

Lessons from Axion Dark Matter for Gravitational-Wave Physics

Max Planck Institute for Gravitational Physics
Albert Einstein Institute

Hannover, Germany
October 8, 2024



Camilo García Cely

Based on

Novel Search for High-Frequency Gravitational Waves with Low-Mass Axion Haloscopes

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



PUBLISHED FOR SISSA BY SPRINGER

PUBLISHED: March 21, 2024

Symmetries and selection rules: optimising axion haloscopes for Gravitational Wave searches

Valerie Domcke ^a, Camilo Garcia-Cely ^b, Sung Mook Lee ^{a,c}
and Nicholas L. Rodd ^a

arXiv > hep-ph > arXiv:2407.18297

High Energy Physics - Phenomenology

[Submitted on 25 Jul 2024]

Complete Gravitational-Wave Spectrum of the Sun

[Camilo García-Cely](#), [Andreas Ringwald](#)

and work in progress with Luca Marsili, Aaron Spector and Andreas Ringwald

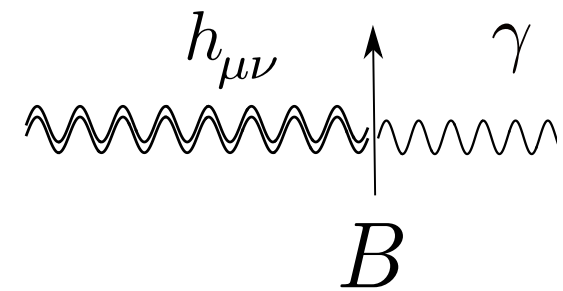
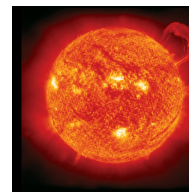
Outline

- Motivation: the Gertsenhstein effect

Axions versus high-frequency gravitational waves

- Standard model backgrounds:

Solar gravitational waves



- Novel experimental approaches:

Axion haloscopes and polarimetry techniques

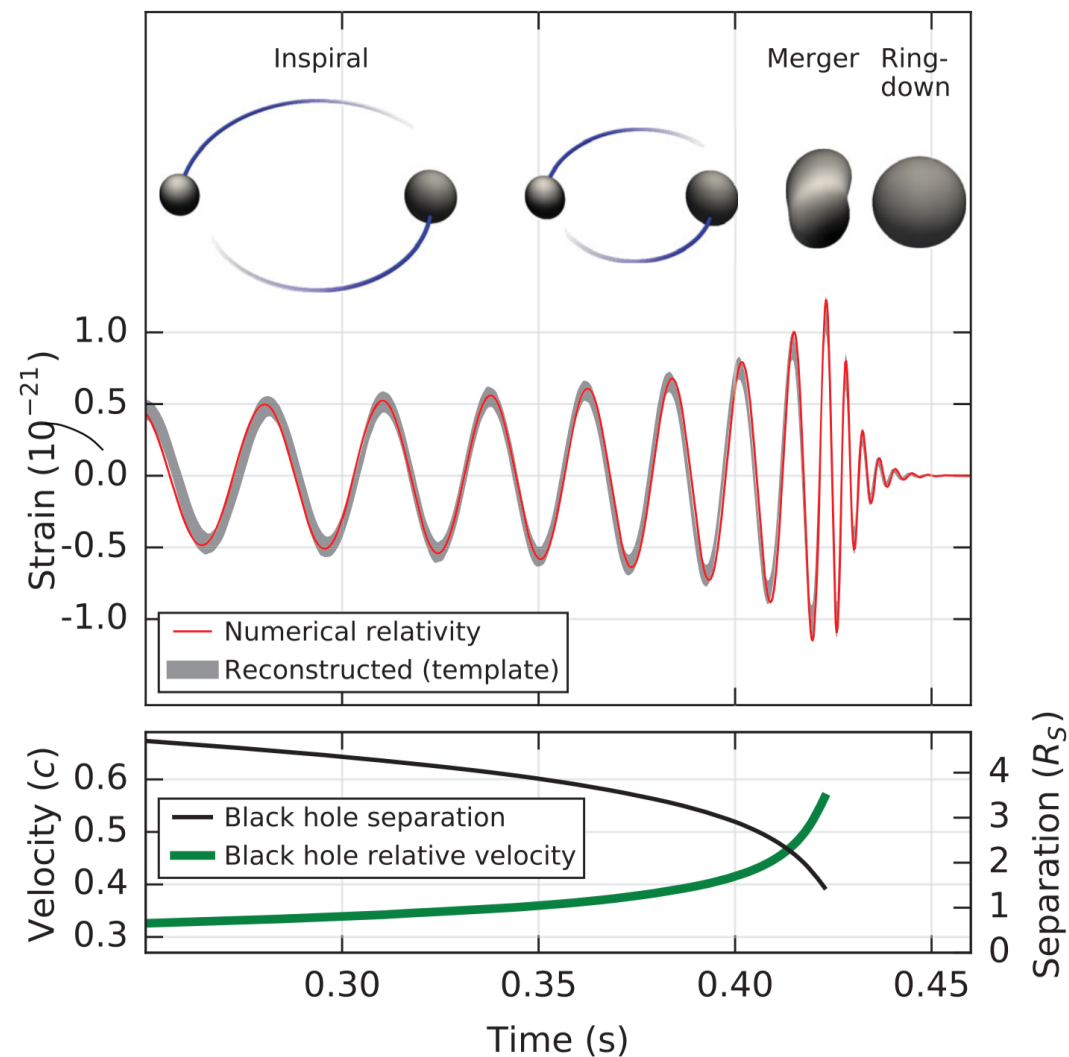
- Conclusions

Motivation

The Gertsenhstein effect

Axions versus high-frequency
gravitational waves

Gravitational waves



PRL **116**, 061102 (2016)

PHYSICAL REVIEW LETTERS

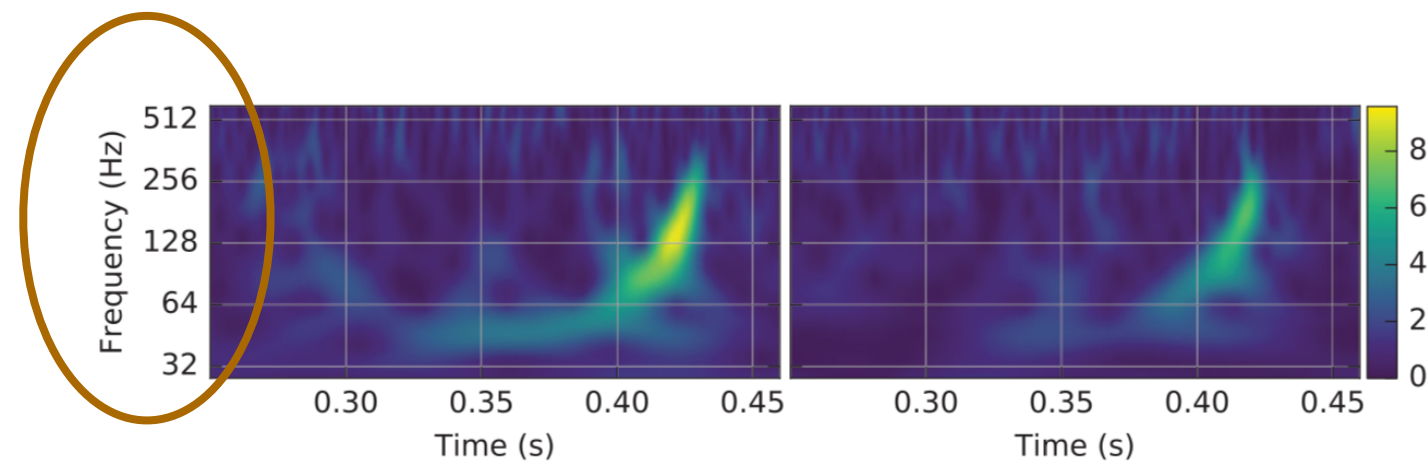
12 FEBRUARY 2016



Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)



$$f \approx \frac{1}{2\pi} \sqrt{\frac{GM}{R^3}} \ll 10 \text{ kHz}$$

No known astrophysical objects are small and dense enough to produce gravitational waves beyond 10 kHz

High-frequency gravitational waves

Part of a collection:

[Gravitational Waves](#)

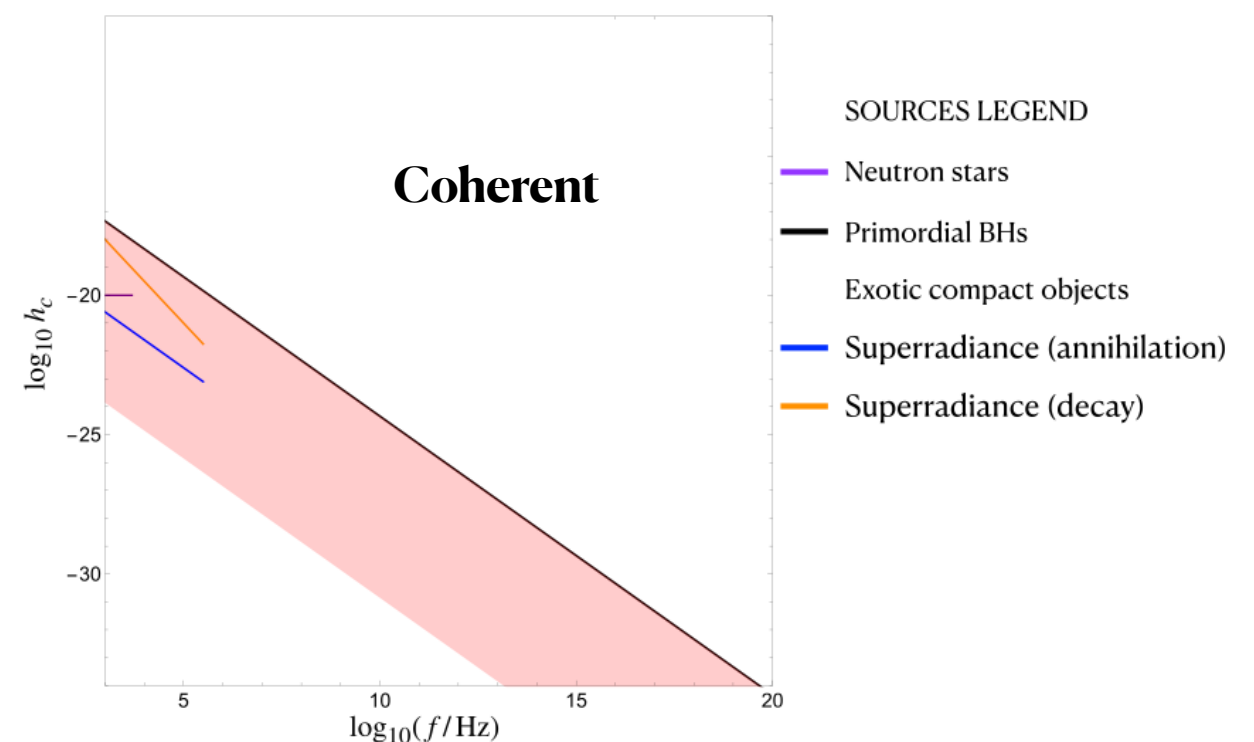
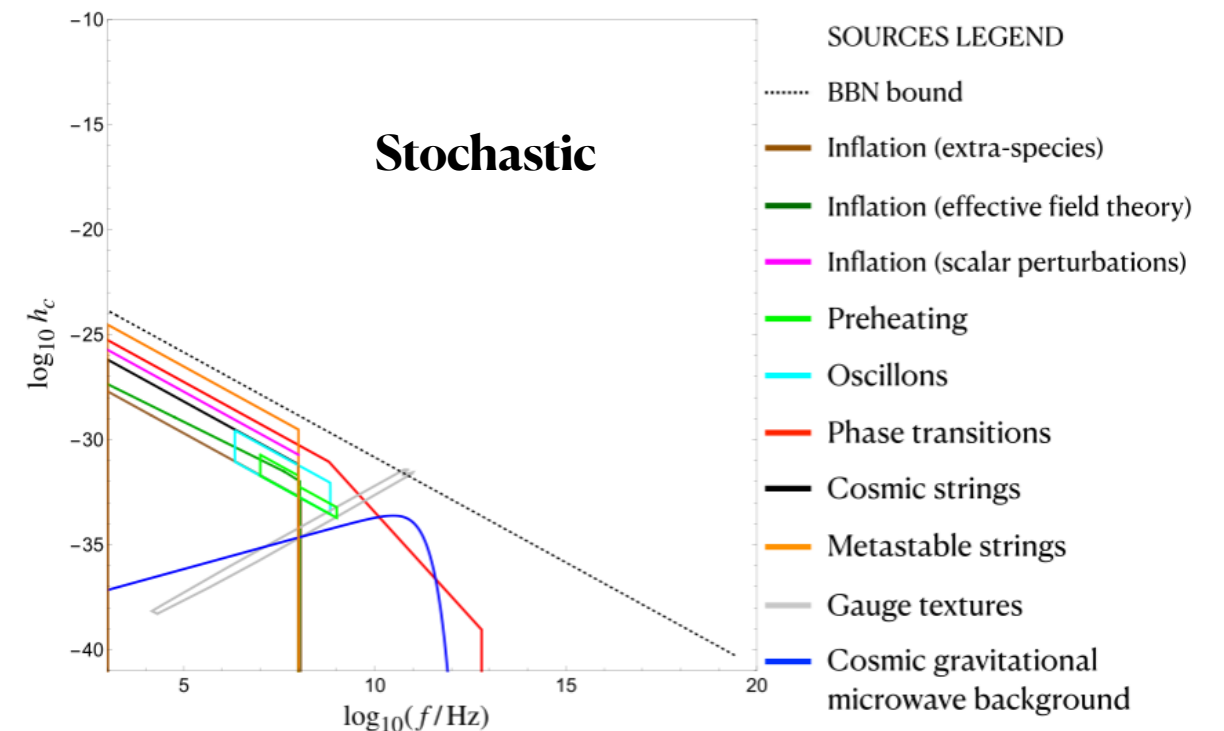
Review Article | [Open Access](#) | [Published: 06 December 2021](#)

Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies

[Nancy Aggarwal](#) , [Odylio D. Aguiar](#), [Andreas Bauswein](#), [Giancarlo Cella](#), [Sebastian Clesse](#), [Adrian Michael Cruise](#), [Valerie Domcke](#) , [Daniel G. Figueroa](#), [Andrew Geraci](#), [Maxim Goryachev](#), [Hartmut Grote](#), [Mark Hindmarsh](#), [Francesco Muia](#) , [Nikhil Mukund](#), [David Ottaway](#), [Marco Peloso](#), [Fernando Quevedo](#) , [Angelo Ricciardone](#), [Jessica Steinlechner](#) , [Sebastian Steinlechner](#) , [Sichun Sun](#), [Michael E. Tobar](#), [Francisco Torrenti](#), [Caner Ünal](#) & [Graham White](#)

[Living Reviews in Relativity](#) **24**, Article number: 4 (2021) | [Cite this article](#)

A growing community is seriously considering the search of high frequency gravitational waves



Revisiting Gertsenhstein's ideas

SOVIET PHYSICS JETP

VOLUME 14, NUMBER 1

JANUARY, 1962

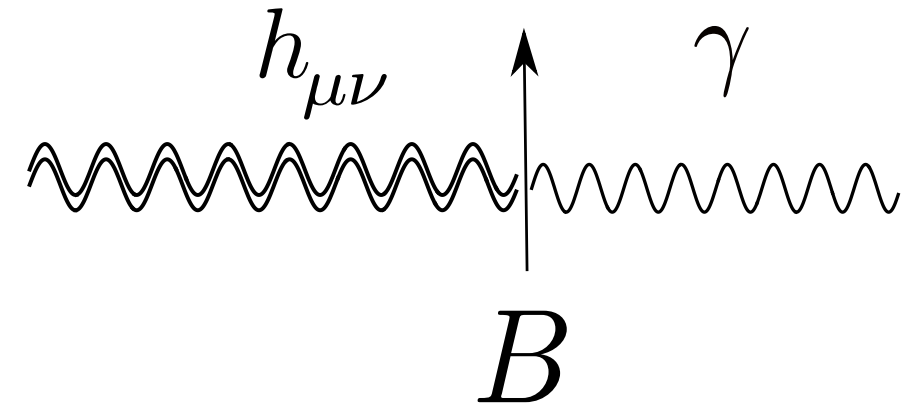
WAVE RESONANCE OF LIGHT AND GRAVITATIONAL WAVES

M. E. GERTSENSHTEĬN

Submitted to JETP editor July 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 113-114 (July, 1961)

The energy of gravitational waves excited during the propagation of light in a constant magnetic or electric field is estimated.



SOVIET PHYSICS JETP

VOLUME 16, NUMBER 2

FEBRUARY, 1963

ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

M. E. GERTSENSHTEĬN and V. I. PUSTOVOĬT

Submitted to JETP editor March 3, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 605-607 (August, 1962)

It is shown that the sensitivity of the electromechanical experiments for detecting gravitational waves by means of piezocrystals is ten orders of magnitude worse than that estimated by Weber.^[1] In the low frequency range it should be possible to detect gravitational waves by the shift of the bands in an optical interferometer. The sensitivity of this method is investigated.

Terrestrial
interferometers



The (inverse) Gertsenhstein Effect

- The conversion of gravitational waves into electromagnetic waves is a classical process. Its rate does not involve $\hbar$

$$P \sim GB^2L^2$$

- Cosmological conversion

Potential of Radio Telescopes as High-Frequency Gravitational Wave Detectors

Valerie Domcke and Camilo Garcia-Cely
Phys. Rev. Lett. **126**, 021104 – Published 14 January 2021

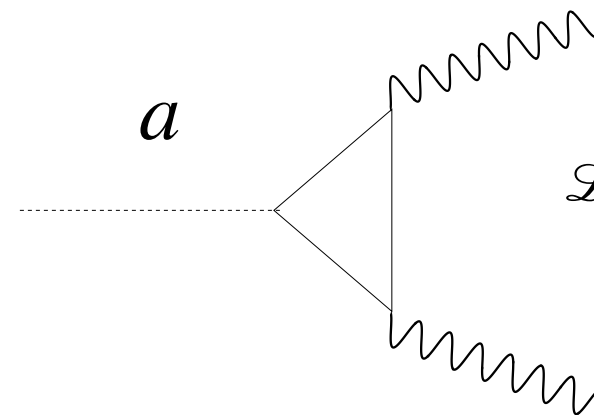


- The process is strictly analogous to axion conversion.

Raffelt, Stodolski'89

QCD axion as dark matter

- Pseudoscalar field



A Feynman diagram illustrating the interaction of an axion field a with two photons. On the left, a horizontal dashed line labeled a enters a triangular loop. The two vertices of the loop are connected by solid lines. From each vertex, a wavy line representing a photon extends to the right.

$$\mathcal{L} = -\frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

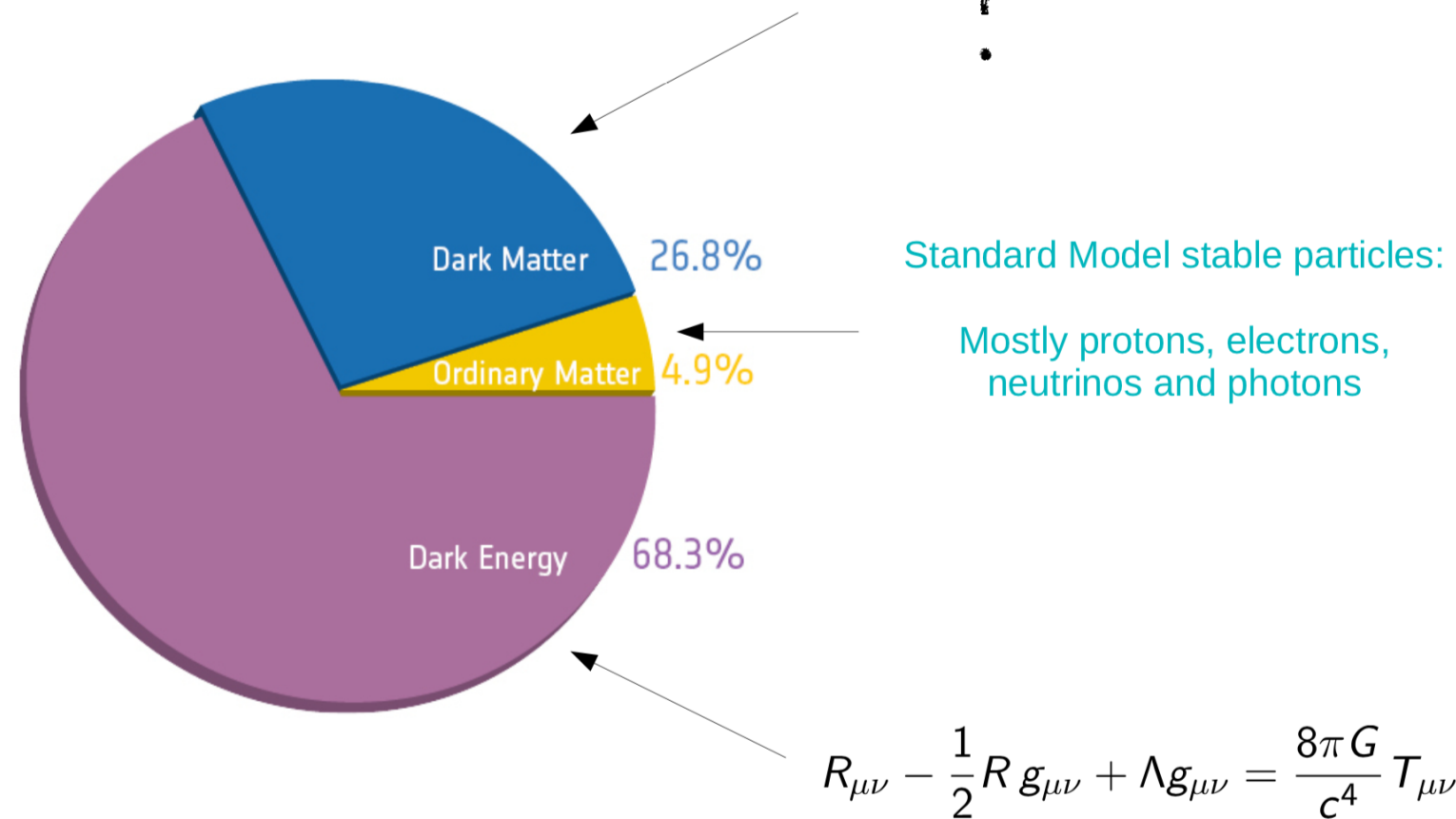
- Solution to the strong CP problem

Peccei, Quinn 1977

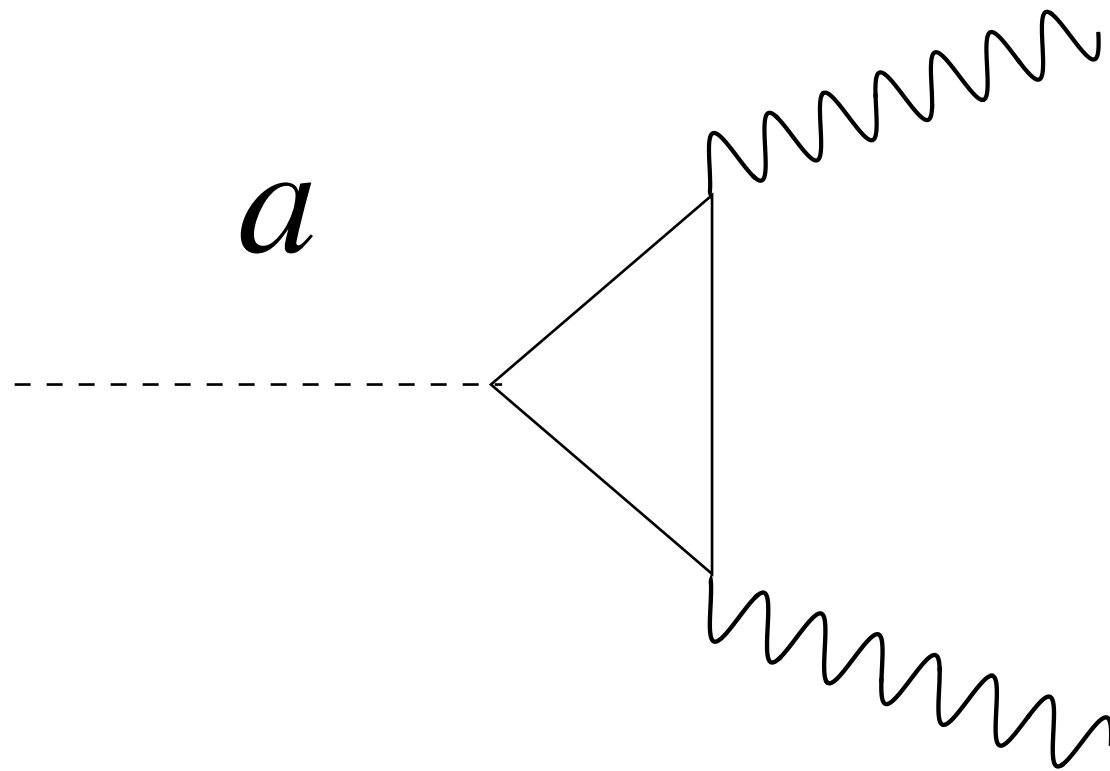
- Excellent dark matter candidate

Weinberg, Wilczek 1978

Collisionless Cold Dark Matter



Axion electrodynamics



$$\mathcal{L} = -\frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

Axion electrodynamics

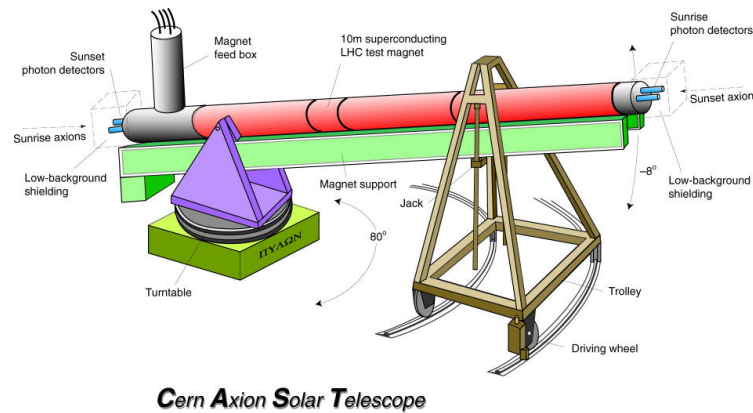
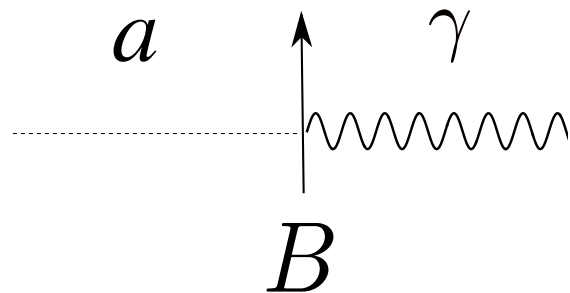
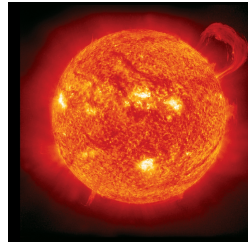
Axions act as a source term to Maxwell's equations, **effectively inducing an electromagnetic current.**

$$\begin{aligned}\nabla \cdot \mathbf{B} &= 0 && \text{Sikivie, 1983} \\ \nabla \times \mathbf{E} + \partial_t \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{E} &= j^0 \\ \nabla \times \mathbf{B} - \partial_t \mathbf{E} &= \mathbf{j}\end{aligned}$$

$$j^0 = -g_{a\gamma\gamma} \nabla a \cdot \mathbf{B} \quad \mathbf{j} = g_{a\gamma\gamma} \left(\nabla a \times \mathbf{E} + \partial_t a \mathbf{B} \right)$$

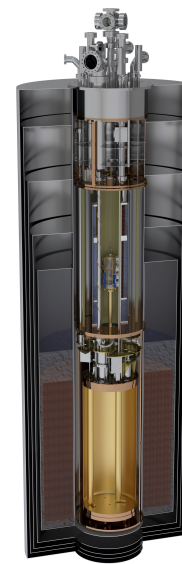
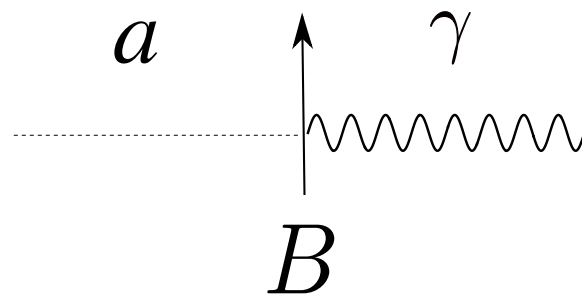
Axion electrodynamics

- Helioscopes (X rays)



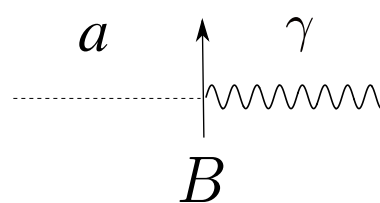
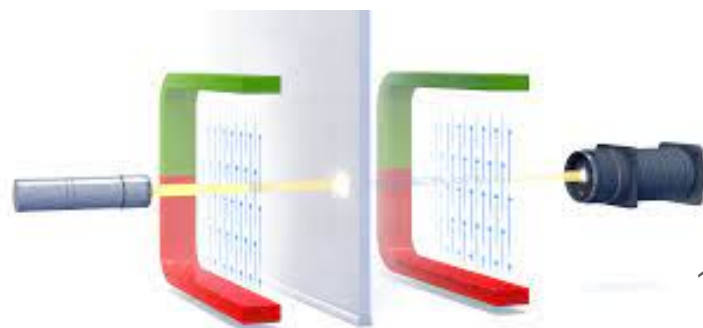
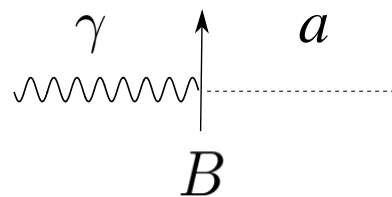
- CAST
- IAXO
-

- Haloscopes (radio frequencies)



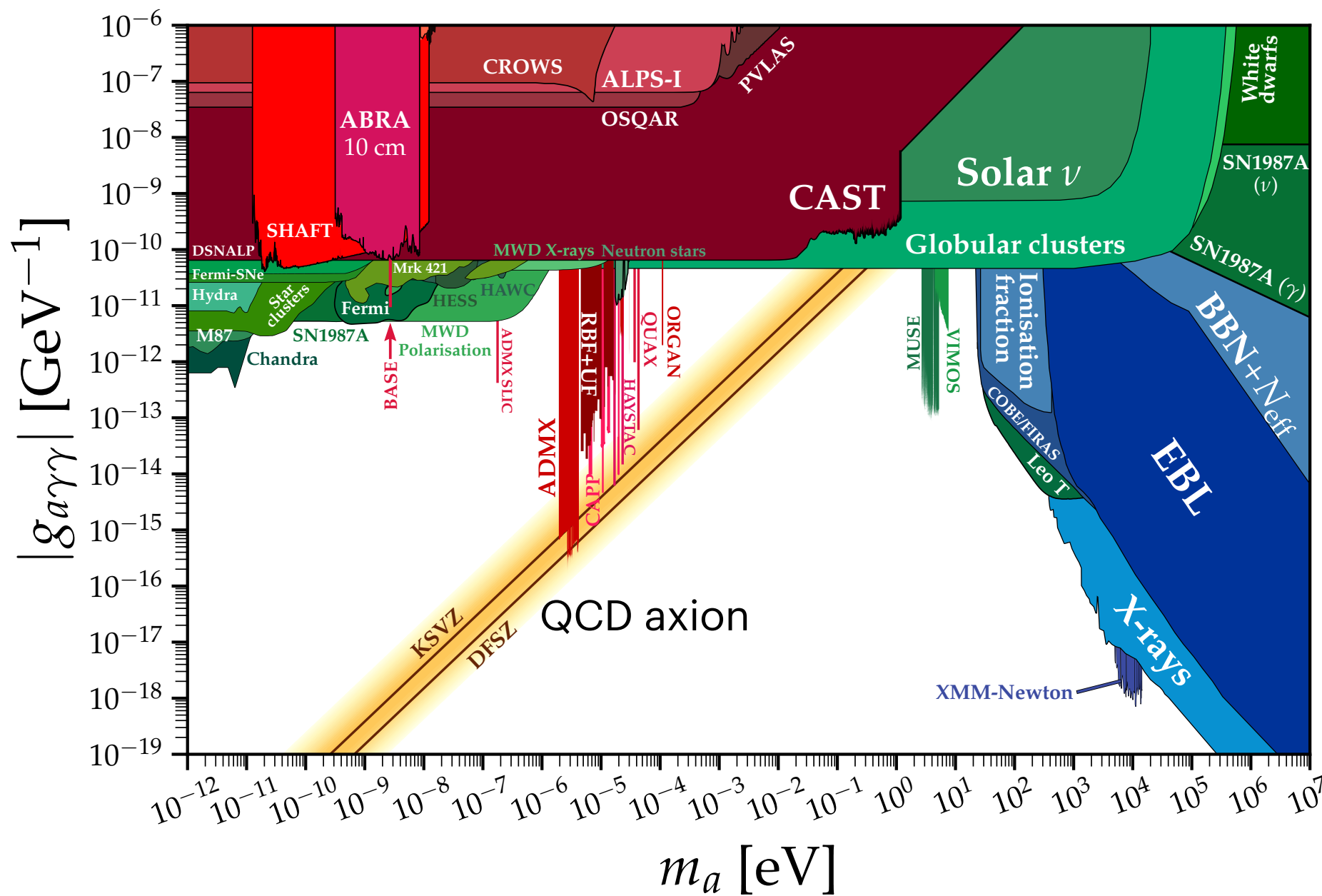
- microwave cavities
- MADMAX
- ADMX
- HAYSTAC
- ABRACADABRA
- Lumped element detectors
- ...

- Purely lab experiments



- Light shining through the walls
- OSCAR
- ALPS II
- ...

Axion electrodynamics



<https://github.com/cajohare/AxionLimits>

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a} \left(\frac{E}{N} - 1.92 \right)$$

DFSZ (Dine, Fischler, Srednicki, Zhitniskii)

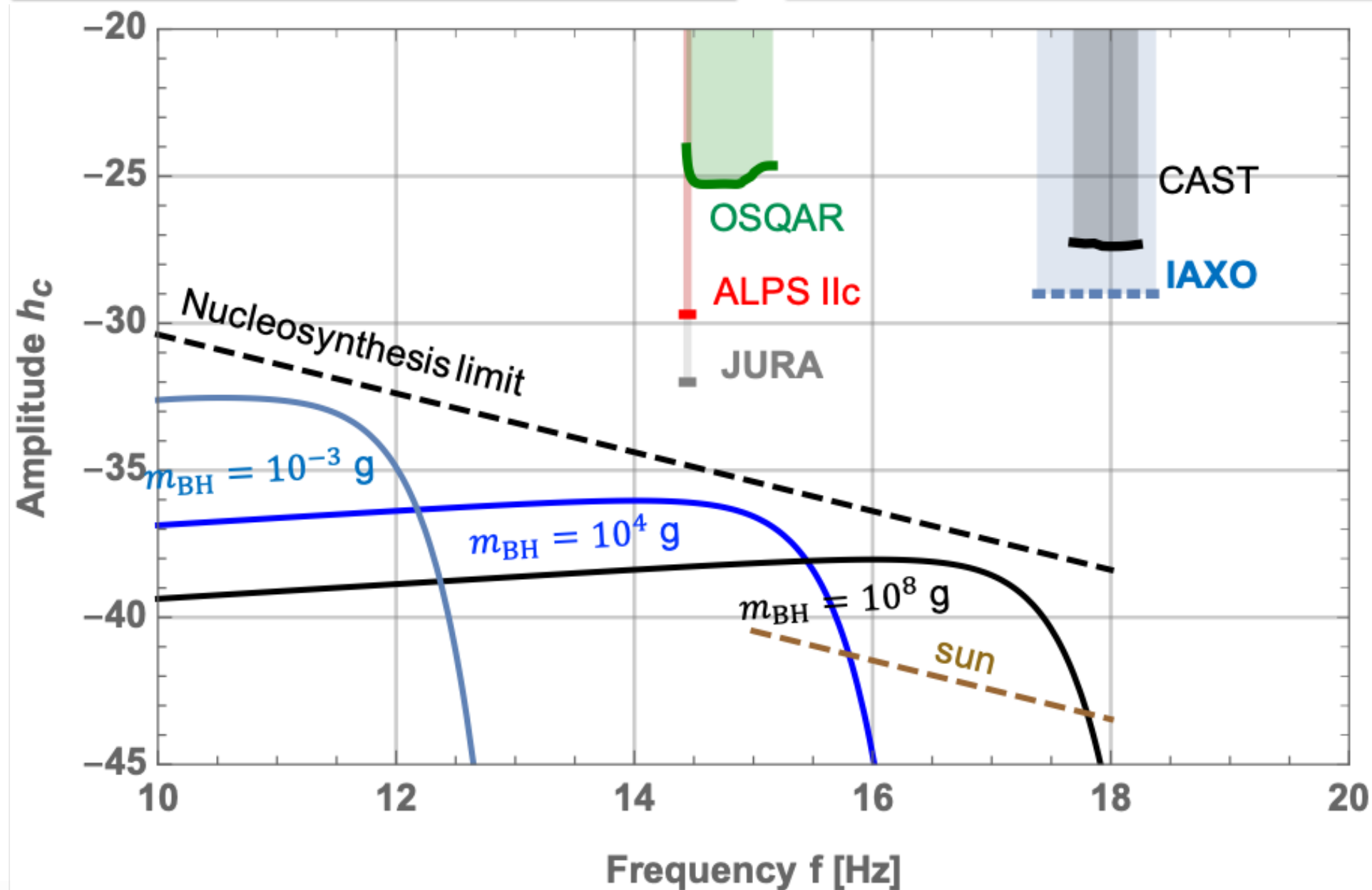
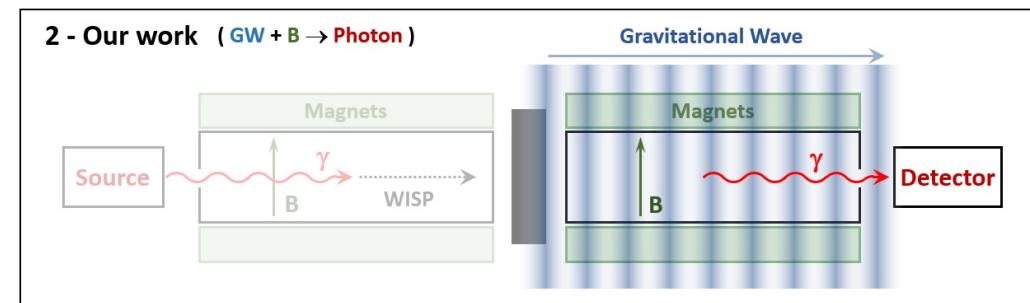
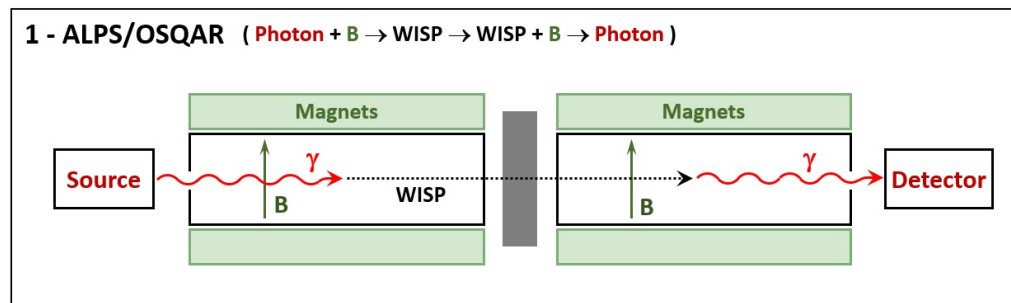
axions couple to fermions

$$E/N = 8/3$$

KSVZ (Kim, Shifman, Vainshtein, Zakharov)

axions couple to exotic heavy quarks only. $E/N = 0$

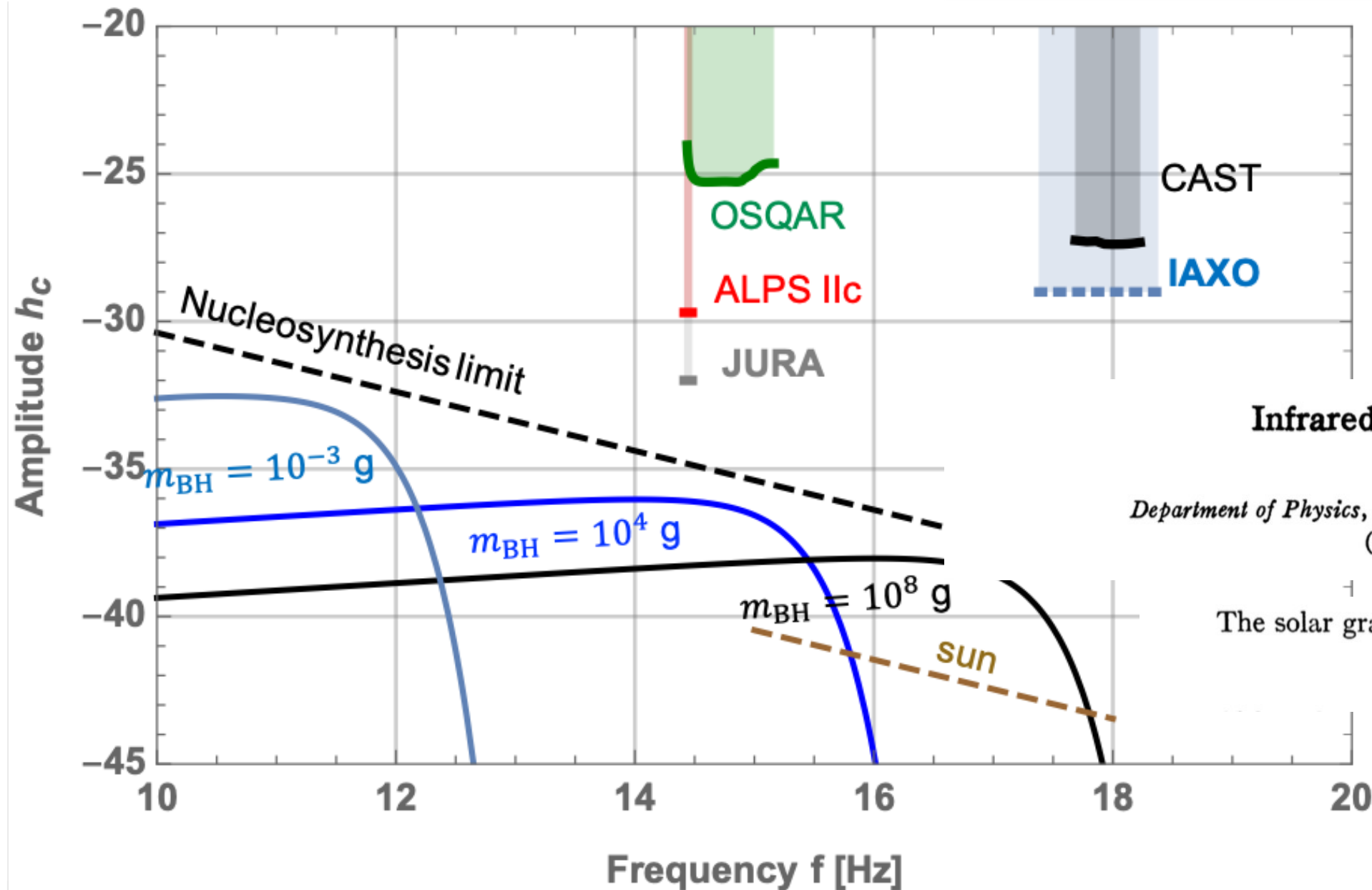
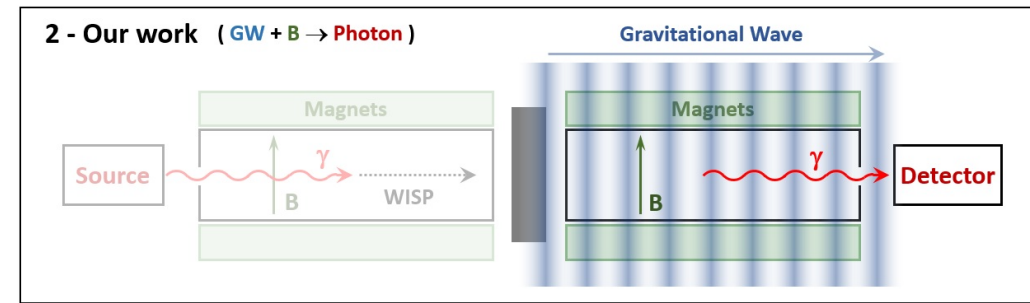
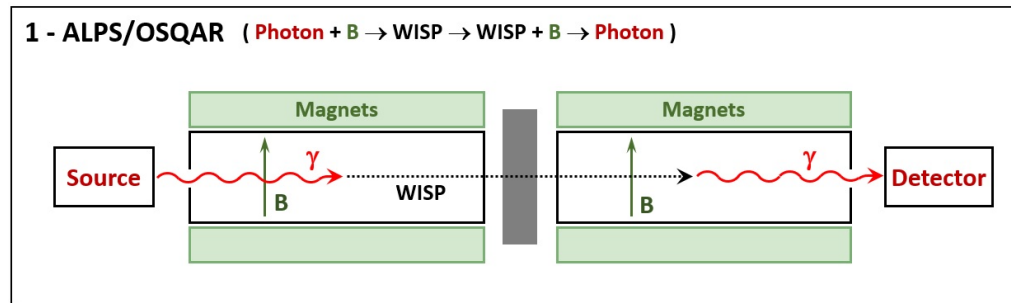
The (inverse) Gertsenhstein Effect



Standard model backgrounds

Solar gravitational waves

The (inverse) Gertsenhstein Effect



Infrared Photons and Gravitons*

STEVEN WEINBERG†

Department of Physics, University of California, Berkeley, California

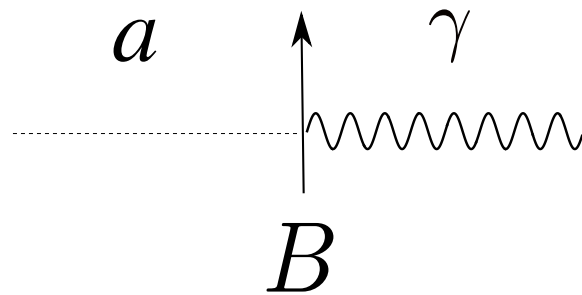
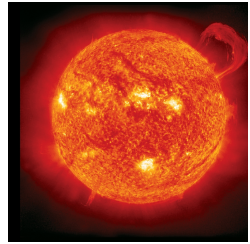
(Received 1 June 1965)

The solar gravitational radiation power is then

$$P_{\odot} \simeq 6 \times 10^{14} \text{ erg/sec.} \quad (4.24)$$

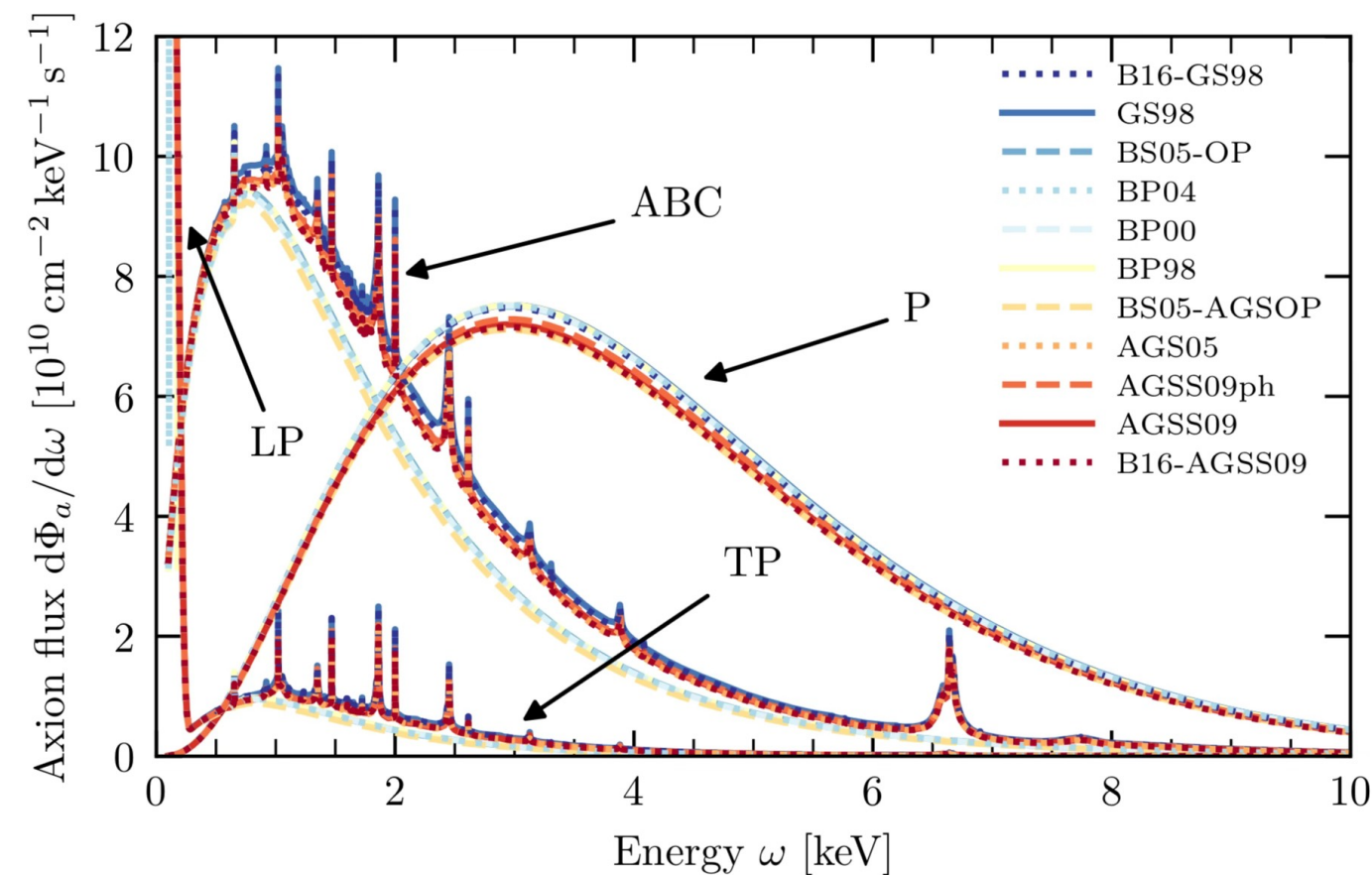
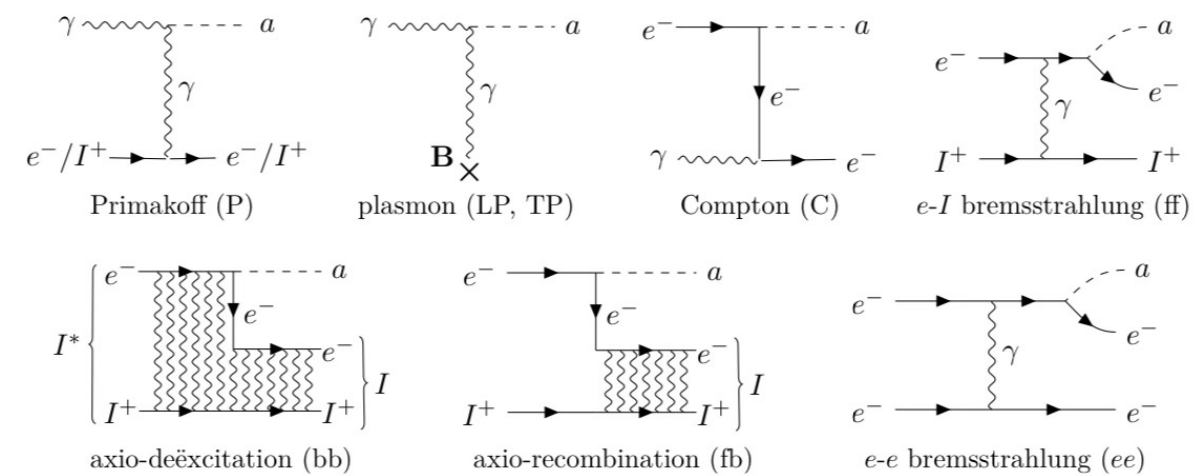
Solar emission of axions

• Helioscopes



- CAST
- IAXO
-

Hoof et al, 2021



PHYSICAL REVIEW D VOLUME 33, NUMBER 4 15 FEBRUARY 1986
Astrophysical axion bounds diminished by screening effects
Georg G. Raffelt

PHYSICAL REVIEW D VOLUME 37, NUMBER 6 15 MARCH 1988
Plasmon decay into low-mass bosons in stars
Georg G. Raffelt

PHYSICAL REVIEW D 102, 043019 (2020)
Axion helioscopes as solar magnetometers
Ciaran A. J. O'Hare^{1,2}, Andrea Caputo^{2,3}, Alexander J. Millar^{3,4,5} and Edoardo Vitagliano^{5,6}

PHYSICAL REVIEW D 101, 123004 (2020)
Revisiting longitudinal plasmon-axion conversion in external magnetic fields
Andrea Caputo^{1,2}, Alexander J. Millar^{2,3,7} and Edoardo Vitagliano^{4,5}

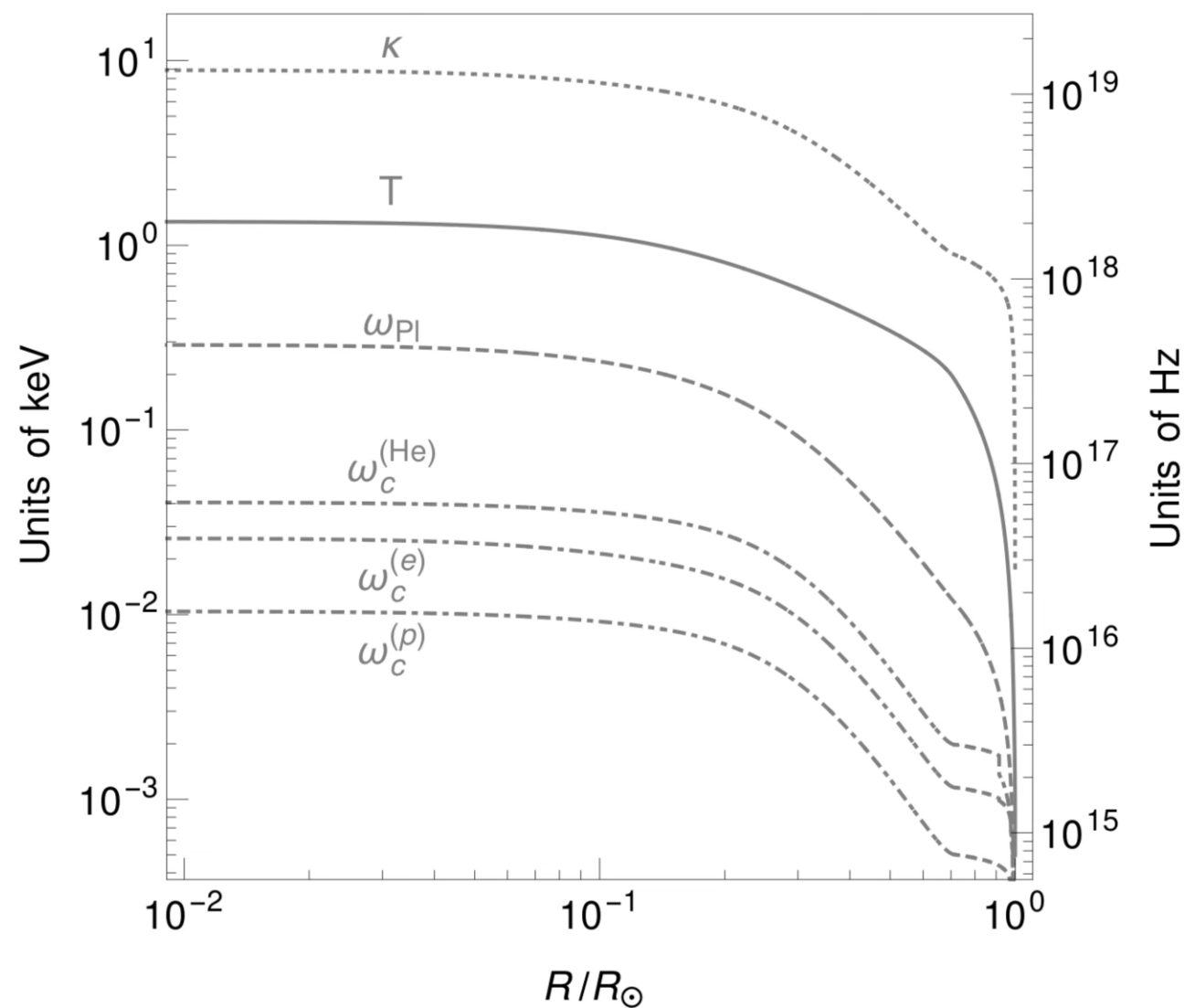
PHYSICAL REVIEW D 102, 123024 (2020)
Production of axionlike particles from photon conversions in large-scale solar magnetic fields
Ersilia Guarni¹, Pierluca Carenza^{1,2}, Javier Galán³, Maurizio Giannotti⁴, and Alessandro Mirizzi^{1,2}

Axion emission by magnetic-field induced conversion of longitudinal plasmons
N. V. Mikheev
Department of Theoretical Physics, Yaroslavl State University, Sovetskaya 14, Yaroslavl 150000, Russia
G. Raffelt
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany
L. A. Vasilievskaya

Solar axion flux from the axion-electron coupling
Javier Redondo (Munich U., ASC and Munich, Max Planck Inst.)
Oct 2, 2013

The (inverse) Gertsenhstein Effect

García-Cely, Ringwald, 2024



Infrared Photons and Gravitons*

STEVEN WEINBERG†

Department of Physics, University of California, Berkeley, California

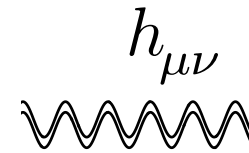
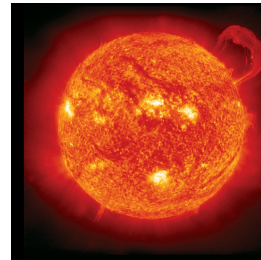
(Received 1 June 1965)

The solar gravitational radiation power is then

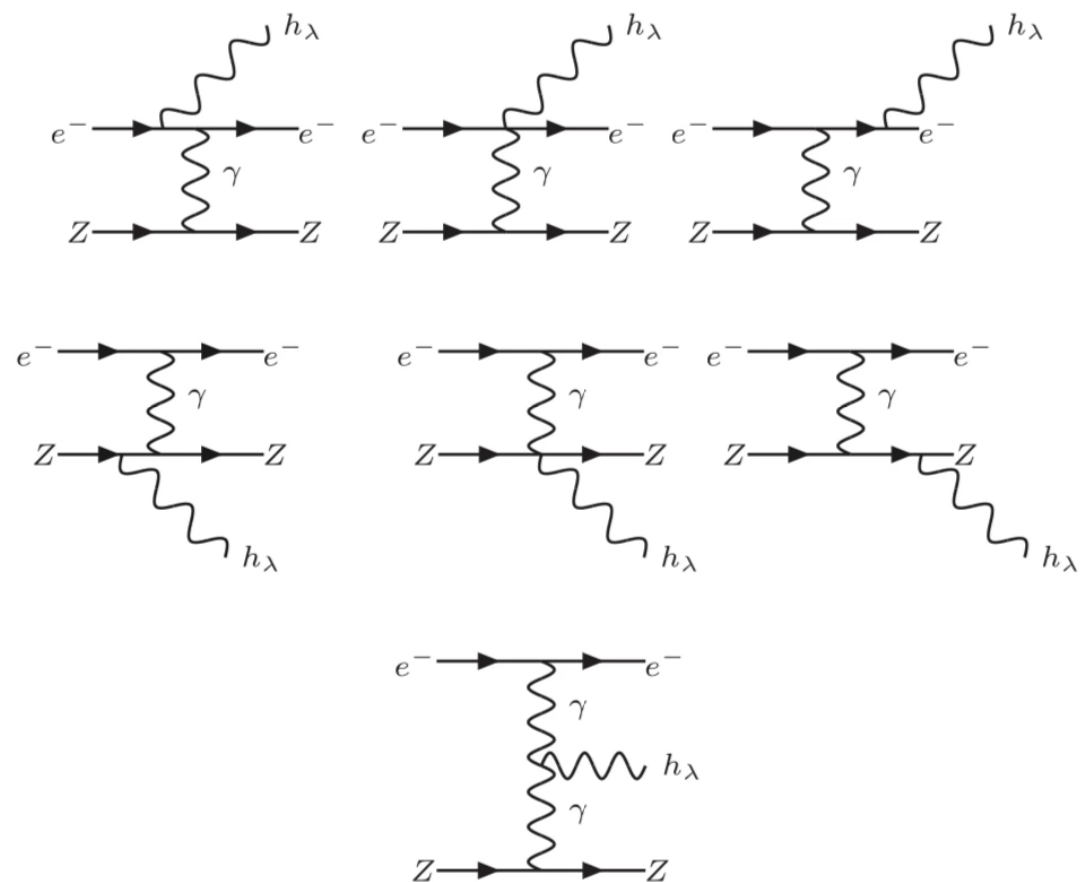
$$P_\odot \simeq 6 \times 10^{14} \text{ erg/sec.} \quad (4.24)$$

Solar emission of gravitational waves

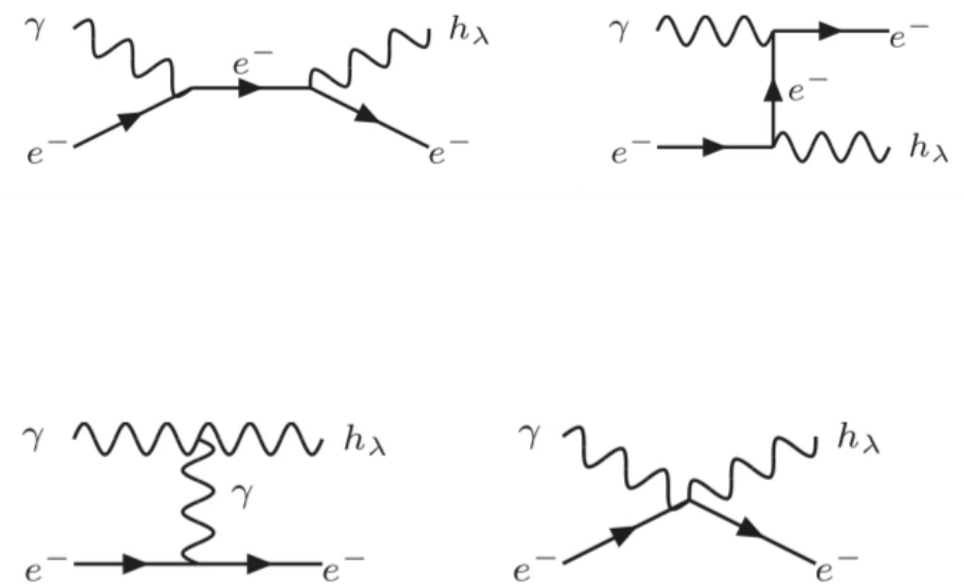
Microscopic contribution: graviton emission



Bremsstrahlung



Photoproduction



García-Cely, Ringwald, 2024

Solar emission of gravitational waves

Macroscopic contribution: Hydrodynamical fluctuations

García-Cely, Ringwald, 2024

Gravitational wave background from Standard Model physics: qualitative features

J. Ghiglieri¹ and M. Laine¹

Published 16 July 2015 • [Journal of Cosmology and Astroparticle Physics](#), Volume 2015, July 2015

Citation J. Ghiglieri and M. Laine JCAP07(2015)022

DOI 10.1088/1475-7516/2015/07/022

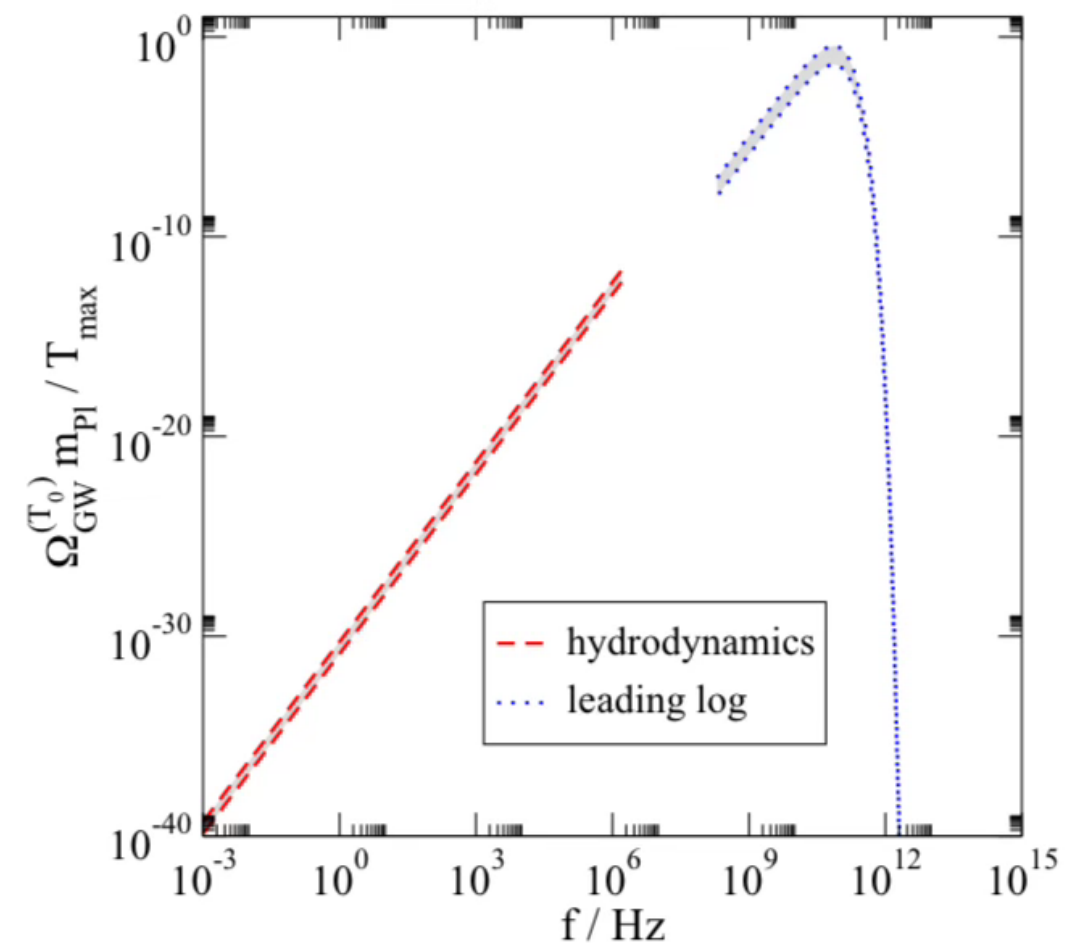
 Article PDF

References ▾

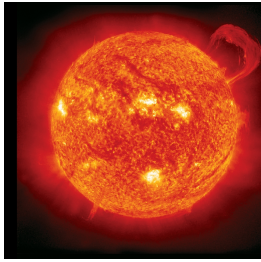
[+ Article and author information](#)

Abstract

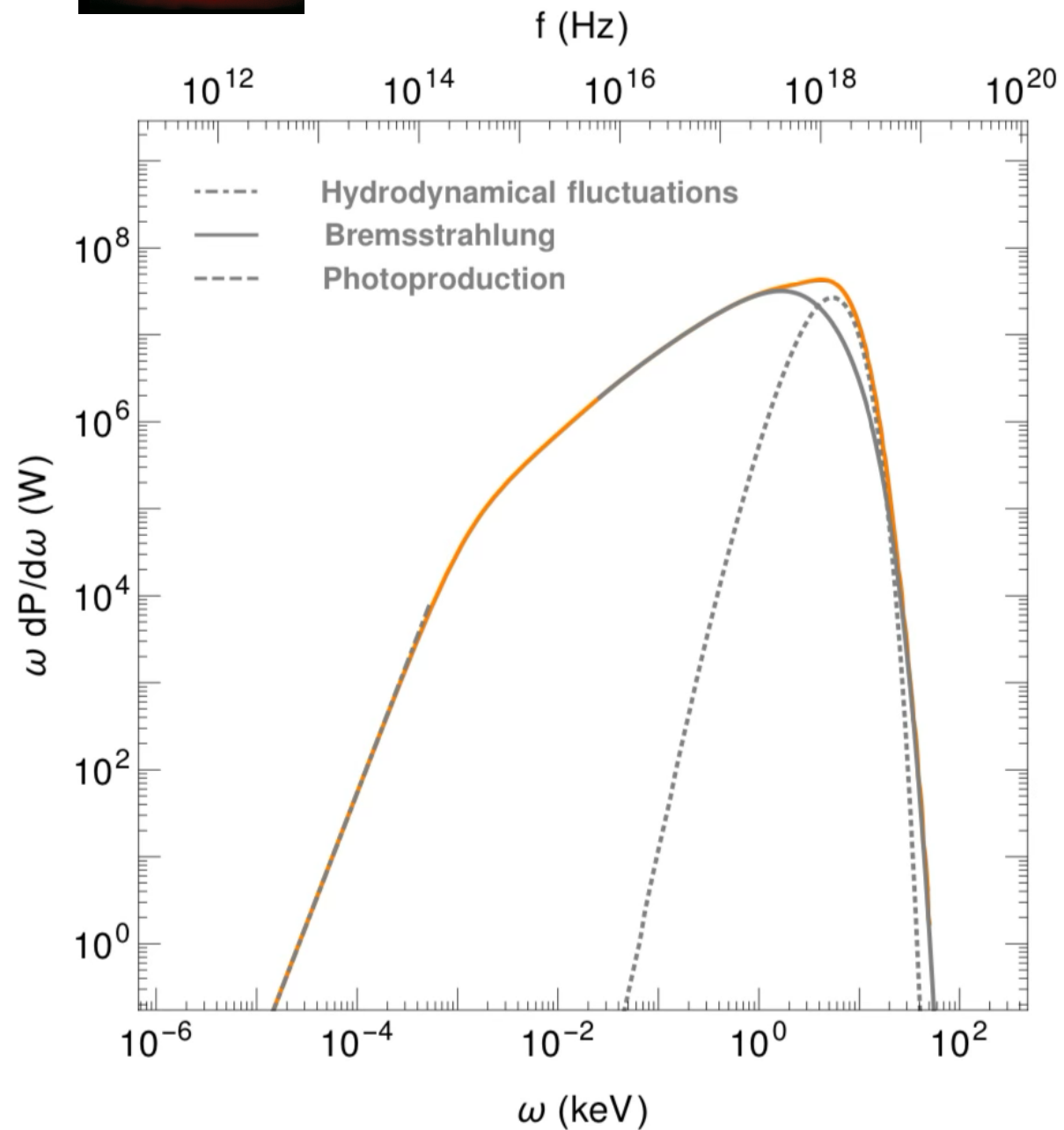
Because of physical processes ranging from microscopic particle collisions to macroscopic hydrodynamic fluctuations, any plasma in thermal equilibrium emits gravitational waves. For the largest wavelengths the emission rate is proportional to the shear viscosity of the plasma. In the Standard Model at



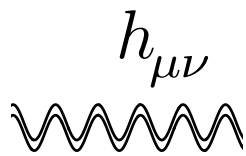
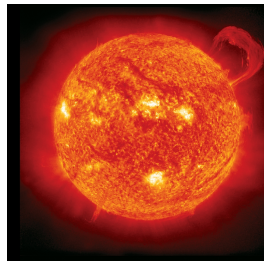
Solar gravitational waves



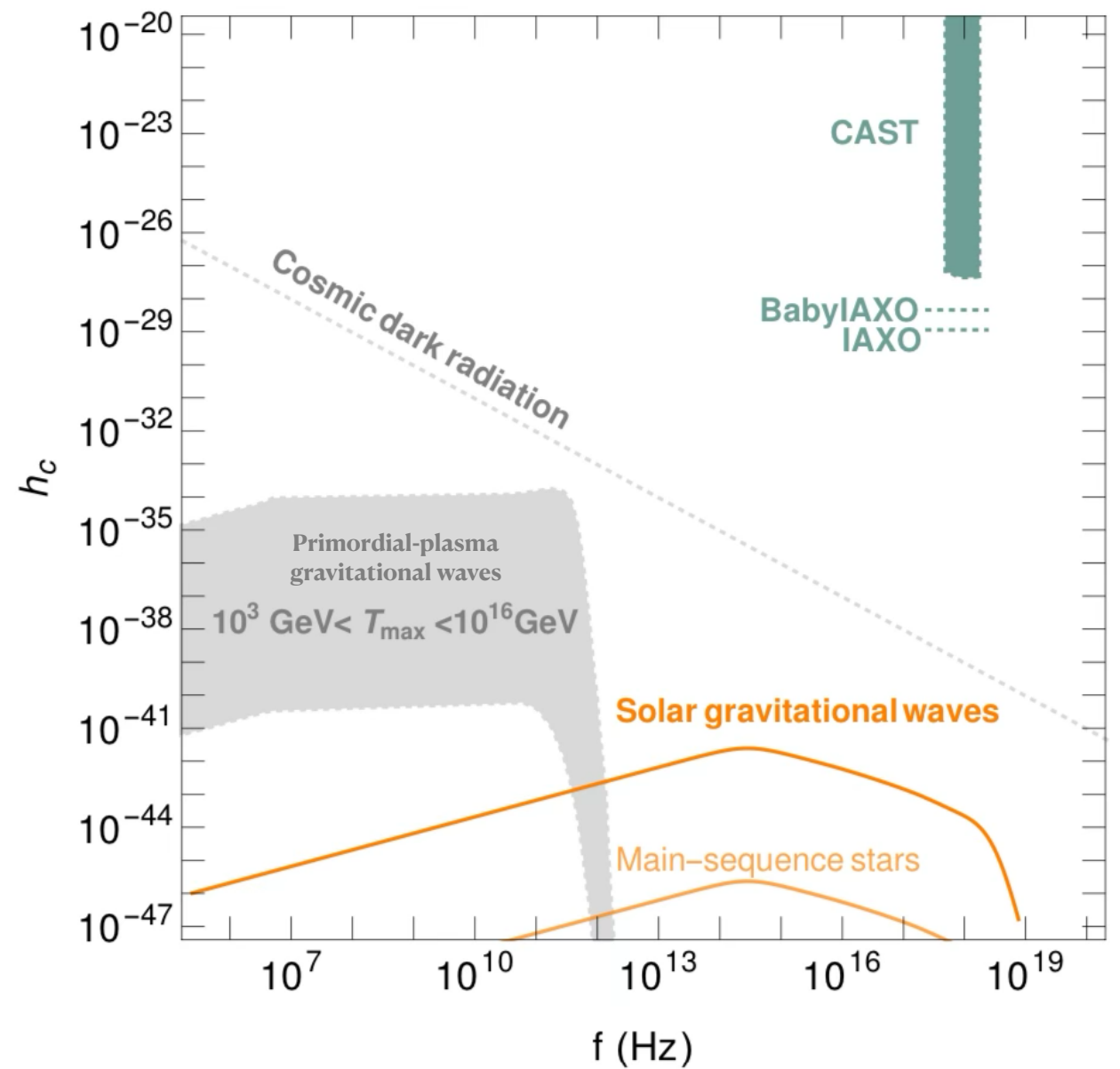
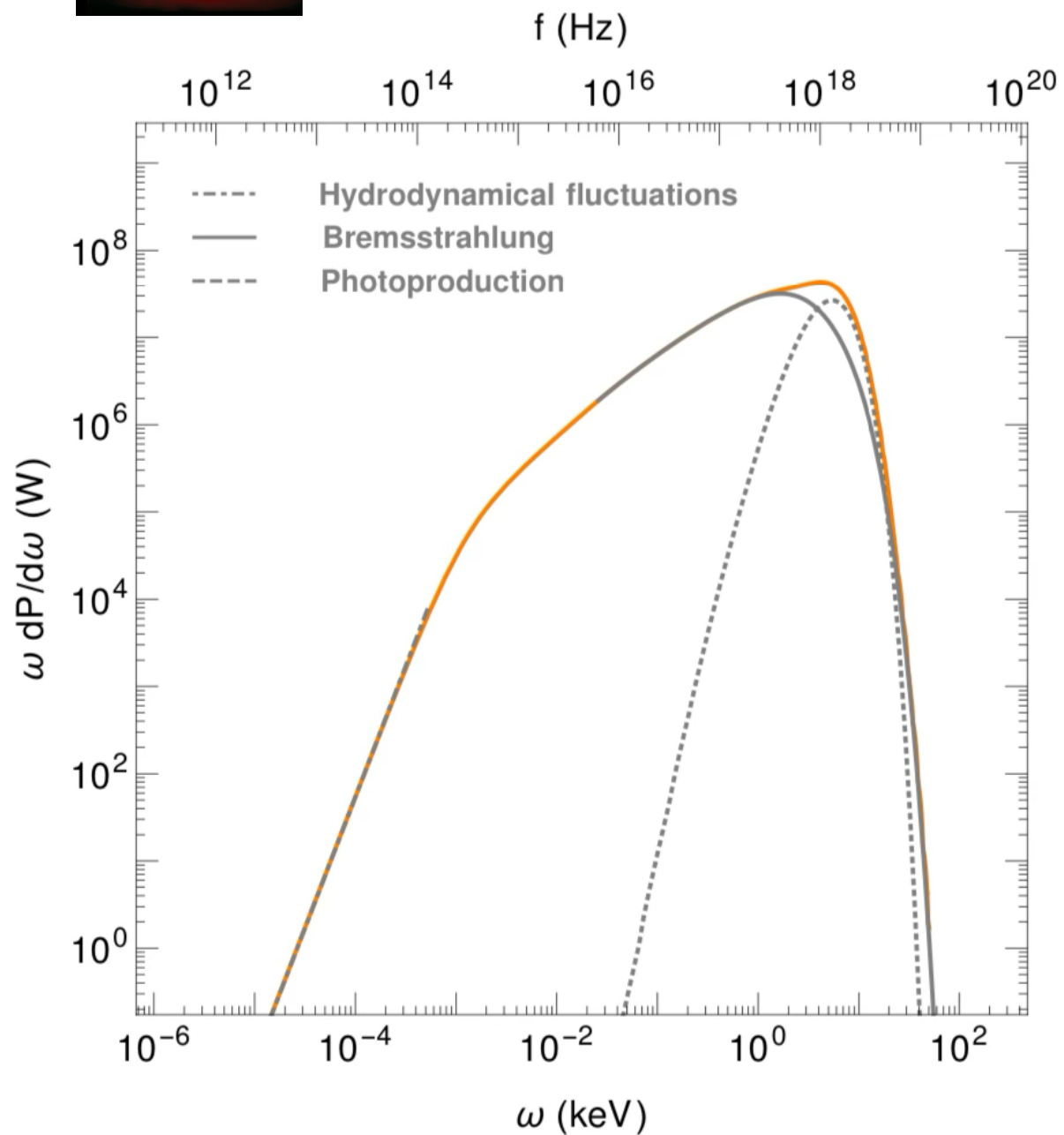
García-Cely, Ringwald, 2024



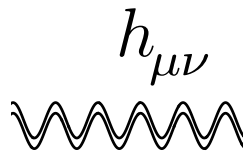
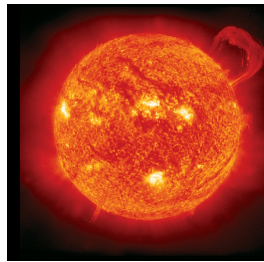
Solar gravitational waves



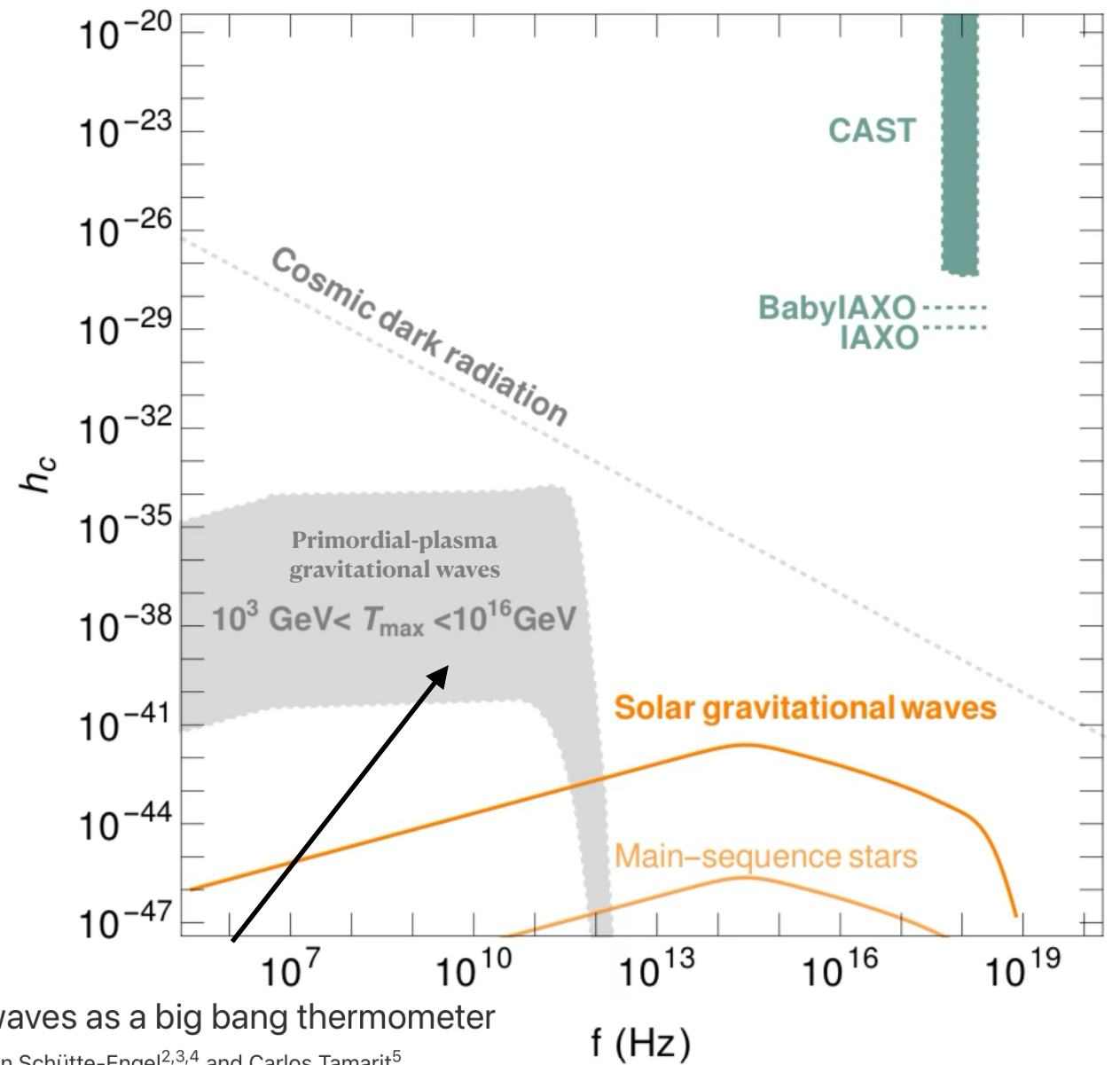
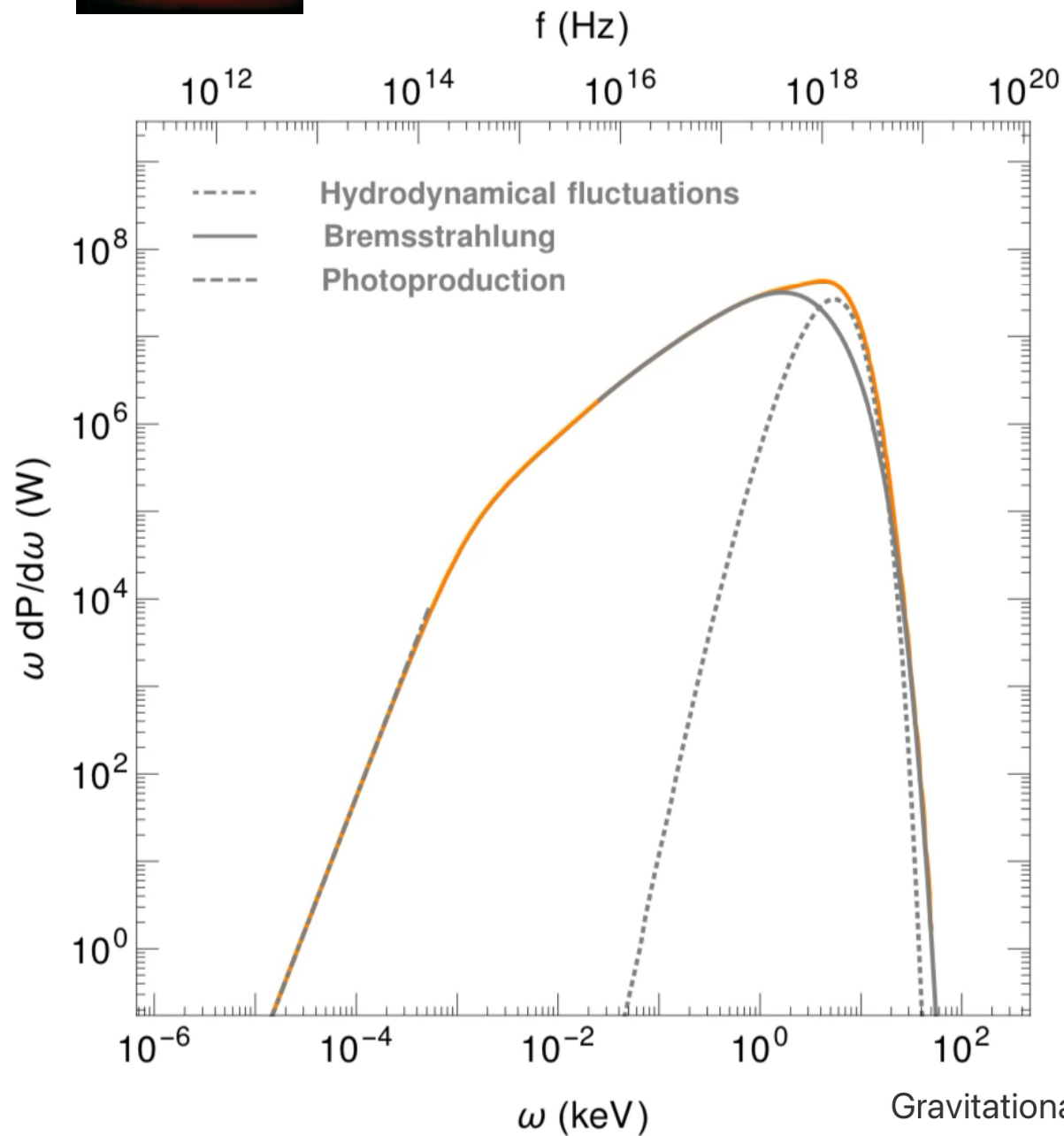
García-Cely, Ringwald, 2024



Solar gravitational waves



García-Cely, Ringwald, 2024



Gravitational waves as a big bang thermometer

Andreas Ringwald¹, Jan Schütte-Engel^{2,3,4} and Carlos Tamarit⁵

Published 17 March 2021 • © 2021 IOP Publishing Ltd and Sissa Medialab

[Journal of Cosmology and Astroparticle Physics, Volume 2021, March 2021](#)

Novel experimental approaches

Axion haloscopes and polarimetry techniques

How does it work?

Axions act as a source term to Maxwell's equations, effectively inducing an electromagnetic current.

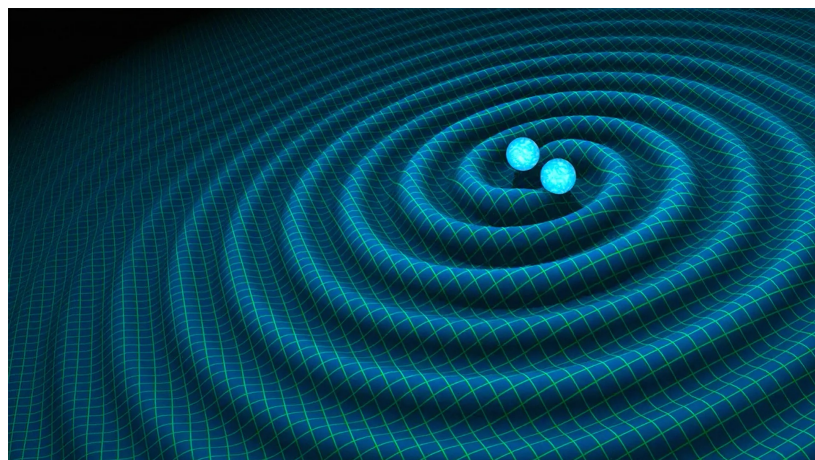
Sikivie, 1983

$$j^0 = -g_{a\gamma\gamma} \nabla a \cdot \mathbf{B}$$

$$\mathbf{j} = g_{a\gamma\gamma} (\nabla a \times \mathbf{E} + \partial_t a \mathbf{B})$$

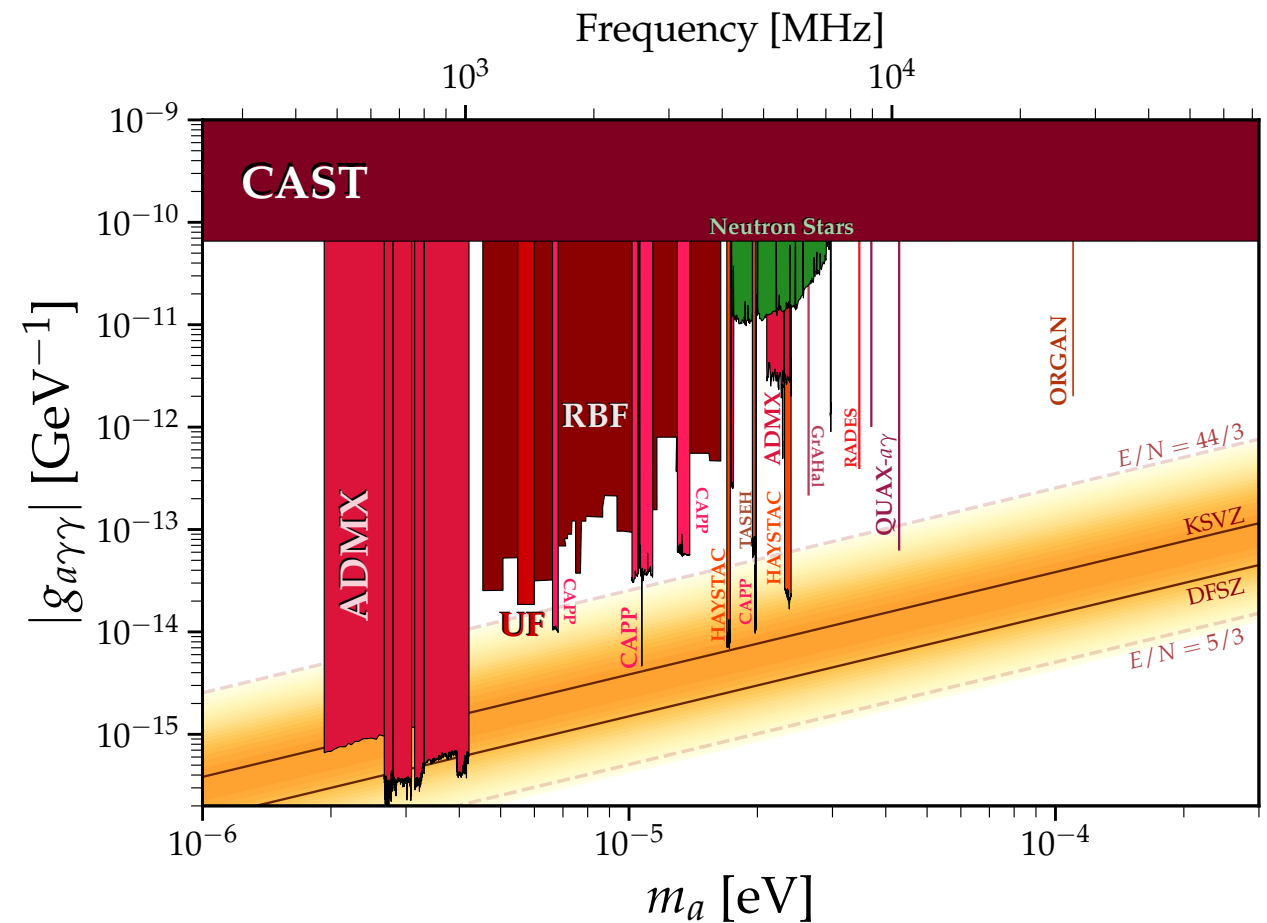
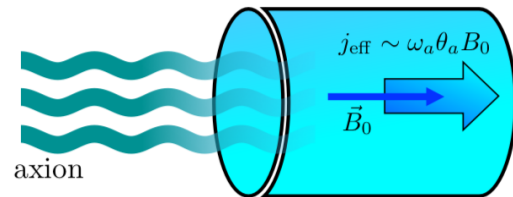
Gravitational waves act as a source term to Maxwell's equations, effectively inducing an electromagnetic current.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1$$



$$j_{\text{eff}}^\mu = \partial_\nu \left(-\frac{1}{2} h F^{\mu\nu} + F^{\mu\alpha} h^\nu{}_\alpha - F^{\nu\alpha} h^\mu{}_\alpha \right)$$

Haloscopes based on microwave cavities



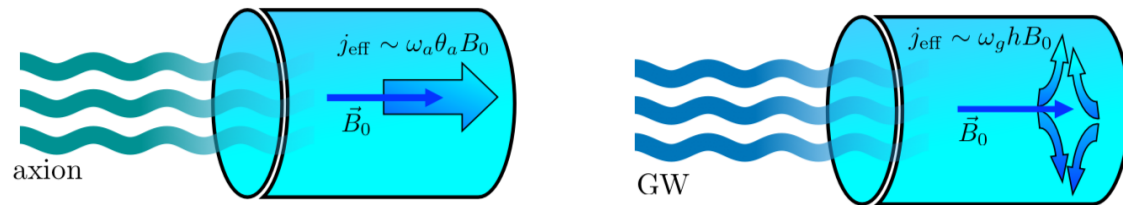
<https://github.com/cajohare/AxionLimits>

It resonates when the axion frequency matches one of the eigenmode frequencies

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) e_n(t) = - \frac{\int_{V_{\text{cav}}} d^3\mathbf{x} \mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\text{eff}}}{\int_{V_{\text{cav}}} d^3\mathbf{x} |\mathbf{E}_n|^2}$$

Eigenmodes $\mathbf{E}(\mathbf{x}, t) = \sum_n e_n(t) \mathbf{E}_n(\mathbf{x})$

Haloscopes based on microwave cavities



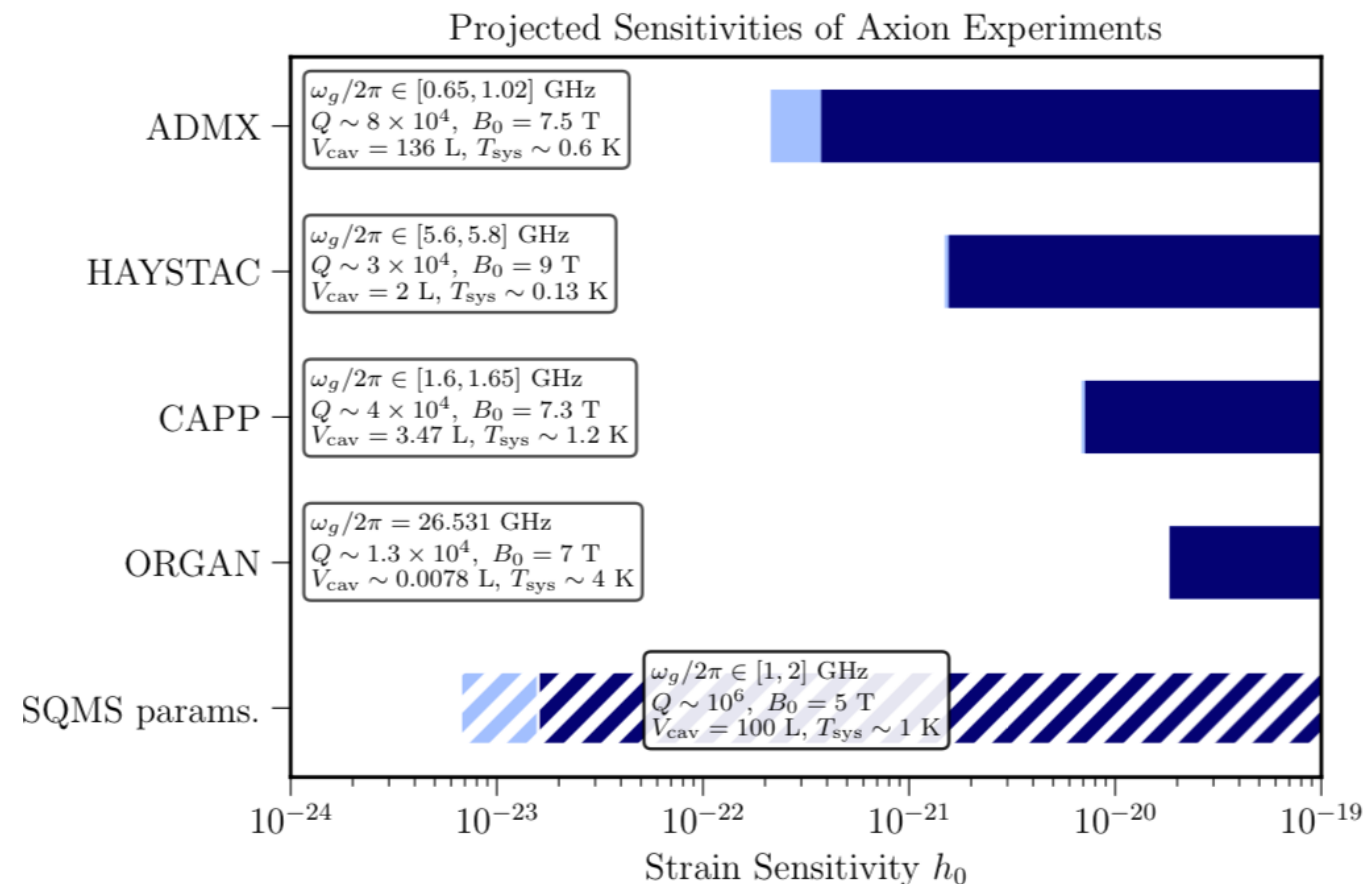
Detecting planetary-mass primordial black holes with resonant electromagnetic gravitational-wave detectors

Nicolas Herman, André Füzfa, Léonard Lehoucq, and Sébastien Clesse
Phys. Rev. D **104**, 023524 – Published 19 July 2021

It resonates when the GW frequency matches one of the eigenmode frequencies

Detecting high-frequency gravitational waves with microwave cavities

Asher Berlin, Diego Blas, Raffaele Tito D'Agnolo, Sebastian A. R. Ellis, Roni Harnik, Yonatan Kahn, and Jan Schütte-Engel
Phys. Rev. D **105**, 116011 – Published 17 June 2022



Towards realistic simulations

Study of a cubic cavity resonator for gravitational waves detection in the microwave frequency range

Pablo Navarro, Benito Gimeno, Juan Monzó-Cabrera, Alejandro Díaz-Morcillo, and Diego Blas
Phys. Rev. D **109**, 104048 – Published 14 May 2024

$$j_{\text{eff}}^{\mu} = \partial_{\nu} \left(-\frac{1}{2} h F^{\mu\nu} + F^{\mu\alpha} h^{\nu}_{\alpha} - F^{\nu\alpha} h^{\mu}_{\alpha} \right)$$

[Submitted on 30 Jul 2024 (v1), last revised 14 Aug 2024 (this version, v2)]

High-frequency gravitational waves detection with the BabyIAXO haloscopes

José Reina Valero, Jose R. Navarro Madrid, Diego Blas, Alejandro Díaz Morcillo, Igor García Irastorza, Benito Gimeno, Juan Monzó Cabrera

We present the first analysis using RADES-BabyIAXO cavities as detectors of high-frequency gravitational waves (HFGWs). In particular, we discuss two configurations for distinct frequency ranges of HFGWs: Cavity 1, mostly sensitive at a frequency range of 252.8 - 333.2 MHz, and Cavity 2, at 2.504 - 3.402 GHz, which is a scaled down version of Cavity 1. We find that Cavity 1 will reach sensitivity to strains of the HFGWs of order $h_1 \sim 10^{-21}$, while Cavity 2 will reach $h_2 \sim 10^{-20}$. These represent the best estimations of the RADES-BabyIAXO cavities as HFGWs detectors, showing how this set-up can produce groundbreaking results in axion physics and HFGWs simultaneously.

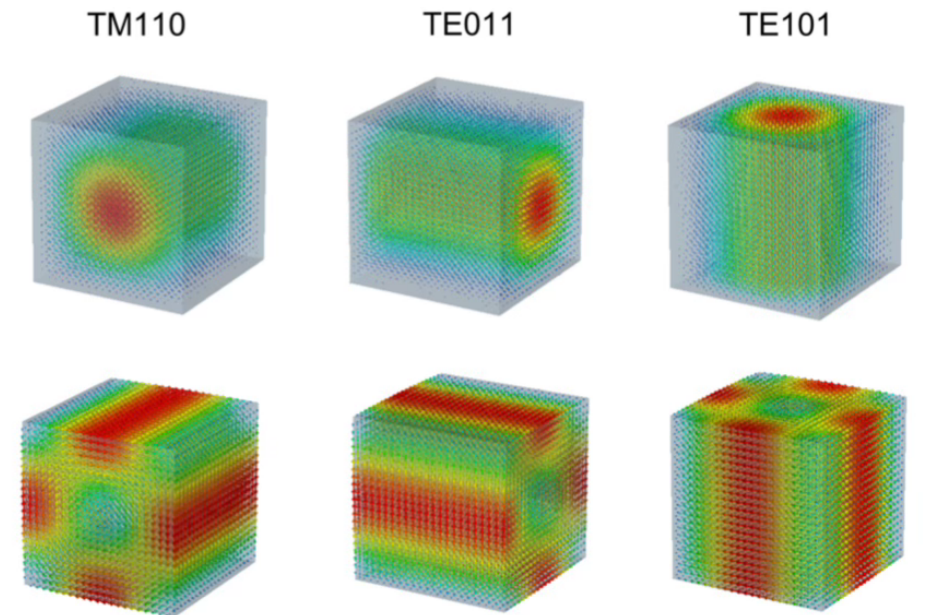
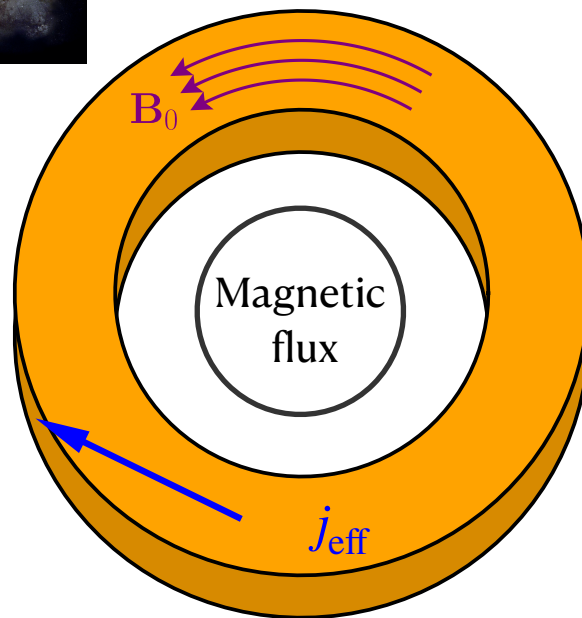
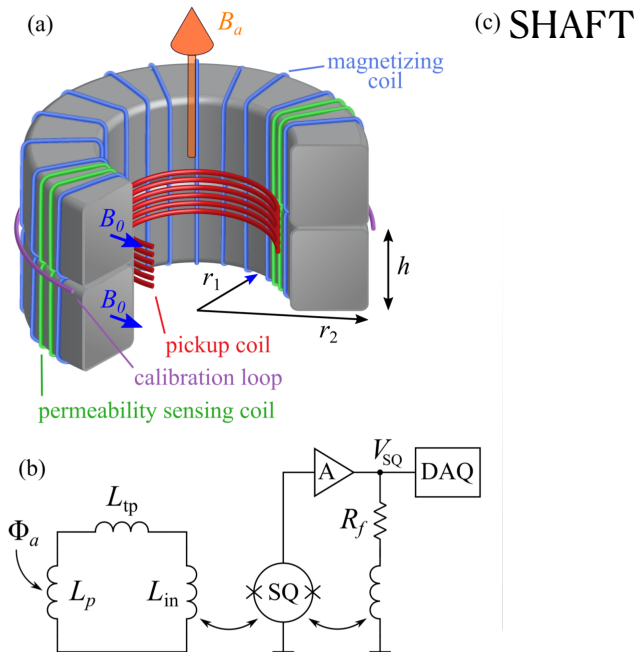


FIG. 3. Electric (up) and magnetic (down) field distributions for the three considered modes TM_{110} , TE_{011} and TE_{101} of cavity C1. The electromagnetic field distributions are identical in the other cavities C2 and C3. The red hue represents the largest values for the modules of electric and magnetic fields, while the blue color represents the lowest values for these modules. The remaining colors in this illustration represent intermediate levels between minima and maxima.

Haloscopes based on lumped-element detectors



$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \underbrace{g_{a\gamma\gamma} \partial_t a \mathbf{B}_0}_{j_{eff}}$$



PRL 117, 141801 (2016)

PHYSICAL REVIEW LETTERS

week ending
30 SEPTEMBER 2016

Broadband and Resonant Approaches to Axion Dark Matter Detection

Yonatan Kahn,^{1,*} Benjamin R. Safdi,^{2,†} and Jesse Thaler^{2,‡}

¹Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

²Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 3 March 2016; published 30 September 2016)

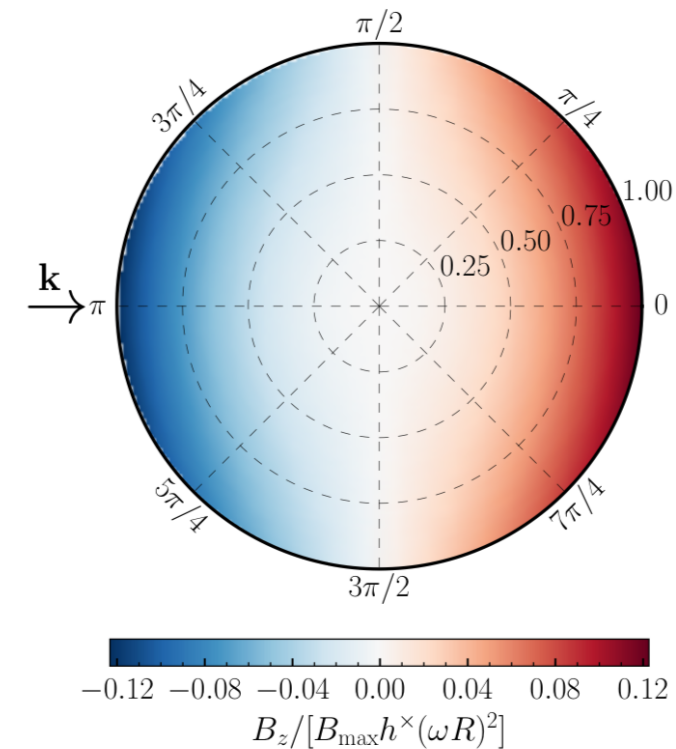
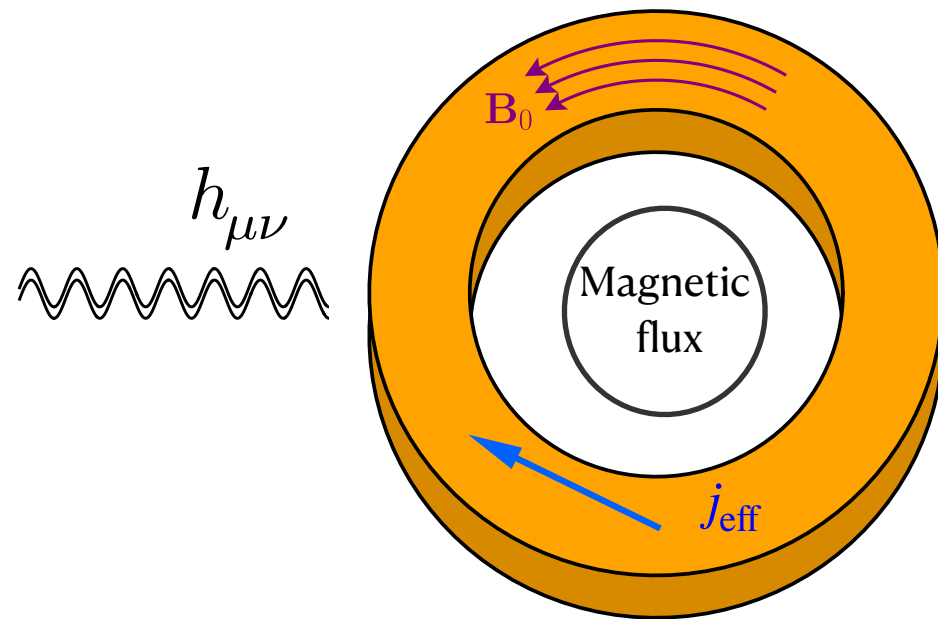
Search for axion-like dark matter with ferromagnets

Alexander V. Gramolin¹, Deniz Aybas^{1,2}, Dorian Johnson¹, Janos Adam¹ and Alexander O. Sushkov^{1,2,3}

The electromagnetic fields produced by the axion drive a current through a pickup coil

Haloscopes based on lumped-element detectors

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



$$\Phi \approx \frac{ie^{-i\omega t}}{16\sqrt{2}} h^{\times} \omega^3 B_{\max} \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

Only one polarization

(selection rules)

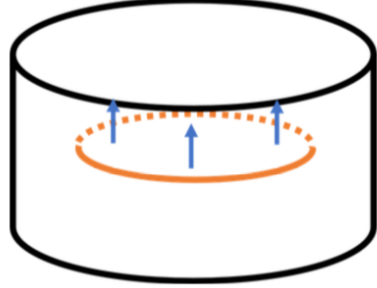
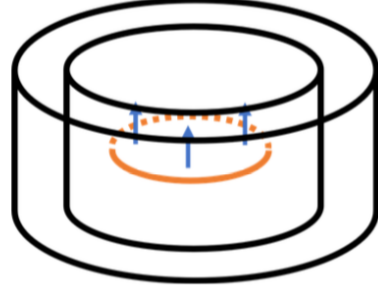
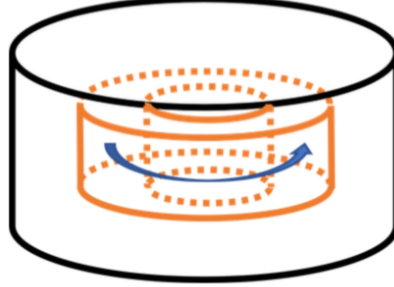
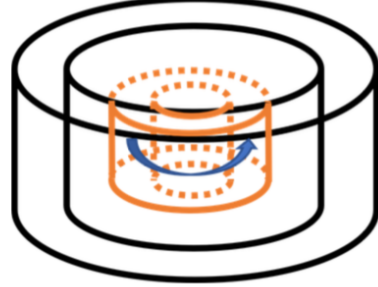
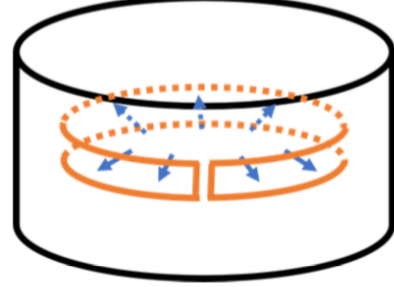
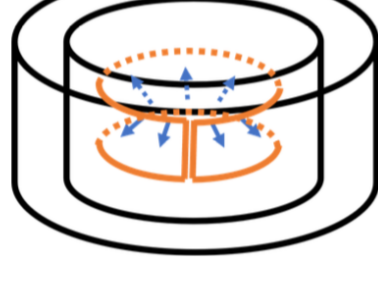
$$\Phi_{\text{axions}} \approx e^{-i\omega t} g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} B_{\max} \pi r^2 R$$

Selection rules

Type of external field

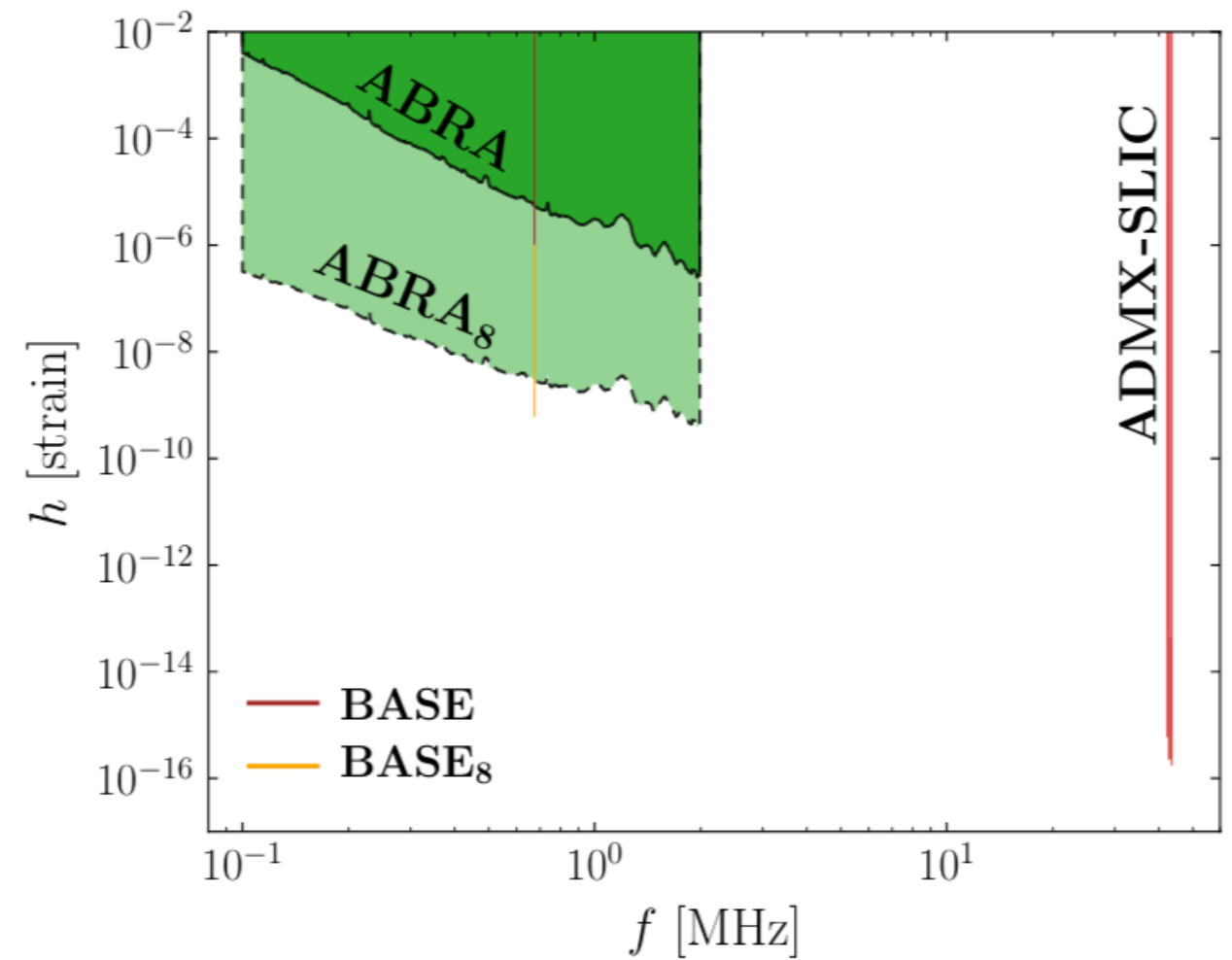
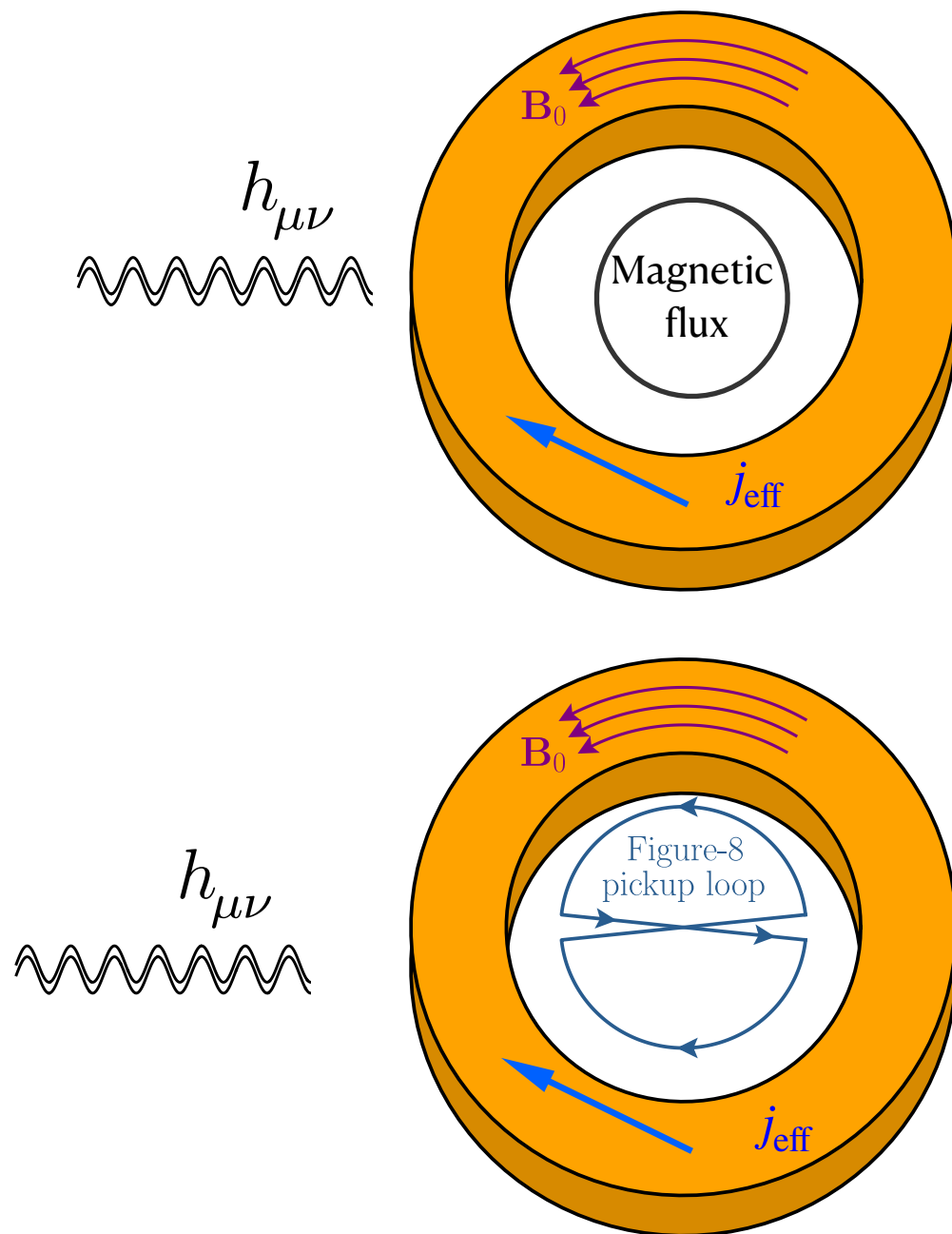
Domcke, CGC, Lee, Rodd, 2023

Pickup loop orientation

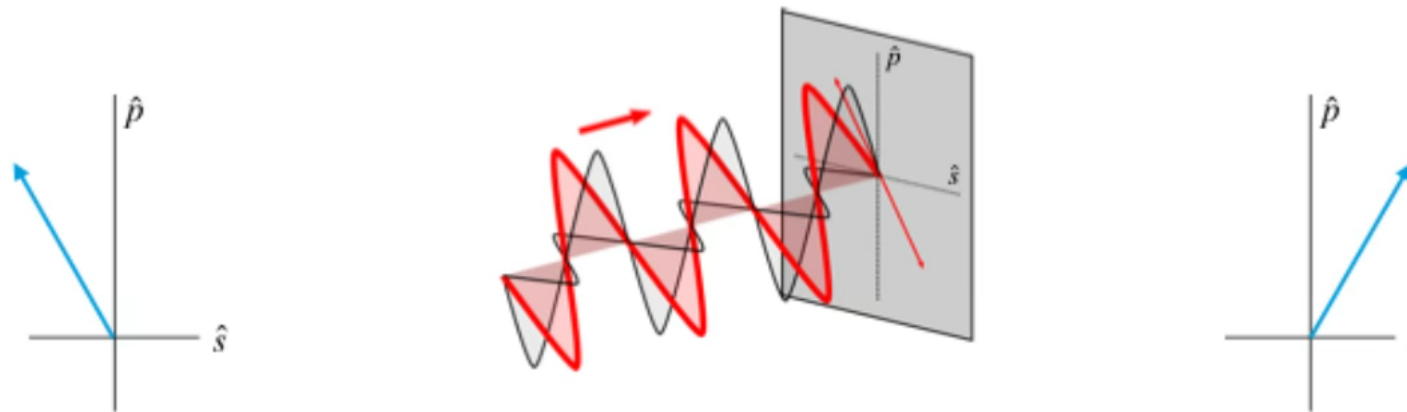
	Solenoid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_z$	Toroid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_\phi$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_z$	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{e^{-i\omega t}}{48\sqrt{2}} h^+ \omega^2 B_0 s_{\theta_h}^2 \pi r^2 (11r^2 + 14R^2 + 16R^2 \ln \frac{R}{H})$ 	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{48\sqrt{2}} h^\times \omega^3 B_{\max} \pi r^2 a R (a + 2R) s_{\theta_h}^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\phi$	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{96\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 l (12R^2 - 5r^2) s_{\theta_h}^2$ 	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{3e^{-i\omega t}}{4\sqrt{2}} h^+ \omega^2 B_{\max} \frac{\pi r^2 a R l (a + 2R)}{H^2} s_{\theta_h}^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\rho$	$h^+, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{96\sqrt{2}} h^+ B_0 \omega^3 c_{\theta_h} s_{\theta_h}^2 \times \pi r^2 l (3l^2 - 22(r^2 + 2R^2) - 36R^2 \ln \frac{R}{H})$ 	$h^\times, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^4]$ $\Phi_h = \frac{e^{-i\omega t}}{32\sqrt{2}} h^\times \omega^4 B_{\max} \pi r^2 a R l (a + 2R) c_{\theta_h} s_{\theta_h}^2$ 

Selection rules

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



Axion birefringence



Geometrical
optics limit



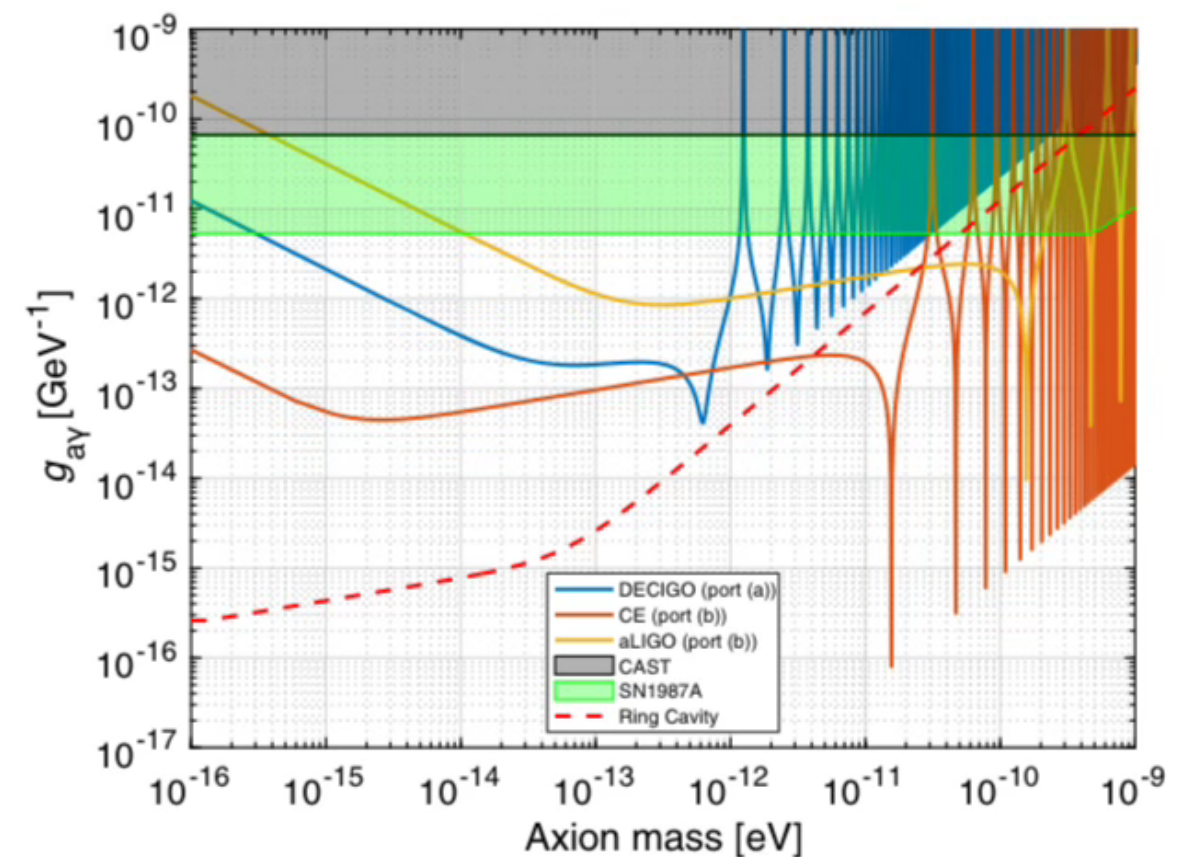
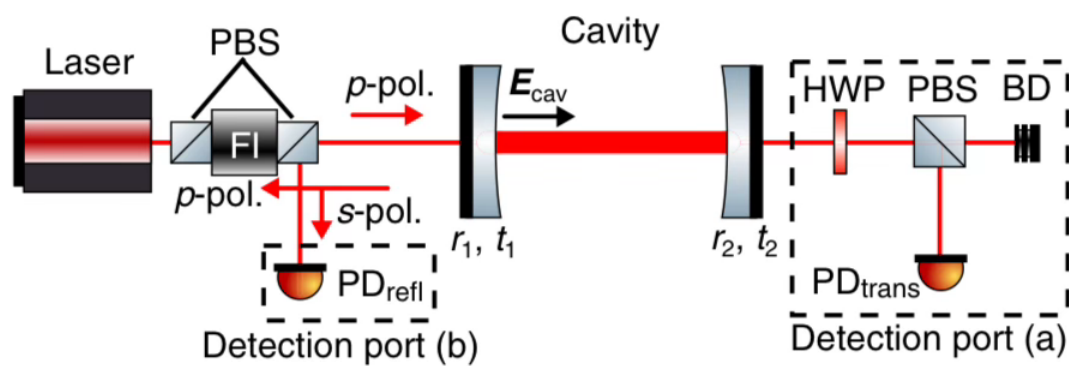
$$\frac{d\mathbf{e}}{dt} = -\frac{1}{2}g_{a\gamma\gamma}\dot{a}(t) \hat{\mathbf{k}} \times \mathbf{e}$$

Axion birefringence

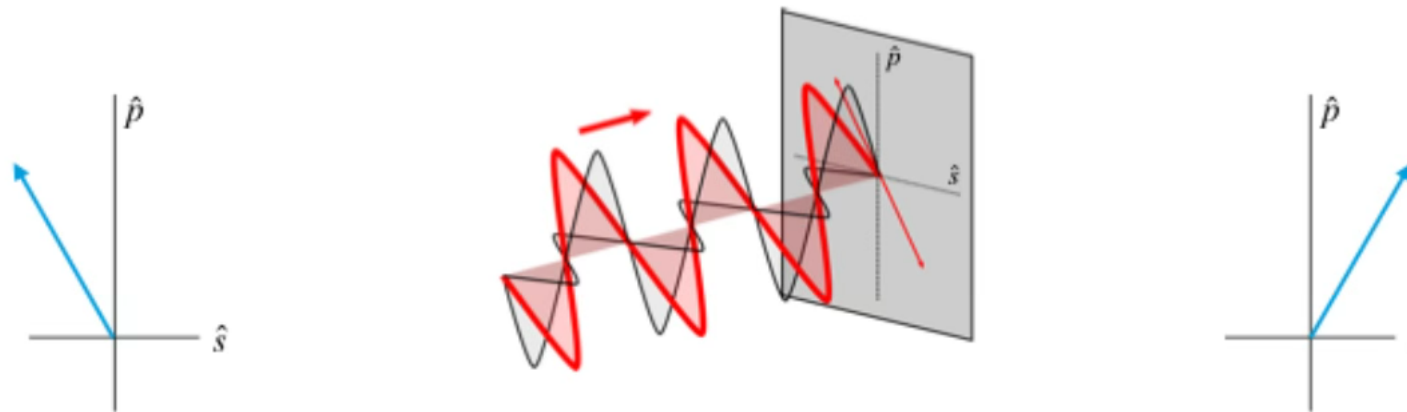
PHYSICAL REVIEW LETTERS **123**, 111301 (2019)

Axion Dark Matter Search with Interferometric Gravitational Wave Detectors

Koji Nagano¹, Tomohiro Fujita^{2,3}, Yuta Michimura⁴, and Ippei Obata¹



Axion birefringence



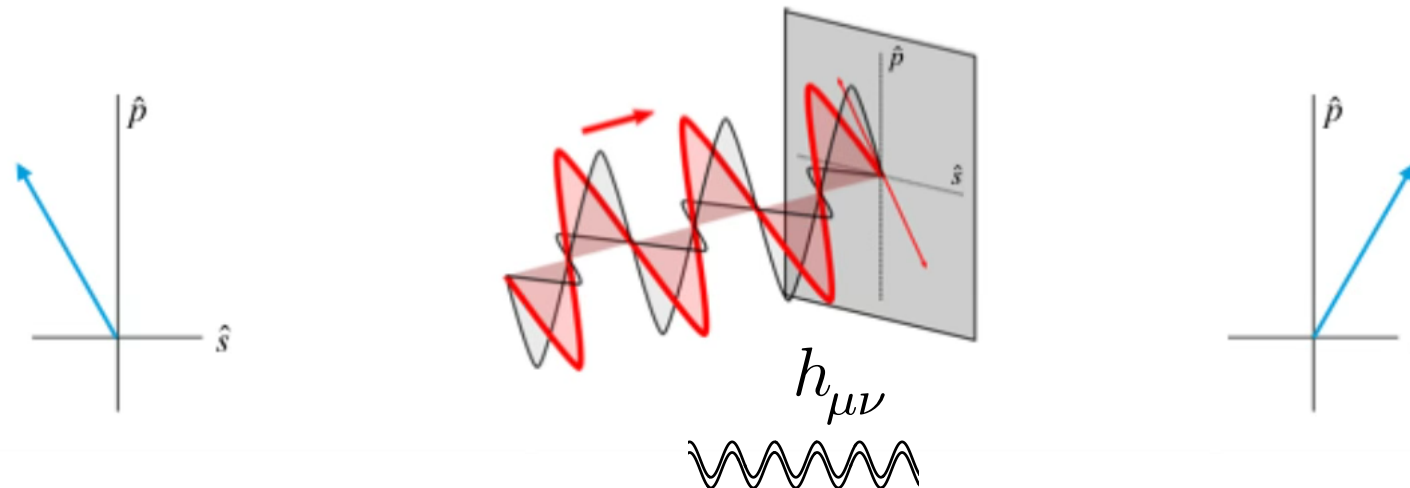
Geometrical
optics limit



$$\frac{d\mathbf{e}}{dt} = -\frac{1}{2}g_{a\gamma\gamma}\dot{a}(t) \hat{\mathbf{k}} \times \mathbf{e}$$

Gravitational Faraday effect

work in progress with Luca Marsili, Aaron Spector and Andreas Ringwald



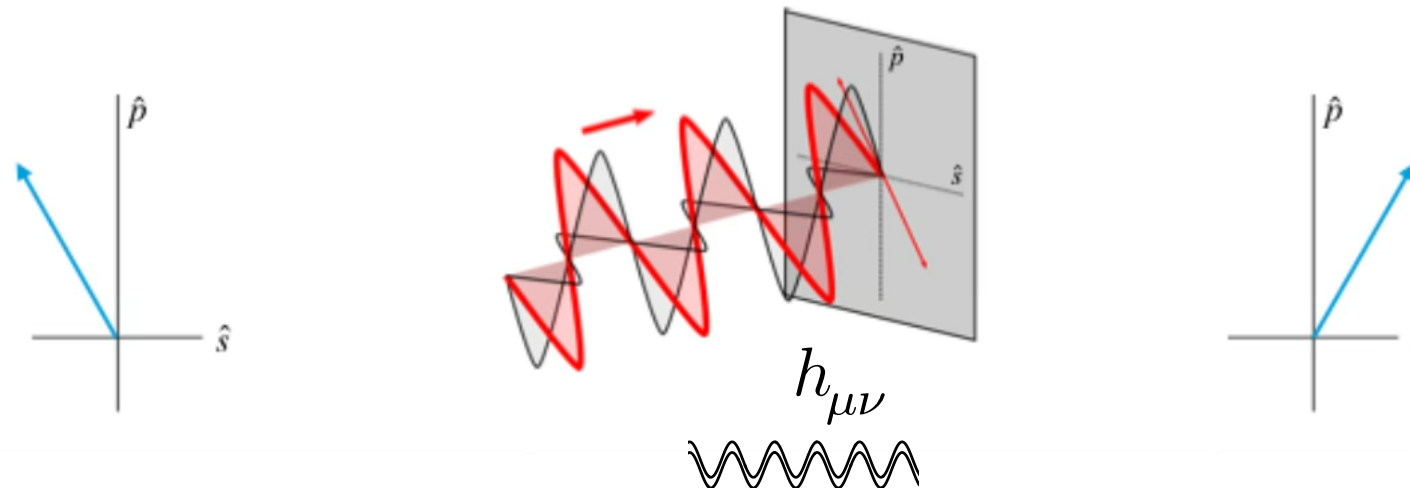
Geometrical
optics limit



$$\frac{de^i}{dt} = \left(\Gamma_{\rho\lambda}^0 \frac{dx^i}{dt} - \Gamma_{\rho\lambda}^i \right) \frac{dx^\rho}{dt} e^\lambda$$

Gravitational Faraday effect

work in progress with Luca Marsili, Aaron Spector and Andreas Ringwald



ALPs experiment at DESY



For GWs coming from the zenith

- The cross polarization has the same cavity response function as axions
- The plus polarization decouples (selection rules)

PRELIMINARY

Competitive bounds at high frequencies

Conclusions

The techniques developed for detecting **axion dark matter** could potentially be used to discover new sources of **gravitational waves**.

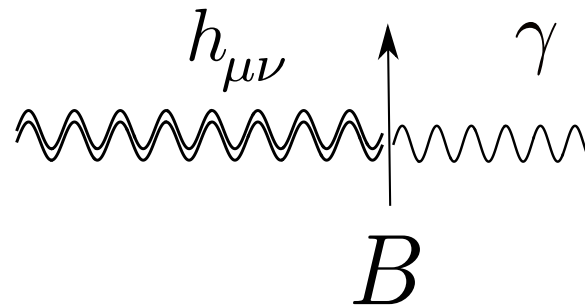
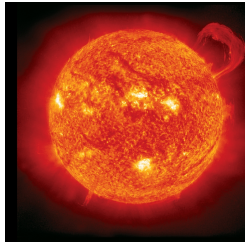
Different experimental proposals have coalesced on a **strain sensitivity of 10^{-22} for MHz GWs**, still orders of magnitude away from signals of the early Universe.

Lots of room for improvement because experiments are not optimized for gravitational wave searches.

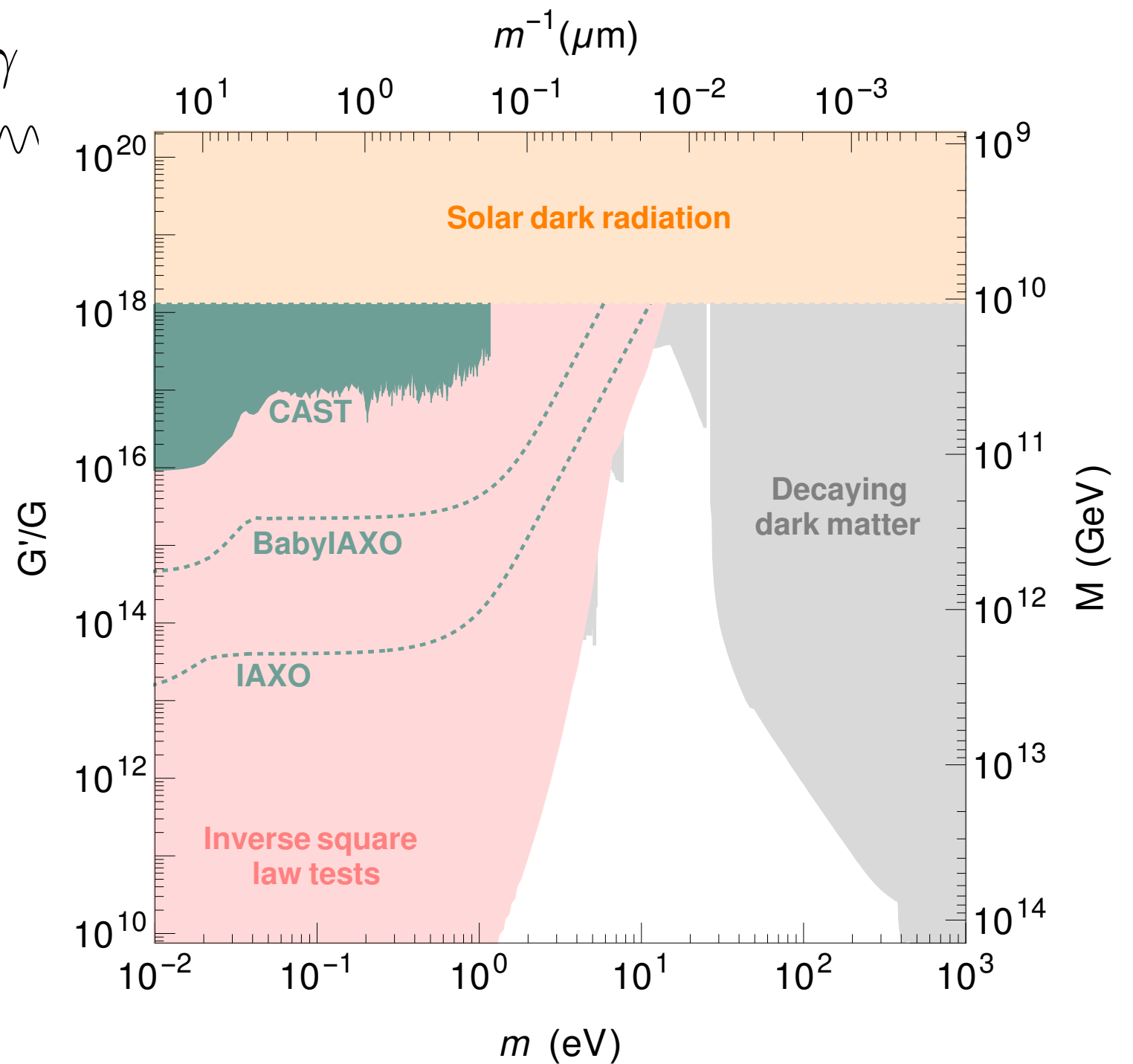
Indeed, theoretical studies indicate that **selection rules** limit the detectability of gravitational waves in highly symmetric detectors.

Simple modifications of readout (such as the figure-8 pickup loop) can overcome this limitation

Solar emission of light spin-2 particles



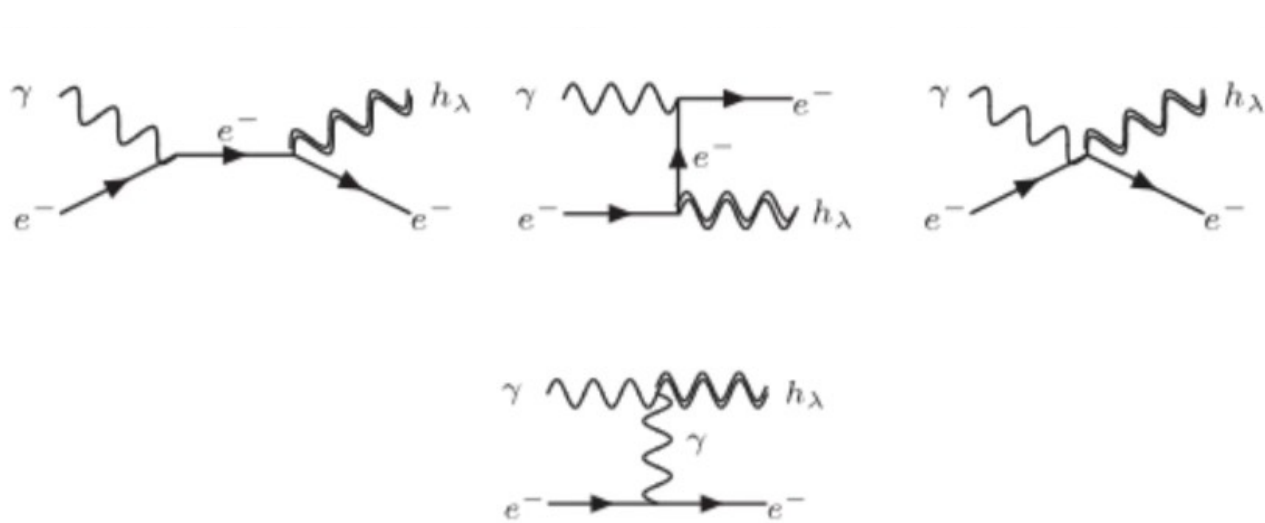
CGC, Ringwald **PRELIMINARY**



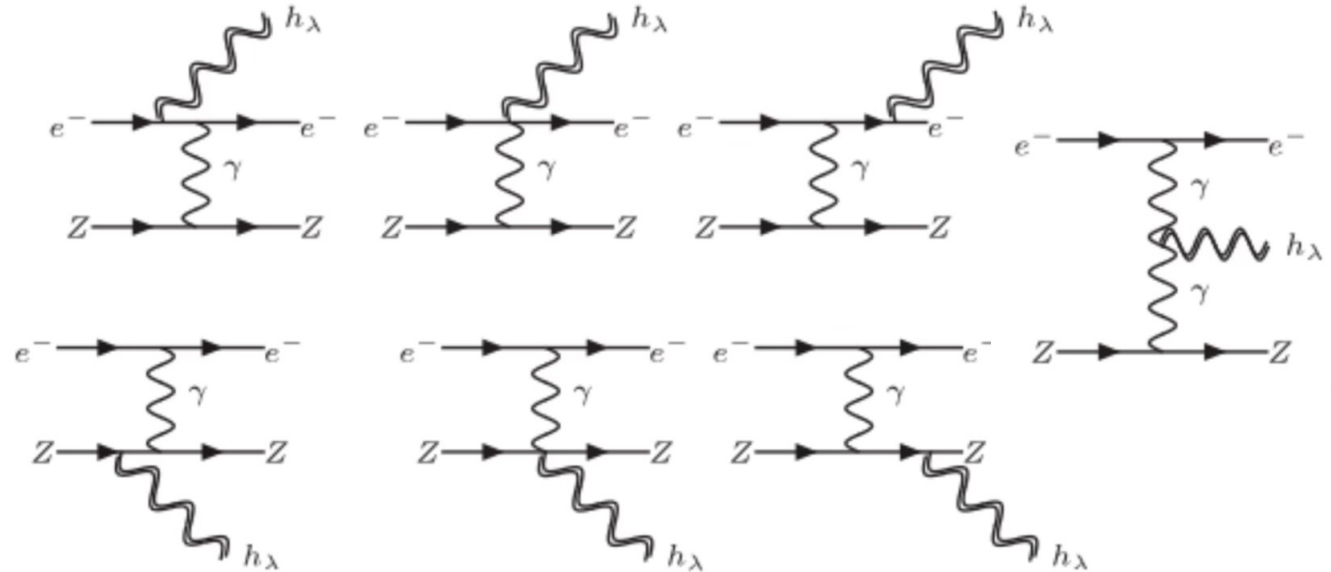
Solar emission of light spin-2 particles

Collision	λ	$\frac{d\Gamma}{d\omega dV}$	CGC, Ringwald PRELIMINARY
Photo-production $\gamma Z \rightarrow Z h_\lambda$	± 2 ± 1 0	$\frac{n_\gamma n_Z G' Z^2 \alpha \pi \delta(\omega - p_i) \int d\cos\theta \cot^2 \frac{\theta}{2} [1 + \cos^2 \theta] F(\theta)}{0}$ $\frac{\frac{4}{3} n_\gamma n_Z G' Z^2 \alpha \pi \delta(\omega - p_i) \int d\cos\theta \cot^2 \frac{\theta}{2} \sin^4 \frac{\theta}{2} F(\theta)}{0}$	$F(\theta) = \frac{(2\omega \sin \frac{\theta}{2})^2}{\kappa^2 + (2\omega \sin \frac{\theta}{2})^2}$
Bremsstrahlung $eZ \rightarrow eZ h_\lambda$	± 2 ± 1 0	$\frac{32 n_e n_Z G' Z^2 \alpha^2 p_i}{15\omega} \left(\frac{1}{m_e} + \frac{1}{m_Z} \right) \left(3(1 + \xi^2) L + 10\xi + \mathcal{O}(\xi_s^2) \right)$ $\frac{16 n_e n_Z G' Z^2 \alpha^2 p_i}{45\omega} \left(\frac{1}{m_e} + \frac{1}{m_Z} \right) \left((1 + \xi^2) L + 30\xi + \mathcal{O}(\xi_s^2) \right)$	$\xi = \frac{p_f}{p_i}, \quad \xi_s = \frac{\kappa}{p_i}$ $\omega = E_i(1 - \xi^2)$
Bremsstrahlung $ee \rightarrow ee h_\lambda$	± 2 ± 1 0	$\frac{16 n_e^2 G' \alpha^2 p_i}{15\omega m_e} \left(\left(6(1 + \xi^2) - \frac{3(1 - \xi^2)^4 + 7(1 - \xi^4)^2}{2(1 + \xi^2)^3} \right) L + 20\xi - \frac{6\xi(1 + \xi^4)}{(1 + \xi^2)^2} + \mathcal{O}(\xi_s^2) \right)$ $\frac{16 n_e^2 G' \alpha^2 p_i}{15\omega m_e} \left(\left(\frac{1}{3}(1 + \xi^2) - \frac{(1 - \xi^2)^4 + 29(1 - \xi^4)^2}{12(1 + \xi^2)^3} \right) L + \frac{29\xi}{3} + \frac{2\xi^3}{3(1 + \xi^2)^2} + \mathcal{O}(\xi_s^2) \right)$	$L = \log \sqrt{\frac{(1 + \xi)^2 + \xi_s^2}{(1 - \xi)^2 + \xi_s^2}}$

1. Photoproduction



2. Bremsstrahlung



Selection rules

Domcke, CGC, Lee, Rodd, 2023

Write down the detector response matrix for a wave coming from an arbitrary direction, and impose **cylindrical symmetry** for both external magnetic field and loop:

Selection Rule 1: For an instrument with azimuthal symmetry, $\Phi_h \propto h^+$ at $\mathcal{O}[(\omega L)^2]$

Selection Rule 2: For an instrument with azimuthal symmetry, the flux is proportional to either h^+ or $h^\times$, but not both. This holds to all orders in (ωL) .

Selection Rule 3: For an instrument with full cylindrical symmetry, Φ_h will contain only even or odd powers of ω .

Proper detector frame

The coordinate system closely matches the intuitive description of an Earth-based laboratory

Fermi, 1922

Manasse and Misner, 1963

Ni and Zimmermann, 1978

- Coordinates given by ideal rigid rulers

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu \text{ for } dx^\mu = (0, dr \hat{\mathbf{r}})$$

- The gravitational wave acts as a Newtonian force.

If negligible, the static fields applied in experiments remain static in the presence of GWs.

- Crucial for haloscopes

Berlin et al 2022

Excitation of mechanical modes

The proper detector frame closely matches the intuitive description of an Earth-based laboratory

Fermi, 1922

Manasse and Misner, 1963

Ni and Zimmermann, 1978

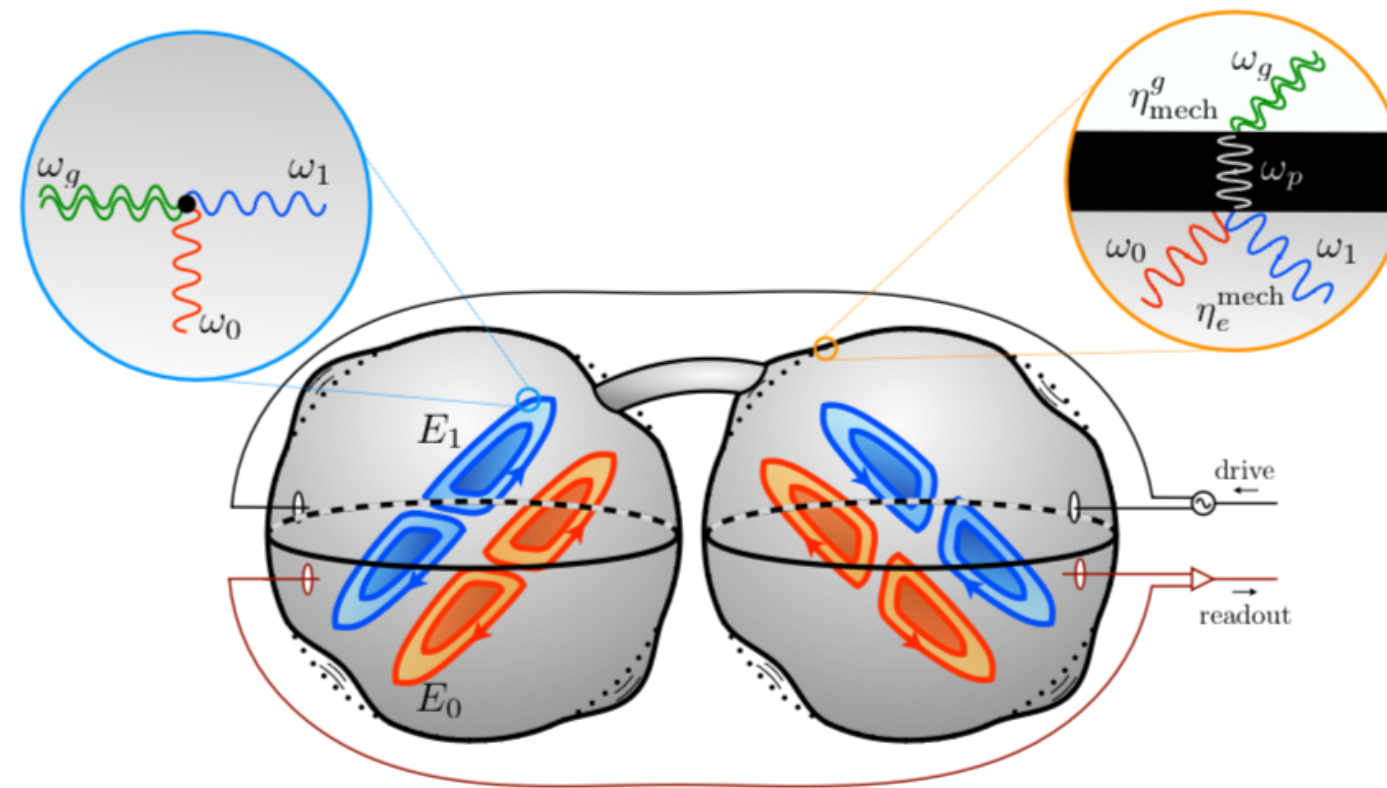
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- The gravitational wave acts as a Newtonian force.
If negligible, the static fields applied in experiments remain static in the presence of GWs.

Berlin et al 2022

Excitation of mechanical modes



- The gravitational wave acts as a Newtonian force.
If not negligible, coupling of the mechanical modes can play an important role (this is certainly the case at frequencies above the first mechanical resonance)
- This can enhance the sensitivity

Berlin et al [2022](#)