Orden → $(d^{o})^{gr}$
Grado

TEMA 1

- separa
- exactas
- f. integrante $\ln|\mu(x)| = \int \frac{1}{B}\left(\frac{\partial A}{\partial y} - \frac{\partial B}{\partial x}\right)dx$ $\quad (y) = \int \frac{1}{A}\left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right)dy$
- lineal → $\frac{dy}{dx} + P(x)y = Q(x)$ → $\mu(x) = e^{\int P(x)dx}$
- isobáricas → $y = vx^{m}$ ... peso y
- ↳ homogénea $y = vx$ $\quad \frac{dy}{dx} = F\left(\frac{y}{x}\right)$ → $\ln x = \int \frac{dv}{F(v)-v}$
- Bernouilli → $\frac{dy}{dx} + P(x)y = Q(x)y^{n}$ $\quad n \neq 0, 1$ (lineal, homog.)
  $v = y^{1-n}$

→ $\frac{dv}{dx} + (1-n)P(x)\cdot v = (1-n)Q(x)$

- Miscelánea
  $\frac{dy}{dx} = F(\underbrace{ax + by + c}_{=v})$

- Clairaut
  $y = px + F(p)$ ... no lineal $\quad p = y'$
  ↳ $y(x) = F(b) + bx$ = arbitraria

UIMP Universidad Internacional Menéndez Pelayo

TEMA 2 - EDO

$$\sum_{k=0}^{n} a_k(x) \frac{d^k y}{dx^k} = f(x)$$

f → $=0$ homog. / → $\neq 0$ inhomog

$a_K(x)$ o no lineal → no sabemos

a_K cte → seguro sí

→ lineal + coef. ctes

Ecuación característica: (ensayo $e^{\lambda x}$)

$$e^{\lambda x}\left[a_n \lambda^n + \dots a_1\lambda + a_0\right] = 0$$

- $\lambda \neq$ → $\in \mathbb{R}$ $\quad y = \sum c_i e^{\lambda_i x}$ ↗, ↘
  $\in \mathbb{C}$
  → imag. puro $y = c_1 e^{i\alpha x} + c_2 e^{-i\alpha x} = d_1 \cos\alpha x + d_2 \operatorname{sen}\alpha x$ (señal periódica)
  → $\beta + i\alpha$ → $\beta > 0$ → crece ↗ (determina amplitud)
  < 0 → decrece ↘
- $\lambda_1 = \lambda_2$
  $y_1 = e^{\lambda_1 x}$ → $y_2 = x e^{\lambda_1 x}$ → cero doble

Particular:

1. $f(x) = a e^{rx}$ → $b e^{rx}$
2. $f(x) = a_1 \sin rx + a_2 \cos rx$ → $b_1 \sin rx + b_2 \cos 3x$
3. $= a_0 + a_1 x + \dots + a_N x^N$ → $b_0 + b_1 x \dots + b_N x^N$
4. Combinaciones, Σ, Π
5. Variación de parámetros

$k_1(x) y_1 + k_2(x) y_2 + \dots = f(x)$

(y_1, y_2: mini soluciones homogénea)

$$\rightarrow \begin{cases} k_1' y_1 + k_2' y_2 = 0 \\ k_1' y_1' + k_2' y_2' = \dfrac{f(x)}{a_n} \end{cases}$$

Δ, Cramer

SISTEMAS

$$\begin{cases} \dot{x} = ax + by \\ \dot{y} = cx + dy \end{cases}$$

$$\vec{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\frac{d\vec{x}}{dt} = \dot{\vec{x}}(t) = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = A \cdot \vec{x} \quad \rightarrow \text{si } \vec{f}(t) \rightarrow \dot{\vec{x}} = A\vec{x} + \vec{f}(t)$$

$\boxed{A}$

→ $\{\lambda_i\}$ valores propios $\quad A\vec{y}_k = \lambda \vec{y}_k$

$\{\vec{y}_k\}$ vectores propios

Soluc. EDO:

$$\vec{x}_h(t) = \sum_{i=1}^{N} c_i e^{\lambda_i t} \vec{y}_i \quad \xleftrightarrow{\text{Proyectores}} \quad \vec{x}_h(t) = \exp[A(t-t_0)] \cdot \vec{x}_0$$

$$= \sum_{n=0}^{\infty} \frac{A^n (t-t_0)}{n!}$$

Exponencial → Taylor Σ

→ Proyect $e^{\lambda_i} \cdot \vec{y}_i$

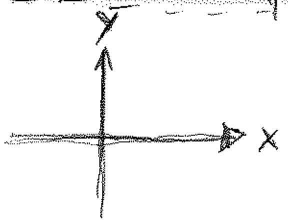
→ Diagonalización

- $U = \begin{pmatrix} \langle e_i \mid e_k \rangle \\ \vdots \end{pmatrix}$ vectores columna
- $\lambda_i \rightarrow D$
- $\exp A = U^{-1} \exp D \, U^{-1}$ (sin normalizar)

$$\vec{x}_p(t) = \exp[A(t-t_0)] \int_{t_0}^{t} \exp[-A(s-t_0)] \cdot \vec{f}(s)\, ds$$

$$\Rightarrow \vec{x}(t) = e^{At}\left(\vec{x}_0 + \int_0^t e^{-As} \cdot \vec{f}(s) \cdot ds\right)$$

Planos de fase



$x(t)$ → diagrama de fase

$y(t)$ → líneas de flujo

→ $\vec{y}_k$ direcciones privilegiadas

predicar comportam., movim., pierdes 'tiempo'

- sin, cos → elipses (imaginarios puros → órbitas cerradas) OAS (vectores propios compl.)
- $e^{\gamma t}$ → amort. espiral decrec. complejos → órbitas no cerradas amort + resonante espiral crec. (vectores propios complejos) } raíz periódica

→ signo frecue → + antihorario, − horario (= sin, cos)

- v. propios reales  direcciones signo λ

No lineal

→ puntos de equilibrio

→ + ε, despreciar ε^2

TEMA 3
EDP

$u(x,y)$

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G \qquad B^2 - 4AC = \Delta$$

$$\begin{cases} \text{Parabólicas} & \Delta = 0 \dashrightarrow \text{Difusión calor} \\ \text{Hiperbólicas} & > 0 \\ \text{Elípticas} & < 0 \end{cases}$$

$\boxed{\text{DIF. CALOR}}$

• Separac de variables

$$\left.\begin{array}{l} u_t = \alpha^2 u_{xx} \\ cc \begin{cases} u(0,t) = 0 \\ u(L,t) = 0 \end{cases} \\ CI \quad u(x,0) = \phi(x) \end{array}\right\} \Rightarrow \begin{array}{l} \exists \ u(x,t) = X(x) \cdot T(t) \\ k \text{ cte de separac } < 0 \ (\text{enfriado}) \\ X(x) = B\cos\lambda x + C\sin\lambda x \\ T(t) = A \cdot e^{-\lambda\alpha^2 t} \end{array}$$

$$\rightsquigarrow u(x,t) = \sum_{n=1}^{\infty} a_n e^{-\lambda_n^2 \alpha^2 t} \sin(\lambda_n x)$$

$$a_n = 2\int_0^1 \sin(n\pi x)\, \phi(x)\, dx \qquad (L=1)$$

λ_n cc2

$$\int_0^1 \sin n\pi x \sin m\pi x = \begin{cases} 0 & m \neq n \\ 1/2 & m = n \end{cases}$$

$$\int_0^L \sin\frac{n\pi x}{L} \sin\frac{m\pi x}{L} \to \begin{cases} 0 \\ L/2 \end{cases}$$

• Cambio de variables

$$cc \begin{cases} u(0,t) = k_1 \\ u(L,t) = k_2 \end{cases} \longrightarrow u(x,t) = \underbrace{\left[k_1 + \frac{x}{L}(k_2 - k_1)\right]}_{\text{estacionario}} + \underbrace{v(x,t)}_{\text{transitoria}}$$

$\Downarrow$

$$v_t = \alpha^2 v_{xx}$$

$$cc \begin{cases} v(0,t) = 0 \\ v(L,t) = 0 \end{cases}$$

$$v(x,0) = \bar{\phi}(x) = \phi(x) - \left[k_1 + \frac{X}{L}(k_2 - k_1)\right]$$

• Variante

$$cc \begin{cases} u(0,t) = 0 \\ u_x(1,t) + u(1,t) = 0 \end{cases} \to \lambda = -\operatorname{tg}\lambda \to \infty\text{s soluc.} \quad (\text{numérico})$$

Propagaα de ondas

$$u_{tt} = c^2 u_{xx} \qquad \longrightarrow \qquad \begin{aligned} \xi &= x + ct \\ \eta &= x - ct \end{aligned}$$

$$CI \begin{cases} u(x,0) = f(x) \\ u_t(x,0) = g(x) \end{cases}$$

Sol. D'Alembert:

$$\Rightarrow \boxed{u(x,t) = \frac{1}{2}\left[f(x-ct) + f(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(z)\,dz}$$

* Cuerda guitarra

$$CC \begin{cases} u(0,t) = 0 \\ u(L,t) = 0 \end{cases} \quad (f) \qquad \longrightarrow \qquad \begin{aligned} T(t) &= A \sin \alpha\beta t + B \cos \alpha\beta t \\ X(x) &= C \sin \beta x + D \cos \beta x \end{aligned}$$

$k \vee \lambda < 0$

$$\rightarrow \beta L = n\pi, \quad D = 0$$

$$u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right) \cdot \left[a_n \sin\left(\frac{n\pi\alpha}{L} t\right) + b_n \cos\left(\frac{n\pi\alpha}{L} t\right)\right] = \sum R_n \sin\frac{n\pi x}{L} \cos\left(\frac{n\pi\alpha}{L}\right. -$$

$$\begin{cases} a_n = \dfrac{2}{n\pi\alpha} \displaystyle\int_0^L g(x) \cdot \sin\frac{n\pi x}{L}\,dx \\ b_n = \dfrac{2}{L} \displaystyle\int_0^L f(x) \sin\frac{n\pi x}{L}\,dx \end{cases}$$

TEMA 4

SOLUCIONES EN SERIE DE POTENCIAS E.D.O.H

$$y''(x) + p(x)y'(x) + q(x)y(x) = 0$$

$$y(x) = c_1 y_1(x) + c_2 y_2(x) \longrightarrow \text{Base } \mathbb{R}^2$$

$$W(y_1, y_2) = W(x) = y_1 y_2' - y_2 y_1' \neq 0 \rightarrow \text{L.I.}$$

$$W(x) = e^{-\int p(x)dx}$$ → no depende de q

$z \in \mathbb{C}$

$$y(z) = \sum_{n=0}^{\infty} a_n z^n$$

puntos z_0:
- ordinarios → $p(z) = \sum p_n (z-z_0)^n$, $q(z) = \sum q_n$
- singulares:
  - regulares $\begin{cases} (z-z_0)p(z) = \sum p_n (z-z_0)^n \\ (z-z_0)^2 q(z) = \sum q_n (z-z_0)^n \end{cases}$
  - irregulares → caso contrario

Si z_0 es ord/reg $\Rightarrow \exists \; y(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n$

$\infty \rightarrow \frac{1}{z} = w \quad \frac{d^2y}{dw^2} + P(w)\frac{dy}{dw} + Q(w)\, y(w) = 0$

FUNCIONES ESPECIALES

Legendre

$(1-x^2)y'' - 2xy' + l(l+1)y = 0 \quad |x| < 1$

$(n+1)P_{n+1} - (2n+1)x P_n + nP_{n-1} = 0$

Asociada de Legendre

$[(1-x^2)y']' + \left[l(l+1) - \frac{m^2}{1-x^2}\right]y = 0$

Bessel

$x^2 y'' + xy' + (x^2 - n^2)y = 0$

Hermite

$y'' - 2xy' + 2\alpha y = 0$

$H_0 = 1$
$H_1 = x$
$H_2 = x^2 - 1$
$H_3 = x^3 - 3x$

Γ de Euler

$\Gamma(n) \equiv \int_0^{\infty} x^{n-1} e^{-x} dx$

$\Gamma(n+1) = n\Gamma(n) = n!$

$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \longrightarrow \left(\frac{1}{2}\right)! = \frac{\sqrt{\pi}}{2}$

$n! = \sqrt{2\pi n}\, n^n e^{-n} \quad n \gg 1$

β de Euler

$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$

$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

Armónicos esféricos

$\nabla^2 u = 0$