$\hat{\tilde{N}}$? Picl?

extras ← C.C. H, B, E, D
$\phi, \vec{E}, \vec{A}, \vec{B}, \vec{H}, \vec{M}, \vec{K}$
$\vec{P}$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\vec{R}}{R^3}\,dV$$

Circuito
Imágenes
Guía

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \frac{d\vec{p}}{dV} \quad \rho_p = -\vec{\nabla}\cdot\vec{P}$$

T. Maxwell
ec. Helmholtz

$\vec{F}$ dipolo q
cond.

$$\vec{p} = \int \vec{P}_1\,dV$$

$$\vec{F}_{2\to1} = -(\vec{p}_1\vec{\nabla})\vec{E}_2$$

$$= (\vec{p}_2\vec{\nabla})\vec{E}_1$$

$$\int \frac{\sigma^2}{2\varepsilon_0}\,d\vec{s}$$

$$= \int \frac{E_{sup}}{2}\cdot dq$$

$$\vec{M} = (\mu_r - 1)\vec{H} \qquad \vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$$

$$\vec{J}_M = \vec{\nabla}\times\vec{M} \qquad \rho_p = -\vec{\nabla}\cdot\vec{P}$$

$$K_m = \vec{M}\times\vec{n} \qquad \sigma_p = \vec{P}\cdot\vec{n}$$

$$\rho_m = -\vec{\nabla}\vec{M}$$
$$\sigma_m = \vec{M}\cdot\vec{n}$$

posibles teoría:
cuest.

- ϕ_p esfera ✓
- unicidad ✓
- Green?
- imág ∞ Green
- + imág. ✓
- energía stma.

$$\int_V \vec{\nabla}\cdot\vec{E}\,dV = \int_S \vec{E}\cdot d\vec{S}$$

45 14 10 / 18 / 24 ...

$$\int \vec{\nabla}\times\vec{E}\cdot d\vec{S} = \int \vec{E}\cdot d\vec{l}$$

- cuasiest.
- corr. fouc.
- cir. estac.
- RC ✓
- leyes Kirch ← cuasiest.
- Poynting circuitos
- Máx. potencia ✓

$$\vec{\nabla}\left(\frac{\vec{R}}{R^3}\right) = 4\pi\delta(\vec{R})$$

$$\frac{\vec{R}}{R^3} = -\vec{\nabla}\left(\frac{1}{R}\right)$$

$$\frac{dp}{dt} = -\frac{\partial}{\partial t}\int \frac{\vec{N}}{c^2}\,dV + \int T\cdot d\vec{s}$$

- OPM Eq ✓
- Helmholtz ✓
- Fresnel ✓
- medios cond.
- $Z = \frac{\varepsilon_0}{\sigma}$ ✓
- $Z = 1 - j$ ✓
- polarizabilidad ✓
- guía
- $\lambda \leftarrow \frac{v_g}{v_p}$ – ✓ vel.

dieltr. esférica
c. radial (intervalo.)
potencias dep
$U(\rho,\phi) \to U(\vec{E},\vec{D})$ ✓
$\frac{1}{2}$. esferas, V

$$\phi = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{p}\cdot\vec{R}}{R^3}\,dV$$

TEORÍA DEL POTENCIAL

$\Delta\phi = 0 \rightarrow$ funciones armónicas sin max-min $\rightarrow \phi_P = \frac{1}{S}\int_S \phi(\vec{r})\cdot dS$

- Condición de Dirichlet $\rightarrow \phi = \phi_s$ en $P \in S$, (2) $(\Delta\phi = 0)$, (2) $\Delta\phi = -\rho/\varepsilon_0 \rightarrow \phi$ unívoco
- Conductores (Q_i conocida, $\rho(\vec{r})$ entre ellos ✓) $\rightarrow \vec{E}$ unívoco (3)
- Condición de Neumann, $\rho(\vec{r})$ conocida, $\frac{\partial\phi}{\partial n}$ en S conocida (4) $\rightarrow \vec{E}$ unív.

Método de Green

$G(\vec{r},\vec{r}') = -\frac{1}{4\pi R} + F(\vec{r},\vec{r}')$ (caso c. puntual) ϕ_1, ϕ_2 / $\phi_1 \Delta\phi_2 dV'$

- Dirichlet $\rightarrow \phi = \int_V -\frac{\rho}{\varepsilon_0} G dV' + \int \phi \frac{\partial G}{\partial n}\cdot dS'$ ($G = 0$ en S)
- Neumann $\rightarrow \phi = \int -\rho/\varepsilon_0\, G dV' + \underset{\text{cte}}{\langle\phi\rangle_S} - \int_S G \frac{\partial\phi}{\partial n} dS'$

Método imágenes $G = -\frac{1}{4\pi R} + F \rightarrow \phi = \phi_p + \phi_i \rightarrow$ buscas ρ_i / $\phi_i = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho_i dV'}{R}$

- Plano conductor $\phi_s = 0$ (Dirichl) $\rightarrow q_i = -q$, $d_i = d$ (solución en espacio real $z>0$)
- " (Neumann) $\frac{\partial\phi}{\partial n} = 0 \rightarrow q_i = q$, $d_i = d$
- Esfera ($\phi_s = 0$, D) $\rightarrow q_i = -\frac{a}{d}q$, $d_i = \frac{a^2}{d}$ tanto fuera como dentro

$\phi_s = V$

$\rightarrow q_i'$ en centro esfera / $q_i' = 4\pi\varepsilon_0 a V$ si q exterior

$\rightarrow \sigma_i'$ en corteza / $\sigma_i' = \frac{\varepsilon_0 V}{a}$ si q interior

- Cilindro ($\phi_s = 0$, D) $\rightarrow \lambda_i(\lambda)$, $d_i(d)$

$\lambda_i = -\lambda$ $d_i = \frac{a^2}{d}$

Separa de variables $\Delta\phi = 0$

- Simetría plana ↳ Cartesianas $\underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2}}_{\alpha^2} + \underbrace{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{\beta^2} + \underbrace{\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}}_{\gamma^2} = 0$

$\alpha^2 + \beta^2 + \gamma^2 = 0 \nearrow \infty$

$X = a e^{\alpha x} + b e^{-\alpha x}$
$= a' \sinh(\alpha x) + b' \cosh(\alpha x)$
$= a'' \operatorname{sen} kx + b'' \cos kx$ ↳ $\alpha = jk$ — periódicas

$\rightarrow$ C. C.

$\rightarrow \vec{v} = A_n \cdot \vec{u_n} \rightarrow \frac{\vec{u_m}\cdot\vec{v}}{\vec{u_m}\cdot\vec{u_m}} = A_n$ $\int_0^a \operatorname{sen} k_m x \operatorname{sen} k_n x = \frac{a}{2}\delta_{nm}$

- Simetría cilíndrica ↳ cilíndricas $\Delta\phi = 0$, $\phi = R\Phi$ $\frac{r}{R}\frac{\partial}{\partial r}\left(r\frac{dR}{dr}\right) + \underbrace{\frac{1}{\Phi}\frac{d^2\Phi}{d\varphi^2}}_{-k\varphi^2}$

$\Phi = A\cos p\varphi + B\operatorname{sen} p\varphi$ $p = 1, \ldots$
$R = C r^p + D r^{-p}$

$\int_0^{2\pi} \operatorname{sen}(m\varphi)\operatorname{sen}(n\varphi)\, d\varphi = \pi\delta_{nm}$ $m, n = 1, 2, \ldots$

- Simetría esférica $\phi = \frac{R(r)}{r}\Theta(\theta)\Omega(\varphi)$

$\left.\begin{array}{l}\Omega(\varphi) = A\operatorname{sen} m\varphi + B\cos m\varphi \\ \Theta(\theta) = P_n^m(\mu)\end{array}\right\} Y_{nm}(\theta,\varphi)$

+ simetría revolución (azimutal)

$\phi = \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}}\right) P_n(\cos\theta)$ $\int_0^{\pi} P_n(\cos\theta) P_m(\cos\theta)\operatorname{sen}\theta\, d\theta = \frac{2}{2n+1}\delta_{nm}$

$P_0(x) = 1$ $P_1 = x$ $P_2 = (3x^2 - 1)/2$

ENERGÍA EN CAMPOS ESTACIONARIOS

$$-\frac{\partial u}{\partial t} = \int_V \vec{J}\cdot\vec{E}\cdot dV + \int \underbrace{\vec{N}}_{\vec{E}\times\vec{H}}\cdot d\vec{S}$$

cargas quietas
corrientes estacionarias

$$u = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\cdot dV + \frac{1}{2}\int \vec{B}\cdot\vec{H}\cdot dV$$

- Energía electrostática

$W_{el} = \int q\cdot\vec{E}\cdot d\vec{l}$ $\qquad T = \int_A^B \vec{F}_{mec.}\cdot d\vec{l} = -q\int \vec{E}\, d\vec{l} = q(V_B - V_A) = W_B - W_A$

$W_i = q_i \cdot V(r_i)$

× N cargas → $W = \frac{1}{2}\sum_{i=1}^{N} q_i V(r_i)$, $V(r_i) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{1}{4\pi\varepsilon_0}\frac{q_j}{R_{ji}}$

- no energía de creación, sólo interacción
- nulo xa 1 carga, > ó < 0 en general

× Distribución

$W = \frac{1}{2}\int_V \rho(\vec{r}\,')V(\vec{r}\,')\,dV$ $\qquad$ En N conductores equip.: $\frac{1}{2}\sum^{N} V_i Q_i$

Lo contiene autoenergía, siempre >0

Condensador: $W = \frac{1}{2}V\cdot Q = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$ $\qquad (Q = CV)$

$W = \frac{\varepsilon_0}{2}\int_V \vec{E}\cdot\vec{E}\,dV$ $\qquad w = \frac{\varepsilon_0}{2}\vec{E}^2$ $\qquad (\vec{\nabla}\vec{E} = \frac{\rho + \rho_p}{\varepsilon}$, $\vec{\nabla}(\varepsilon_0\vec{E} + \vec{P}) = \rho_{reales})$

torque $\vec{\tau} = \vec{p}\times\vec{E}$

Medios lineales e isótropos:

$W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\cdot dV$ $\qquad w = \frac{1}{2}\vec{D}\cdot\vec{E}$

- Energía magnética

$W = \sum_{n=1}^{N}\int i_n\, d\phi_n$ $\qquad$ lineal → $W = \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} M_{kj} I_j I_k$

1 circuito: $W = \frac{1}{2}LI^2$ $\qquad$ E de formación de N circuitos
$P_E = RI^2$

$$W = \frac{1}{2}\int_V \vec{J}\cdot\vec{A}\,dV = \frac{1}{2}\int_{\mathbb{R}^3} \vec{B}\cdot\vec{H}\,dV$$

$W_{\substack{\text{imanación} \\ \text{medio}}} = \frac{1}{2}\int_V \vec{M}\cdot\vec{B}\,dV$

T. 7

Soluciones cuasiestacionarias

$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$ despreciables, no la inducción, p.ej σ >>, ω bajas

$$\boxed{\begin{aligned} \vec{\nabla}\vec{D} &= \rho \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \vec{J} \end{aligned}}$$

$$\boxed{\begin{aligned} \vec{\nabla}^2\vec{H} &= \sigma\mu\frac{\partial \vec{H}}{\partial t} \\ \vec{\nabla}^2\vec{E} &= \sigma\mu\frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

Ecua de difusión

$\tau = \frac{\varepsilon}{\sigma}$, $\rho \to 0$

Campos armónicos

$\tilde{H} = \tilde{H}_0\, e^{-j\omega t}$ $\quad \vec{H} = \mathrm{Re}(\tilde{\vec{H}})$ $\quad$, $\vec{\nabla}^2\tilde{H} = -j\omega\sigma\mu\vec{H}$

Corrientes inducidas

$\vec{J}\,(\vec{E})$

$\nabla^2\vec{J} = \sigma\mu\frac{\partial \vec{J}}{\partial t}$

$\vec{H} \propto e^{-z/\delta}$ $\qquad \delta = \sqrt{\frac{2}{\omega\sigma\mu}}$

$\vec{E}, \vec{J} \propto e^{-z/\delta}$ $\qquad \vec{J} \cdot e^{j\omega t} \cdot e^{(1+j)z/\delta}$

δ = longitud de penetración

Efecto Pinch: $\delta \propto \frac{1}{\sqrt{\omega\sigma}}$ → pasan ω + bajas

σ menores ... agua mar

$\sigma(\omega)$

: absorción : $\mathrm{Im}(n)$, agua

Coeficientes de inducción

$M_{21} = M_{12} = \frac{\mu_0}{4\pi}\oint_2\oint_1 \frac{d\vec{l}_1 d\vec{l}_2}{R}$ (dos cables cerrados)

Realmente:

$M_{21} = \frac{\Phi_2}{I_1}$ $\qquad \Phi_2 = M_{21}I_1$

$L_{bob} = \mu_0 N^2 S/l = L_{tor}$

$\Phi = L\cdot I$

$\mathcal{E} = -L\frac{dI}{dt}$

Corrientes estacionarias

$\vec{\nabla}\cdot\vec{J} = 0 \qquad \vec{\nabla}\times\vec{J} = 0$

↳ $\Delta\phi = 0$

2 medios homog $\begin{cases} \phi_1 = \phi_2 \quad P\in S \\ J_{n1} = J_{n2} = \sigma E_{n1} = \sigma_1\frac{\partial\phi_1}{\partial n} = \sigma_2\frac{\partial\phi_2}{\partial n} \end{cases}$

- - - - . .

2 conductores: Q_1 ε, σ Q_2

$C = \frac{Q_1}{\phi_1 - \phi_2}$ $\qquad \sigma \neq 0 \to R = \frac{\phi_1 - \phi_2}{I} = \frac{\int_1^2 \vec{E}\cdot d\vec{l}}{\int \vec{J}\cdot d\vec{S}}$

$RC = \frac{\varepsilon}{\sigma}$

Leyes de Kirchhoff (a partir ec. Maxwell)

3D, vol. finito, R, C

Red: conjunto componentes discretos conectados entre sí x hilos ideales ($R\simeq 0$)

Nudo: punto de unión de 3 ó más hilos

Rama: conjunto de elementos conectados entre sí entre 2 nudos consecutivos

Malla: conjunto de ramas que forman un circuito cerrado sin pasar 2 veces por la misma rama

1ª LK: Ley de los nudos

$$\oint_S \vec{J}\cdot d\vec{S} = \int_V \vec{\nabla}\cdot\vec{J}\cdot dV = 0 \cdot \int_S \underbrace{\sigma}_{\sigma_{ext}=0}\vec{E}\cdot d\vec{S} = \boxed{\sum_K \pm I_K = 0}$$

2ª LK: Ley de las mallas

$$\oint \vec{E}\cdot d\vec{l} = \sum \pm \mathcal{E}_K = \oint \vec{E}\cdot d\vec{l} = R\int \vec{J}\cdot d\vec{S} = \boxed{\sum_K \pm R_K I_K = \sum_K \pm \mathcal{E}_K}$$

criterio signos: $\xrightarrow{I}\ \big|\big|_{\mathcal{E}} \rightarrow\ \ \rightarrow +\mathcal{E} = +RI$ $\qquad \vec{E} = \vec{E}_{est} + \vec{E}_{gen}$

Corrientes cuasiestacionarias (inducción, no J_D)

$\hookrightarrow \sum \pm \mathcal{E}_K = \sum_K R_K I_K + \frac{d\Phi}{dt}$ $\quad L\frac{dI}{dt}$

$$\vec{E} = \vec{E}_c + \vec{E}_{ind} + \vec{E}_g$$

$$\hookrightarrow \int \vec{E}\cdot d\vec{l} = \mathcal{E}_K - \frac{\partial\Phi}{\partial t}$$

1 malla ⇒

$$\mathcal{E} = RI(t) + L\frac{dI(t)}{dt}$$

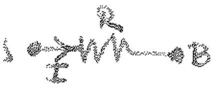
Si condensador: (aquí ya no desprecias J_D x $\oint J = 0$)

$$V_A - V_B = \frac{Q}{C} \quad \rightarrow \quad \mathcal{E} = RI + L\frac{dI}{dt} + \frac{1}{C}\int_0^t I(t)\,dt + V_0$$

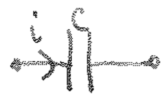
$$\rightarrow \frac{d\mathcal{E}}{dt} = R\frac{dI}{dt} + L\frac{d^2I}{dt^2} + \frac{I(t)}{C}$$

Parámetros localizados

$\sigma_{ext}\simeq 0$, volum. finito



$V_A - V_B = R\cdot I(t)$ $\qquad P(t) = RI^2(t)$ $\qquad R = \frac{1}{S}\frac{L}{\sigma}$



$i(t) = C\cdot\frac{dV_{AB}}{dt}$ $\qquad U_E = \frac{1}{2}CV^2(t)$

$V_A - V_B = L\frac{dI}{dt}$ $\qquad U_B = \frac{1}{2}LI^2(t)$

x retardos temporales despreciables (volumen pequeño)

Transitorio $i = i_p + i_h$ $\sim \mu s$

RLC → ihom → transitorio
ipart → solución estacionaria

- Carga y descarga de un C Descarga

$Q_p = C \cdot \varepsilon$ $Q_h = -C\varepsilon e^{-t/\tau}$

$Q(t) = C\varepsilon(1 - e^{-t/\tau})$ $\tau = RC$

Bobina

$i(t) = \frac{\varepsilon}{R}(1 - t/\tau)$ $\tau = L/R$

Análogo en alterna $\tilde{\varepsilon}(t) = \varepsilon_0 e^{j\varphi_e} e^{j\omega t} = \tilde{\varepsilon}_0 e^{j\omega t}$

$\varepsilon(t) = Re[\tilde{\varepsilon}(t)]$

—R— $\tilde{V} = z_R \cdot \tilde{i}(t)$ → $z_R = R$

—C— $\tilde{V} = z_C \tilde{i}$ → $z_C = 1/j C\omega$

—L— $\tilde{V} = z_L \cdot \tilde{i}$ → $z_L = jL\omega$

$(R + z_C + z_L)\tilde{i} = \tilde{\varepsilon}$ → serie/paralelo análogo

$\hat{=}$ Leyes Kirchoff $\sum I_k = 0$; $\sum \varepsilon_k = \sum \pm z_k I_k$

Método mallas

$$\left(\sum_k z_{mk}\right)\tilde{i}_m - \sum_k z_{mk}\tilde{i}_k = \sum_k \tilde{\varepsilon}_{mk}$$

N mallas : $\begin{pmatrix} r_{11} & r_{12} & \cdots \\ \vdots & & \\ \vdots & & \end{pmatrix}\begin{pmatrix} \tilde{i}_1 \\ \vdots \\ \tilde{i}_n \end{pmatrix} = \begin{pmatrix} \tilde{\varepsilon}_1 \\ \\ \tilde{\varepsilon}_n \end{pmatrix}$ $\tilde{\varepsilon}_n = \sum_k^{\pm} \tilde{\varepsilon}_{nk}$

$r_{mn} = \sum_k z_{mk}$

$r_{mk} = -z_{mk}$ $m \neq k$

comprobar $P = \varepsilon \cdot I = \sum R_i I_i^2$

Cada vez q eliges 1 malla, borras 1 rama

Método nudos

$$\sum_k \frac{V_m - V_k}{R_{mk}} + \frac{\varepsilon_{mk}}{R_{mk}} = 0$$

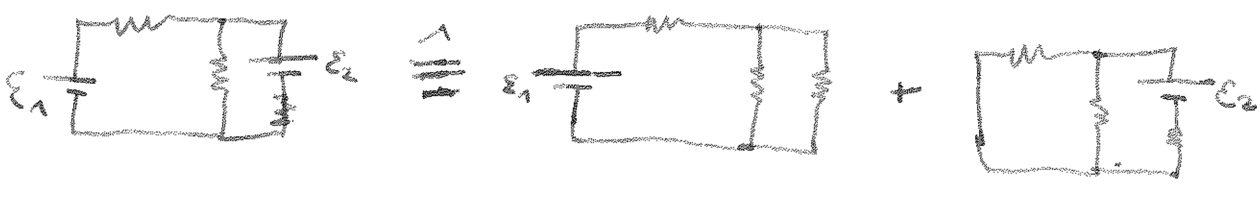
1 nudo origen de V

V nudo

TEOREMAS DE CIRCUITOS

Teorema de superposición

combinads $R_k \times I_k = \varepsilon_m$ → una $\varepsilon_i \to I_{ki}$ ⇒ $I_k = \sum I_{ki}$



Teorema de sustitución

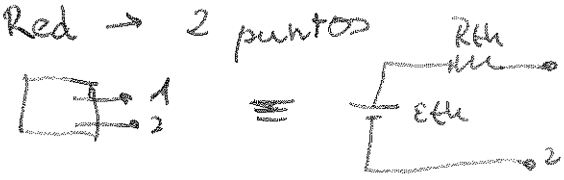
Dada una red, nos fijamos en una rama

$\mathcal{E}_{eq} = V_B - V_A$
no modifica corrientes ni potenciales

Corolario
↳ Siempre podemos añadir rama suplementaria sin que se modifiquen los potenciales y corrientes del resto del circuito con $\mathcal{E} = V_A - V_B$ y una Z cualquiera | $R = \infty \rightarrow \hat{=}$ $\frac{1}{I}$ $\mathcal{E} = \frac{V_A - V_B}{R}$

Tma. de Thevenin:

Red → 2 puntos

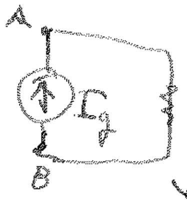


$R_{th} = \frac{\mathcal{E}_{th}}{I_{cc}}$

I_{cc} → amperímetro ideal
$\mathcal{E}_{th}$ → voltímetro
R variable

R_{th} → R_{eq} sin $\mathcal{E}$ caja

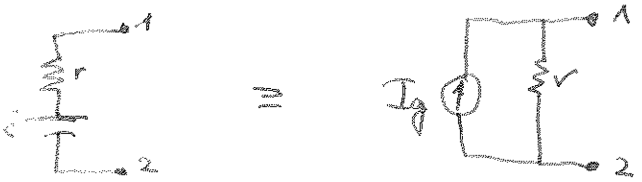
Generador corriente ideal $\quad r \to \infty \; / \; \frac{\mathcal{E}}{r} = I_g$ finito cte



tensión $\quad I = \frac{\mathcal{E}}{R+r}$ $\quad \mathcal{E}$ cte, I variable, $V_A - V_B = I \cdot R - \frac{\mathcal{E} R}{R+r}$

$V_A - V_B = R I_g$ ↳ variable

Tma. Norton:



$I_g = \frac{\mathcal{E}}{r}$ $\quad$ sustituciones equivalentes

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Potencia en corriente alterna

$P(t) = V(t) \cdot I(t) \quad \rightarrow \quad \langle P \rangle = \frac{1}{2} V_0 I_0 \cos\phi$

$\boxed{\tilde{P} = \frac{1}{2} \tilde{V} \cdot \tilde{I}^*}$, $\langle P \rangle = \mathrm{Re}\{\tilde{P}\}$

—R— $\tilde{P}_R = \frac{1}{2} R I_0^2 = P_R$

—C— $\tilde{P}_C = \frac{1}{2} \tilde{V} \left(\frac{\tilde{V}}{Z_C}\right)^* = \frac{1}{2}(-j\omega C)|\tilde{V}|^2 \Rightarrow \mathrm{Re}\{\tilde{P}_C\} = 0$

—L— $\tilde{P}_L = j\omega \frac{1}{2} L I^2$ $\quad \mathrm{Re}\{\tilde{P}_L\} = 0$

$U_E = \frac{1}{2} C V^2$
$\langle U_E \rangle = \frac{1}{4} C V_0^2$
$\tilde{P}_C = -j 2\omega \langle U_E \rangle$

$\langle U_B \rangle = \frac{1}{4} L I_0^2$
$\tilde{P}_L = j 2\omega \langle U_B \rangle$

$\langle U_{tot} \rangle = 2\langle U_E \rangle = \frac{1}{2} L I_0^2$

$Q_R = \omega_R \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_R}{\delta\omega}$ $\quad (R \ll \ldots R)$

si $R = 0 \rightarrow |Z| \rightarrow 0$

$X_{ef} = \frac{X_0}{\sqrt{2}}$

ω tales que $\phi(\mathcal{E}) - \phi(i) = 0$ → ω resonancia $\quad$ $\tilde{V} = Z \cdot \tilde{I}$, $\phi_Z = 0$

RLC serie → $\omega_r = \frac{1}{\sqrt{LC}}$

$I \hat{=} \frac{I_0}{\sqrt{2}}$

$\mathrm{Im}(\tilde{P}) = 0$
$\langle U_E \rangle = \langle U_B \rangle$
$\mathrm{Re}(\tilde{P}) = \frac{1}{2} R I_0^2$
$Q = \frac{\langle U_{total} \rangle \cdot \omega_R}{\langle P \rangle}$

$R_1 L \| R_2 C$

RLC paralelo
$\omega_R = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$
$R_1, R_2 \to 0$
$Z = \infty \rightarrow$ circuito abierto
$Q = \omega_R L / \ldots \simeq \omega_R / \delta\omega$

$C, L = \frac{\ldots \langle U_{E,B} \rangle}{I_{ef}^2}$

→ cambiando ω conectados a otros puntos circuito

Máxima potencia
$\mathcal{E} - Z - Z_g$
si $Z = Z_g^*$

T. 8

$\vec\nabla\cdot\vec D=\rho \quad \vec\nabla\vec B=0 \quad \vec\nabla\times\vec E=-\frac{\partial\vec B}{\partial t} \quad \vec\nabla\times\vec H=\vec J+\sigma\cdot\vec E+\frac{\partial\vec D}{\partial t} \quad \oplus\ \vec B=\mu\vec H \quad \oplus\ \vec D=\varepsilon\vec E$

Solución general → ondas electromagnéticas

$$\nabla^2\vec E-\mu\varepsilon\frac{\partial^2\vec E}{\partial t^2}-\sigma\mu\frac{\partial\vec E}{\partial t}=\frac{\vec\nabla\rho}{\varepsilon}+\mu\frac{\partial\vec J}{\partial t}$$

$= 0$ fuera fuentes

si $\sigma=0$ → ec. ondas $v=\sqrt{\frac{1}{\mu\varepsilon}}$

$$\nabla^2\vec H-\mu\varepsilon\frac{\partial^2\vec H}{\partial t^2}-\sigma\mu\frac{\partial\vec H}{\partial t}=-\vec\nabla\times\vec J$$

OPHLP

$\vec E=\vec E(\xi,t)$ → $\vec\nabla\vec D=0=\hat u\frac{\partial\vec D}{\partial\xi}$ $\quad \hat u\times\frac{\partial\vec E}{\partial\xi}=-\frac{\partial\vec B}{\partial t}$ → $E_\xi=0$ ...transitorio

$\vec\nabla\vec B=0=\hat u\frac{\partial\vec B}{\partial\xi}$ $\quad \hat u\times\frac{\partial\vec H}{\partial\xi}=\sigma\vec E+\frac{\partial\vec D}{\partial t}$ → $B_\xi=0$ } transversales

Medios aislantes $\sigma=0$

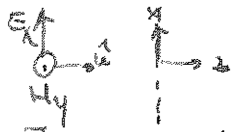
$\vec\nabla^2\vec E-\mu\varepsilon\frac{\partial^2\vec E}{\partial t^2}$, $\vec\nabla^2\vec H-\mu\varepsilon\frac{\partial^2\vec H}{\partial t^2}$ → $\tilde E=\tilde E_0e^{j\omega t}$, $\hat H=\hat H_0e^{j\omega t}$

→ $\vec\nabla^2\tilde{\vec E}+\underbrace{K^2}_{\omega^2\mu\varepsilon}\vec E=0$, $\vec\nabla^2\vec H+K^2\vec H=0$ ⇒ $\vec H=\frac{1}{Z}(\hat u\times\vec E)$ $\quad Z=\sqrt{\frac{\mu}{\varepsilon}}$

$Z_0=377\,\Omega$

$$\tilde E=\tilde E_0\,e^{j(\underbrace{k\xi}_{\vec k\cdot\vec r}-\omega t)}$$

Interfases



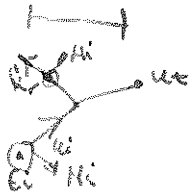
$r=\frac{1-Z_1/Z_2}{1+Z_1/Z_2}$ $\quad$ si $\mu_i=\mu_0$

$t=\frac{2}{1+Z_1/Z_2}$ $\quad$ ↳ $\frac{Z_i}{Z_j}=\frac{n_i}{n_j}$

$\vec N=\vec E\times\vec H$ → instant.

$\tilde N=\frac{1}{2}\vec E\times\vec H^*$ promedio

↳ $\sigma=0$ → dieléctrico puro $\frac{1}{2}\frac{E_0^2}{Z}\hat u$ → $R=r^2$ $\quad T=\frac{Z_1}{Z_2}\cdot t^2$



$\vec k_{i,r,t}$ en mismo plano

$\theta_r=\theta_i$

$n_1\operatorname{sen}\theta_i=n_2\operatorname{sen}\theta_t$

RTF → $K_{zt}=j\alpha$

$\vec E$ tangente continuo

$\vec H$ tangente "

$$r_\perp=\frac{1-\frac{Z_1}{Z_2}\frac{c_t}{c_i}}{1+\frac{Z_1}{Z_2}\frac{c_t}{c_i}} \qquad t_\perp=\frac{2}{1+\frac{Z_1}{Z_2}\frac{c_t}{c_i}}$$

$$r_\parallel=\frac{\frac{Z_1}{Z_2}-\frac{c_t}{c_i}}{\frac{Z_1}{Z_2}+\frac{c_t}{c_i}} \qquad t_\parallel=\frac{2}{\frac{Z_1}{Z_2}+\frac{c_t}{c_i}}$$

↳ $r_\parallel=0$ → $\theta_B:\ \operatorname{sen}^2\theta_B=\frac{1-\left(\frac{Z_1}{Z_2}\right)^2}{\left(\frac{n_1}{n_2}\right)^2-\left(\frac{Z_1}{Z_2}\right)^2}$ → si $\mu_i=\mu_0$ → $\operatorname{tg}\theta_B=\frac{n_2}{n_1}$

Medios conductores

(8)

$$\nabla^2 \tilde{E} - \sigma\mu \frac{\partial \tilde{E}}{\partial t} - \varepsilon\mu \frac{\partial^2 \tilde{E}}{\partial t^2} \xrightarrow{\text{armónica}} \nabla^2 \tilde{E} + \underbrace{j\omega\mu\sigma \tilde{E} + \omega^2 \varepsilon\mu \tilde{E}}_{\tilde{k}^2} \quad , \quad \tilde{k} = \underset{\text{fase}}{k} + j\underset{\text{atenua}}{k'}$$

$$\nabla^2 \tilde{E} + \omega^2\mu \underbrace{\left(\varepsilon + j\frac{\sigma}{\omega}\right)}_{\tilde{\varepsilon}} \tilde{E} = 0$$

$\hookrightarrow \tilde{E} = \tilde{E}_0 \, e^{-k'\xi} \, e^{j(k\xi - \omega t)}$

$$k' = \pm\, \omega \sqrt{\frac{\varepsilon\mu}{2}} \left\{ \pm \sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon^2}} - 1 \right\}^{1/2} \qquad k = \underbrace{\omega\sqrt{\varepsilon\mu}}_{k_{id}(\sigma=0)} \left\{ \frac{\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}} - 1}{2} \right\}^{1/2}$$

$$\tilde{Z} = \underbrace{\sqrt{\frac{\mu}{\varepsilon}}}_{Z_{id}} \sqrt{\frac{1}{1 + j\frac{\sigma}{\omega\varepsilon}}} \qquad \Delta\varphi = \varphi_Z \rightarrow \text{efecto joule, atenuación}$$

$\rightarrow k' = 0$ si $\sigma = 0$

$\rightarrow$ pierdes triedro

$\sigma, \varepsilon \rightarrow \rho = \rho_0 \cdot e^{-t/\tau} \qquad \tau = \frac{\varepsilon}{\sigma}$

$$\frac{\sigma^2}{\omega^2 \varepsilon^2} = \left(\frac{T}{2\pi\tau}\right)^2$$

a) Dieléctricos con pequeñas pérdidas $\sigma \ll 1$, $\tau \gg T$

$$k = \omega\sqrt{\varepsilon\mu}\left(1 + \frac{1}{8}\frac{\sigma^2}{\omega^2\varepsilon^2}\right) \quad ; \quad k' = \sqrt{\frac{\mu}{\varepsilon}}\,\frac{\sigma}{2}$$

$\hookrightarrow$ dispersión

b) Buenos conductores $\sigma \gg 1$, $\tau \ll T$

$$k = k' = \sqrt{\frac{\omega\sigma\mu}{2}} = \frac{1}{\delta} \rightarrow \tilde{k} = \frac{1+j}{\delta}$$

$\delta \rightarrow$ longitud de penetración

$|Z| = \sqrt{\frac{\mu}{\varepsilon}}\left(\frac{2\pi\tau}{T}\right)^{1/2} \rightarrow H$ grande $\propto \sigma$, ganancia $\quad H_{ot} = 2H_{oi}$ $\times$ inducción

$|H| = \frac{1}{|Z|}|E|$ $\qquad \hookrightarrow +$ relevante

$$\tilde{J} = \tilde{J}_0 \cdot e^{-\frac{1-j}{\delta}z} \, e^{-j\omega t}$$

$\rightarrow$ corrientes pegadas a la superficie $\sim \mu m$

Dispersión

$$\left.\begin{aligned} v_f &= \frac{\omega}{k} \rightarrow n_f = \frac{c}{v_f} \\ v_g &= \frac{\partial\omega}{\partial k} \rightarrow n_g = \frac{c}{v_g} \end{aligned}\right\} \quad n_g = n + \omega\frac{\partial n}{\partial\omega}$$

$k = k_0 n$, $\quad \frac{\partial k}{\partial k_0} = n + k_0\frac{\partial n}{\partial k_0}$

$v_f \cdot v_g = v_0^2$

$\rightarrow \frac{\partial n}{\partial\omega} > 0$, $\quad n_g > n$, $v_g < v_f$ $\quad$ Dispersión normal

$\rightarrow \frac{\partial n}{\partial\omega} < 0$, $\quad n_g < n$, $v_g > v_f$ $\quad$ " anómala (ω)

Polarizabilidad átomo

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{k}{m}x = \frac{q \cdot E_0}{m}\cos\omega t \quad , \quad F_R = kd$$

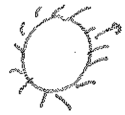
$$k = \frac{q^2}{4\pi\varepsilon_0 r^3} \qquad \left(\frac{1}{\pi a^3}\, e^{-2r/a}\right) \quad a: \text{Bohr}$$

$p = \alpha E \rightarrow \alpha = 4\pi\varepsilon_0 R^3 \quad (g)$

Espectro electromagnético

γ	1 pm
X	1 nm
UV	100 nm
VIS	500 nm $\rightarrow$ 400 - 760 nm
IR	10 μ
MW	1 cm
TV	10 m
FM	
AM	1 km
10^3 Hz RF	100 km

GUÍAS



$\hat{\varepsilon}(x,y)$ simetría traslación en z

$\nabla^2\vec{E} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

$E_i = E_i(x,y) f(z) e^{-j\omega t} \longrightarrow \frac{1}{e_i}\nabla_t^2 e_i + \frac{1}{f}\frac{\partial^2 f}{\partial z^2} + k^2 = 0$

$-k^2$ … $-\beta^2$ … β^2

$(\nabla_t^2 + k^2) e_i = \beta^2 e_i \qquad f(z) = e^{\pm j\beta z}$

$E_i = e_i(x,y)\, e^{\pm j\beta z} e^{-j\omega t}$

Modos → Base del espacio vectorial

→ se propagan con v_f definida $\phi(z,t) = \omega t - \beta z \rightarrow v_f = \frac{\partial z}{\partial t} = \frac{\omega}{\beta}$

→ Σ modos → frente se dispersa

- 2 indep → E_z, H_z

$E_x/H_x, E_y/H_y$ (↓)

$\beta \neq \omega\sqrt{\mu\varepsilon} = K$

- TE → $E_z = 0$ $\qquad Z_{TE} = \frac{\omega\mu}{\beta}$

$\vec{H_t} = \frac{j\beta}{K^2-\beta^2}\vec{\nabla}_t H_z$

$\vec{E_t} = -Z_{TE}\, \hat{u_z} \times \hat{H_t}$ … tenemos sólo transv.

- TM → $H_z = 0$ $\qquad Z_{TM} = \frac{\beta}{\omega\varepsilon}$

$\vec{E_t} = \frac{j\beta}{K^2-\beta^2}\vec{\nabla}_t E_z$

$\vec{H_t} = -\frac{1}{Z_{TM}}\hat{u_z} \times \hat{E_t}$

- híbridos $E_z \neq 0, H_z \neq 0$

- $E_z = 0 / H_z = 0$

TEM

→ $\beta = K$ $\qquad \vec{H_t} = \frac{1}{Z_{TEM}}\hat{u_z} \times \vec{E_t} \qquad Z_{TEM} = Z_{medio} = \sqrt{\frac{\mu}{\varepsilon}}$

electrostática bidimensional $\vec{\nabla}_t \times \vec{E_t} = 0$

$\vec{\nabla}_t \cdot E_t = 0$

Guía de planos ∞s

- TEM $\vec{E_t} = e_0 \hat{u_x} e^{+j(\beta z - \omega t)}$ $\qquad P_z = \frac{1}{2}Re(E_t \times H_t^*)$

- TM $e_z = B_n \operatorname{sen}\frac{n\pi x}{a}$ $\qquad \beta_n = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2} \longrightarrow v_f = \frac{\omega}{\beta}$

$h_y(e_x)$ $\qquad v_g = \frac{\partial\omega}{\partial\beta}$

$\beta \in Im$ $\qquad v_g = \frac{c^2}{v_f}$

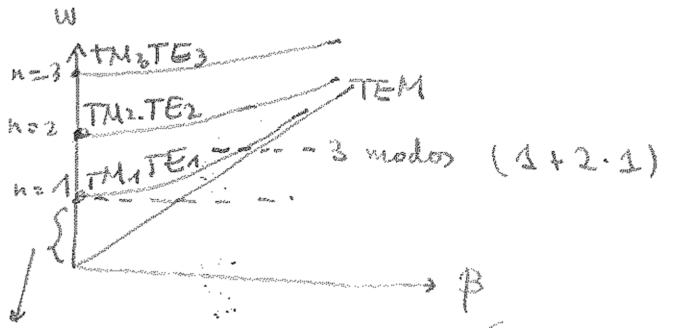
↓

modos evanescente

$P_z = \frac{1}{4 Z_0} K \frac{\beta}{K_x^2} B^2 ab$

$\omega_c = c\frac{n\pi}{a}$

- TE $h_z = A\cos\frac{n\pi}{a}x$ $\qquad \beta_n = \sqrt{k^2 - \left(\frac{n\pi}{a}\right)^2}$ → degenerado TM



Intervalo (ω) de propagación monomodo → modo fundamental TEM

$v_f = v_g$ / sin dispersión

Guía rectangular

$e_z(x,y) = f(x)\cdot g(y)$

$$\underbrace{\frac{1}{f}\frac{\partial^2 f}{\partial x^2}}_{-k_x^2} + \underbrace{\frac{1}{g}\frac{\partial^2 g}{\partial y^2}}_{-k_y^2} + (k^2-\beta^2) = 0$$

$\beta = \sqrt{k - k_x^2 - k_y^2}$

$\rightarrow k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$ $n, m = 1, 2, \ldots$

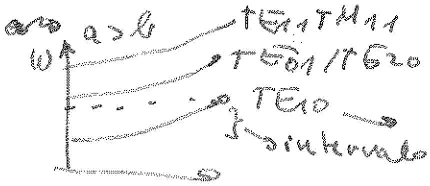
$$TM_{mn} \begin{cases} \beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \\ e_z(x,y) = A \operatorname{sen} \frac{m\pi x}{a} \operatorname{sen} \frac{n\pi y}{b} \\ E_z = e_z(x,y)\, e^{i(\beta z - \omega t)} \end{cases}$$

$\}$ degenerados salvo $(0,n)$ $(m,0)$

TE $e_z = 0$, $h_z \neq 0$

$$TE_{mn} \begin{cases} \beta_{mn} \hat{=} \\ h_z = B \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \end{cases}$$ $m, n = \underline{0}, 1, 2, \ldots$ $\hookrightarrow$ 1 sólo

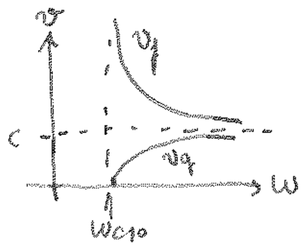
$\nexists$ solución TEM ($\phi = cte$) Dirichlet



$\rightarrow$ modo fundamental / dominante TE_{10}

$\}$ intervalo de propagac monomodo

$$\omega_{cmn} = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$



$v_g < c$

$v_f > c$

$P = \frac{1}{2} \operatorname{Re}\left(\int \vec{E} \times \vec{H}^* \cdot d\vec{S}\right)$

$= \frac{Z_{TE}}{2} \int |h_x|^2 \, dx\,dy$

$\frac{U_E}{z} = \frac{\varepsilon_0}{4} \int |e_y|^2 dx\,dy = \frac{\varepsilon_0}{2} Z_{TE} P$

$U_E \propto P_{modo}$

$\frac{U_B}{z} = \frac{U_E}{z}$

$U = 2U_E = \varepsilon_0 Z_{TE} P \rightarrow P = v_g \cdot U$

Cavidades

$$\vec{\nabla} \times \left\{\frac{1}{\varepsilon_r} \vec{\nabla} \times \vec{H}\right\} = \left(\frac{\omega}{c}\right)^2 \mu \vec{H}$$

Cavid paralelepípeda $\varepsilon_r = 1$

$H_x = f(x)\cdot g(y)\, h(z)$

$\Rightarrow$ guía rect + 2 C.C. $\rightarrow$ resonancias subconjunto a partir TE, TM

superponer $\pm e_z^{TE}$, $\beta^- = -\beta^+$ $\quad \vec{e_t}^- = -\vec{e_t}^+$ $\quad \vec{h_t}^- = \vec{h_t}^+$ $\quad h_z = -h_z^+$

TM $\cancel{h_z}$ $\vec{e_t}^+$ $\quad e_z = e_z^+$

$\beta^+ d = l\pi$

$$\frac{\omega_{mnl}}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

TE_{mn} $H_z = 2jB \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \operatorname{sen}\frac{l\pi z}{d}$ $\quad l = 1, \ldots$ $\quad m, n = \underline{0}, 1, \ldots$ ($\hookrightarrow$ 1 solo)

TM_{mn} $E_z = 2A \operatorname{sen}\frac{m\pi x}{a} \operatorname{sen}\frac{n\pi y}{b} \cos\frac{l\pi z}{d}$ $\quad l = 0, 1, \ldots$ $\quad m, n = 1, 2, \ldots$

RADIACIÓN ELECTROMAGNÉTICA

Ec. ondas no homogénea ε, μ

$$\boxed{\begin{aligned} &\nabla^2 \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{\nabla}\left(\frac{\rho_e}{\varepsilon}\right) + \mu \frac{\partial \vec{J}_e}{\partial t} \\ &\nabla^2 \vec{H} - \mu\sigma \frac{\partial \vec{H}}{\partial t} - \varepsilon\mu \frac{\partial^2 \vec{H}}{\partial t^2} = -\vec{\nabla} \times \vec{J}_e \end{aligned}}$$

$$\boxed{\begin{aligned} &\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \\ &\vec{B} = \vec{\nabla} \times \vec{A} \end{aligned}}$$

$\vec{A}' = \vec{A} + \vec{a}$
$\phi' = \phi + \Psi$ $\rightarrow$ Transformaciones de contraste $\quad \vec{A}' = \vec{A} - \vec{\nabla}\chi \quad \phi' = \phi + \frac{\partial}{\partial t}\chi \quad \forall \chi(r,t)$ escalar

$\hookrightarrow \nabla^2 \phi + \frac{\partial (\vec{\nabla}\cdot\vec{A})}{\partial t} = -\frac{\rho_e}{\varepsilon_0}$

$\hookrightarrow \nabla^2 \vec{A} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{A}}{\partial t^2} - \vec{\nabla}\left\{ (\vec{\nabla}\cdot\vec{A}) + \varepsilon_0\mu_0 \frac{\partial \phi}{\partial t} \right\} = -\mu_0 \vec{J}$

Condición de Coulomb: $\chi / \vec{\nabla}\cdot\vec{A} = 0$

$\vec{\nabla}^2 \phi = -\rho/\varepsilon_0 \rightarrow$ electrostática $\quad \phi = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{R}$

$\vec{\nabla}^2 \vec{A} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{A}}{\partial t^2} - \varepsilon_0\mu_0 \frac{\partial}{\partial t}(\vec{\nabla}\phi) = -\mu_0 \vec{J}$

Condición de Lorentz: $\chi / \vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0$

$\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = -\rho/\varepsilon_0$ $\qquad$ 4 ecuaciones de ondas, desacopladas

$\vec{\nabla}^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$ $\qquad \downarrow$ resolver 1

Método Green $\rightarrow L\, g(x-x') = \delta(x-x')$

$\rightarrow$ Potenciales retardados

$\phi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{dV'}{R}\, \rho\left(\vec{r}', t - \frac{R}{c}\right)$

$\vec{A}(r,t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}', t - R/c)\, dV'}{R}$

Campos de radiación

$\vec{B}_{rad} = \frac{\mu_0}{4\pi c}\int \frac{\dot{\vec{J}} \times \vec{R}}{R^2}\, dV' \qquad \vec{E}_{rad}(\vec{r},t) = \frac{\mu_0}{4\pi}\int_{V'} \frac{(\dot{\vec{J}} \times \vec{R}) \times \vec{R}}{R^3}\, dV'$ en t retardado !! $\qquad \propto \frac{1}{r}$

$\vec{E}_{rad} \times \vec{B}_{rad} = \frac{|\vec{E}_{rad}|^2}{c}\hat{u}_r$ $\quad$ tricho a dichos a gran dist. $\quad E_\theta, H_\varphi$

Dipolo oscilante

$\vec{J}_i\, dV = i\, d\vec{l}_i \rightarrow \vec{B}_{rad} = \frac{\mu_0}{4\pi}\frac{\ddot{\vec{p}} \times \vec{R}}{cR^2} \qquad E_{rad} = c(\vec{B}_{rad} \times \hat{u}_r) = \frac{1}{4\pi\varepsilon_0 c^2}\frac{1}{c^2}\frac{(\ddot{\vec{p}} \times \vec{R}) \times \vec{R}}{R^3}$

$\vec{N} = \frac{\mu_0 \ddot{p}^2 \sin^2\theta}{(4\pi)^2 c R^2}\hat{u}_r \rightarrow P = \frac{\ddot{p}^2}{6\pi\varepsilon_0 c^3}$

Oscilad. armónica $\rightarrow \vec{N} = \frac{1}{2}(\vec{E} \times \vec{H}^*) = \frac{1}{32}\left\{\frac{\mu_0 \omega^4 \sin^2\theta}{\pi^2 c R^2}\right\}\hat{u}_R \rightarrow \langle P \rangle = \mathrm{Re}\left(\oint \vec{N}\cdot d\vec{s}\right)$

$\langle P \rangle = \frac{\mu_0 \omega^4 p_0^2}{12\pi c}$ $\qquad \omega \sim \mathrm{MHz}$

potencial 2 cargas $\rightarrow \phi = \frac{\vec{p}\cdot\vec{R}}{4\pi\varepsilon_0 R^3} + \frac{\dot{\vec{p}}\cdot\vec{R}}{4\pi\varepsilon_0 c R^2} \rightarrow \vec{E} = \vec{E}_{dip} + \vec{E}_{rad}$

ρ simetría esférica $\rightarrow$ no radia

$\vec{J} = J\hat{u}_r \rightarrow \vec{B}_{rad} = 0$

$\vec{E}_{rad} = 0$