

$$I = \int_0^{\infty} x^{\alpha} e^{-x^2}\, dx = \frac{1}{2}\Gamma\left(\frac{\alpha+1}{2}\right)$$

Campo complejo:

$$\underset{z\in\mathbb{C}}{\Gamma(z)} = \int_0^{\infty} t^{z-1} e^{-t}\, dt \quad Re\{z\}>0 \quad \to \; <0 \to \; \Gamma(z) = \frac{\Gamma(z+1)}{z}$$

Función Beta

$$B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}\, dt = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \qquad Re\{p\}>0 \quad Im\{p\}>0$$

$$t = \operatorname{sen}^2\varphi \quad = 2\int_0^{\pi/2} \operatorname{sen}^{2p-1}\varphi \cos^{2q-1}\varphi\, d\varphi$$

Propiedades

1) $B(p,q) = B(q,p)$
2) $p B(p,q+1) = q B(p+1,q)$
3) $B(p,q) = B(p+1,q) + B(p,q+1)$
4) $B(p,1-p) = \Gamma(p)\Gamma(1-p) = \dfrac{\pi}{\operatorname{sen} p\pi}$ → Fórmula de los complementos

Transformada de Laplace

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} \underbrace{e^{-st}}_{K(s,t)\,\equiv\,\text{kernel}} \cdot f(t)\,dt \qquad t \in \mathbb{R}$$

∃ si: i) $f(t)$ continua "a trozos" a lo largo $\mathrm{Re}\{t\}$

ii) $f(t)$ debe crecer a lo sumo como exp. $\sim e^{+\sigma_0 t}$

↳ $\mathrm{Re}\{s\} \geq \sigma_0$

- $\mathcal{L}\{e^{t^2}\} \nexists$
- $\mathcal{L}\{t^{\alpha}\} = \dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}}$ $\quad \mathrm{Re}\{s\} > 0$, $\alpha > -1$
- $\mathcal{L}\{\mathrm{sen}\,\omega t\} = \dfrac{\omega}{\omega^2 + s^2}$

Propieds

① Linealid $\mathcal{L}\{\sum \alpha_i f_i(t)\} = \sum \alpha_i \mathcal{L}\{f_i(t)\} = \sum \alpha_i F_i(s)$

② ↳ $\sigma_0 = \mathrm{Max}\{\sigma_{01}, \sigma_{02}, \dots \sigma_{0N}\}$

③ $\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$ $\quad$ esp. t → exp → traslα en espacio S_s

④ $\mathcal{L}\{f^{n}(t)\} = s^n \mathcal{L}\{f(t)\} - \sum_{i=0}^{n-1} f^{(i)}(0) \cdot s^{n-1-i}$

$= s^n \mathcal{L}\{f(t)\} - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0)$

$\mathcal{L}\{f'\} = s F(s) - f(0)$

$\mathcal{L}\{f''\} = s^2 F(s) - s f(0) - f'(0)$

Función Heaviside o escalón

$$H(t-a) = \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases}$$

discontinuid 1ª especie

$$\mathcal{L}\{H(t-a)\} = \int_a^{\infty} e^{-st}\,dt = \frac{1}{s} e^{-sa}$$

"Función" Delta de Dirac

$$\delta(t) = \frac{d}{dt} H(t) = \begin{cases} = 0, & t \neq c \\ = \infty, & t = c \end{cases}$$

↳ $\int_a^b f(t)\,\delta(t-c)\,dt = \begin{cases} f(c) & \text{si } c \in]a,b[\\ 0 & \text{si } c \notin]a,b[\end{cases}$

$\mathcal{L}\{\delta(t-a)\} = e^{-sa}$; $\mathcal{L}\{\delta(t)\} = 1$

Propieds

① $\delta(t) = \delta(-t)$

② $\delta(t-a) = \delta(a-t)$

③ $\delta(at) = \dfrac{1}{|a|}\delta(t)$

④ $\delta(t^2 - a^2) = \dfrac{1}{2|a|}[\delta(t+a) + \delta(t-a)]$

⑤ $\delta(h(t)) = \sum_{i=1}^{n} \dfrac{\delta(t-t_i)}{|h'(t_i)|}$ — n polos simples t_i

Transformada inversa

$f(t) \xrightarrow{T.L} F(s)$, $F(s) \xrightarrow{T.I} f(t)$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$ → tablas

1	$1/s$
t	$1/s^2$
$\mathrm{sen}\,\omega t$	$\dfrac{\omega}{\omega^2+s^2}$

- Linealid $\mathcal{L}^{-1} \sum \alpha_i f_i = \sum \alpha_i \mathcal{L}^{-1} f_i$
- Traslα

$$\mathcal{L}^{-1}\{F(s+a)\} = e^{-at} \underbrace{\mathcal{L}^{-1}\{F(s)\}}_{f(t)}$$

Aplicα EDOL.

$$\mathcal{L}\{\ddot{x}\} + \frac{k}{m}\mathcal{L}\{x\} = \mathcal{L}\left\{\frac{F_0}{\omega}\delta(t)\right\} \to (s^2 + \omega^2)\mathcal{L}\{x\} = \frac{F_0}{m\omega} \cdot 1 \dots$$

Series de Fourier

$f(x)$ periódica $f(x+2L)=f(x)$

cond. sufic $\rightarrow$ $f(x)$ y $f'(x)$ continuas "a trozos"

$$f(x)=\sum_{n=0}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \operatorname{sen}\frac{n\pi x}{L} \longrightarrow a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx$$

Lo ptos. de continuid

Si $f(x)$ continua $\rightarrow$ nº finito components

dis " $\rightarrow \infty$

impar $\rightarrow$ sólo senos

par $\rightarrow$ " cosenos

Fenómeno Gibbs

Impide descripción perfecta en discontinuids, oscilas arbitrarias grandes

Notación compleja (no pide)

$$f(x)=\sum_{n=-\infty}^{+\infty}\alpha_n e^{\frac{in\pi x}{L}} \qquad \alpha_n\in\mathbb{C} \qquad \alpha_n=\frac{1}{2L}\int_{-L}^{L} f(x)e^{-\frac{in\pi x}{L}}dx$$

$$\alpha_1=\frac{a_1-ib_1}{2}$$

$$\alpha_{-1}=\frac{a_1+ib_1}{2}$$

Transformada de Fourier

$$\mathcal{F}\{f(x)\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\cdot \underbrace{e^{ikx}}_{K(x,k)}dx = F(k)$$

$x\longrightarrow k$

$f(x)\longrightarrow F(k)$

Laplace + robuste

$$\mathcal{F}^{-1}\{F(k)\}=f(x)$$

i) $f(x), f'(x)$ continuas a trozos

ii) $\int_{-\infty}^{+\infty}|f(x)|dx$ $\exists$ $\equiv$ Abs. Int.

$$\mathcal{F}\{\delta(x)\}=\frac{1}{\sqrt{2\pi}}, \quad \mathcal{F}^{-1}\left\{\frac{1}{\sqrt{2\pi}}\right\}=\delta(x)$$

Propieds

i) Linealid $\mathcal{F}\{\sum c_i f_i\}=\sum c_i \mathcal{F}\{f_i\}$

ii) $\mathcal{F}\{f(x)e^{-iax}\}=F(k+a)$

$$-\mathcal{F}\{\underset{\theta(\,)\theta(\,)}{\_\_\!-\!\_\_}\}=\frac{\sqrt{2}a}{\sqrt{\pi}}\frac{\operatorname{sen} ka}{ka}$$

$$-\mathcal{F}\{e^{-\frac{x^2}{2\sigma^2}}\}=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\sigma^2k^2} \quad \tilde{\sigma}=\frac{1}{\sigma}$$

Convolución

$$f(x)*g(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\cdot g(x-y)dy$$

$$\mathcal{F}\{f(x)*g(x)\}=F(k)\cdot G(k)$$