J.A. Oteo. Departamento de Física Teórica (UVEG).[MMF3-B:2008-9]

TEMA 1: EDO primer orden*

20 de enero de 2009

Resolver las siguientes EDO de primer orden:

1. `//Oteo//` $y'^2 + 2xy' + x^2 = 0$
2. `//Erica [Rosa]//` $y' = (x+1)/y^3 + xy/[2(x^2+4)]$
3. `//Rosa [Erica]//` $2y'x\sqrt{y} + \sqrt{y^3} = x^2$
4. `//David [Fdo.H.]//` $(3x^2y^2 + 4xy)dx + (3x^3y + 4x^2)dy = 0$
5. `//Carlos R. [Yanis]//` Anulado
6. `//José Luís [Marta T.]//` $y' = -1/(x \exp y)$ (por factor integrante)
7. `//Enrique [Aitor]//` $3x(xy-2)dx/2 + (x^3/2 + y)dy = 0$
8. `//Almudena [Isabel]//` $y' = (2x+2y+4)/([3(x+y)]$
9. `//Isabel [Almudena]//` $y' = -(1+4y)/(1+4x)$
10. `//Patricia [Marta M.]//` $y' + 3x^2y = x^2y^2/15$
11. `//Marta M. [Patricia]//` $y' + 2y/x = y^3/x^2$
12. `//Laura [Yolanda]//` $6xy + 3x = 2y'$
13. `//Yolanda [Laura]//` $y' = 7xy^4 - xy/(3x^2+2)$
14. `//Roberto [Esther]//` $y' = -(3x^2y^2 + 3 + 4y^3)/(2x^3y - 2 + 12xy^2 + \exp(-y))$
15. `//Esther [Roberto]//` $1/y' = 3x - y + 2$
16. `//Jesús [Fdo.S.]//` $y' = 1/(\exp(y) - x)$
17. `//Fdo.S. [Jesús]//` $(x^4yx^{-15/20}/2)dx - x^{-1/4}dy = 0$
18. `//Carlos H. [Luis C.]//` $y' = (2x^2 + y)/(x - x^2y)$
19. `//Alberto [Damián]//` $y' + y = 1 + x\exp(-x)$
20. `//Damián [Alberto]//` $y' = -(3xy^2 + 1)/(2x^2y)$
21. `//Alejandro [Luis M.]//` $y' = -(x^3 + y)/(3x^2)$
22. `//Luis M. [Alejandro]//` $y' = 1/[xy(1+xy^2)]$

*Preguntas y soluciones contrastadas por [...]

23. `//Eduardo [Juan Ramón]//` $(2x^2 + y)dx + (x^2 y - x)dy = 0$
24. `//Rubén [Pablo]//` $(y^3/3x)dx + y^2 dy = 0$
25. `//Javier [Carlos C.]//` $y' = (x^4 + 4y)/(2x)$
26. `//Carlos C. [Javier]//` $y' = 2x/y + y/x$
27. `//Luis C. [Carlos H.]//` $6ydx + 2xdy = 0$

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TEMA 2: EDO orden superior y sistemas lineales*

24 de noviembre de 2008

1. `//Oteo//` Resolver $y'' + 2\gamma y' + \omega_0^2 y = F\,\sin(\omega t)$, $(\gamma, \omega_0, \omega :\ ctes.)$ en los casos siguientes:
   a) $\gamma = 0$, $\omega_0 \neq \omega$
   b) $\gamma = 0$, $\omega_0 = \omega$
   c) $0 < \gamma < \omega_0$, $\omega^2 \neq \omega_0^2 - \gamma^2$
   d) $0 < \gamma < \omega_0$, $\omega^2 = \omega_0^2 - \gamma^2$
2. `//Erica [Roberto]//` $y''' - 3y' + 2y = 4x^2$
3. `//Rosa [Esther]//` $4y'' - 4y' + y = (4x^2 + 4x + 1)\exp x$
4. `//Esther [Rosa]//` $y'' - y = 4x \exp x$
5. `//Javier [Fdo. H.]//`
   $y'' + y' + y = x^2(6\sin x + 7\cos x) + x(4\sin x + 3\cos x) + 2\sin x + \cos x$
6. `//Roberto [Erica]//` $y'' + 6y' - 12y = \sin 2x$
7. `//Pablo [Fdo. S.]//` $y'' - 4y = \sinh 2x + \cosh 2x$
8. `//Fdo. S. [Pablo]//` $y''' + 2y'' + y' = \exp(-x)$
9. `//Carlos H. [Luis C.]//` $y'' + 3y' + 4y = 3\sin 3x + 2\cos 2x$
10. `//Almudena [Yolanda]//` $y'' + 4y = (2x^2 + x)\exp x$
11. `//Yolanda [Almudena]//` $y'' - 9y' = x^2 \exp(-3x)$
12. `//Jesús [Alberto]//` $y'' + 4y = 3x^2 \exp x$
13. `//Alberto [Jesús]//` $\ddot{x} + \omega_0^2 x = 5\sin(\omega t)$
14. `//Ander [Damián]//` $y'' - 4y = (\exp(ix) + \exp(-ix))^2$
15. `//Alejandro [Luis M.]//` $y'' - 8y' + 12y = 3x^2 \exp x$
16. `//Luis M. [Alejandro]//` $y'' - y = 2/(1 + \exp x)$
17. `//Fdo. H. [Carlos C.]//` $(y'')^2 + 8yy'' + 16y^2 = x^2(2y'' + 8y - x^2\exp(-x))\exp(-x)$
18. `//Carlos C. [Javier]//` $y'' - 2y' + y = \exp(x)/\sqrt{x^2 - 1}$
19. `//Damián [Ander]//` $y'' - 4y = (\pi - 2)\exp(\pi x)$

*Preguntas y soluciones contrastadas por [...]

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(1) $\boxed{y'' + 2\gamma y' + W_0^2 y = F \sin wt}$

~~f(t)~~ F = 0 (hom.) $\rightarrow \sim e^{\lambda t}$

$$\lambda^2 + 2\gamma\lambda + W_0^2 = 0$$

$$\hookrightarrow \lambda_{1,2} = -\gamma \pm \sqrt{\gamma^2 - W_0^2} \quad !$$

a) $\gamma = 0$

$\rightarrow \lambda = \pm i W_0$

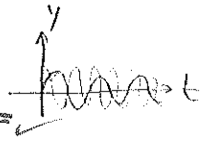
$$y_h = c_1 \cdot e^{iW_0 t} + c_2 e^{-iW_0 t} \rightarrow \text{reales } c_1 = c_2^* \qquad \rightarrow -W_0^2 + W_0^2 = 0 \checkmark$$

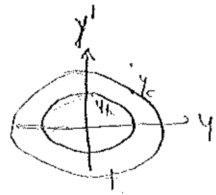
$$= A \sin(W_0 t + \delta)$$

$$y_p = B \sin wt + C \cos wt \qquad C = 0 \qquad \text{res: } W_0 = W$$

$$\sin wt\, B(-W^2 + W_0^2) = F \sin wt \rightarrow B = \frac{F}{W_0^2 - W^2} \qquad W \neq W_0$$

$$y = A \sin(W_0 t + \delta) + \frac{F}{W_0^2 - W^2} \sin Wt \rightarrow \frac{-W^2 F}{W_0^2 - W^2} + \frac{W_0^2 F}{W_0^2 - W^2} = F \checkmark$$





b) $\gamma = 0$, $W_0 = W$ $\rightarrow y \rightarrow \infty \rightarrow$ Resonancia (no am.)

$$y_h = A \sin(wt + \delta)$$

$$y_p = Bt \sin(wt\ \cancel{\ }) + Ct \cos wt$$

$$y_p' = B \sin(\ \ ) + BWt \cos(\ \ ) + C\cos - CWt \sin$$

$$y_p'' = + BW\cos(\ \ ) + BW\cos(\ \ ) - BW^2 t \sin(\ \ ) - 2CW \sin - CW^2 t \cos$$

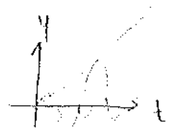
$$\cos(\ \ )\left(2BW - \cancel{CW^2 t} + \cancel{W_0^2} Ct\right) + \sin(\ \ )\left(-2CW - \cancel{BW^2 t} + W^2 Bt\right)$$

$$= F \sin wt \rightarrow B = 0, \quad C = -\frac{F}{2W}$$

$$y = A \sin(Wt + \delta) - \frac{F}{2W} t \cos wt$$

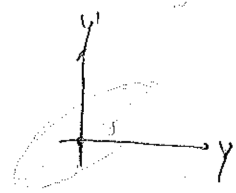
$t \to \infty$ $y \to \pm\infty$ $\omega > \omega_0$

Resonancia



$$-\frac{F}{2W} c + \frac{Ft}{2} s \rightarrow \frac{F}{2} s + \frac{F}{2} s - \frac{FWt}{2} c \rightarrow + \frac{FWt}{2} c \Rightarrow Fs \checkmark$$

(13) $\gamma = 0$, $F = 5$

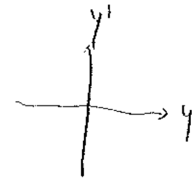


$$y_h = A \sin(W_0 t + \delta) + \frac{5}{W_0^2 - W^2} \sin Wt$$

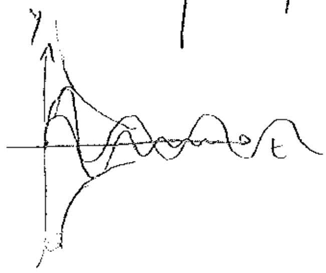
c) $0 < \gamma < \omega_0$

$\omega^2 \neq \omega_0^2 - \gamma^2$

$\lambda = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2} = -\gamma \pm i\sqrt{\omega_0^2 - \gamma^2}$ $\neq 0$



$y_h = e^{-\gamma t}\left[c_1 e^{i\sqrt{\ }t} + c_2 e^{-i\sqrt{\ }t}\right] \rightarrow c_1 = c_2^*$



$= A e^{-\gamma t} \sin(\sqrt{\ }\, t + \delta)$

$y_h' = e^{-\gamma t}(A\sqrt{\ }\cos) - \gamma y_h$

$y_h'' = -\gamma(y_h' + \gamma y_h) + e^{-\gamma t}(-A(\ )\sin) - \gamma y_h' = -2\gamma y_h' - \gamma^2 y_h - A(\ )y_h$

$\rightarrow = 0$ ✓

$y_p = B\sin\omega t + C\cos\omega t$

$y_p' = B\omega\cos - C\omega\sin$

$y_p'' = -\omega^2 y_p$

$\rightarrow \sin\ ((\omega_0^2 - \omega^2)B - 2\gamma C\omega)$

$+ \cos\ (C(\omega_0^2 - \omega^2) + 2\gamma B\omega) = F\sin\omega t$

$\rightarrow$ s.ec.

$= 0 \rightarrow B = \frac{C(\omega^2 - \omega_0^2)}{2\gamma\omega}$

$y_p = B\sin(\omega t + \delta)$

$y_p' = \omega B\cos$

$y_p'' = -\omega^2 y_p$

$\rightarrow 2\gamma\omega B\cos(\omega t + \delta) = F\sin\omega t$

$+ B(\omega_0^2 - \omega^2)\sin(\omega t + \delta)$

$F = \frac{(\omega_0^2 - \omega^2)^2 C}{2\gamma\omega} - 2\gamma\omega C \rightarrow C = \frac{F\cdot 2\gamma\omega}{(\omega_0^2 - \omega^2)^2 - (2\gamma\omega)^2} \rightarrow B = \frac{F\cdot(\omega^2 - \omega_0^2)}{(\omega_0^2 - \omega^2)^2 - (2\gamma\omega)^2}$

$2\gamma\omega B(\cos\omega t\cos\delta - \sin\omega t\sin\delta)$

$B(\omega_0^2 - \omega^2)(\sin\omega t\cos\delta + \sin\delta\cos\omega t) = F\sin\omega t$

$2\gamma\omega B\cos\delta + B(\omega_0^2 - \omega^2)\sin\delta = 0$

$\delta = \arctan\left(-\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)$

$\frac{F}{B} = -2\gamma\omega\sin\delta + (\omega_0^2 - \omega^2)\cos\delta \rightarrow \frac{\sin\delta}{\cos\delta}\cdot\frac{(\omega_0^2 - \omega^2)}{-2\gamma\omega} = 1$

$\hookrightarrow B = \frac{F}{\cos\delta\left((2\gamma\omega)^2/(\omega_0^2 - \omega^2) + \omega_0^2 - \omega^2\right)} = \frac{F}{\frac{\cos\delta}{\omega_0^2 - \omega^2}\left((2\gamma\omega)^2 + (\omega_0^2 - \omega^2)^2\right)}$

$1 = \frac{\sin^2\delta}{\cos^2\delta}\frac{(\omega_0^2 - \omega^2)^2}{(2\gamma\omega)^2} = \frac{1}{\cos^2\delta}(\quad) - (\quad) \rightarrow 1 + (\ ) = \frac{1}{\cos^2\delta}(\ )$

$\cos^2\delta = \frac{1}{1 + \frac{1}{(\ )}} = \frac{(\ )}{\frac{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}{(2\gamma\omega)^2}}$ $\cos^2\delta = \frac{(\omega_0^2 - \omega^2)^2}{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}$

$\rightarrow B = \frac{F}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}$

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$y = A e^{-\gamma t}\sin\left(\sqrt{\omega_0^2 - \gamma^2}\, t + \phi\right) + \frac{F}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}\sin\left(\omega t + \arctan\left(-\frac{2\gamma\omega}{\omega_0^2 - \omega^2}\right)\right)$

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d) $\omega^2 = \omega_0^2 - \gamma^2$ → ¿Resonancia? ;

$\lambda = -\gamma \pm i\omega$

$\rightarrow y_h = A \cdot e^{-\gamma t} \cdot \sin(\omega t + \varphi)$

$y_p = B \sin(\omega t + \varphi)$ → porque $e^{-\gamma t} \sin \omega t \not\propto \sin \omega t$

si $f(t) = F \cdot e^{-\gamma t} \sin \omega t$ → sí

$$y = A \cdot e^{-\gamma t} \sin(\omega t + \varphi) + \frac{F}{\sqrt{\gamma^4 + 4\gamma^2 \omega^2}} \sin\left(\omega t + \operatorname{arctg}\left(-\frac{2\omega}{\gamma}\right)\right)$$

2 $\boxed{y''' - 3y' + 2y = 4x^2}$

$$(\lambda^3 - 3\lambda + 2) = 0$$

	1	0	-3	2
1		1	1	-2
	1	1	-2	0
1		1	2	
$\lambda_{1,2} = 1$	1	2	0	

$\lambda_3 = -2$

$(\lambda^2 - 2\lambda + 1)(\lambda + 2)$

$\lambda^3 - 2\lambda^2 + \lambda + 2\lambda^2 - 4\lambda + 2$

$$(\lambda - 1)(\lambda - 1)(\lambda + 2)$$

$$(\lambda - 1)^2(\lambda + 2)$$

$$y_h = c_1 \cdot e^{x} + c_2 x e^{x} + c_3 e^{-2x}$$

$$y_p = A + Bx + Cx^2$$

$$y_p' = B + 2Cx$$

$$y_p'' = 2C$$

$$y_p''' = 0$$

$\rightarrow -3B - 6Cx + 2A + 2Bx + 2Cx^2 = 4x^2$

$C = 2$

$-3B + 2A = 0 \qquad A = 9$

$-6C + 2B = 0 \rightarrow B = 6$

$$y = (c_1 + c_2 x)e^x + c_3 e^{-2x} + 2x^2 + 6x + 9$$

$$y' = (c_1 + c_2 x)e^x + c_2 e^x - 2c_3 e^{-2x} + 4x + 6$$

$$y'' = (c_1 + c_2 x)e^x + 2c_2 e^x + 4c_3 e^{-2x} + 4$$

$$y''' = (c_1 + c_2 x)e^x + 3c_2 e^x - 8c_3 e^{-2x}$$

$$e^x(\cancel{c_1} + \cancel{c_2 x} + \cancel{3c_2} - \cancel{3c_1} - \cancel{3c_2 x} - \cancel{3c_2} + \cancel{2c_1} + \cancel{2c_2 x})$$

$$+ e^{-2x}(-\cancel{8c_3} + \cancel{6c_3} + \cancel{2c_3})$$

$$-\cancel{12x} - \cancel{18} + 4x^2 + \cancel{12x} + \cancel{18} = 4x^2 \checkmark$$

③ $\boxed{4y'' - 4y' + y = (4x^2+4x+1)e^x}$

$4\lambda^2 - 4\lambda + 1 = 0 \rightarrow \lambda_{1,2} = \dfrac{4 \pm \sqrt{16-16}}{8} = \dfrac{1}{2}$ (doble)

VALOR DEGENERADO

$y_h = Ae^{x/2} + Bxe^{x/2}$

$y_h' = \frac{1}{2}Ae + Be + \frac{Bx}{2}e$ $\rightarrow 2Ae + 2Bxe + 4Be$

$y_h'' = \frac{1}{4}Ae + \frac{B}{2}e + \frac{Bx}{4}e + \frac{B}{2}e$ $\quad -2Ae - 4Be - 2Bxe = 0$ ✓

$y_p = (Ax^2+Bx+C)e^x \rightarrow C, B \rightarrow \neq 0 \quad e^x \leftrightarrow e^{x/2}$

NO HAY RESONANCIA

$= (Ax^3 + Bx^2)e^x$

$y_p' = (3Ax^2 + 2Bx + Ax^3 + Bx^2)e^x = (\quad) + y_p$

$y_p'' = (6Ax + 2B + 3Ax^2 + 2Bx + 3Ax^2 + 2Bx + Ax^3 + Bx^2)e^x = (\;) + y_p'$

$4 \cdot (6Ax + 2B)e^x + (Ax^3 + Bx^2)e^x = (4x^2+4x+1)e^x$

$\rightarrow A = 0$

$y_p' = e^x(2Ax + B) + y_p$

$y_p'' = (2Ax + 2Ax + B)e^x + y_p'$

$\rightarrow e^x(8A + Ax^2 + Bx + C + 8Ax + 4B) = (4x^2+4x+1)e^x$

$A = 4$ $\quad 8A + B = 4 \rightarrow B = -28$

$C = 81$ $\quad 8A + 4B + C = 1 \rightarrow C = 81$

$y_p = (4x^2 - 28x + 81)e^x$

$y_p' = (8x - 28)e^x + y_p$

$y_p'' = (8 + 8x - 28)e^x + y_p'$

$e^x(32 - 112 + 81 + 32x - 28x + 4x^2) = (4x^2+4x+1)e^x$ ✓

$y = Ae^{x/2} + Bxe^{x/2} + e^x(4x^2 - 28x + 81)$

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$$\boxed{y'' - y = 4xe^{x}}$$

$\lambda^2 - 1 = 0 \rightarrow \lambda_{12} = \pm 1$

$y_h = Ae^{x} + Be^{-x}$

$y_h' = Ae^{x} - Be^{-x}$ $\quad \rightarrow y_h + y_h'' = 0$ ✓

$y_h'' = Ae^{x} + Be^{-x}$

$\rightarrow$ Resonancia

$\rightarrow y_p = (Ax^2 + Bx)e^{x}$

$y_p' = (2Ax + B)e^{x} + y_p$ $\quad \rightarrow (2A + 4Ax + 2B)e^{x} = 4xe^{x}$

$y_p'' = (2A + 2Ax + B)e^{x} + y_p'$ $\quad A = 1$, $2A + 2B = 0 \rightarrow B = -1$

$y_p = (x^2 - x)e^{x}$

$y_p' = (2x - 1 + x^2 - x)e^{x}$ $\quad \rightarrow y_p\ 4x$ ✓

$y_p'' = (2 + 2x - 1 + 2x - 1 + x^2 - x)e^{x}$

$y = Be^{-x} + e^{x}(x^2 - x + A)$

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$$\boxed{y'' + 6y' - 12y = \sin 2x}$$

$(\lambda^2 + 6\lambda - 12) = 0 \quad \rightarrow \lambda_{12} = \dfrac{-6 \pm \sqrt{36 + 48}}{2} = -3 \pm \sqrt{21}$

$y_h = Ae^{-(3+\sqrt{21})x} + Be^{(-3+\sqrt{21})x}$

$y_h' = -(\quad)Ae^{-(\quad)} + (\quad)Be^{(\quad)x}$

$y_h'' = (\quad)^2Ae^{-(\quad)} + (\quad)^2Be^{(\quad)x}$

$Ae^{-(\quad)}\left((+3+\sqrt{21})^2 - 6(+3+\sqrt{21}) - 12\right) = 0$ ✓

$Be^{(\quad)}\left((-3+\sqrt{21})^2 + 6(-3+\sqrt{21}) - 12\right) = 0$ ✓

$y_p = A\cos 2x + B\sin 2x$

$y_p' = -2A\sin + 2B\cos$

$y_p'' = -4A\cos - 4B\sin$

$\cos 2x(-4A + 12B - 12A) \rightarrow 12B = 16A$, $B = \frac{4}{3}A$

$+ \sin 2x(-4B - 12A - 12B) = \sin 2x$

$-16 \cdot \frac{4}{3}A - 12A = 1$

$A = -\frac{3}{100}$ $\quad B = -\frac{4}{100}$

~~$y_p = -\left(\frac{67}{36}\cos 2x + \frac{67}{27}\sin 2x\right) = -\frac{67}{9}\left(\frac{1}{4}\cos 2x + \frac{1}{3}\sin 2x\right)$~~

~~$y_p' = -\ (-\frac{1}{2}\ \ \frac{2}{3}$~~ $\quad -1 + \frac{16}{3}$ $\quad -4/3 = \frac{6}{2}$

$y_p''\ \ (-1\ \ -4/3$

$$y_p = -\frac{3}{100}\cos 2x - \frac{4}{100}\sin 2x$$

$$y_p' = +\frac{6}{100}\sin 2x - \frac{8}{100}\cos 2x$$

$$y_p'' = +\frac{12}{100}\cos 2x + \frac{16}{100}\sin 2x$$

$$\left(+\frac{16}{100} + \frac{36}{100} + \frac{48}{100}\right) = 1 \checkmark$$

$$\left(\frac{12}{100} - \frac{48}{100} + \frac{36}{100}\right) = 0 \checkmark$$

$$y = Ae^{-(3+\sqrt{21})x} + Be^{(-3+\sqrt{21})x} - \frac{1}{100}\left(3\cos 2x + 4\sin 2x\right)$$

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$$\boxed{y'' - 4y = \sinh 2x + \cosh 2x}$$

$$\lambda^2 - 4 = 0 \rightarrow \lambda = \pm 2$$

$$y_h = Ae^{2x} + Be^{-2x} \qquad y_h'' - 4y_h = 0 \checkmark$$

$$y_h' = 2(Ae^{2x} - Be^{-2x})$$

$$y_h'' = 4(Ae^{2x} + Be^{-2x})$$

$\leftarrow$ rezon.

$$y_p = Ax\sinh 2x + Bx\cosh 2x$$

$$y_p' = A\sinh + Ax\cos + B\cos + Bx\sin$$

$$y_p'' = A\cosh + Ax\sin + A\cos + B\sin + Bx\cos + B\sin$$

$$= \cosh(2A + Bx) + \sinh(Ax + 2B) \times$$

$$y_p'' - 4y_p = e^{2x}$$

$$y_p = Axe^{2x}$$

$$y_p' = e^{2x}(2Ax + A) \rightarrow A(4x + 4 - 4x) = 1 \rightarrow A = 1/4$$

$$y_p'' = Ae^{2x}(2 + 4x + 2) \rightarrow y_p = \frac{xe^{2x}}{4}$$

$$y_p' = e^{2x}\left(\frac{x}{2} + \frac{1}{4}\right)$$

$$y_p'' = e^{2x}\left(\frac{1}{2} + x + \frac{1}{2}\right) \rightarrow x + 1 - x = 1 \checkmark$$

$$y = e^{2x}\left(\frac{x}{4} + A\right) + Be^{-2x}$$

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(8) $\boxed{y''' + 2y'' + y' = e^{-x}}$

$\lambda^3 + 2\lambda^2 + \lambda = 0$

$\lambda(\lambda^2 + 2\lambda + 1) = \lambda(\lambda+1)^2 = 0 \rightarrow \lambda_1 = 0$

$\lambda_{2,3} = -1$ (doble)

$y_h = A + \cancel{Bx} + Cxe^{-x} + De^{-x}$

$y_h' = -De^{-x} - Cxe^{-x} + Ce^{-x} \rightarrow$ 3BL BC4 + 3B + 2a

$y_h'' = De^{-x} + Cxe^{-x} - Ce^{-x} - Ce^{-x}$

$y_h''' = -De^{-x} - Cxe^{-x} + Ce^{-x} + Ce^{-x} + Ce^{-x} = -De^{-x} - Cxe^{-x} + 3Ce^{-x}$

$-D + 3C - 4C + 2D - D + C = 0$ ✓

$-C + 2C - C = 0$ ✓

$y_p = Ax^2e^{-x}$

$y_p' = 2Axe^{-x} - y_p \longrightarrow (-4A + \cancel{2Ax} + 2A - \cancel{2Ax}) = e^{-x}$

$-2A = 1$

$A = -1/2$

$y_p'' = (2A - 2Ax)e^{-x} - y_p'$

$y_p''' = (-2A - 2A + 2Ax)e^{-x} - y_p''$

$y_p = -\frac{1}{2}x^2e^{-x}$

$y_p' = (-x + \frac{1}{2}x^2)e^{-x}$

$y_p'' = (-1 + x + x - \frac{1}{2}x^2)e^{-x}$

$y_p''' = (2 - x + 1 - 2x + \frac{1}{2}x^2)e^{-x}$

$3 - \cancel{3x} + \frac{1}{2}x^2 - 2 + \cancel{4x} - \cancel{x^2} - \cancel{x} + \cancel{\frac{1}{2}x^2}$
$= 1$ ✓

$y = A + (B + Cx - \frac{1}{2}x^2)e^{-x}$

⑨ $\boxed{y'' + 3y' + 4y = 3\sin 3x + 2\cos 2x}$

$\lambda^2 + 3\lambda + 4 = 0$

$\lambda_{12} = \frac{-3 \pm \sqrt{9-16}}{2} = -\frac{3 \pm 7i}{2}$

$y_h = Ae^{-\frac{3+7i}{2}x} + Be^{-\frac{(3+7i)}{2}x}$

$y_h' = \frac{-3+7i}{2} A e^{( )} + -(\frac{\ }{2}) B e^{( )}$ $\rightarrow ( )^2 + 3( ) + 4$

$\frac{9-42i-7}{4} + \frac{-9+21i}{2} + 4 = 0$ ✓

$y_h'' = ( )^2 A e^{( )} + ( )^2 B$

$\frac{(3+7i)^2}{2} + \frac{-9-21i}{2} + 4$ ✓

$y_p = A\sin 3x + B\cos 3x + C\sin 2x + D\cos 2x$

$y_p' = 3A\cos 3x - 3B\sin 3x + 2C\cos 2x - 2D\sin 2x$

$y_p'' = -9A\sin 3x - 9B\cos 3x - 4C\sin 2x - 4D\cos 2x$

$B = -\frac{27}{106}$

$y_p'' + 3y_p' + 4y_p = (-9A - 9B + 4A)\sin 3x$
$+ (-9B + 9A + 4B)\cos 3x$
$+ (-4C - 6D + 4C)\sin 2x$
$+ (-4D + 6C + 4D)\cos 2x$

$+5B = 9A$

$-3C - \frac{4}{3}C = 2$

$= -\frac{13}{3}C$

$A = -\frac{24}{13}$ $B = -\frac{8}{13}$

$C = -\frac{6}{13}$

$C = 0$ $D = -\frac{4}{13}$

$D = -\frac{1}{3}$

$y_p = -\frac{15}{106}\sin 3x - \frac{27}{106}\cos 3x - \cancel{\frac{1}{13}\sin 2x} - \frac{1}{3}\cos 2x$

$y_p' = -\frac{3\cdot 15}{106}\cos 3x + \frac{27\cdot 3}{106}\sin 3x - \frac{1}{3}\cos 2x + \frac{2}{3}\sin 2x$

$y_p'' = +\frac{9\cdot 15}{106}\sin 3x + \frac{381}{106}\cos 3x + \frac{4}{3}\cos 2x$

$\frac{16}{13} - \frac{36}{13} - \frac{16}{13}$

$\sin 2x$ ✓ $\cos 2x$ ✓ $\cos 3x$ ✓

$\sin 3x$ $243 + 135 - 60 = 318/106 = 3$ ✓

$y = Ae^{-\frac{3+7i}{2}x} + Be^{-\frac{(3+7i)}{2}x} - \frac{15}{106}\sin 3x - \frac{27}{106}\cos 3x - \frac{1}{3}\cos 2x$

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(10) $\boxed{y'' + 4y = (2x^2+x)e^x}$

$\lambda^2 + 4 = 0 \quad \lambda = \pm 2i$

s.i.h ?

$y_h = Ae^{2ix} + Be^{-2ix}$

$y_h' = 2iAe^{2ix} - 2iBe^{-2ix} \rightarrow 4y_h + y_h'' = 0$ ✓

$y_h'' = -4Ae^{2x} - 4Be^{-2x}$

$y_p = (A + Bx + Cx^2)e^x$

$y_p' = (2Cx + B)e^x + y_p \rightarrow (2C + 4Cx + 2B + 5A + 5Bx + 5Cx^2)e^x = (2x^2+x)e^x$

$y_p'' = (2C + 2Cx + B)e^x + y_p'$

$5C = 2 \rightarrow C = 2/5$

$4C + 5B = 1 \quad B = -8/25$

$2C + 2B + 5A = 0 \quad A = -4/125$

$\frac{4}{5} - \frac{16}{25}$

$y_p = \left(\frac{2}{5}x^2 - \frac{8}{25}x - 4/125\right)e^x$

$y_p' = \left(\frac{4}{5}x - \frac{8}{25} + \frac{2}{5}x^2 - \frac{8}{25}x - \frac{4}{125}\right)e^x$

$y_p'' = \left(\frac{4}{5} + \frac{4}{5}x - \frac{8}{25} + \frac{12}{25}x - \frac{44}{125} + \frac{2}{5}x^2\right)e^x$

$100 - 40 - 44 = 16$ ✓

$32 - 32 = 0$ ✓

$\frac{2}{5} + \frac{8}{5} = 2$ ✓

$y = Ae^{2ix} + Be^{-2ix} + \frac{1}{5}\left(2x^2 - \frac{8}{5}x - \frac{4}{25}\right)e^x$

(11) $\boxed{y'' - 9y' = x^2 e^{-3x}}$

$\lambda^2 - 9\lambda = 0 : \lambda(\lambda - 9) = 0 \quad \lambda_1 = 0 \quad \lambda_2 = 9$

$y_h = A + Be^{9x}$

$y_h' = 9Be^{9x} \rightarrow y_h'' - 9y' = 0$ ✓

$y_h'' = 81Be^{9x}$

$y_p = (Ax^2 + Bx + C)e^{-3x}$

$y_p' = (2Ax + B - 3Ax^2 - 3Bx - 3C)$

$y_p'' = (2A - 6Ax - 3B - 6Ax - 3B + 9Ax^2 + 9Bx + 9C)$

$\rightarrow (-18Ax - 9B + 27Ax^2 + 27Bx + 27C)$

$36A = 1 \rightarrow A = 1/36$

$-30A + 36B = 0 \quad B = +\frac{30}{36\cdot 36} = +\frac{5}{6\cdot 36} = +\frac{5}{216}$

$2A - 6B + 9C - 9B + 27C = 2A - 15B + 36C = 0$

$\frac{12}{6\cdot 36} - \frac{15\cdot 5}{6\cdot 36} + 36C = 0$

$-\frac{63}{6\cdot 36} = 36C$

$C = +\frac{63}{6\cdot 36\cdot 36} = +\frac{7}{4\cdot 36}$

$$y_p = \left(\frac{x^2}{36} + \frac{5x}{216} + \frac{7}{6\cdot 23 6^4}\right)e^{3x}$$

$$y_p' = \left(\frac{2x}{36} + \frac{5}{216} + \frac{3x^2}{36} + \frac{15x}{216} + \dots\right)e^{-3x}$$

$$y_p'' = \left(\frac{2}{36} - \frac{6x}{36} + \frac{15}{216} + \frac{9\cdot 7/4}{6\cdot 36} + \frac{9x^2}{36} + \frac{9}{216}x\right)$$

$$\frac{9}{36} + \frac{27}{36} = 1 \checkmark \qquad \frac{-36+9}{216} + \frac{27}{216} = 0 \checkmark \qquad -30\cdot 6 + 2\cdot 36 + 24\cdot 3$$

$$-48\cdot 6 + 24\cdot 3$$

$$\cancel{-30\cdot 6 + 2\cdot 36 + 9\cdot 63 - 9\cdot 5\cdot 6 + 27\cdot 63} = 2\cdot 6 - 30 + 9\cdot 7/4 + 27\cdot 7/4 - 45 = 0 \checkmark$$

$$y = A + Be^{3x} + \left(\frac{x^2}{36} + \frac{5x}{216} + \frac{7}{864}\right)$$

12

$$y'' + 4y = -e^{-x^2}$$

$$\lambda^2 + 4 = 0 \to \lambda = \pm 2i$$

sin, cos

$$y_h = Ae^{2ix} + Be^{-2ix}$$
$$y_h' = 2iAe^{2ix} - 2iBe^{-2ix}$$
$$y_h'' = -4Ae^{2ix} - 4Be^{-2ix}$$

$\to y'' + 4y = 0 \checkmark$

$$y_p = Ce^{-x^2}$$

$$\boxed{y'' + 4y = 3x^2e^x}$$

$$\lambda^2 + 4 = 0$$

$$\lambda = \pm 2i$$

$$y_h = A\cos 2t + B\sin 2t$$

$$y_h' = -2A\sin 2t + 2B\cos 2t$$

$$y_h'' = -4A\cos 2t - 4B\sin 2t = -4y_h \quad + 4y_h = 0 \checkmark$$

$$y_p = (A + Bx + Cx^2)e^x$$

$$y_p' = (2Cx + B)e^x + y_p$$

$$y_p'' = (2C + 2Cx + B)e^x + y_p'$$

$$\to (2C + 2Cx + B + 2Cx + B + 5A + 5Bx + 5Cx^2) = x^2$$

$$5C = 1 \to C = 1/5$$

$$4C + 5B = 0 \to B = -\frac{4}{25}$$

$$2B + 5A + 2C = 0 \qquad A = \frac{2}{125}$$

$$y = A\sin 2x + B\cos 2x + \frac{1}{5}\left(x^2 - \frac{4}{5}x + \frac{2}{125}\right)e^x$$

$$y_p' = \frac{1}{5}\left(2x - \frac{4}{5} + x^2 - \frac{4}{5}x + \frac{2}{25}\right)e^x$$

$$y_p'' = \frac{1}{5}\left(2 + 4x - \frac{8}{5} + x^2 - \frac{4}{5}x + \frac{2}{25}\right)e^x$$

$$y = A\sin 2x + B\cos 2x + \frac{1}{5}\left(x^2 - \frac{4}{5}x - \frac{2}{125}\right)$$

$13 \equiv 1$

(14) $\boxed{y'' - 4y = (e^{ix} + e^{-ix})^2}$

$y'' - 4y = |\cos x + i\,\mathrm{sen}\,x + \cos(-x) + i\,\mathrm{sen}(-x)|^2 = (2\cos x)^2 = 4\cos^2 x$

$= 2 + 2\cos 2x$

$\lambda^2 - 4 = 0 \to \lambda = \pm 2$

$$\left.\begin{array}{l} y_h = Ae^{2x} + Be^{-2x} \\ y_h' = 2Ae^{2x} - 2Be^{-2x} \\ y_h'' = 4Ae^{2x} + 4Be^{-2x} \end{array}\right\} \to 4A + 4B - 4A - 4B = 0 \checkmark$$

$$\left.\begin{array}{l} y_p = A\cos 2x + B\sin 2x + C \\ y_p' = -2A\sin 2x + 2B\cos 2x \\ y_p'' = -4A\cos x - 4B\sin x \end{array}\right\} \begin{array}{l} -4A - 4A = 2 \to A = -1/4 \\ -B - 4B = 0 \to B = 0 \\ -4C = 2 \to C = -1/2 \end{array}$$

$y_p = -\frac{1}{4}\cos 2x - 1/2$

$y_p' = \frac{1}{2}\sin 2x \qquad 1 - (-1) = 2 \checkmark$

$y_p'' = \cos 2x \qquad 0 - (-1/2) = 2 \checkmark$

$y = Ae^{2x} + Be^{-2x} - \frac{1}{4}\cos x - 1/2$

(15) $\boxed{y'' - 8y' + 12y = 3x^2 e^x}$

$\lambda^2 - 8\lambda + 12 = 0 \qquad \lambda_{12} = \frac{8 \pm \sqrt{64 - 4\cdot 12}}{2} = 4 \pm 2$

$$\begin{array}{c|ccc} & 1 & -8 & 12 \\ 6 & & 6 & -12 \\ \hline & 1 & -2 & \end{array}$$

$(\lambda - 6)(\lambda - 2) = 0$

$$\left.\begin{array}{l} y_h = Ae^{6x} + Be^{2x} \\ y_h' = 6Ae^{6x} + 2Be^{2x} \\ y_h'' = 36Ae^{6x} + 4Be^{2x} \end{array}\right\} \to \begin{array}{l} 36A - 48A + 12A = 0 \checkmark \\ 4B - 16B + 12B = 0 \checkmark \end{array}$$

$y_p = (Ax^2 + Bx + C)e^x$

$y_p' = (2Ax + B)e^x + y_p$

$y_p'' = (2A + 2Ax + B)e^x + y_p'$

$2A + 2Ax + B - 14Ax - 7B + 5Ax^2 + 5Bx + 5C = 3x^2$

$5A = 3 \to A = 3/5$

$-12A + 5B = 0 \to B = \frac{12}{5}A = \frac{36}{25}$

$2A + \cancel{B} - 6B + 5C = 0 : \frac{6}{5} - \frac{36}{25} + 5C = 0 \to C = \frac{186}{125}$

$$y_p = \left(\frac{3}{5}x^2 + \frac{36}{25}x + \frac{186}{125}\right)e^x$$

$$y_p' = \left(\frac{6}{5}x + \frac{36}{25} + \frac{3}{5}x^2 + \frac{36}{25}x + \frac{186}{125}\right)e^x$$

$$y_p'' = \left(\frac{6}{5} + \frac{6}{5}x + \frac{36}{25} + \frac{6}{5}x + \frac{36}{25} + \frac{3}{5}x^2 + \frac{36}{25}x + \frac{186}{125}\right)e^x$$

$$+ \frac{12}{5}x + \frac{\cancel{42}}{25} - \frac{186}{125}$$

$$- \frac{8\cdot 6}{5}x - \frac{8\cdot 36}{25} - \frac{24}{5}x^2 - \frac{8\cdot 36}{25}x + \frac{8\cdot 186}{125}$$

$$+ \frac{12\cdot 3}{5}x^2 + \frac{12\cdot 36}{25}x - \frac{12\cdot 186}{125} \qquad - \frac{186}{25}$$

$$\Rightarrow \frac{12\cdot 3 - 8\cdot 3 + 1\cdot 3}{5} = 3 \checkmark \qquad \frac{96}{25} - 8\cdot 30 - 8\cdot 36 + 12\cdot 36 = 0 \checkmark$$

$$\frac{402}{25} - \frac{8\cdot 36}{25} + \frac{186}{125} = 0 \checkmark$$

$$y = Ae^{6x} + Be^{2x} + \left(\frac{3}{5}x^2 + \frac{36}{25} + \frac{186}{125}\right)e^x$$

16 $\boxed{y'' - y = \frac{2}{1+e^x}}$

$\lambda^2 - 1 = 0 \rightarrow \lambda = \pm 1$

$y_h = Ae^x + Be^{-x}$

$y_h' = Ae^x - Be^{-x} \rightarrow A + B - A - B = 0 \checkmark$

$y_h'' = Ae^x + Be^{-x}$

$y_p = k_1(x)e^x + k_2(x)e^{-x}$

$$k_1' e^x + k_2' e^{-x} = 0$$

$$k_1' e^x - k_2' e^{-x} = \frac{2}{1+e^x}$$

$$\Delta = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -1 - 1 = -2$$

$z = e^{+x} \quad dz = +z\,dx$

$$k_1' = -\frac{1}{2}\begin{vmatrix} 0 & e^{-x} \\ \frac{2}{1+e^x} & -e^{-x} \end{vmatrix} = \frac{e^{-x}}{1+e^x} \Rightarrow k_1 = \int \frac{e^{-x}}{1+e^x}dx = \int \frac{z^{-1} z^{-1}}{1+z}dz$$

$$k_2' = -\frac{1}{2}\begin{vmatrix} e^x & 0 \\ e^x & \frac{2}{1+e^x} \end{vmatrix} = -\frac{e^x}{1+e^x} \qquad k_2 = -\ln|1+e^x|$$

$$\int z^{-2}(1+z)^{-1}dz = \int \frac{1}{z^2(z+1)}dz \Rightarrow \frac{A}{z^2} + \frac{B}{z} + \frac{C}{z+1} \rightarrow \underbrace{A(z+1) + Bz(z+1) + Cz^2}_{=1}$$

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$B+C=0$

$A+B=0$

$A=1 \quad B=-1 \quad C=1$

$\frac{1}{z^2}-\frac{1}{z}+\frac{1}{z+1} = z+1-z(z+1)+z^2=1$ ✓

$\int \frac{1}{z^2}dz=-\frac{1}{z} \qquad -\int\frac{1}{z}dz=-\ln z \qquad \int\frac{1}{z+1}dz=\ln|z+1|$

$K_1=-\frac{1}{z}+\ln\frac{z+1}{z}=-e^{x}+\ln|e^{x}+1|-x$

$y_p=(-e^{x}-\cancel{x}+\ln|e^{x}+1|)e^{x}+e^{-x}(-\ln|1+e^{x}|)$

$y_p'=\cancel{\ }-e^{x}-xe^{x}+e^{x}\ln|e^{x}+1|+\frac{e^{2x}}{e^{x}+1}+e^{-x}\ln|1+e^{x}|-\frac{1}{1+e^{x}}$

$y_p''=\cancel{\ }-e^{x}-e^{x}-xe^{x}+e^{x}\ln|e^{x}+1|+\frac{e^{2x}}{e^{x}+1}+\frac{2e^{2x}}{e^{x}+1}-\frac{e^{3x}}{(e^{x}+1)^2}$

$-e^{-x}\ln|1+e^{x}|+\frac{1}{1+e^{x}}+\frac{e^{x}}{(1+e^{x})^2}$

$y_p''-y=\cancel{\ }-2e^{x}+\frac{3e^{2x}(e^{x}+1)+1+e^{x}+e^{x}-e^{3x}}{(e^{x}+1)^2}+1$

$=\cancel{\ }\frac{-2e^{x}(e^{2x}+2e^{x}+1)+3e^{3x}+3e^{2x}+1+2e^{x}-e^{3x}}{(e^{x}+1)^2}+1$

$=\cancel{\ }+\frac{-4e^{2x}-2e^{x}+3e^{2x}+2e^{x}+1}{e^{2x}+2e^{x}+1}=-3e^{2x}+\frac{-e^{2x}+1}{e^{2x}+2e^{x}+1}+1$

$=\cancel{\ }+\frac{1-e^{x}}{1+e^{x}}+1=\frac{2}{1+e^{x}}$ ✓

$y=(A-x+\ln|e^{x}+1|)e^{x}-\ln|1+e^{x}|-1$

(18)

$$\boxed{y'' - 2y' + y = \frac{e^x}{\sqrt{x^2-1}}}$$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda_{1,2} = 1$ (doble)

$y_h = (Ax+B)e^x$

$y_h' = (A+B+Ax)e^x$

$y_h'' = (2A+B+Ax)e^x$

$\left.\right\} 2A+B+Ax-2A-2B-2Ax+Ax+B = 0$ ✓

$y_p = k_1(x)e^x + k_2(x)xe^x$

$k_1'(x)e^x + k_2'xe^x = 0$

$k_1'e^x + k_2'(xe^x+e^x) = \dfrac{e^x}{\sqrt{x^2-1}}$

$\Delta = \begin{vmatrix} e^x & xe^x \\ e^x & e^x(x+1) \end{vmatrix} = e^{2x}$ $\quad k_1 = \cancel{k_2(x-1)}$

$\Delta k_1' = \begin{vmatrix} 0 & xe^x \\ \frac{e^x}{\sqrt{x^2-1}} & \end{vmatrix} = -\dfrac{xe^{2x}}{\sqrt{x^2-1}} \to k_1' = -\dfrac{x}{\sqrt{x^2-1}} \qquad k_1 = -\sqrt{x^2-1}$

$\Delta k_2' = \begin{vmatrix} e^x & 0 \\ e^x & \frac{e^x}{\sqrt{x^2-1}} \end{vmatrix} = \dfrac{e^{2x}}{\sqrt{x^2-1}} \to k_2' = \dfrac{1}{\sqrt{x^2-1}}$

$k_2 = \displaystyle\int \frac{1}{\sqrt{x^2-1}}dx = \ln|x+\sqrt{x^2-1}|$

$y_p = -\sqrt{x^2-1}\,e^x + x\ln|x+\sqrt{x^2-1}|\,e^x$

$y_p' = e^x\left(-\dfrac{x}{\sqrt{\ }} + \ln| \quad | + \dfrac{x}{x+\sqrt{\ }}\cdot\left(1+\dfrac{x}{\sqrt{\ }}\right)\right) + y_p$

$y_p'' = y_p' + y_p' - y_p + e^x\left(-\dfrac{1}{\sqrt{\ }} + \cancel{\dfrac{x^2}{(\ )^{3/2}}} + \dfrac{1}{x+\sqrt{\ }}\cdot\left(1+\dfrac{x}{\sqrt{\ }}\right) + x\left(\dfrac{1}{\sqrt{x^2-1}} - \cancel{\dfrac{x^2}{(x^2-1)^{3/2}}}\right)\right)$

$\to$ ~~$= \dfrac{e^x}{\sqrt{x^2-1}}\left(-1 + \dfrac{x^2}{x^2-1} + 3x - \dfrac{x^3}{x^2-1}\right) = \dfrac{e^x}{\sqrt{x^2-1}(x^2-1)}\cdot\left(-x^2+1+x^2+3x^3-3x-x^3\right)$~~

~~$= \dfrac{e^x}{\sqrt{x^2-1}}\left(\dfrac{2x^3-3x+1}{x^2-1}\right) = \dfrac{e^x}{\sqrt{\ }}\dfrac{(x-1)(}{(x+1)(x-1)}$~~

$= \dfrac{e^x}{\sqrt{x^2-1}}$

J.A. Oteo. Departamento de Física Teórica (UVEG).[MMF3-B:2008-9]

TEMA 2: EDO Sistemas lineales*

11 de diciembre de 2008

1. //Oteo// Representa el diagrama de fase en el primer cuadrante

   a)

   $$\begin{aligned} \dot{y_1} &= y_1^2 + y_1 y_2 - y_1 \\ \dot{y_2} &= y_2^2 + y_1 y_2 - 2y_2 \end{aligned}$$

   b)

   $$\begin{aligned} \dot{x} &= 4x - xy \\ \dot{y} &= 2xy - 6y \end{aligned}$$

2. //Erica [Fernando]// En una planta infestada de pulgones viven mariquitas, que se alimentan de ellos. También viven hormigas, que cuidan de ellos, para conseguir a cambio una sustancia dulce que producen. Estas hormigas, además de tener rebaños de pulgones, de vez en cuando se los comen, sobre todo si están enfadadas. Comérselos las hace felices, por lo que cuanto ms enfafadas estn, ms pulgones se comen y ms felices se vuelven. Las mariquitas, cuando son felices, cazan mejor a los pulgones, y son an ms felices. Por esto, la felicidad de las mariquitas molesta a las hormigas. Del mismo modo, cuanto ms contentas estn las hormigas, menos pulgones se comen y ms felices son las mariquitas. Por ltimo, las hormigas tienen otras fuentes de alimentacin que las hace felices. De este modo, siendo H la felicidad de las hormigas y M la de las mariquitas:

   $$\begin{aligned} \dot{H} &= -2H - 3M + 1 \\ \dot{M} &= H + 2M \end{aligned}$$

   Cómo evolucionará la felicidad de los insectos de esta planta? Será un ecosistema en equilibrio? Para averiguarlo dibuja un diagrama de flujo en el plano HM.

3. //Fernando [Erica]//

   $$\begin{aligned} \dot{y} &= y + z \\ \dot{z} &= 6y + 2z + t^2 \exp(-t) \end{aligned}$$

   a) Resuelve el sistema por el método más corto.

   b) Haz un diagrama de fase ignorando el término inhomogéneo

   c) Cómo cambiaría el diagrama de fase si el término inhomogéneo en $\dot{z}$ fuese una constante?

   d) Misma cuestión en el caso del sistema propuesto

*Preguntas y soluciones contrastadas por [...]

4. `//Esther [Rosa]//` Resuelve y representa el diagrama de fase
$$\begin{aligned}\dot{x} &= -3x - y\\ \dot{y} &= 7x - 3y\end{aligned}$$

5. `//Rosa [Esther]//` Resuelve: a) como sistema y b) transformando en una EDO de segundo orden.
$$\begin{aligned}\dot{x} &= 2y + 3\\ \dot{y} &= 2x - 2t\end{aligned}$$

6. `//Almudena [Isabel]//` Resuelve y representa el diagrama de fase
$$\begin{aligned}\dot{x} &= 3y + 3x\\ \dot{y} &= 3x + y\end{aligned}$$

7. `//Isabel [Almudena]//` Resuelve
$$\begin{aligned}\dot{x} &= 4y + 1\\ \dot{y} &= x + 3\end{aligned}$$

8. `//Pablo [Fdo. S.]//` Resuelve y representa el diagrama de fase
$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= 3x - 2y\end{aligned}$$

9. `//Fdo. S. [Pablo]//` Resuelve, como sistema, en $t = 42$, con $y(0) = 1/3$, $z(0) = 0$
$$\begin{aligned}\dot{y} &= z + \sinh t \cosh t\\ \dot{z} &= y\end{aligned}$$

10. `//Luis [Alejandro]//` Resuelve
$$\frac{d\mathbf{x}(t)}{dt} = \begin{pmatrix} 4 & 2\\ 3 & -1\end{pmatrix}\mathbf{x}(t) - \begin{pmatrix} 15\\ 4\end{pmatrix} t\exp(-2t), \qquad \mathbf{x}(0) = \begin{pmatrix} 1\\ -1\end{pmatrix}$$

11. `//Alejandro [Luis]//` Resuelve, con $y(0) = z(0) = 1$
$$\begin{aligned}\dot{y} &= z + \exp(-t)\\ \dot{z} &= y + \text{sech}\, t\end{aligned}$$

12. `//Jesús [Alberto]//` Resuleve con $x(0) = z(0) = 0,\ y(0) = 1$
$$\begin{aligned}\dot{x} &= x + 2y\\ \dot{y} &= 2x + y + 2z + 3\\ \dot{z} &= z\end{aligned}$$

13. `//Alberto [Jesús]//` Resuelve con $y(0) = \alpha, z(0) = \beta$
$$\begin{aligned}\dot{y} &= z\\ \dot{z} &= y + 2t\end{aligned}$$

14. `//Yanis [Aitor]//` Resuelve con $y(0) = 1, z(0) = 1$
$$\begin{aligned}\dot{y} &= 3z + 2\\ \dot{z} &= 3y + 4\end{aligned}$$

15. `//Aitor [Yanis]//` Resuelve y representa el diagrama de fase
$$\begin{aligned}\dot{x} &= 6x + 5y\\ \dot{y} &= 2x + 3y\end{aligned}$$

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(1)

a) $\boxed{\begin{cases} \dot y_1 = y_1^2 + y_1 y_2 - y_1 \\ \dot y_2 = y_2^2 + y_1 y_2 - 2y_2 \end{cases}}$

$\dot y_1 = 0 \;,\; \dot y_2 = 0$

$\dot y_1 = y_1(y_1 + y_2 - 1) \rightarrow y_1 = 0 \;\vee\; y_1 + y_2 = 1$

$\dot y_2 = y_2(y_2 + y_1 - 2) \rightarrow y_2 = 0 \;\vee\; y_2 + y_1 = 2$

$A(0/0) \qquad B(0/2) \qquad C(1/0)$

A:

$\dot y_1 = \dot\varepsilon_1 = \varepsilon_1(\varepsilon_1 + \varepsilon_2 - 1) \approx -\varepsilon_1 \qquad \rightarrow A = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

$\dot y_2 = \dot\varepsilon_2 = \varepsilon_2(\varepsilon_2 + \varepsilon_1 - 2) \approx -2\varepsilon_2$

$\rightarrow y_h = c_1 e^{-x}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-2x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

B:

$\dot y_1 = \varepsilon_1(\varepsilon_1 + \varepsilon_2 + 1) \approx \varepsilon_1 \qquad \rightarrow A = \begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}$

$\dot y_2 = (2 + \varepsilon_2)(\varepsilon_2 + \varepsilon_1) \approx 2\varepsilon_2 + 2\varepsilon_1$

$\det = (1-\lambda)(2-\lambda) \rightarrow \lambda_1 = 1 \quad \lambda_2 = 2 \quad \rightarrow (1,-2)\;;\;(1,0)$

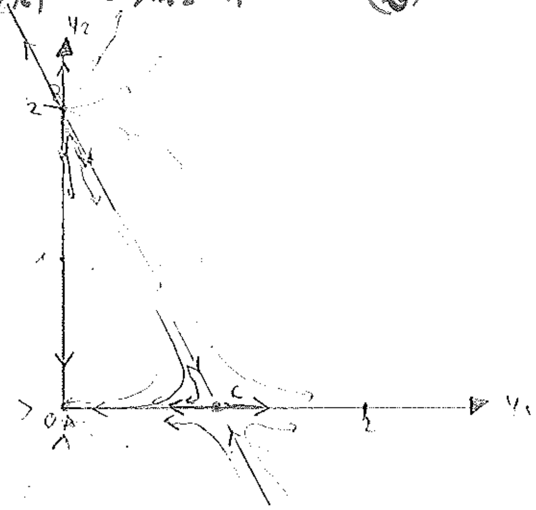
$y_h = c_1 e^{x}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2 e^{2x}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

C:

$\dot y_1 = (1+\varepsilon_1)(\varepsilon_1 + \varepsilon_2) \approx \varepsilon_1 + \varepsilon_2 \qquad A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \qquad y_h = c_1 e^{x}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^{-x}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\dot y_2 = \varepsilon_2(\varepsilon_2 + \varepsilon_1 - 1) \approx -\varepsilon_2$

$\rightarrow (1-\lambda)(-1+\lambda) \rightarrow \lambda_1 = 1 \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \lambda_2 = -1 \rightarrow \begin{pmatrix} 1 \\ -2 \end{pmatrix}$



6) $\boxed{\begin{cases} \dot{x} = 4x - xy \\ \dot{y} = 2xy - 6y \end{cases}}$

$\dot{x} = x(4-y) = 0 \rightarrow x = 0 \vee y = 4$ → $A(0/0)$

$\dot{y} = y(2x-6) = 0 \rightarrow y = 0 \vee x = 3$ → $B(3/4)$

A: $\dot{\varepsilon}_1 : \dot{x} = \varepsilon_1(4-\varepsilon_2) \approx 4\varepsilon_1$

$\dot{\varepsilon}_2 : \dot{y} = \varepsilon_2(2\varepsilon_1 - 6) \approx -6\varepsilon_2 \rightarrow A = \begin{pmatrix} 4 & 0 \\ 0 & -6 \end{pmatrix}$

$y_h = c_1 e^{4t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{-6t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

B: $\dot{x} = (3+\varepsilon_1)(\varepsilon_2) \approx 3\varepsilon_2$

$\dot{y} = (4+\varepsilon_2)(2\varepsilon_1) \approx 8\varepsilon_1$

$A = \begin{pmatrix} 0 & 3 \\ 8 & 0 \end{pmatrix}$ $\quad \lambda^2 - 24 = 0$

$\lambda = \pm\sqrt{24} = 2\sqrt{6}$

$D = \begin{pmatrix} \sqrt{24} & 0 \\ 0 & -\sqrt{24} \end{pmatrix}$

$\lambda_1 = \sqrt{24}$

$-\sqrt{24}x + 3y = 0 \quad x = 3$

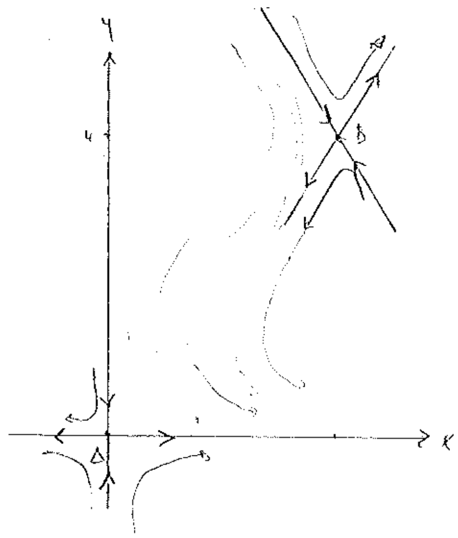
$8x - \sqrt{24}y = 0 \quad y = \sqrt{24}$

$\lambda_2 = -\sqrt{24}$

$\sqrt{24}x + 3y = 0$

$x = 3 \quad y = -\sqrt{24} \quad \begin{pmatrix} 3 \\ -2\sqrt{6} \end{pmatrix}$

$y_h = c_1 e^{2\sqrt{6}t}\begin{pmatrix} 3 \\ 2\sqrt{6} \end{pmatrix} + c_2 e^{-2\sqrt{6}t}\begin{pmatrix} 3 \\ -2\sqrt{6} \end{pmatrix}$



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$$\boxed{\begin{cases}\dot x = -3x - y\\ \dot y = 7x - 3y\end{cases}}$$

$$\to \begin{pmatrix}-\sqrt7 i & -1\\ 7 & -\sqrt7 i\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$A = \begin{pmatrix}-3 & -1\\ 7 & -3\end{pmatrix}$$

$$x\sqrt7 i + y = 0$$

$$(3+\lambda)^2 + 7 = 0 \qquad \lambda = -3 \pm \sqrt7 i$$

$$x = 1 \quad y = -\sqrt7 i \qquad \begin{pmatrix}1\\ -\sqrt7 i\end{pmatrix}$$

$$\vec{y_h} = e^{-3t}\left[c_1 e^{\sqrt7 i t}\begin{pmatrix}1\\ -\sqrt7 i\end{pmatrix} + c_2 e^{-\sqrt7 i t}\begin{pmatrix}1\\ \sqrt7 i\end{pmatrix}\right.$$

$$\sqrt7 i x - 1y = 0$$
$$7 \qquad \sqrt7 i$$

$$x = 1 \quad y = \sqrt7 i\} \begin{pmatrix}1\\ \sqrt7 i\end{pmatrix}$$

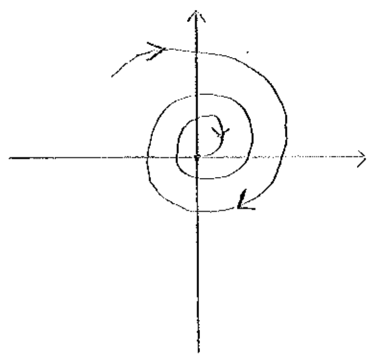
$$x = e^{-3t}\left(A\,\text{sen}\,\sqrt7 t + B\cos\sqrt7 t\right) = e^{-3t}\left(A\cos(\sqrt7 t + \varphi)\right)$$

$$y = e^{-3t}\left(C\,\text{sen}\,\sqrt7 t + D\cos\sqrt7 t\right) \qquad \cancel{A, B\,?}$$

$$A = B^*$$

$$x = A\cos(\sqrt7 t + \varphi)$$

$$y = \sqrt7 A\sin(\sqrt7 t + \varphi)$$



$$\dot x = -3x + \underbrace{\sqrt7 c_1 \cos - \sqrt7 c_2\,\text{sen}}_{y} \quad ✓$$

$$\dot y = -3y - \underbrace{7c_1\,\text{sen} - 7c_2\cos}_{7x} \quad ✓$$

a

$$\boxed{\begin{cases} \dot H = -2H - 3M + 1 \\ \dot M = H + 2M \end{cases}}$$

$$A = \begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix} \quad f = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} -2-\lambda & -3 \\ 1 & 2-\lambda \end{vmatrix} = -(2+\lambda)(2-\lambda)+3 = \lambda^2 - 4 + 3 = \lambda^2 - 1 = 0$$

$$\lambda_{1,2} = \pm 1$$

$$\begin{pmatrix} -3 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow \vec{y}_h = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot e^{t} + c_2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} e^{-t} = \begin{pmatrix} H \\ M \end{pmatrix}$$

$$\begin{pmatrix} -1 & -3 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\dot H = c_1 e^t - 3c_2 e^{-t} = 3c_1 e^t + 3c_2 e^{-t} - 2c_1 e^t - 6c_2 e^{-t} \checkmark$$

$$\dot M = -c_1 e^t + c_2 e^{-t} = c_1 e^t + 3c_2 e^{-t} - 2c_1 e^t - 2c_2 e^{-t} \checkmark$$

$$A = \begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix} \quad \det A\, A^{-1} = \begin{pmatrix} +2 & -1 \\ +3 & -2 \end{pmatrix}^T = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$

$$\det A = -1 \qquad A^{-1} = \begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix}$$

$$A \begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A = A^{-1} = A^3 = A,\ A^2 = I$$

$$U = \begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix} \quad \det U = 2 \quad U^{-1} = \frac{1}{2}\begin{pmatrix} -1 & +1 \\ -3 & +1 \end{pmatrix}^T = \frac{1}{2}\begin{pmatrix} -1 & -3 \\ 1 & 1 \end{pmatrix} = -\frac{1}{2} U$$

$$-\frac{1}{2}\begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \cdot \left(-\frac{1}{2}\right) = I$$

$$\exp As = \frac{1}{2}\begin{pmatrix} -1 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^s & 0 \\ 0 & e^{-s} \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -e^s & -3e^{-s} \\ e^s & e^{-s} \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -e^s + 3e^{-s} & -3e^s + 3e^{-s} \\ e^s - e^{-s} & 3e^s - e^{-s} \end{pmatrix}$$

$$\exp[-As]\cdot\vec{f}(s) = \frac{1}{2}\begin{pmatrix} -e^{-s}+3e^{+s} & -3e^{-s}+3e^{+s} \\ e^{-s}-e^{+s} & 3e^{-s}-e^{+s}\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -e^{-s}+3e^{+s} \\ e^{-s}-e^{+s}\end{pmatrix}$$

$$\rightarrow \int_0^t \text{——}\, ds = \frac{1}{2}\begin{pmatrix} e^{-s}+3e^{s} \\ -e^{-s}-e^{s}\end{pmatrix}\Bigg|_0^t = \frac{1}{2}\begin{pmatrix} e^{-t}+3e^{t} - 4 \\ -e^{-t}-e^{t}+2\end{pmatrix}$$

$$y_p = e^{At}\cdot\int = \frac{1}{4}\begin{pmatrix} -e^{t}+3e^{-t} & -3e^{t}+3e^{-t} \\ e^{t}-e^{-t} & 3e^{t}-e^{-t}\end{pmatrix}\begin{pmatrix} e^{-t}+3e^{t}-4 \\ -e^{-t}-e^{t}+2\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} -1-3\cancel{e^{2t}}+4e^{t}+3\cancel{e^{-2t}}-9-12e^{-t}+\cancel{3}+3\cancel{e^{2t}}-6e^{t}-3\cancel{e^{-2t}}-\cancel{3}+6e^{-t} \\ 1+3\cancel{e^{2t}}-4e^{t}-\cancel{e^{-2t}}-3+4e^{-t}+3-3\cancel{e^{2t}}+6e^{t}+\cancel{e^{-2t}}+1-2e^{-t}\end{pmatrix}$$

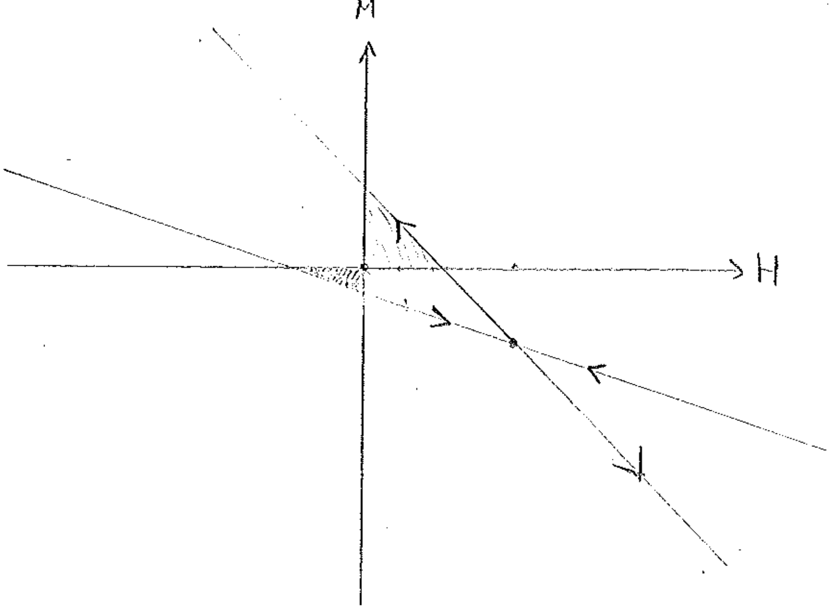
$$= \frac{1}{4}\begin{pmatrix} 8+\cancel{6e^{2t}}-2e^{t}-6e^{-t} \\ -4-\cancel{6e^{2t}}+2e^{t}+2e^{-t}\end{pmatrix}$$

$$\dot{H} = \frac{1}{4}\left(\cancel{\quad}-2e^{t}+6e^{-2t}\right) = \frac{1}{4}\left(-16+4e^{t}+12e^{-t}+12-6e^{t}-6e^{-t}+4\right)$$

$$\dot{M} = \frac{1}{4}\left(2e^{t}-2e^{-t}\right) = \frac{1}{4}\left(8-2e^{t}-6e^{-t}+4e^{t}+4e^{-t}-8\right)$$

$$\Rightarrow \vec{y} = c_1 e^{t}\begin{pmatrix}1\\-1\end{pmatrix} + c_2 e^{-t}\begin{pmatrix}3\\-1\end{pmatrix} + \begin{pmatrix}2\\-1\end{pmatrix}$$

$\Rightarrow$



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$\dot{x} = 2y + 3$
$\dot{y} = 2x \quad - 2t$

$A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad \vec{f} = \begin{pmatrix} 3 \\ -2t \end{pmatrix} \quad \begin{pmatrix} x \\ y \end{pmatrix}$

$\lambda^2 - 4 = 0$

$\lambda = \pm 2$

~~$A^{-1} = \frac{1}{2} I = \frac{1}{2} A$~~ ~~$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $U^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$~~

~~$\exp As =$~~

$\lambda = 2 \quad -2x + 2y = 0 \quad \to \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = -2 \quad 2x + 2y \quad \to \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \det U = -2 \quad \mathrm{d}.U^{-1} = \begin{pmatrix} -1 & -1 \\ -1 & +1 \end{pmatrix}$

$U^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} U$

$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = I$ ✓

$\exp At = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{2t} & 0 \\ 0 & e^{-2t} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{2t} & e^{-2t} \\ e^{2t} & -e^{-2t} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} e^{2t} + e^{-2t} & e^{2t} - e^{-2t} \\ e^{2t} - e^{-2t} & e^{2t} + e^{-2t} \end{pmatrix}$ $\begin{matrix} 3 \\ -2t \end{matrix}$

$\int 2x e^{2x} = x e^{2x} - \int e^{2x} dx$
$= x e^{2x} - \frac{1}{2} e^{2x}$

$\exp -As \cdot f(s) ds = \frac{1}{2}\begin{pmatrix} e^{-2s} + e^{2s} & e^{-2s} - e^{2s} \\ e^{-2s} - e^{2s} & e^{-2s} + e^{2s} \end{pmatrix}\begin{pmatrix} 3 \\ -2s \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 3e^{-2s} + 3e^{2s} - 2s e^{-2s} + 2s e^{2s} \\ 3e^{-2s} - 3e^{2s} - 2s e^{-2s} - 2s e^{2s} \end{pmatrix}$

$\int 2x e^{-2x} dx = -x e^{-2x} + \int e^{-2x} dx$
$u = 2x \quad dv = e^{-2x} dx$
$du = 2dx \quad v = -\frac{1}{2} e^{-2x}$
$= -x e^{-2x} - \frac{1}{2} e^{-2x}$

$\int_0^t \exp -As\, f(s) ds = \frac{1}{2} \cdot \begin{pmatrix} -\frac{3}{2}e^{-2s} + \frac{3}{2}e^{2s} + s e^{-2s} + \cancel{\frac{1}{2}e^{-2s}} + s e^{2s} - \cancel{\frac{1}{2}e^{2s}} \\ -\frac{3}{2}e^{-2s} - \frac{3}{2}e^{2s} + s e^{-2s} + \frac{1}{2}e^{-2s} - s e^{2s} + \frac{1}{2}e^{2s} \end{pmatrix}\Bigg|_0^t$

$$= \frac{1}{2}\begin{pmatrix} -\cancel{3}e^{-2t} + \cancel{3}e^{2t} + t\cdot e^{2t} + t\cdot e^{-2t} \\ -e^{-2t} - e^{2t} + t e^{-2t} - t e^{2t} + 2 \end{pmatrix}$$

$$\frac{1}{4}\begin{pmatrix} e^{2t}+e^{-2t} & e^{2t}-e^{-2t} \\ e^{2t}-e^{-2t} & e^{2t}+e^{-2t} \end{pmatrix}\begin{pmatrix} -\frac{3}{2}e^{-2t}+\frac{3}{2}e^{2t} & \dots \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} -\cancel{1} + \cancel{1}\cancel{e^{4t}} + t\cancel{e^{4t}} + t - \cancel{1}\cancel{e^{-4t}} + \cancel{1} + t + \cancel{te^{-4t}} \\ + -\cancel{1} - \cancel{e^{4t}} + t - t\cancel{e^{4t}} + 2e^{2t} + \cancel{e^{-4t}} + \cancel{1} - \cancel{te^{-4t}} + t - 2e^{-2t} \\ -\cancel{1} + \cancel{1}\cancel{e^{4t}} + \cancel{te^{4t}} + \cancel{t} + \cancel{1}\cancel{e^{-4t}} - \cancel{1} - \cancel{t} - \cancel{te^{-4t}} \\ -1 - \cancel{e^{4t}} + \cancel{t} - \cancel{te^{4t}} + 2e^{2t} - \cancel{e^{-4t}} - 1 + t\cancel{e^{-4t}} - \cancel{t} + 2e^{-2t} \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} \cancel{1}\cancel{e^{4t}} - \cancel{1}\cancel{e^{-4t}} + 2e^{2t} - 2e^{-2t} + 4t \\ +\cancel{\tfrac{1}{2}e^{4t}} + \tfrac{1}{2}\cancel{e^{4t}} + 2e^{2t} + 2e^{-2t} - \cancel{8}\,4 \end{pmatrix}$$

$$\dot{x} = \frac{1}{4}\left(\cancel{6e^{4t}} + \cancel{6e^{-4t}} + 4e^{2t} + 4e^{-2t} + 4\right) = \frac{1}{4}\left(\cancel{e^{4t}} + \cancel{e^{-4t}} + 4e^{2t} + 4e^{-2t} - 8\,12\right)$$

$$\dot{y} = \frac{1}{4}\left(-\cancel{2e^{4t}} + \cancel{2e^{-4t}} + 4e^{2t} - 4e^{-2t}\right) = \frac{1}{4}\left(4e^{2t} - 4e^{-2t} + 8t - 8t\right)$$

b) $y = \frac{\dot{x} - 3}{2}$ $\quad \dot{y} = \frac{\ddot{x}}{2}$

$\frac{\ddot{x}}{2} = 2x - 2t$

$\ddot{x} - 4x = -4t \rightarrow \lambda^2 - 4 = 0 \rightarrow \lambda = \pm 2$

$x_h = c_1 e^{2t} + c_2 e^{-2t}$ $\quad \rightarrow x = c_1 e^{2t} + c_2 e^{-2t} + t$

$x_p = (A + Bt)$ $\quad y = \frac{2c_1 e^{2t} - 2c_2 e^{-2t} + 1 - 3}{2} = c_1 e^{2t} - c_2 e^{-2t} - 1$

$x_p' = B$

$x_p'' = 0$

$-4(A + Bt) = -4t \rightarrow A = 0,\ B = 1$

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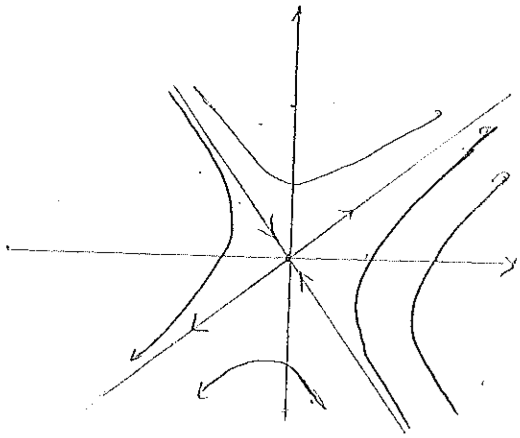
$\dot{x} = 3y + 3x$
$\dot{y} = 3x + y$

$A = \begin{pmatrix} 3 & 3 \\ 3 & 1 \end{pmatrix} \quad \begin{vmatrix} 3-\lambda & 3 \\ 3 & 1-\lambda \end{vmatrix} = (3-\lambda)(1-\lambda) - 9 = 3 - 4\lambda + \lambda^2 - 9$
$= \lambda^2 - 4\lambda - 6$

$\lambda_{12} = \dfrac{4 \pm \sqrt{16+24}}{2} = 2 \pm \sqrt{10}$

$-2, \quad 3 \quad -4, \quad \begin{pmatrix} 3 \\ 2 \end{pmatrix} \qquad 4, \; 3 \quad \begin{pmatrix} 3 \\ -4 \end{pmatrix}$



$\vec{r} = c_1 e^{(2+\sqrt{10})t} \begin{pmatrix} 3 \\ -2+\sqrt{10} \end{pmatrix} + c_2 e^{(2-\sqrt{10})t} \begin{pmatrix} -3 \\ 1+\sqrt{10} \end{pmatrix}$

$\dot{x} = (2+\sqrt{10})3c_1 e^{2+} - (2-\sqrt{10})3c_2 e^{2-}$

$= 9c_1 e^{2+} - 9c_2 e^{2-} + 3c_1(\sqrt{10}-2)e^{2+} + 3(1+\sqrt{10})c_2 e$ ✓

$\dot{y} = (-1+\sqrt{10})(2+\sqrt{10}) c_1 e^{2+} + (1+\sqrt{10})(2-\sqrt{10}) c_2 e^{2-}$

$= 9c_1 e^{2+} - 9c_2 e^{2-} + (\sqrt{10}-1) c_1 e^{2+} + (1+\sqrt{10}) c_2 e^{2-}$ ✓

$\dfrac{\dot{y} - y}{3} = x \qquad \dot{x} = \dfrac{\ddot{y} - \dot{y}}{3}$

$\dfrac{\ddot{y} - \dot{y}}{3} = 3y + \dot{y} - y \rightarrow \quad \ddot{y} = \dot{y} + 9y + 3\dot{y} - 3y = 4\dot{y} + 6y \; \hat{=}$

...

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$$\boxed{\begin{aligned}\dot{x} &= 4y + 1\\ \dot{y} &= x + 3\end{aligned}}$$

$$A = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix} \qquad \vec{f} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad \begin{vmatrix} -\lambda & 4 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 4 \to \lambda_{12} = \pm 2$$

$$-2x + 4y = 0 \to \begin{pmatrix} 4 \\ 2 \end{pmatrix} \hat{=} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad 2\,4 \to \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \qquad \det U = -4$$

$$\Lambda = V^{-1} A V$$
$$A = V \Lambda V^{-1}$$

$$d \cdot U^{-1} = \begin{pmatrix} +-1 & -1 \\ -2 & +2 \end{pmatrix}^T = \begin{pmatrix} -1 & -2 \\ -1 & 2 \end{pmatrix} \qquad V^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} \quad e^A = V e^\Lambda V^{-1}$$

$$\frac{1}{4} \cdot \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \checkmark$$

$$\exp As = \underbrace{\frac{1}{4}\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}}_{V} \begin{pmatrix} e^{2t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \underbrace{\begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix}}_{V^{-1}} = \frac{1}{4}\begin{pmatrix} 2e^{2t} & 2e^{-2t} \\ e^{2t} & -e^{-2t} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 2e^{2t} + 2e^{-2t} & 4e^{2t} - 4e^{-2t} \\ e^{2t} - e^{-2t} & 2e^{2t} + 2e^{-2t} \end{pmatrix} = \begin{pmatrix} \cosh 2t & 2\sinh 2t \\ \frac{\sinh 2t}{2} & \cosh 2t \end{pmatrix}$$

$$\exp -As \cdot f(s) = \begin{pmatrix} \cosh 2t & -2\sinh 2t \\ -\frac{\sinh 2t}{2} & \cosh 2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \cosh 2t - 6\sinh 2t \\ -\frac{\sinh 2t}{2} + 3\cosh 2t \end{pmatrix}$$

$$\int \text{——} = \frac{1}{2}\begin{pmatrix} \sinh 2t - 6\cosh 2t \\ -\frac{\cosh 2t}{2} + 3\sinh 2t \end{pmatrix}\Bigg|_0^t = \frac{1}{2}\begin{pmatrix} \sinh 2t - 6\cosh 2t + 6 \\ 3\sinh 2t - \frac{\cosh 2t}{2} + \frac{1}{2} \end{pmatrix}$$

$$\exp \cdot \int = \frac{1}{2}\begin{pmatrix} \cosh 2t & 2\sinh 2t \\ \frac{\sinh 2t}{2} & \cosh 2t \end{pmatrix}\begin{pmatrix} 2\alpha + \sinh 2t - 6\cosh 2t + 6 \\ 2\beta + 3\sinh 2t - \frac{\cosh 2t}{2} + \frac{1}{2} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} (2\alpha+3)\cosh 2t + \cancel{\sinh\cosh} - 6\cosh^2 + 2(2\beta + \frac{1}{2})\sinh 2t + 6\sinh^2 - \cancel{3\sinh\cosh} \\ (2\alpha+3)\frac{\sinh 2t}{2} + \frac{\sinh^2}{2} - 3\cancel{\sinh\cosh} + (2\beta+\frac{1}{2})\cosh 2t + 3\sinh\cosh - \frac{\cosh^2}{2} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} (2\alpha+3)\cosh 2t + 2(2\beta + \frac{1}{2})\sinh 2t - 6 \\ (2\alpha+3)\frac{\sinh 2t}{2} + (2\beta+\frac{1}{2})\cosh 2t - 1/2 \end{pmatrix} \qquad t_0 = 0$$

$$\dot{x} = (2\alpha+3)\sinh 2t + 2(2\beta + \tfrac{1}{2})\cosh 2t = 4y + 1 \checkmark$$

$$\dot{y} = (2\alpha+3)\cosh 2t + (2\beta + \tfrac{1}{2})\sinh 2t = x + 3 \checkmark$$

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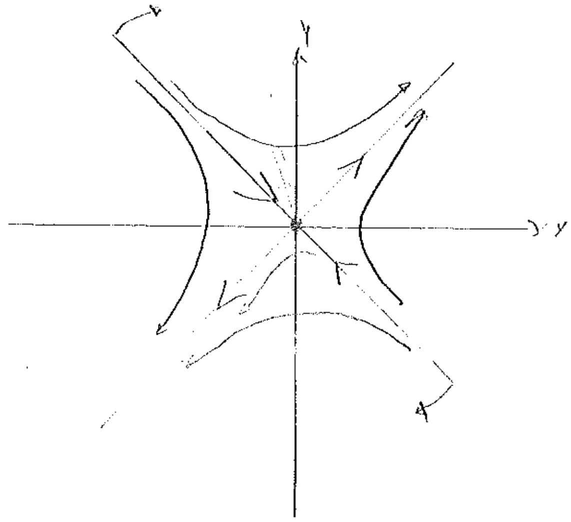
$$\begin{cases} \dot{x} = y \\ \dot{y} = 3x - 2y \end{cases}$$

$$A = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix} \qquad \begin{vmatrix} -\lambda & 1 \\ 3 & -(2+\lambda) \end{vmatrix} = \lambda^2 + 2\lambda - 3 = \lambda^2 - 1 = 0 \to \lambda_1 = 1, \quad \lambda_2 = -3$$

Y_h ... $1 \to \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad -3 \to \begin{pmatrix} 1 \\ -3 \end{pmatrix}$



$$Y_h = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 e^{-3t} \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$\dot{x} = c_1 e^{t} - 3c_2 e^{-3t} = y$ ✓

$\dot{y} = c_1 e^{t} + 9c_2 e^{-3t} = 3x - 2y$ ✓

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$$\begin{cases} \dot{y} = z + \sinh t \cosh t \\ \dot{z} = y \end{cases}$$

$t = \ln 2 \quad y(0) = 1/3 \quad z(0) = 0$

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad A^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad A \cdot A^{-1} = I$ ✓ ; $\quad \lambda^2 - 1 = 0 \to \lambda = \pm 1$

$+1 \to \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix} \leftarrow -1$

$$Y_h = c_1 \cdot e^{t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} \dot{y} = c_1 - c_2 = z \checkmark \\ \dot{z} = c_1 + c_2 = y \checkmark \end{cases}$$

$U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\quad \det U = -2 \quad \text{Adj}\, U^{-1} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}^T = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \to U^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} U$

$\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

$$\exp As = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^{s} & 0 \\ 0 & e^{-s} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{s} & e^{-s} \\ e^{s} & -e^{-s} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{s}+e^{-s} & e^{s}-e^{-s} \\ e^{s}-e^{-s} & e^{s}+e^{-s} \end{pmatrix}$$

$$= \begin{pmatrix} \cosh s & \sinh s \\ \sinh s & \cosh s \end{pmatrix}$$

$\exp -As \cdot \begin{pmatrix} \sinh t \cosh t \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} \cosh & -\sinh \\ -\sinh & \cosh \end{pmatrix} \begin{pmatrix} \sinh s \cosh s \\ 0 \end{pmatrix} = \begin{pmatrix} \sinh s \cosh^2 s \\ -\sinh^2 \cosh s \end{pmatrix}$$

$$\int \rightarrow \frac{1}{3} \begin{pmatrix} \cosh^3 s \\ -\sinh^3 s \end{pmatrix} \Big|_0^t = \frac{1}{3} \begin{pmatrix} \cosh^3 t - 1 \\ -\sinh^3 t \end{pmatrix}$$

$$\frac{1}{3} \begin{pmatrix} \cosh & \sinh \\ \sinh & \cosh \end{pmatrix} \begin{pmatrix} \cosh^3 t - 1 + 3\alpha \\ -\sinh^3 t + 3\beta \end{pmatrix} = \frac{1}{3} \begin{pmatrix} (3\alpha - 1)\cosh t + \cosh^4 t + 3\beta \sinh t - \sinh^4 t \\ (3\alpha - 1)\sinh t + \sinh \cosh^3 t - \cosh \sinh^3 t + 3\beta \cosh t \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} (3\alpha - 1)\cosh t + \cosh^2 t + \sinh^2 t + 3\beta \sinh t \\ (3\alpha - 1)\sinh t + \sinh t \cosh t + 3\beta \cosh t \end{pmatrix}$$

$$\dot{y} = \frac{1}{3}\left((3\alpha - 1)\sinh t + 4 \sinh t \cosh t + 3\beta \cosh t\right) = z + \sinh t \cosh t \checkmark$$

$$\dot{z} = y \checkmark$$

$$\alpha = 1/3 \quad \beta = 0 \quad t = 42 \rightarrow \vec{r} = \begin{pmatrix} \cosh^2 42 + \sinh^2 42 \\ \sinh 42 \cdot \cosh 42 \end{pmatrix} = \begin{pmatrix} 1{,}5 \\ 0{,}75 \end{pmatrix} \cdot 10^{36}$$

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$$\boxed{A = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} \quad \vec{f} = \begin{pmatrix} 15 \\ 4 \end{pmatrix} t e^{-2t} \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 3 & -(1+\lambda) \end{vmatrix} = -(4-\lambda)(1+\lambda) - 6 = 0 = 4 - \lambda + 4\lambda - \lambda^2 + 6$$
$$= -\lambda^2 + 3\lambda + 10 = (\lambda - 5)(\lambda + 2)$$

$\lambda_1 = 5$ $\qquad$ $\lambda_2 = -2$

$\begin{matrix} -1 & 2 \\ 3 & -6 \end{matrix} \rightarrow \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\qquad$ $\begin{matrix} 6 & 2 \\ 3 & 1 \end{matrix} \rightarrow \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$$\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} \rightarrow \det = -7 \qquad d\,U^{-1} = \begin{pmatrix} -3 & -1 \\ -1 & 2 \end{pmatrix}^T = \begin{pmatrix} -3 & -1 \\ -1 & 2 \end{pmatrix} \rightarrow U^{-1} = \frac{1}{7} \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix}$$

$$\frac{1}{7} \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = I \checkmark$$

$$\exp As = \frac{1}{7} \begin{pmatrix} 3 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} e^{5t} & \\ & e^{-2t} \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 3e^{5t} & e^{-2t} \\ e^{5t} & -2e^{-2t} \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} 6e^{5t} + e^{-2t} & 3e^{5t} - 3e^{-2t} \\ 2e^{5t} - 2e^{-2t} & e^{5t} + 6e^{-2t} \end{pmatrix}$$

(11) $\boxed{\begin{aligned}\dot y &= z + e^{-t}\\ \dot z &= y + \operatorname{sech} t\end{aligned}}$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \lambda^2 - 1 = 0 \qquad \lambda = \pm 1$$

$$1 \to \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad -1 \to \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \det U = -2 \qquad d\,U^{-1} = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \qquad U^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \checkmark$$

$$\exp As = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^s & \\ & e^{-s} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^s & e^{-s} \\ e^s & -e^{-s} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} e^s + e^{-s} & e^s - e^{-s} \\ e^s - e^{-s} & e^s + e^{-s} \end{pmatrix} = \begin{pmatrix} \cosh s & \sinh s \\ \sinh s & \cosh s \end{pmatrix}$$

$$\exp As \cdot f(s) = \begin{pmatrix} \cosh s & -\sinh s \\ -\sinh s & \cosh s \end{pmatrix}\begin{pmatrix} e^{-s} \\ \frac{1}{\cosh s} \end{pmatrix} = \begin{pmatrix} e^{-s}\cosh s - \tanh s \\ -e^{-s}\sinh s + 1 \end{pmatrix}$$

$$\int \begin{pmatrix} \frac{1}{2} + \frac{e^{-2s}}{2} - \tanh s \\ \frac{e^{-2s}}{2} - \frac{1}{2} + 1 \end{pmatrix} ds = \begin{pmatrix} \frac{1}{2}s - \frac{e^{-2s}}{4} - \ln\cosh s \\ \frac{1}{2}s - \frac{e^{-2s}}{4} \end{pmatrix}\Bigg|_0^t$$

$$= \begin{pmatrix} \frac{1}{2}t - \frac{e^{-2t}}{4} - \ln\cosh t + \frac{1}{4} \\ \frac{1}{2}t - \frac{e^{-2t}}{4} + \frac{1}{4} \end{pmatrix}$$

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$$\exp \cdot As \cdot g(s) = \frac{1}{7}\begin{pmatrix} 6e^{-5s}+e^{2s} & 3e^{-5s}-3e^{2s} \\ 2e^{-5s}-2e^{2s} & e^{-5s}+6e^{2s} \end{pmatrix}\begin{pmatrix} 15s\,e^{-2s} \\ 4s\,e^{2s} \end{pmatrix}$$

$$= \frac{1}{7}\begin{pmatrix} 90s\,e^{-7s} + 15s + 12s\,e^{-7s} - 12s \\ 30s\,e^{-7s} - 30s + 4s\,e^{-7s} + 24s \end{pmatrix}$$

$$\mapsto \frac{1}{7}\left(\begin{matrix} \cancel{-\frac{20}{7}e^{-7s}} + \cancel{\frac{15}{2}s^2} - \frac{102}{7}s\,e^{-7s} - \frac{102}{49}e^{-7s} - \frac{3}{2}s^2 \\ 3s^2 \quad -\frac{34}{7}s\,e^{-7s} - \frac{34}{49}e^{-7s} \end{matrix}\right)\Bigg|_0^t$$

$$= \frac{1}{7}\begin{pmatrix} -\frac{102}{7}t\cdot e^{-7t} - \frac{102}{49}e^{-7t} - \frac{3}{2}t^2 + \frac{102}{49} \\ 3t^2 - \frac{34}{7}t\,e^{-7t} - \frac{34}{49}e^{-7t} + \frac{34}{49} \end{pmatrix}$$

$$\frac{1}{49}\begin{pmatrix} 6e^{5t}+e^{-2t} & 3e^{5t}-3e^{-2t} \\ 2e^{5t}-2e^{-2t} & e^{5t}+6e^{-2t} \end{pmatrix}\begin{pmatrix} \; \end{pmatrix}$$

$$= \Big(-\frac{6\cdot 102}{7}t\,e^{-2t} - \frac{6\cdot 102}{49}\cdot e^{-2t} - \cancel{\frac{9}{2}t^2 e^{5t}} + \left(\frac{102}{49}+7\right)e^{5t}$$

$$- \cancel{\frac{102}{7}t\,e^{-9t}} - \cancel{\frac{102}{49}e^{-9t}} - \frac{3}{2}t^2 e^{-2t} + \left(\frac{102}{49}+7\right)e^{-2t}$$

$$+ \cancel{9e^{5t}t^2} - \frac{34\cdot 3}{7}e^{-2t} - \frac{34\cdot 3}{49}e^{-2t} + \left(\frac{34}{49}-7\right)\cdot 3e^{5t}$$

$$-9t^2e^{-2t} + \cancel{\frac{3\cdot 34}{7}t\,e^{-9t}} + \cancel{\frac{34\cdot 3}{49}e^{-9t}} - \left(\frac{34\cdot 2}{49}-7\right)\cdot 3e^{-2t}$$

$$-\frac{2\cdot 102}{7}t\,e^{-2t} - \frac{2\cdot 102}{49}e^{-2t} - \cancel{3t^2 e^{5t}} + \left(\frac{102}{49}+7\right)\cdot 2e^{5t}$$

$$+ \cancel{\frac{2\cdot 102}{7}t\,e^{-9t}} + \cancel{\frac{2\cdot 102}{49}e^{-9t}} + 3t^2 e^{-2t} - \left(\frac{102}{49}+7\right)2e^{-2t}$$

$$+ \cancel{3t^2 e^{5t}} - \frac{34}{7}t\,e^{-2t} - \frac{34}{49}e^{-2t} + \left(\frac{34}{49}-7\right)e^{5t}$$

$$+ 18t^2 e^{-2t} - \cancel{6\cdot\frac{34}{7}t\cdot e^{-9t}} - \cancel{\frac{6\cdot 34}{49}e^{-9t}} + \left(\frac{34}{49}-7\right)\cdot 6e^{-2t} \Big)$$

$$\begin{pmatrix}\cosh t & \sinh t\\ \sinh t & \cosh t\end{pmatrix}\begin{pmatrix}\frac{1}{2}t - \frac{e^{-2t}}{4} - \ln\cosh t + \frac{5}{4}\\ \frac{1}{2}t - \frac{e^{-2t}}{4} \qquad + \frac{5}{4}\end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}t\cosh t - \frac{e^{-2t}}{4}\cosh t - \cosh t\ln\cosh t + \frac{5}{4}\cosh t + \frac{1}{2}t\sinh t - \frac{e^{-2t}}{4}\sinh t + \frac{5}{4}\sinh t \\ \frac{1}{2}t\sinh t - \frac{e^{-2t}}{4}\sinh t - (\ln\cosh t)\sinh t + \frac{5}{4}\sinh t + \frac{1}{2}t\cosh t - \frac{e^{-2t}}{4}\cosh t + \frac{5}{4}\cosh t \end{pmatrix}$$

$$\dot{y} \Rightarrow \frac{1}{2}\sinh + \frac{1}{2}\cosh + \frac{e^{-2t}}{2}\cosh - \sinh + \frac{e^{-2t}}{2}\sinh = \frac{3}{2}e^{t} + e^{-t} - \sinh = \frac{e^{t}}{2} + \frac{3}{2}e^{-t}$$

$$= \underbrace{(\sinh + \cosh)}_{e^{t}}\frac{1}{2}\cdot(1+e^{-2t}) = \frac{e^{t}}{2} + \frac{e^{-t}}{2} - \frac{\sinh}{2} + \frac{e^{-t}}{2} = \frac{3}{2}e^{-t} \checkmark$$

$$\dot{z} \to \frac{e^{2t}+1}{2}(\sinh + \cosh) - \frac{\sinh^2 t}{\cosh t} = \frac{\cosh^2 - \sinh^2}{\cosh t} = \frac{1}{\cosh t} \checkmark$$

(12)
$$\boxed{\begin{aligned}\dot{x} &= x + 2y\\ \dot{y} &= 2x + y + 3\end{aligned}}$$

$$A = \begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix} \qquad (1-\lambda)^2 - 4 = 0 \qquad \begin{aligned}\lambda_1 &= 3\\ \lambda_2 &= -1\end{aligned}$$

$$-2 \quad 2 \quad 3 \to \begin{pmatrix}1\\1\end{pmatrix} \qquad 2 \quad 2 \quad -1 \to \begin{pmatrix}1\\-1\end{pmatrix} \qquad U = \begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \qquad U^{-1} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$\exp At = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\cdot\begin{pmatrix}e^{3t} & \\ & e^{-t}\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}e^{3t} & e^{-t}\\ e^{3t} & -e^{-t}\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}e^{3t}+e^{-t} & e^{3t}-e^{-t}\\ e^{3t}-e^{-t} & e^{3t}+e^{-t}\end{pmatrix}$$

$$\exp A(-s)\, f = \frac{1}{2}\begin{pmatrix}e^{-3t}+e^{t} & e^{-3t}-e^{t}\\ e^{-3t}-e^{t} & e^{-3t}+e^{t}\end{pmatrix}\begin{pmatrix}0\\3\end{pmatrix} = \frac{1}{2}\begin{pmatrix}3e^{-3t}-3e^{t}\\ 3e^{-3t}+3e^{t}\end{pmatrix}$$

$$\int \to \frac{1}{2}\begin{pmatrix} -e^{-3t} & -3e^{t} \\ -e^{-3t} & +3e^{t}\end{pmatrix}\Big|_0^t = \frac{1}{2}\begin{pmatrix} -e^{-3t} - 3e^{t} + 4 \\ -e^{-3t} + 3e^{t} - 2\end{pmatrix}$$

$$\frac{1}{4}\begin{pmatrix} e^{+3t} + e^{-t} & e^{+3t} - e^{-t} \\ e^{+3t} - e^{-t} & e^{+3t} + e^{-t}\end{pmatrix}\begin{pmatrix} (4+\cancel{\ }) - e^{-3t} - 3e^{t} \\ (\cancel{2\beta}-2) - e^{-3t} + 3e^{t}\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 4e^{3t} - 1 - \cancel{3e^{4t}} + 4e^{-t} - \cancel{e^{-4t}} - 3 - 1 + \cancel{3e^{4t}} + \cancel{e^{-4t}} - 3 \\ 4e^{3t} - 1 - \cancel{3e^{4t}} - 4e^{-t} + \cancel{e^{-4t}} + 3 - 1 + \cancel{3e^{4t}} - \cancel{e^{-4t}} + 3\end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 4e^{3t} + 4e^{-t} - 8 \\ 4e^{3t} - 4e^{-t} + 4\end{pmatrix} = \begin{pmatrix} e^{3t} + e^{-t} - 2 \\ e^{3t} - e^{-t} + 1\end{pmatrix}$$

$\dot{x} = 3e^{3t} - e^{-t} = \underset{(x+2y)}{3e^{3t}} - e^{-t}$ ✓

$\dot{y} = 3e^{3t} + e^{-t}$ ✓✓

(13)
$$\boxed{\begin{aligned}\dot{y} &= z \\ \dot{z} &= y + 2t\end{aligned}}$$

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}$ $\quad \lambda^2 - 1 \to \lambda = \pm 1$

$\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$

$$\exp As = \begin{pmatrix} \cosh s & \sinh s \\ \sinh s & \cosh s\end{pmatrix}$$

$$\exp -As \cdot f = \begin{pmatrix} \cosh s & -\sinh s \\ -\sinh s & \cosh s\end{pmatrix}\begin{pmatrix} 0 \\ 2s\end{pmatrix} = \begin{pmatrix} -2s\sinh s \\ 2s\cosh s\end{pmatrix}$$

$$\int \to \begin{pmatrix} -2s\cosh s + 2\sinh s \\ +2s\sinh s - 2\cosh s\end{pmatrix}\Big|_0^t = \begin{pmatrix} -2t\cosh t + 2\sinh t \\ +2t\sinh t - 2\cosh t + 2\end{pmatrix}$$

$$\begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t\end{pmatrix}\begin{pmatrix} \alpha + 2\sinh t - 2t\cosh t \\ (\beta+2) - 2\cosh t + 2t\sinh t\end{pmatrix}$$

$$= \begin{pmatrix} \alpha\cosh t + 2\cancel{\sinh t}\cosh t - 2t\cosh^2 t + (\beta+2)\sinh t - \cancel{2\sinh t\cosh t} + 2t\sinh^2 t \\ \alpha\sinh t + 2\sinh^2 t - 2t\cancel{\sinh t}\cosh t + (\beta+2)\cosh t - 2\cosh^2 t + \cancel{2t\sinh t\cosh t}\end{pmatrix}$$

$$= \begin{pmatrix} \alpha\cosh t + \cancel{4\sinh t\cosh t} - 2t\,(\cancel{\cosh^2 t - \sinh^2 t}) + (\beta+2)\sinh t \\ \alpha\sinh t - \cancel{4t\sinh t\cosh t} - 2\,(\cancel{\cosh^2 t - \sinh^2 t}) + (\beta+2)\cosh t\end{pmatrix}$$

$\cancel{2\sinh^2 + 2\cosh^2 - 8t\sinh\cosh}$

$\dot{y} = z$ ✓

$\dot{z} = y + 2t$ ✓

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$$\boxed{\begin{aligned}\dot{y} &= 3z+2\\ \dot{z} &= 3y+4\end{aligned}}$$

$$A = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \qquad \lambda^2 - 9 \to \lambda = \pm 3$$

$$\exp As = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} e^{3s} & 0 \\ 0 & e^{-3s} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{3s} & e^{-3s} \\ e^{3s} & -e^{-3s} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh 3s & \sinh 3s \\ \sinh 3s & \cosh 3s \end{pmatrix} \checkmark \qquad \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$-As \cdot f = \begin{pmatrix} 2\cosh 3s - 4\sinh 3s \\ -2\sinh 3s + 4\cosh 3s \end{pmatrix}$$

$$\int \to \frac{1}{3}\begin{pmatrix} 2\sinh 3s - 4\cosh 3s \\ -2\cosh 3s + 4\sinh 3s \end{pmatrix}\Bigg|_0^t = \frac{1}{3}\begin{pmatrix} 2\sinh 3t - 4\cosh 3t + 4 \\ -2\cosh 3t + 4\sinh 3t + 2 \end{pmatrix}$$

$$+\alpha \quad = \frac{1}{3}\begin{pmatrix} \cosh 3t & \sinh 3t \\ \sinh 3t & \cosh 3t \end{pmatrix}\begin{pmatrix} 2\sinh 3t - 4\cosh 3t + 7 \\ -2\cosh 3t + 4\sinh 3t + 5 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 2\cancel{\sinh}\cos - 4\cos^2 + 7\cos - 2\cancel{\sin}\cos + 4\sin^2 + 5\sin \\ 2\sin^2 - 4\cancel{\sin}\cos + 7\sin - 2\cos^2 + 4\cancel{\sin}\cos + 5\cos \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 7\cosh 3t + 5\sinh 3t - 4 \\ 7\sinh 3t + 5\cosh 3t - 2 \end{pmatrix} \qquad \begin{aligned}\dot{y} &= 3z+2 \checkmark\\ \dot{z} &= 3y+4 \checkmark\end{aligned}$$

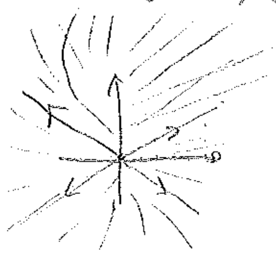
(18) $\boxed{\begin{aligned} \dot{x} &= 6x + 5y \\ \dot{y} &= 2x + 3y \end{aligned}}$

$$A = \begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix} \qquad \begin{vmatrix} (6-\lambda) & 5 \\ 2 & (3-\lambda) \end{vmatrix} = (6-\lambda)(3-\lambda) - 10 = 18 - 9\lambda + \lambda^2 - 10$$

$$= \lambda^2 - 9\lambda + 8$$

$$= (\lambda - 1)(\lambda - 8)$$

$1 \to \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad 8 \to \begin{pmatrix} 5 \\ +2 \end{pmatrix}$



$$\vec{r} = c_1 e^{t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{8t} \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$\dot{x} = c_1 e^{t} + 40 c_2 e^{8t} = 6c_1 + 30c_2 - 5c_1 + 10c_2$ ✓

$\dot{y} = -c_1 + 16c_2 = 2c_1 + 10c_2 - 3c_1 + 6c_2$ ✓

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TEMA 3: Ecuaciones en derivadas parciales *

23 de diciembre de 2008

Resolver las EDP siguientes:

1. //Oteo//

EDP	$u_t = u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0$
CI	$u(x,0) = 1 \qquad (0 < x < 1)$

2. //Oteo//

EDP	$u_t = u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0$
CI	$u(x,0) = x^2 - x \qquad (0 < x < 1)$

3. //Oteo//

EDP	$u_{tt} = u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(L,t) = 0$
CI	$u(x,0) = \sin(3\pi x/L) \qquad (0 < x < L)$ $u_t(x,0) = (3\pi\alpha/L)\sin(3\pi x/L)$

4. //Oteo// Problema de la cuerda de guitarra vibrando

EDP	$u_{tt} = u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0 \qquad (0 < x < 1)$
CI	$u(x,0) = \begin{cases} 2hx & x \leq 1/2 \\ 2h(1-x) & 1/2 < x \leq 1 \end{cases}$ $u_t(x,0) = 0 \qquad h : cte.$

5. //Alejandro [Luis]//

EDP	$u_t = \alpha^2 u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 2$
CI	$u(x,0) = 2x + \sin 3\pi x \qquad (0 < x < 1)$

*Preguntas y soluciones contrastadas por [...]

6. `//Jesús [Fabián]//`

EDP	$u_{tt} = c^2 u_{xx}$
CI	$u(x,0) = \exp(-x^3/5) \qquad (-\infty < x < \infty)$ $u_t(x,0) = x\exp(-x^2) \qquad (0 < t < \infty)$

7. `//Fabián [Jesús]//`

EDP	$u_{tt} = c^2 u_{xx}$
CI	$u(x,0) = \sin^3 x \qquad (-\infty < x < \infty)$ $u_t(x,0) = \cos^2 x \qquad (0 < t < \infty)$

8. `//Fernando [Erica]//`

EDP	$u_{tt} = c^2 u_{xx}$
CI	$u(x,0) = \sin x \qquad (-\infty < x < \infty)$ $u_t(x,0) = x \sin x \cos x \qquad (0 < t < \infty)$

9. `//Pablo [Miguel Angel]//`

EDP	$u_{tt} = c^2 u_{xx}$
CI	$u(x,0) = \exp(-x^2) \qquad (-\infty < x < \infty)$ $u_t(x,0) = 1,\ si\ \lvert x\rvert < 1;\ = 0,\ si\ \lvert x\rvert > 1 \qquad (0 < t < \infty)$

10. `//Miguel Angel [Fernando]//`

EDP	$u_{tt} = c^2 u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0$
CI	$u(x,0) = 0 \qquad (0 < x < 1)$ $u_t(x,0) = \cos(\pi x)$

11. `//Luis [Alejandro]//`

EDP	$u_{tt} = 4u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0$
CI	$u(x,0) = (\sin^3 x)/10 \qquad (0 < x < \pi)$ $u_t(x,0) = 0$

12. `//Erica [Esther]//`

EDP	$u_t = \alpha^2 u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0$
CI	$u(x,0) = \begin{cases} x & x \le 1/2 \\ 1-x & 1/2 < x \le 1 \end{cases} \qquad (0 < x < 1)$

13. `//Carlos [Javier]//`

EDP	$u_t = \alpha^2 u_{xx}$
CC	$u(0,t) = 1 \qquad (0 < t < \infty)$ $u(1,t) = 1/e$
CI	$u(x,0) = \exp(-x) \qquad (0 < x < 1)$

14. `//Erica [Fernando]//`

EDP	$u_{tt} = c^2 u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0 \qquad (0 < x < 1)$
CI	$u_t(x,0) = \begin{cases} x & x \leq 1/2 \\ 1-x & 1/2 < x \leq 1 \end{cases}$ $u(x,0) = -\sin \pi x$

15. `//Fernando [Pablo]//`

EDP	$u_{tt} = c^2 u_{xx}$
CC	$u(0,t) = 0 \qquad (0 < t < \infty)$ $u(1,t) = 0 \qquad (0 < x < 1)$
CI	$u(x,0) = \begin{cases} x & x \leq 1/2 \\ 1-x & 1/2 < x \leq 1 \end{cases}$ $u_t(x,0) = \begin{cases} 1-x & x \leq 1/2 \\ x & 1/2 < x \leq 1 \end{cases}$

(1)

$u_t = u_{xx}$

cc $\begin{cases} u(0,t)=0 \\ u(1,t)=0 \end{cases}$ $0<t<\infty$

ci $u(x,0)=1$ $0<x<1$

$u(x,t) = X(x)\cdot T(t)$

$X(x)\cdot T'(t) = X''(x)\cdot T(t)$

$$\frac{T'(t)}{T(t)} = \frac{X''(x)}{X(x)} = -k^2$$

$X(x) = A\sin kx + B\cos kx$

$T' + k^2 T = 0 \;\rightarrow\; \lambda + k^2 = 0 \; \lambda = -k^2$

$\ln T = -k^2 t + \text{cte} \quad T = A\cdot e^{-k^2 t}$

$u(x,t) = \sum_{n=1}^{\infty} \cancel{(\alpha_n)} \cdot e^{-k_n^2 t} \cdot (A_n \sin k_n x + B_n \cos k_n x)$ $\quad u_t = -k_n^2 \ldots$ $u_{xx} = -k_n^2 \ldots$

$u(0,t)=0 \rightarrow B=0$
$u(1,t)=0 \rightarrow k_n = n\pi$

$u(x,t) = \sum A_n e^{-n^2\pi^2 t} \sin n\pi x$

$u(x,0) = \sum_{n=1}^{\infty} a_n \sin n\pi x = 1 \qquad |\cdot \sin m\pi x$

$$\sum_{n=1}^{\infty} a_n \sin n\pi x \sin m\pi x = \sin m\pi x \qquad |\int dx$$

$$\frac{a_m}{2} = \int_0^1 \sin m\pi x \, dx$$

$$a_m = 2\cdot\frac{\cos m\pi x}{-m\pi}\Big|_0^1 = \frac{2\cdot(-1)^{m+1}}{+m\pi} + \frac{2}{m\pi} = \frac{2}{m\pi}\left((-1)^{m+1}+1\right) = \frac{2}{m\pi} \begin{cases} 0 & m \text{ par} \\ 2 & m \text{ impar} \end{cases}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} \cdot e^{-((2n+1)\pi)^2 t} \cdot \sin(2n+1)\pi x$$

Sumar 9 ... 11

② $\boxed{\phi = x^2 - x}$

$$a_m = 2\int_0^1 \sin m\pi x \cdot (x^2 - x)\,dx$$

$$\int x \sin m\pi x\,dx = -\frac{x\cos m\pi x}{m\pi} + \frac{1}{m\pi}\int \cos m\pi x = -\frac{x\cos m\pi x}{m\pi} + \frac{\sin m\pi x}{(m\pi)^2}\Big|_a^b$$

$u = x \quad dv = \sin m\pi x\,dx$
$du = dx \quad v = -\frac{\cos m\pi x}{m\pi}$

$$\int x^2 \sin m\pi x = -x^2\frac{\cos m\pi x}{m\pi} + \int \frac{2x}{m\pi}\cos m\pi x\,dx$$

$u = x^2$
$du = 2x\,dx$

$$\Rightarrow \int x\cos m\pi x = \frac{x\sin m\pi x}{m\pi} - \int \frac{\sin m\pi x}{m\pi}\,dx$$

$x = u \quad dv = \cos m\pi x$
$dx = du \quad v = \frac{\sin m\pi x}{m\pi}$

$$= \frac{x\sin m\pi x}{m\pi} + \frac{\cos m\pi x}{(m\pi)^2}$$

$$\frac{a_m}{2} = -\frac{x^2\cos m\pi x}{m\pi} + \frac{2x\sin m\pi x}{(m\pi)^2} + \frac{2}{(m\pi)^3}\cdot\cos m\pi x + \frac{x\cos m\pi x}{m\pi} - \frac{\sin m\pi x}{(m\pi)^2}\Big|_a^b$$

$$= \frac{\cos m\pi x}{m\pi}(x - x^2) + \frac{\sin m\pi x}{(m\pi)^2}(2x-1) + \frac{2}{(m\pi)^3}\cos m\pi x\Big|_0^1$$

$$= \frac{(-1)^m\cdot 2}{(m\pi)^3} - \frac{2}{(m\pi)^3} = \frac{2}{(m\pi)^3}\left((-1)^m - 1\right) \begin{cases} 0 & m \text{ par} \\ & m \text{ impar}\end{cases}$$

$$\neq \cancel{\frac{8}{((2k)\pi)^3}} \neq \cancel{\frac{1}{(k\pi)^3}} \quad a_{2k+1} = -\frac{4}{((2k+1)\pi)^3}$$

$$a_{2k+1} = -\frac{8}{((2k+1)\pi)^3}$$

$$u(x,t) = -\sum_{k=0}^{\infty}\frac{8}{((2k+1)\pi)^3}\cdot e^{-((2k+1)\pi)^2 t}\cdot\sin(2k+1)\pi x$$

3

$$\begin{cases} u_{tt} = u_{xx} \\ u(0,t)=0 & 0<t<\infty \\ u(L,t)=0 \\ u(x,0)= \sin\left(\frac{3\pi x}{L}\right) & 0<x<L \\ u_t(x,0) = \frac{3\pi\alpha}{L}\sin\left(\frac{3\pi x}{L}\right) \end{cases}$$

$$u(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\cdot\left[a_n \sin\frac{n\pi\alpha}{L}t + b_n\cos\frac{n\pi\alpha}{L}t\right]$$

$$a_n = \frac{2}{n\pi\alpha}\int_0^L \frac{3\pi\alpha}{L}\sin\frac{3\pi x}{L}\cdot\sin\frac{n\pi x}{L} = \ldots \Rightarrow a_3 = 1$$

$$b_n = \frac{2}{L}\int_0^L \sin\left(\frac{3\pi x}{L}\right)\cdot\sin\frac{n\pi x}{L}\,dx \Rightarrow b_3 = 1$$

$$u(x,t) = \sin\frac{3\pi x}{L}\left(\cos\frac{3\pi\alpha}{L}t + \sin\frac{3\pi\alpha}{L}t\right)$$

4

$$\begin{cases} u_{tt} = u_{xx} \\ u(0,t)=0 \\ u(1,t)=0 \end{cases} \qquad \begin{cases} u(x,0) = \begin{cases} 2hx & x\le 1/2 \\ 2h(1-x) & 1/2 < x \le 1 \end{cases} \\ u_t(x,0) = 0 \end{cases}$$

$$u(x,t) = X(x)\cdot T(t) = (A\sin kt + B\cos kt)(C\sin kx + D\cos kx)$$

$$u(0,t)=0 \rightarrow D=0$$

$$u(1,t)=0 \rightarrow k=n\pi$$

$$u(x,t) = \sum_{n=0}^{\infty}\sin n\pi x\left[a_n\sin n\pi t + b_n\cos n\pi t\right]$$

$$f(x) = u(x,0) = \sum_{n=0}^{\infty} b_n \sin n\pi x$$

$$b_n = 2\int_0^1 f(x)\sin n\pi x\,dx$$

$$g(x) = u_t(x,0) = \sum_{n=0}^{\infty}\sin n\pi x\left[n\pi a_n \cos\cancel{n\pi t} - n\pi \cancel{b_n \sin n\pi t}\right]$$

$$a_n = \frac{2}{n\pi}\int_0^1 g(x)\cdot\sin n\pi x\,dx \rightarrow a_n = 0$$

$$b_n = 2\int_0^{1/2} 2hx\sin n\pi x\,dx + 2\int_{1/2}^1 2h(1-x)\sin n\pi x\,dx$$

$$= 4h\int_0^{1/2} x\sin n\pi x\,dx + 4h\int_{1/2}^1 \sin n\pi x\,dx - 4h\int_{1/2}^1 x\sin n\pi x\,dx$$

$$\frac{4\, b_n}{4h} = \int_0^{1/2} x \sin n\pi x\, dx - \int_{1/2}^{1} x \sin n\pi x\, dx + \int_{1/2}^{1} \sin n\pi x\, dx$$

$u = x \quad dv = \sin$
$du = dx \quad v = -\frac{\cos n}{n\pi}$

$$= -\frac{x \cos n\pi x}{n\pi} + \int \frac{\sin n\pi x}{(n\pi)^2} \Big|_0^{1/2} \Big|_1^{1/2} - \frac{\cos n\pi x}{n\pi}\Big|_{1/2}^{1}$$

$$= \frac{\sin \frac{n\pi}{2}}{(n\pi)^2} + \frac{\sin \frac{n\pi}{2}}{(n\pi)^2} \cancel{\frac{(-1)^n}{n\pi}} - \cancel{\frac{(-1)^n}{n\pi}} + \cancel{\cos \frac{n\pi/2}{n\pi}}$$
$$- \frac{1}{2} \cancel{\cos \frac{n\pi/2}{n\pi}} - \frac{1}{2} \cancel{\cos \frac{n\pi/2}{n\pi}}$$

$$b_n = \frac{8h \sin \frac{n\pi}{2}}{(n\pi)^2} \to b_{2k+1} = \frac{8h \sin (2k+1)\frac{\pi}{2}}{\left((2k+1)\frac{\pi}{\cancel{2}}\right)^2}$$

$$u(x,t) = \sum_{k=0}^{\infty} \sin(2k+1)\pi x \cdot \frac{8h \sin (2k+1)\pi/2}{[(2k+1)\pi]^2} \cdot \cos (2k+1)\pi t$$

---

5) $\boxed{u_t = \alpha^2 u_{xx}} \quad u(0,t) = 0 \quad u(1,t) = 2 \quad u(x,0) = 2x + \sin 3\pi x$

$\frac{T'(t)}{\alpha^2 T} = \cancel{\frac{X''(x)}{X(x)}} = -\lambda^2 \to X(x)$

$X(0) = A \sin 0 + B \cos = 0$

$u(x,t) = 2x + v(x,t)$

$v_t = u_t$
$v_{xx} = u_{xx} \to v_t = \alpha^2 v_{xx}$

$u(0,t) = v(0,t) = 0$
$u(1,t) = 2 + v(1,t) = 2 \to v(1,t) = 0$

$v(x,t) = \sum a_n \cdot e^{-(n\pi\alpha)^2 t} \sin n\pi x$

$v(x,0) = u(x,0) - 2x = \sin 3\pi x = \sum a_n \sin n\pi x \quad \to a_3 = 1$

$\to v(x,t) = e^{-(3\pi\alpha)^2 t} \cdot \sin 3\pi x$

$u(x,t) = 2x + e^{-(3\pi\alpha)^2} \sin 3\pi x$

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(6) $u_{tt} = c^2 u_{xx} \quad u(x,0) = e^{-x^3/5} \quad u_t(x,0) = x e^{-x^2}$

$$u(x,t) = \frac{1}{2}\left[e^{-\frac{(x+ct)^3}{5}} + e^{-\frac{(x-ct)^3}{5}} \right] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(z)\,dz$$

$$\int_0^t z e^{-z^2} dz = -\frac{1}{2} e^{-z^2}\Big|_{x-ct}^{x+ct} = -\frac{1}{2}\dots$$

$$u(x,t) = \frac{1}{2}\left[e^{-\frac{(x+ct)^3}{5}} + e^{-\frac{(x-ct)^3}{5}} \right] + \frac{1}{4c}\left[e^{-(x-ct)^2} - e^{-(x+ct)^2} \right]$$

(7)

$u_{tt} = c^2 u_{xx}$

$u(x,0) = \sin^3 x$

$u_t(x,0) = \cos^2 x$

$$\int_{x-ct}^{x+ct} \cos^2 z\,dz = \int_0^t \frac{1}{2}(1+\cos 2z)\,dz = \frac{1}{2}z + \frac{1}{4}\sin 2z \Big|_{x-ct}^{x+ct}$$

$$= \frac{1}{2}(x+ct) - \frac{1}{2}(x-ct) + \frac{1}{4}\left(\sin(2(x+ct)) - \sin(2(x-ct))\right)$$

$$u(x,t) = \frac{1}{2}\left(\sin^3(x+ct) + \sin^3(x-ct)\right) + \frac{t}{2} + \frac{1}{8c}\left(\sin 2(x+ct) - \sin(2(x-ct))\right)$$

(8)

$\sin^2 x + \cos^2 x = 1$

$\cos 2x = \cos^2 x - \sin^2 x$

$u_{tt} = c^2 u_{xx}$

$u(x,0) = \sin x$

$u_t(x,0) = x \sin x \cos x$

$$\int x \sin x \cos x\,dx = \frac{1}{2}x\sin^2 x - \int \frac{\sin^2 x}{2}dx = \frac{1}{2}x\sin^2 x - \frac{1}{4}\int(1-\cos 2x)\,dx$$

$$= \frac{1}{2}x\sin^2 x - \frac{1}{4}x + \frac{1}{8}\sin 2x \Big|_{x-ct}^{x+ct} = \frac{1}{2}x\dots - \frac{1}{4}(x+ct) + \frac{1}{4}(x-ct)\dots$$

$$u(x,t) = \frac{1}{2}\left(\sin(x-ct) + \sin(x+ct)\right) + \frac{1}{2c}\left[\frac{1}{2}\left((x+ct)\sin^2(x+ct) - (x-ct)\sin^2(x-ct)\right) + \frac{1}{8}\left(\sin 2(x+ct) - \sin 2(x-ct)\right)\right] - \frac{t}{4}$$

$\sin 2x + 2ct = \sin 2x \cos 2ct + \sin 2ct \cos 2x$

$-\sin 2x - 2ct = \underline{\sin 2x \cos 2ct} - \sin 2ct \cos 2x$

$2 \sin 2ct \cos 2x$

(9)

$u_{tt} = c^2 u_{xx}$

$u(x,0) = e^{-x^2}$

$u_t(x,0) = 1 \quad \text{si } |x| < 1$

$\quad = 0 \quad \text{si } |x| > 1$

$$\int_{x-ct}^{x+ct} g(x)\, dx = \int_{x-ct}^{x} 0\, dx + \int_{x}^{x+ct} 0\, dx + \int_{-x}^{x} dz = z\Big|_{-x}^{x} = 2x$$

$$u(x,t) = \frac{1}{2}\left[e^{-(x-ct)^2} + e^{-(x+ct)^2}\right] + \frac{x}{c}$$

(10)

$$\begin{cases} u_{tt} = c^2 u_{xx} \\ u(0,t) = 0 \\ u(L,t) = 0 \\ u(x,0) = 0 \\ u_t(x,0) = \cos \pi x \end{cases}$$

$b_n = 0$

$$\left(a_n = \frac{2}{n\pi\alpha} \int^{L} \cos \pi x \cdot \sin \frac{n\pi x}{L}\, dx = \right)$$

$$u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right) \cdot \left[a_n \sin \frac{n\pi\alpha t}{L} + b_n \left(\cos \frac{n\pi\alpha t}{L}\right)\right]$$

$$u_t = \sum_{n=1}^{\infty} \sin n\pi x \cdot \left[a_n n\pi\alpha \cos n\pi\alpha t - \cancel{b_n}^{0} \sin n\pi\alpha t\right]$$

$$u_t(x,0) = \sum_{n=1}^{\infty} \sin n\pi x \; a_n n\pi\alpha = \cos \pi x$$

$$I = \int \cos \pi x \cdot \sin n\pi x\, dx = \frac{\sin n\pi x \cdot \sin \pi x}{\pi} - \int n \frac{\sin \pi x}{\pi} \cos n\pi x\, dx$$

$u = \sin \qquad dv = \cos \pi x\, dx$

$du = n\pi \cos \qquad v = \frac{\sin \pi x}{\pi}$

$u = \cos n\pi x \qquad dv = \sin \pi x\, dx$

$du = -n\pi \sin n\pi x \qquad v = \frac{n}{\pi} \cos \pi x$

$$\ldots + \frac{n}{\pi} \cos \pi x \cos n\pi x + \int \underbrace{n^2 \sin n\pi x \cos \pi x}_{n^2 I}$$

$$I(1-n^2) = \frac{\sin n\pi x \sin \pi x}{\pi} + \frac{n}{\pi}\cos \pi x \cos n\pi x$$

$$a_n = \frac{2}{n\pi\alpha}\cdot\frac{1}{(1-n^2)}\cdot\left[\frac{\sin n\pi x \sin \pi x}{\pi} + \frac{n}{\pi}\cos\pi x \cos n\pi x\right]_0^1$$

$$= \frac{2}{n\pi\alpha}\cdot\frac{1}{1-n^2}\left[0 - \frac{n}{\pi}\cos n\pi - \frac{n}{\pi}\right]$$

$$= -\frac{2}{\pi^2\alpha}\frac{1}{(1-n^2)}\cdot(\cos n\pi + 1) = \frac{2}{\pi^2\alpha}\cdot\frac{1}{(1-n^2)}\left(1 + (-1)^n\right)$$

$$a_{2k} = \frac{4}{\pi^2\alpha}\cdot\frac{1}{1-4k^2}$$

$$u(x,t) = \sum_{k=1}^{\infty} \sin 2k\pi x \cdot \frac{4}{\pi^2\alpha}\frac{1}{1-4k^2}\cdot \sin 2k\pi\alpha t$$

11

$u_{tt} = 4u_{xx}$

$u(0,t)=0$, $u(\pi,t)=0$ $\qquad$ $u(x,0) = \frac{\sin^3 x}{10}$, $u_t(x,0)=0$

$$XT'' = 4X''T$$

$$\frac{T''}{4T} = \frac{X''}{X} = -\lambda^2$$

$$u(x,t) = \sum_{n=1}^{\infty}(A\sin 2\lambda_n t + B\cos 2\lambda_n t)(C\sin\lambda_n x + D\cos\lambda_n x)$$

$u(0,t)=0 \rightarrow D=0$

$$u(\pi,t) = 0 = \sum_{n=1}^{\infty}\sin \lambda_n\pi\cdot(a_n \sin 2\lambda_n t + b_n\cos 2\lambda_n t) = 0$$

$\rightarrow \lambda_n\pi = n\pi \rightarrow \lambda_n = n$

$$= \sum_{n=1}^{\infty}\sin nx\cdot(a_n\sin 2nt + b_n\cos 2nt)$$

$$u_t(x,0) = \sum_{n=1}^{\infty}\sin nx(2na_n\cos - 2nb_n \sin) = 0 \Rightarrow a_n = 0$$

$$u(x,0) = \sum_{n=1}^{\infty}\sin nx\cdot b_n = \frac{\sin^3 x}{10} = \frac{1}{20}\sin x - \frac{1}{20}\sin x\cos 2x$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}\frac{\sin^3 x}{10}\cdot \sin nx\, dx$$

$\int \sin^3 x \sin nx = -\cancel{\sin^3}\frac{\cos}{n}\Big|_0^\pi + \int_0^\pi 3\sin^2 \frac{\cos}{n}\cdot\cos$ $\qquad \sin^3 x = \frac{1}{2}\sin x(1-\cos 2x)$

$u = \sin^3 \quad dv = \sin nx$

$du = 3\sin^2 \cos^0 = -\frac{\cos nx}{n}$ $\qquad u = 3\sin^2 \quad du =$ $\qquad = \frac{1}{2}\sin x - \frac{1}{2 \sin x}\cos 2x$

$\int \frac{3}{n}\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)\cos^{2x}_{}nx = \cancel{\frac{3}{2n^2}\sin nx\Big|_0^\pi} - \frac{3}{2}\int_0^\pi \cos 2x \cdot \cos nx$

$\int_0^\pi (\sin x - \overset{\sin x}{\cos} 2x)\cdot \sin nx \; dx =$ $\qquad \frac{1}{2}$

$\int \sin nx \underset{\sin x}{\cos} 2x = +\sin nx \frac{\sin 2x}{2n} - \int n\cos nx \frac{\sin 2x}{2}dx$

$u = -n\cos n \quad dv = \frac{\sin 2x}{2}dx$

$du = n^2 \sin n \quad v = -\frac{\cos 2x}{4}$

$= \ldots + \frac{n}{4}\cos nx \cos 2x + \int \frac{n^2}{4}\sin nx \cos 2x \, dx$

$I\left(1 - \frac{n^2}{4}\right) = \frac{n}{4}(\cos n\pi - 1)$

$b_n = \frac{1}{10\pi}\cdot\left(\frac{n}{4}(\cos n\pi - 1) + \int_0^\pi \sin x \sin nx \, dx\right) \qquad b_1 = \frac{1}{5\pi}\left(\frac{1}{2} + \frac{\pi}{2}\right)$

$\int \sin x \sin nx \, dx \qquad \int \sin x \cos 2x \sin nx \, dx$

$\Rightarrow n = 1 \Rightarrow \pi/2 \qquad n=1$

$\Rightarrow$

$u = \sin$

$\frac{2}{10\pi}\int \sin^3 x \sin nx \, dx$

$\sin^3 x = \frac{1}{2}\sin x - \frac{1}{2}\sin 3x + \frac{1}{2}\sin 2x \cos 2x + \sin x \cos^2 x$

$\Rightarrow \sin 3x = \sin 2x \cos x + \cos 2x \sin x$

$\sin x = \sin 2x \cos x - \sin x \cos 2x$

$\sin 3x + \sin x = 2\cos x \sin 2x$

$\sin^3 x = \frac{1}{2}\sin x - \frac{1}{2}\sin 3x + \frac{1}{4}\sin 3x + \frac{1}{4}\sin x$

$= \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$

$b_1 = \frac{3}{40} \quad b_3 = -\frac{1}{40}$

$u(x,t) = \frac{3}{40}\sin x \cdot \cos 2t - \frac{1}{40}\sin 3x \cos 6t$

(12) $u_t = \alpha^2 u_{xx}$

$$\begin{cases} u(0,t)=0 \\ u(1,t)=0 \end{cases}$$

$$u(x,0) = \begin{cases} x & x \le 1/2 \\ 1-x & 1/2 < x \le 1 \end{cases}$$

$$XT' = \alpha^2 X''T$$

$$\frac{T'}{\alpha^2 T} = \frac{X''}{X} = -\lambda^2$$

$$T = C \cdot e^{-(\lambda\alpha)^2 t}$$

$$X = A \sin \lambda x + B \cos \lambda x$$

$$X(0)=0 \to B=0$$

$$X(1)=0 \to \lambda = n\pi$$

$$u(x,t) = \sum_{n=1}^{\infty} a_n \cdot e^{-(n\pi\alpha)^2 t} \sin n\pi x$$

$$u(x,0) = \sum_{n=1}^{\infty} a_n \sin n\pi x = \big\{$$

$$a_n = 2\int_0^1 \sin n\pi x \cdot \ldots = 2\int_0^{1/2} x \sin n\pi x \, dx + 2\int_{1/2}^1 (1-x) \sin n\pi x$$

$$= 2\left(-\frac{x\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\right)\Bigg|_0^{1/2} + 2\left[-\frac{\cos n\pi x}{n\pi}\right]_{1/2}^1 - 2\left[-\frac{x\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\right]_{1/2}^1$$

$$= 2\left(-\frac{1}{2n\pi}\cos n\pi/2 + \frac{\sin n\pi/2}{(n\pi)^2}\right) + 2\left[\frac{-\cos n\pi + \cos\frac{n\pi}{2}}{n\pi}\right] - 2\left[-\frac{\cos n\pi}{n\pi} + \frac{1}{2n\pi}\cos\frac{n\pi}{2} - \frac{\sin n\pi/2}{(n\pi)^2}\right]$$

$$= \cancel{-\frac{2\cos n\pi/2}{n\pi}} + 4\frac{\sin n\pi/2}{(n\pi)^2} - \frac{2\cos n\pi}{n\pi} + \frac{2\cos n\pi/2}{n\pi} + \cancel{\frac{2\cos n\pi}{n\pi}}$$

$$a_n = 4\frac{\sin n\pi/2}{(n\pi)^2} \to a_{2k+1} = \frac{4(-1)^k}{((2k+1)\pi)^2}$$

(13)

$$\begin{cases} u_t = \alpha^2 u_{xx} \\ u(0,t) = 1 \\ u(1,t) = 1/e \\ u(x,0) = e^{-x} \end{cases}$$

$$u(x,t) = \frac{1/e - 1}{1} \cdot x + 1 + v(x,t)$$

$$u(0,t) = 1 = 1 + v(0,t) \rightarrow v(0,t) = 0 \checkmark$$

$$u(1,t) = 1/e - 1 + 1 + v(1,t) = 1/e \rightarrow v(1,t) = 0 \checkmark$$

$$u(x,0) = \frac{1/e - 1}{1} x + 1 + v(x,0)$$

$$v_t = \alpha^2 v_{xx}$$
$$v(0,t) = 0$$
$$v(1,t) = 0$$

$$v = \sum_{n=1}^{\infty} a_n \cdot e^{-(n\pi\alpha)^2 t} \sin n\pi x$$

$$v(x,0) = e^{-x} - \frac{1/e - 1}{1} x - 1$$

$$a_n = 2\int_0^1 \left(e^{-x} + x - \frac{x}{e} - 1\right) \sin n\pi x \, dx \Rightarrow$$

$$\int e^{-x} \sin n\pi x \, dx = -e^{-x} \sin n\pi x + n\pi \int e^{-x} \cos n\pi x \, dx$$

$u = \sin n\pi x \quad dv = e^{-x}$ $\qquad u = n\pi \cos n\pi x \quad dv = e^{-x}$

$du = n\pi \cos n\pi x \quad v = -e^{-x}$ $\qquad du = -(n\pi)^2 \sin n\pi x \quad v = -e^{-x}$

$$I = -e^{-x} \sin n\pi x - n\pi \cos n\pi x \, e^{-x} - \int (n\pi)^2 \sin n\pi x \, e^{-x}$$

$$I\left(1 + (n\pi)^2\right) = -e^{-x} \sin n\pi x - n\pi \cos n\pi x \, e^{-x} \Big|_0^1$$

$$I = \frac{-n\pi \cos n\pi \, e^{-1} + n\pi}{1 + (n\pi)^2}$$

$$\int x \sin n\pi x \, dx = -\frac{x \cos n\pi x}{n\pi} + \int \frac{\cos n\pi x}{n\pi} dx = -\frac{x \cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\Big|_0^1$$

$u = x \quad dv = \sin n\pi x$

$du = dx \quad v = \frac{\cos n\pi x}{-n\pi}$

$$= -\frac{\cos n\pi}{n\pi}$$

$$\int \sin n\pi x = -\frac{\cos n\pi x}{(n\pi)}\Big|_0^1 = \frac{-\cos n\pi + 1}{n\pi}$$

$$a_n = 2 \cdot \left[\frac{-n\pi \cos n\pi \cdot 1/e + n\pi}{1 + (n\pi)^2} + \left(1 - \frac{1}{e}\right) \cdot \left(-\frac{\cos n\pi}{n\pi}\right) + \frac{1 - \cos n\pi}{n\pi}\right]$$

$$= 2 \cdot \left[\frac{-n\pi (-1)^n 1/e + n\pi}{1 + (n\pi)^2} - \frac{2}{n\pi}(-1)^n + \frac{1}{n\pi e}(-1)^n + \frac{1}{n\pi}\right]$$

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(14)

$$\cancel{a_k = \frac{2}{n\pi c}\int_0^1 (x \sin n\pi x)}\cdot \qquad u_{tt} = c^2 u_{xx} \qquad u_t(x,0) = \begin{cases} x & x \le 1/2 \\ 1-x & 1/2 < x \le 1 \end{cases}$$

$$u(0,t) = 0 \qquad u(x,0) = -\sin \pi x$$

$$u(1,t) = 0$$

$$T'' X = c^2 T X'' \quad \rightarrow \quad X = C \sin \lambda x + D \cos \lambda x$$

$$\frac{T''}{c^2 T} = \frac{X''}{X} = -\lambda^2 \qquad X(0) = 0 \rightarrow D = 0$$

$$X(1) = 0 \rightarrow \lambda = n\pi$$

$$T = A \sin c\lambda t + B \cos c\lambda t$$

$$u(x,t) = \sum_{n=1}^{\infty} \sin n\pi x \cdot (a_n \sin n\pi c t + b_n \cos n\pi c t)$$

$$u(x,0) = \sum_{n=1}^{\infty} \sin n\pi x \cdot b_n = -\sin \pi x \rightarrow b_1 = -1$$

$$u_t(x,0) = \sum_{n=1}^{\infty} \sin n\pi x \cdot n\pi a_n$$

$$a_n = \frac{2}{n\pi c}\int_0^1 \dots = \frac{2}{n\pi c}\cdot\left(\frac{-\cos n\pi}{\cancel{n\pi}} + \cancel{\cos\frac{n\pi}{2}} - \frac{1}{2}\frac{\cancel{\cos n\pi/2}}{n\pi} + \frac{2\sin\frac{n\pi}{2}}{(n\pi)^2} + \frac{\cancel{\cos n\pi}}{n\pi}\right)$$

$$\int x \sin n\pi x \, dx = -x \frac{\cos n\pi x}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2} \Big|_0^{1/2} \Big|_1^{1/2}$$

$$\int \sin n\pi x = -\frac{\cos n\pi x}{n\pi} \Big|_{1/2}^{1}$$

$$a_n = \frac{4}{n\pi c}\cdot\frac{\sin n\pi/2}{(n\pi)^2} \rightarrow a_{2k} = 0 \qquad a_{2k+1} = \frac{4\cdot(-1)^k}{((2k+1)\pi)^3 c}$$

(18) $u_{tt} = c^2 u_{xx}$

$u(0,t) = 0$

$u(1,t) = 0$

$u(x,0) = \begin{cases} x & x \leq 1/2 \\ 1-x & \frac{1}{2} < x \leq 1 \end{cases}$

$u_t(x,0) = \begin{cases} 1-x & x \leq 1/2 \\ x & \frac{1}{2} < x \leq 1 \end{cases}$

$$b_{2k+1} = \frac{4}{((2k+1)\pi)^2}(-1)^k$$

$$a_n = \frac{2}{n\pi c}\left[\int_0^{1/2} (1-x)\sin n\pi x \, dx + \int_{1/2}^{1} x \sin n\pi x \, dx\right]$$

$$= \frac{2}{n\pi c}\cdot\left[\cancel{-\frac{\cos n\pi/2}{n\pi}} + \frac{1}{n\pi} + \cancel{\frac{3}{2}\frac{\cos n\pi/2}{(n\pi)}} - 2\frac{\sin n\pi/2}{(n\pi)^2} + \cancel{\frac{\sin n\pi}{(n\pi)^2}} - \frac{\cos n\pi}{n\pi}\right]$$

$$= \frac{2}{n\pi c}\left[\frac{1-(-1)^n}{n\pi} - \frac{2\sin n\pi/2}{(n\pi)^2}\right] \qquad a_{2k} = 0$$

$$a_{2k+1} = \frac{2}{(2k+1)\pi c}\left[\frac{2}{n\pi} - \frac{2}{(n\pi)^2}\right] = \frac{4}{((2k+1)\pi)^2 c}\left[1 - \frac{1}{n\pi}\right]$$