

# Gravitational wave detection with haloscopes

**Galileo Galilei Institute**

**Axions across boundaries between Particle Physics, Astrophysics,  
Cosmology and forefront Detection Technologies**

**June 6, 2023**

**Camilo García Cely**

*Ramón y Cajal Researcher*



# Based on

## Novel Search for High-Frequency Gravitational Waves with Low-Mass Axion Haloscopes

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd  
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022

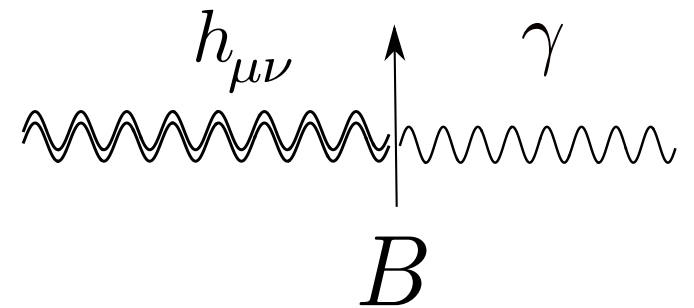
## Symmetries and Selection Rules: Optimising Axion Haloscopes for Gravitational Wave Searches

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Valerie Domcke,<sup>1</sup> Camilo Garcia-Cely,<sup>2</sup> Sung Mook Lee,<sup>1,3</sup> Nicholas L. Rodd<sup>1</sup>

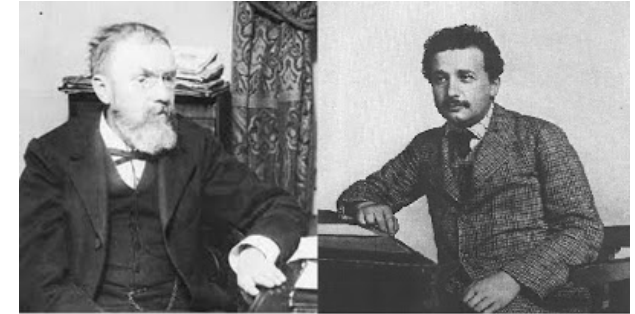
# Outline

- Why high-frequency gravitational waves and ideas to detect them
- Detection in haloscopes
- Selection rules
- Conclusions



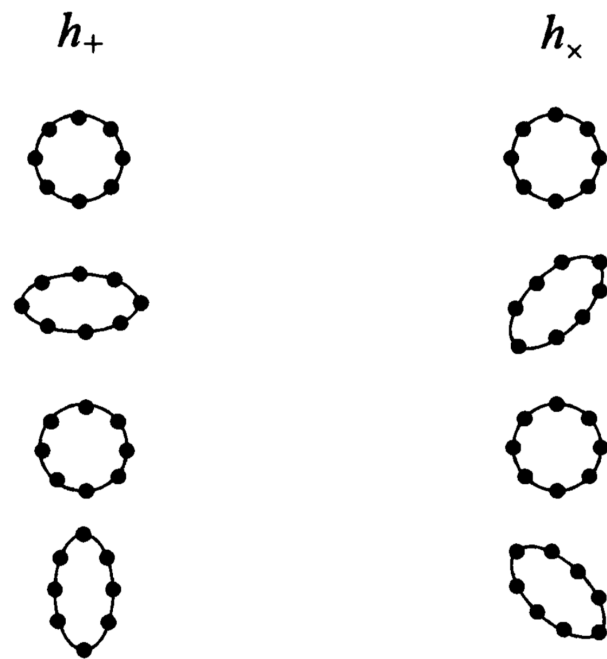
# Gravitational waves

- Speculation by Poincaré (1905)
- Einstein provided a firm theoretical background for them (1916)



$$\square h_{\mu\nu} = -16\pi G T_{\mu\nu}$$

wave equation  
describing two  
polarization modes



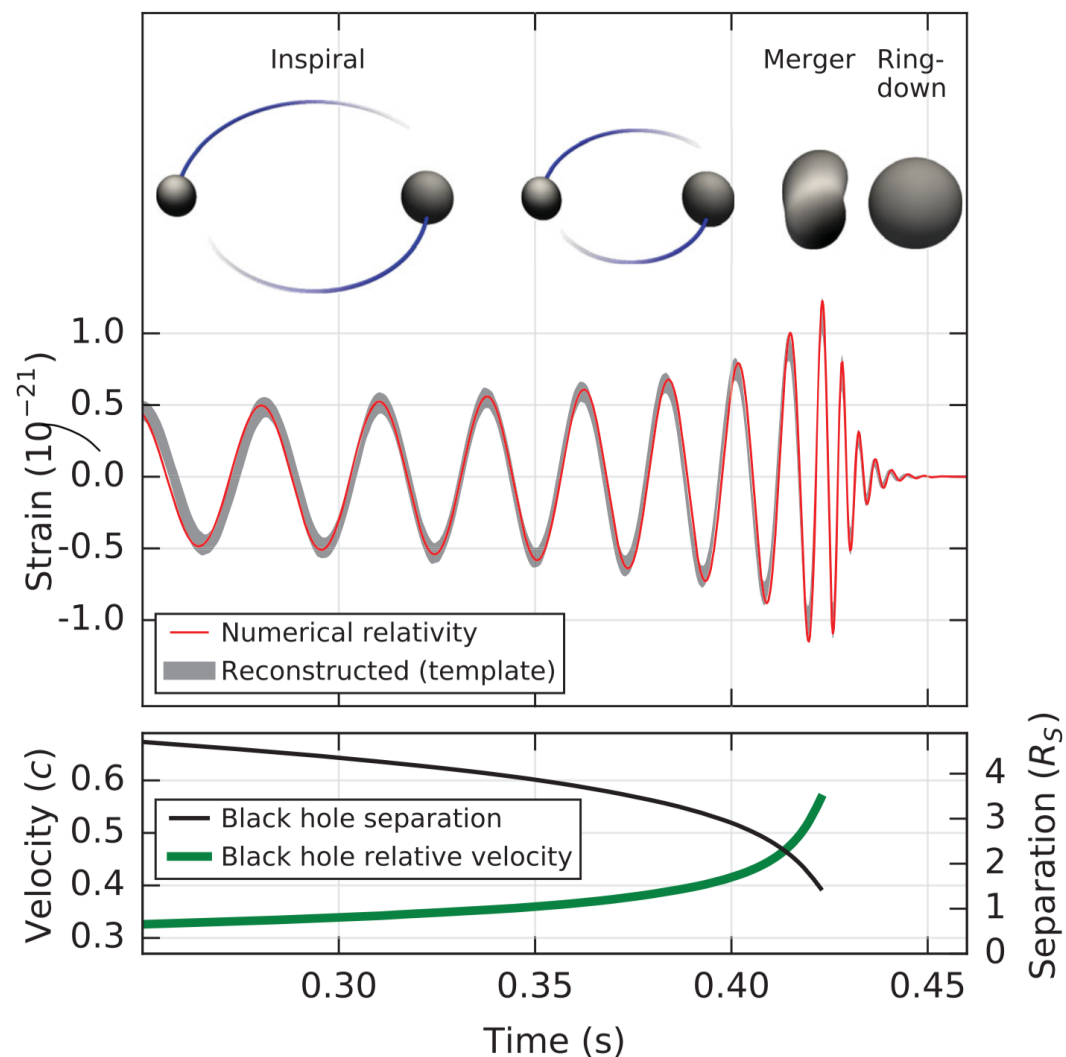
The deformation of a ring of test masses  
due to the different polarization



# Gravitational waves

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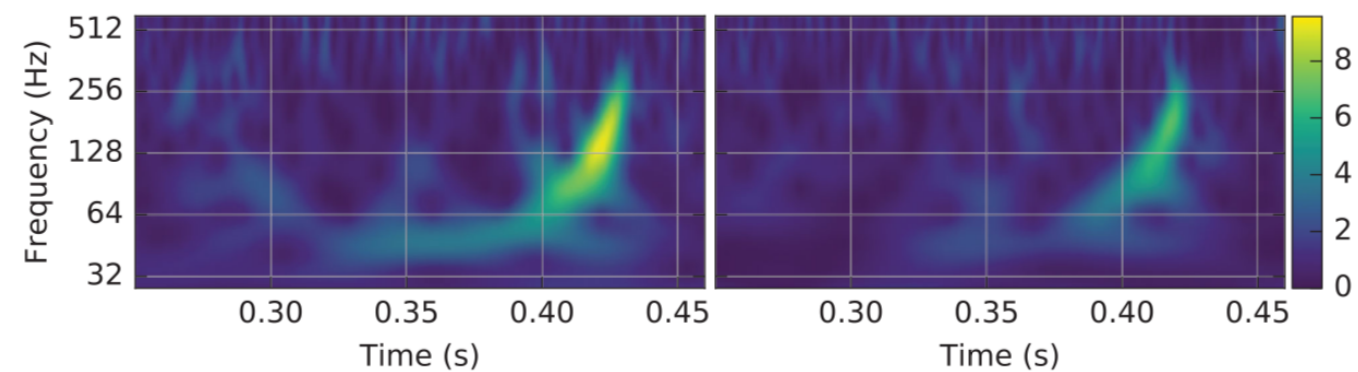
PRL 116, 061102 (2016)

PHYSICAL REVIEW LETTERS

12 FEBRUARY 2016

## Observation of Gravitational Waves from a Binary Black Hole Merger

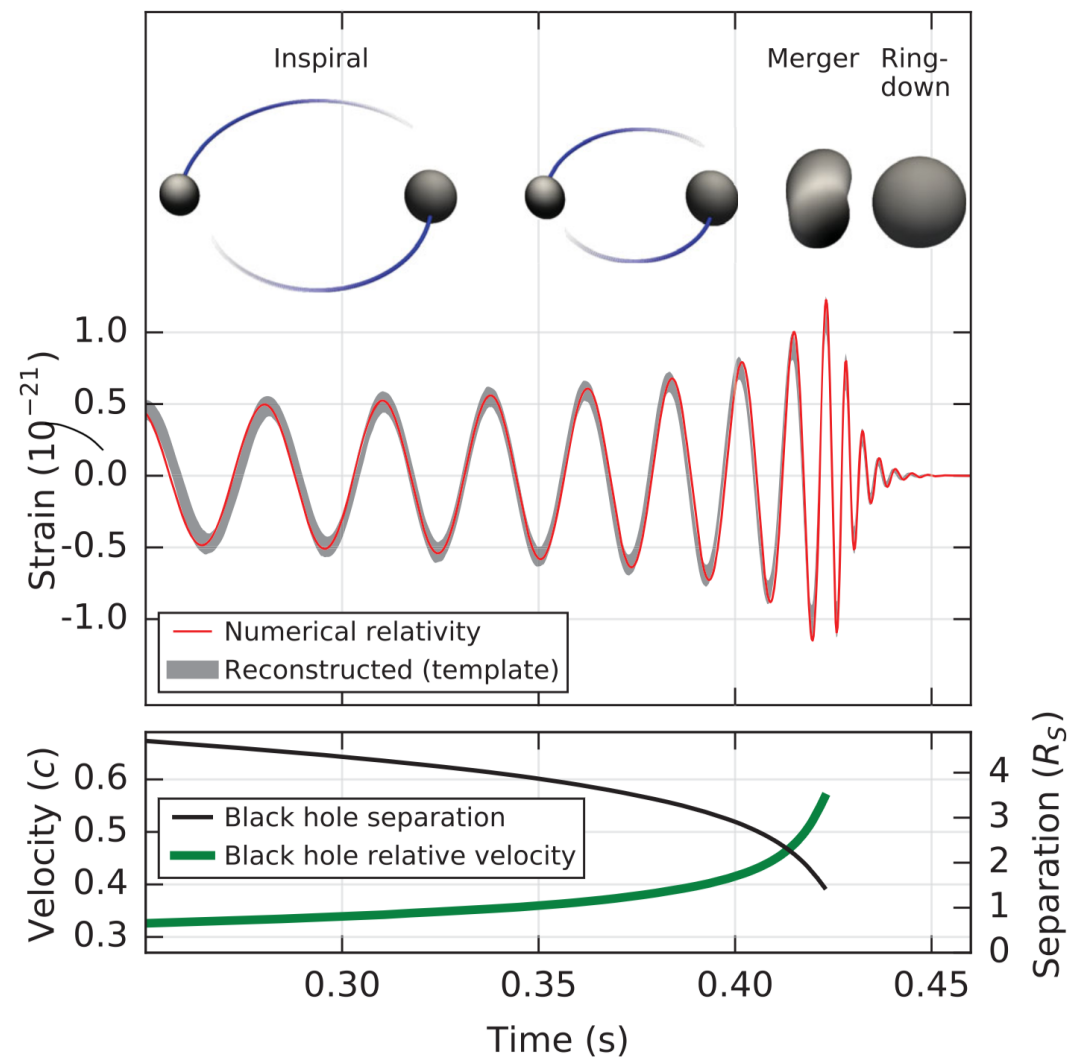
B. P. Abbott *et al.*  
(LIGO Scientific Collaboration and Virgo Collaboration)



interferometers



# High-frequency gravitational waves



PRL **116**, 061102 (2016)

PHYSICAL REVIEW LETTERS

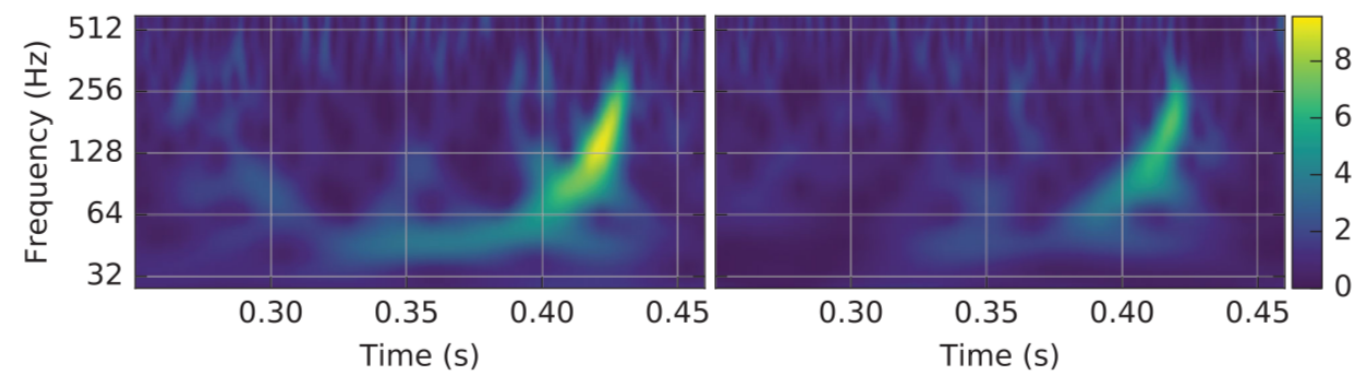
12 FEBRUARY 2016



## Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*\*

(LIGO Scientific Collaboration and Virgo Collaboration)



$$f \approx \frac{1}{2\pi} \sqrt{\frac{GM}{R^3}} \ll 10 \text{ kHz}$$

No known astrophysical objects are small and dense enough to produce gravitational waves beyond 10 kHz

# High-frequency gravitational waves

Part of a collection:

[Gravitational Waves](#)

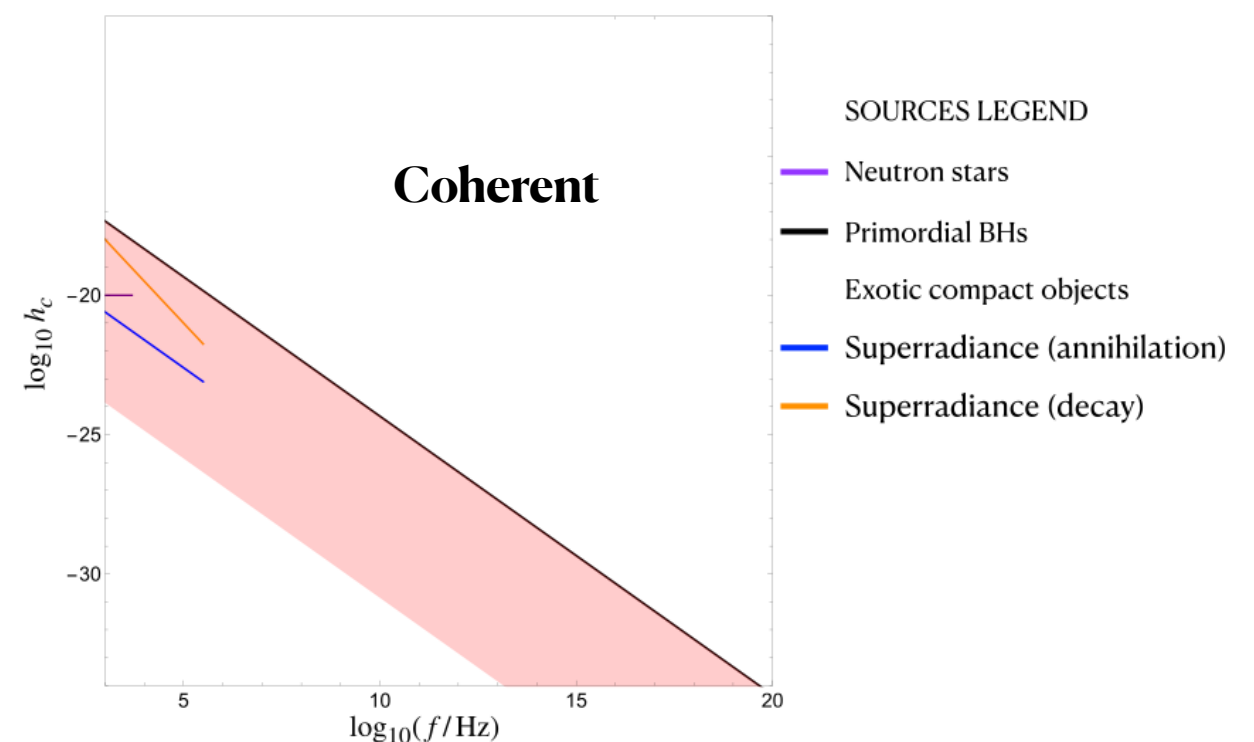
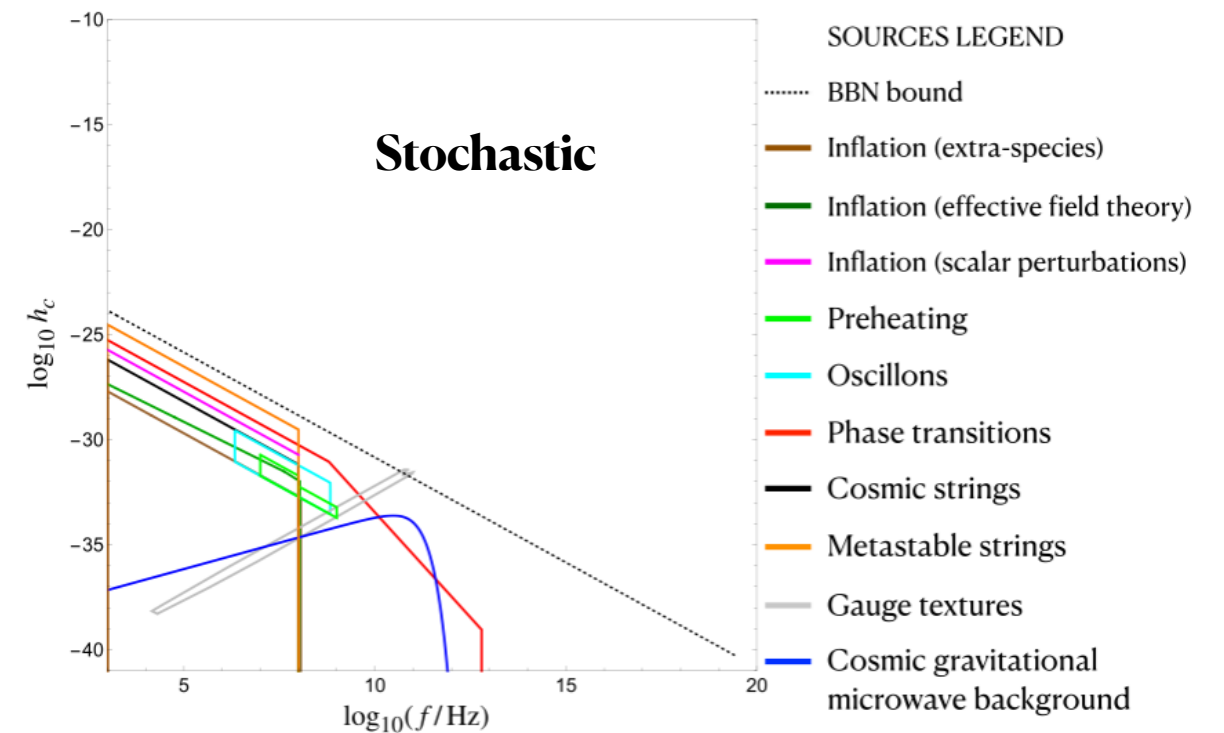
Review Article | [Open Access](#) | [Published: 06 December 2021](#)

## Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies

[Nancy Aggarwal](#) , [Odylio D. Aguiar](#), [Andreas Bauswein](#), [Giancarlo Cella](#), [Sebastian Clesse](#), [Adrian Michael Cruise](#), [Valerie Domcke](#) , [Daniel G. Figueroa](#), [Andrew Geraci](#), [Maxim Goryachev](#), [Hartmut Grote](#), [Mark Hindmarsh](#), [Francesco Muia](#) , [Nikhil Mukund](#), [David Ottaway](#), [Marco Peloso](#), [Fernando Quevedo](#) , [Angelo Ricciardone](#), [Jessica Steinlechner](#) , [Sebastian Steinlechner](#) , [Sichun Sun](#), [Michael E. Tobar](#), [Francisco Torrenti](#), [Caner Ünal](#) & [Graham White](#)

[Living Reviews in Relativity](#) **24**, Article number: 4 (2021) | [Cite this article](#)

A growing community is seriously considering the search of high frequency gravitational waves



# The (inverse) Gertsenhstein Effect

SOVIET PHYSICS JETP

VOLUME 14, NUMBER 1

JANUARY, 1962

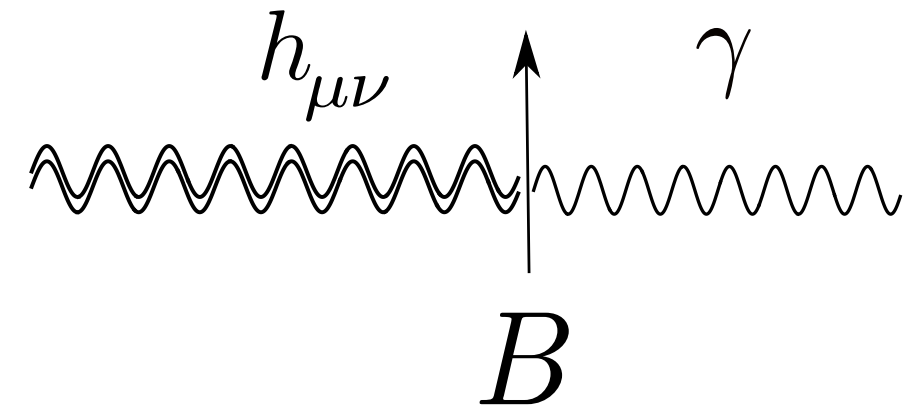
## WAVE RESONANCE OF LIGHT AND GRAVITATIONAL WAVES

M. E. GERTSENSHTEĬN

Submitted to JETP editor July 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 113-114 (July, 1961)

The energy of gravitational waves excited during the propagation of light in a constant magnetic or electric field is estimated.



SOVIET PHYSICS JETP

VOLUME 16, NUMBER 2

FEBRUARY, 1963

## ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

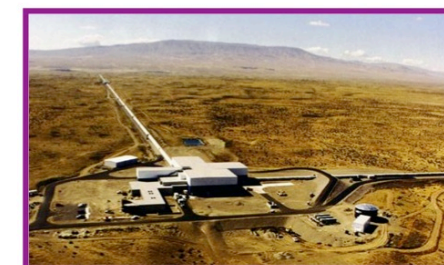
M. E. GERTSENSHTEĬN and V. I. PUSTOVOĬT

Submitted to JETP editor March 3, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 605-607 (August, 1962)

It is shown that the sensitivity of the electromechanical experiments for detecting gravitational waves by means of piezocrystals is ten orders of magnitude worse than that estimated by Weber.<sup>[1]</sup> In the low frequency range it should be possible to detect gravitational waves by the shift of the bands in an optical interferometer. The sensitivity of this method is investigated.

Terrestrial  
interferometers





# The (inverse) Gertsenhstein Effect

- The conversion of gravitational waves into electromagnetic waves is a classical process. Its rate does not involve  $\hbar$

$$P \sim GB^2L^2$$

- Cosmological conversion

Potential of Radio Telescopes as High-Frequency Gravitational Wave Detectors

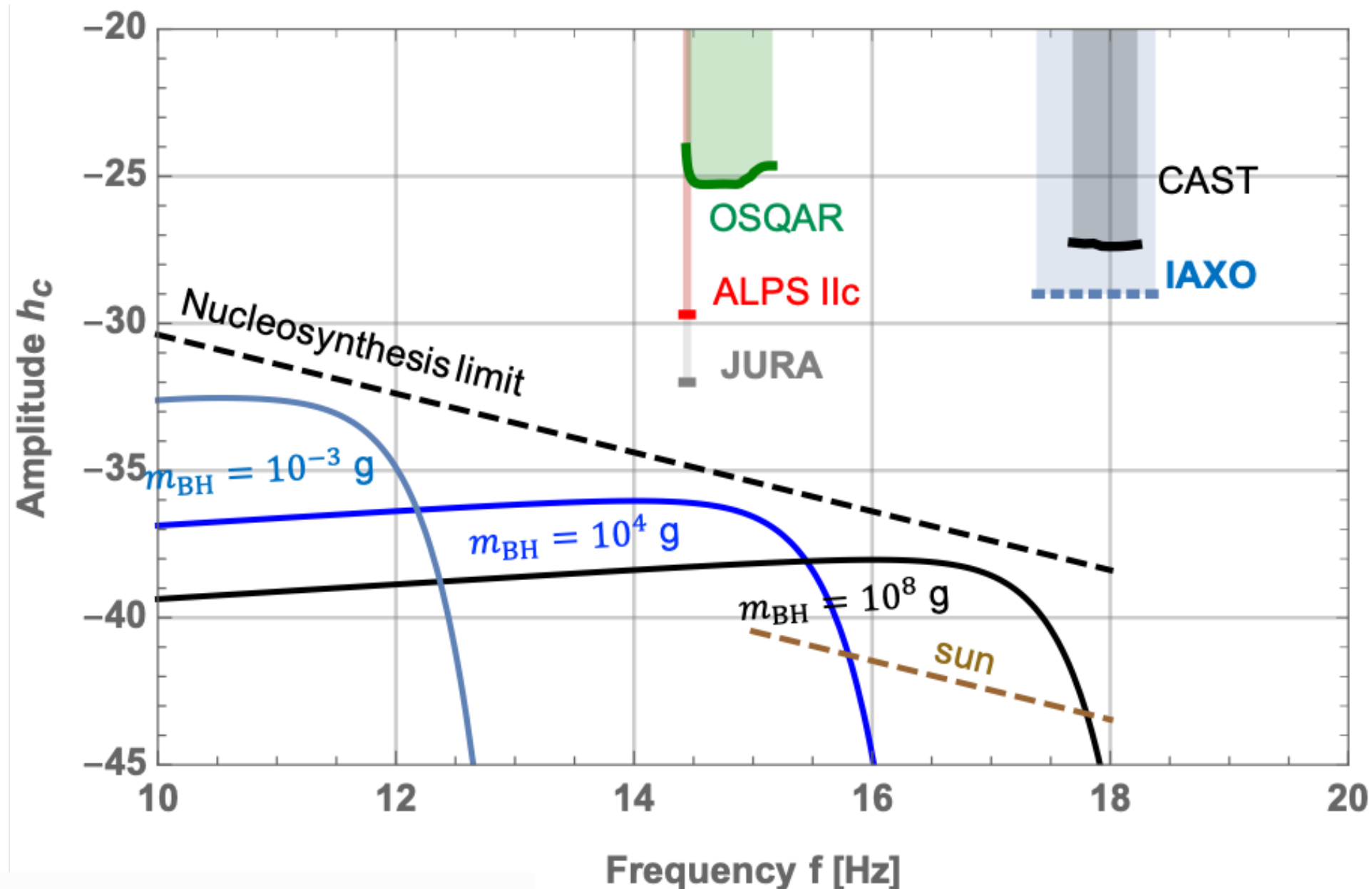
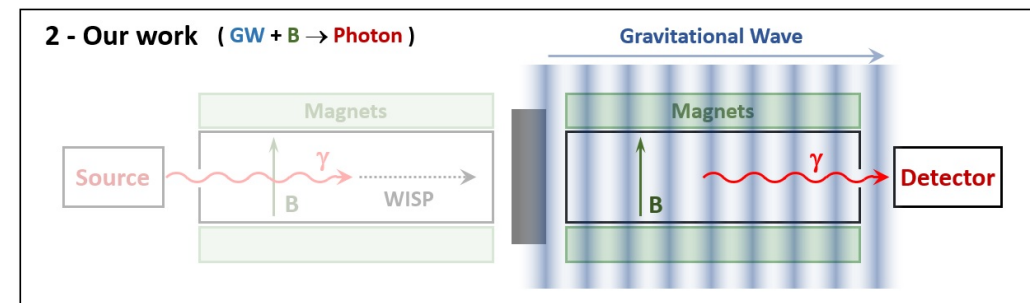
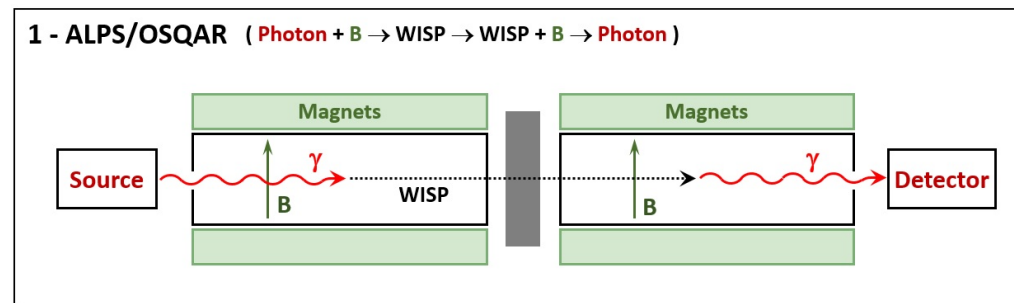
Valerie Domcke and Camilo Garcia-Cely  
Phys. Rev. Lett. **126**, 021104 – Published 14 January 2021



- The process is strictly analogous to axion dark matter conversion.

Raffelt, Stodolski'89

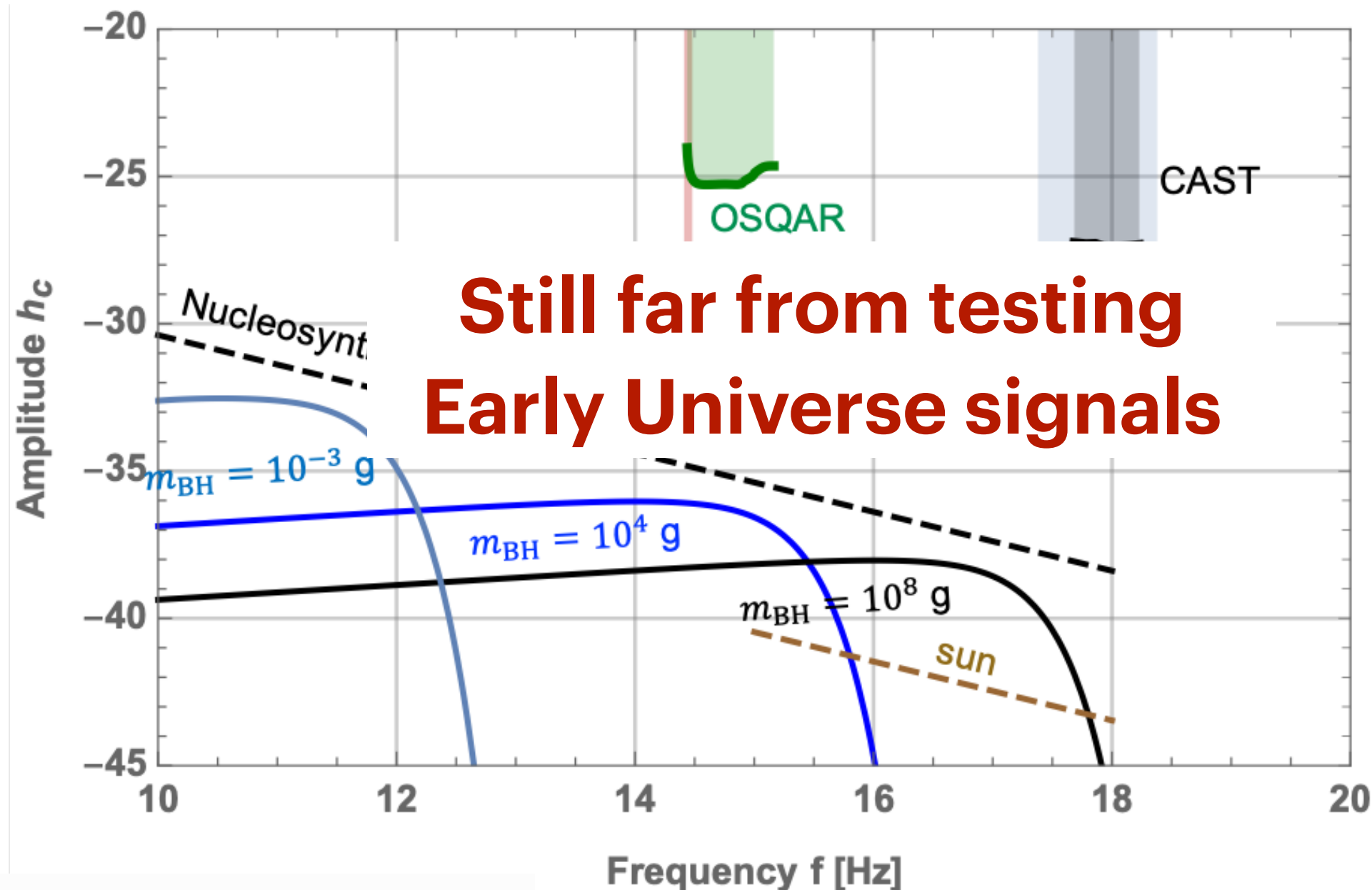
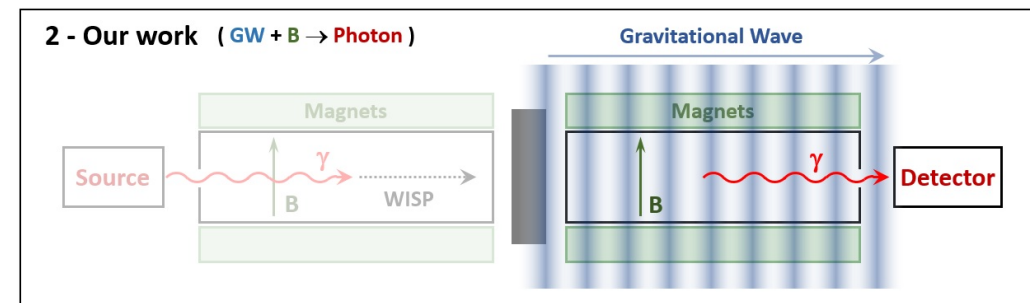
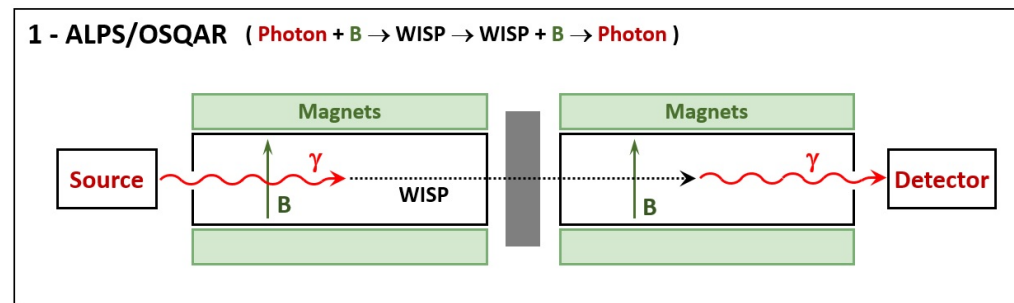
# The (inverse) Gertsenhstein Effect



A. Ejlli , D. Ejlli, A. M. Cruise, G. Pisano & H. Grote

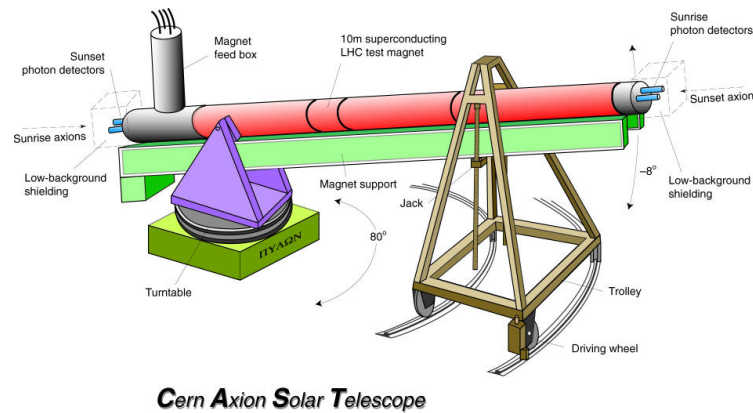
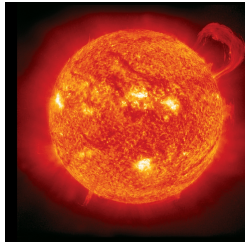
*The European Physical Journal C* **79**, Article number: 1032 (2019)

# The (inverse) Gertsenhstein Effect



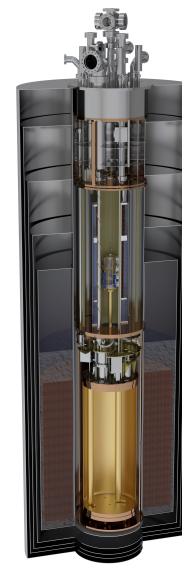
# More possibilities

- Helioscopes (X rays)



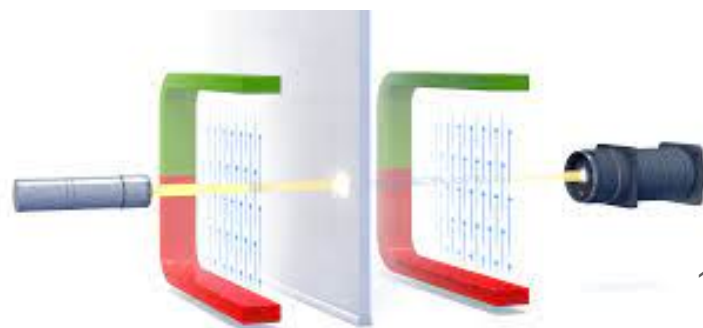
- CAST
- IAXO
- ....

- Haloscopes (radio frequencies)



- microwave cavities
- MADMAX
- ADMX
- HAYSTAC
- ABRACADABRA
- Lumped element detectors
- ...

- Purely lab experiments



- Light shining through the walls
- OSCAR
- ALPS II
- ...



# Detection in haloscopes

# Effective current

Axions act as a source term to Maxwell's equations, **effectively inducing an electromagnetic current.**

*Sikivie, 1983*

$$j^0 = -g_{a\gamma\gamma} \nabla a \cdot \mathbf{B} \qquad \mathbf{j} = g_{a\gamma\gamma} ( \nabla a \times \mathbf{E} + \partial_t a \mathbf{B} )$$

# Effective current

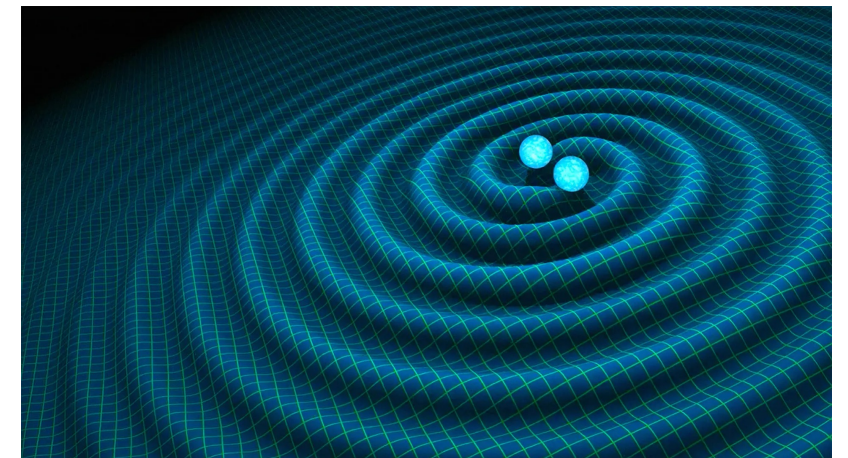
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Sikivie, 1983

$$j^0 = -g_{a\gamma\gamma} \nabla a \cdot \mathbf{B}$$

$$\mathbf{j} = g_{a\gamma\gamma} (\nabla a \times \mathbf{E} + \partial_t a \mathbf{B})$$

**Effectively, the same is true for gravitational waves in haloscopes**



# Proper detector frame

In the proper detector frame the coordinate system closely matches the intuitive description of an Earth-based laboratory

Fermi, 1922

Manasse and Misner, 1963

Ni and Zimmermann, 1978

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Ni and Zimmermann, 1978

- Coordinate distances to the origin match the proper distance  
They coincide with those measured by ideal rigid rulers.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu \text{ for } dx^\mu = (0, dr \hat{\mathbf{r}})$$

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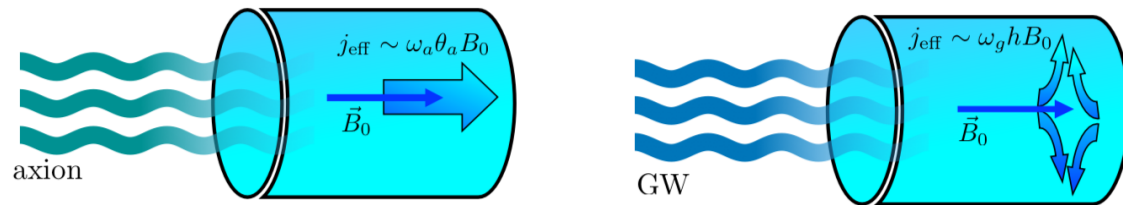
- The gravitational wave acts as a Newtonian force.  
When they are negligible, the static fields applied in experiments remain static in the presence of GWs.
- Crucial for haloscopes

Berlin et al 2022

	Axion electrodynamics	Gravitational wave electrodynamics
For example	Axion-Photon conversion	Gertsenshtein effect
Effective current  $j_{\text{eff}}^\mu = (-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P})$	$\mathbf{P} = g_{a\gamma\gamma} a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma} a \mathbf{E}$  <a href="#">McAllister et al, 1803.07755</a> <a href="#">Tobar et al, 1809.01654</a> <a href="#">Ouellet et al, 1809.10709</a>	$P_i = -h_{ij} E_j + \frac{1}{2} h E_i + h_{00} E_i - \epsilon_{ijk} h_{0j} B_k$  $M_i = -h_{ij} B_j - \frac{1}{2} h B_i + h_{jj} B_i + \epsilon_{ijk} h_{0j} E_k$  <a href="#">Domcke, CGC, Rodd, 2202.00695</a>
Benchmark	QCD axion  $g_{a\gamma\gamma} a \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_a f_a} \sim \frac{\alpha \sqrt{\rho_{\text{DM}}}}{2\pi m_\pi f_\pi} \sim 10^{-22}$	$h \sim 10^{-22}$



# Haloscopes based on microwave cavities



Detecting high-frequency gravitational waves with microwave cavities

Asher Berlin, Diego Blas, Raffaele Tito D'Agnolo, Sebastian A. R. Ellis, Roni Harnik, Yonatan Kahn, and Jan Schütte-Engel

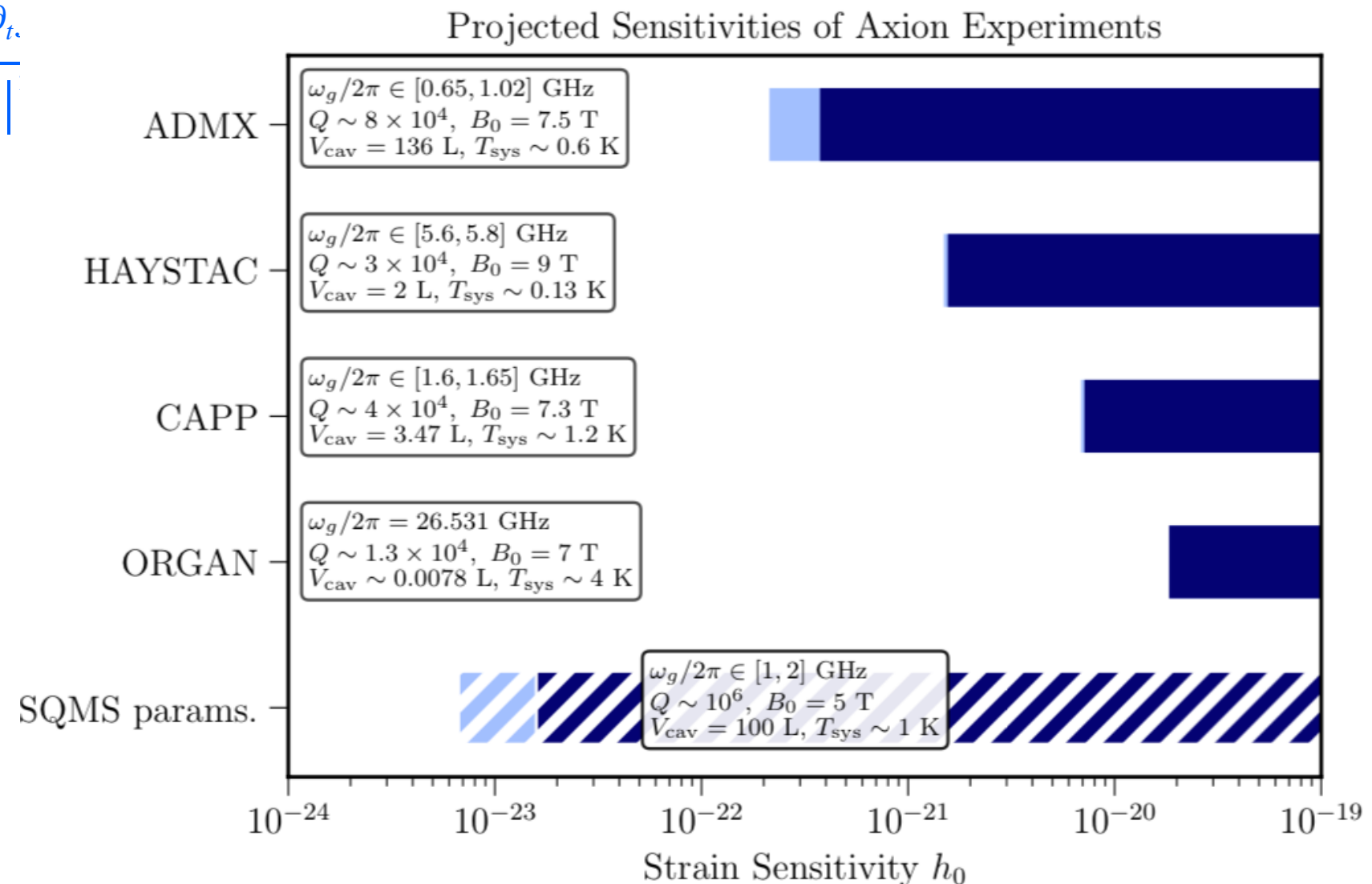
Phys. Rev. D **105**, 116011 – Published 17 June 2022

$$\left( \partial_t^2 + \frac{\omega_n}{Q_n} \partial_t + \omega_n^2 \right) e_n(t) = - \frac{\int_{V_{\text{cav}}} d^3\mathbf{x} \mathbf{E}_n^* \cdot \partial_t \mathbf{E}}{\int_{V_{\text{cav}}} d^3\mathbf{x} |\mathbf{E}_n|}$$

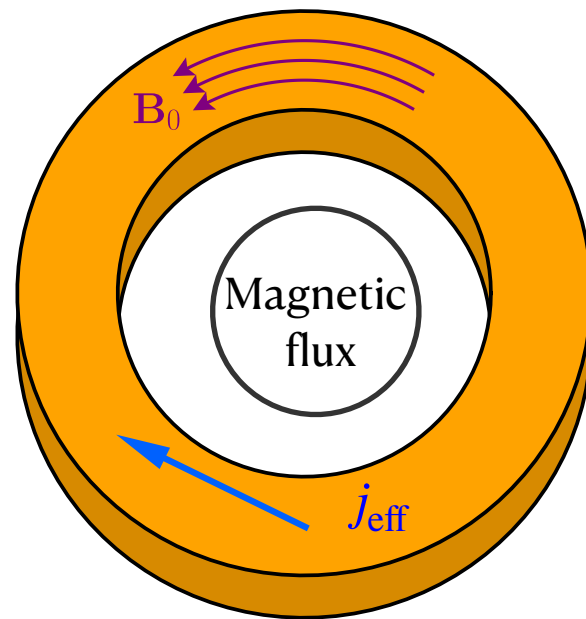
Eigenmodes

$$\mathbf{E}(\mathbf{x}, t) = \sum_n e_n(t) \mathbf{E}_n(\mathbf{x})$$

It resonates when the GW frequency matches one of the eigenmode frequencies



# Haloscopes based on lumped-element detectors



# Haloscopes based on lumped-element detectors

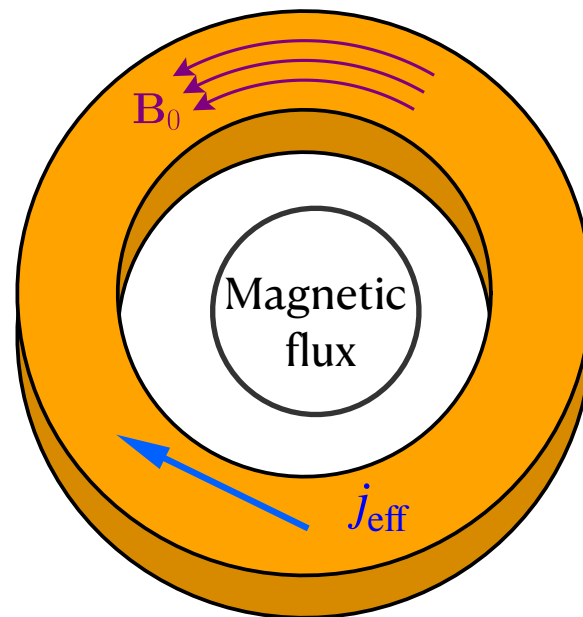
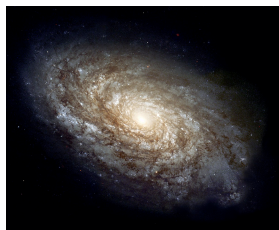
PRL 117, 141801 (2016)

PHYSICAL REVIEW LETTERS

week ending  
30 SEPTEMBER 2016

## Broadband and Resonant Approaches to Axion Dark Matter Detection

Yonatan Kahn,<sup>1,\*</sup> Benjamin R. Safdi,<sup>2,†</sup> and Jesse Thaler<sup>2,‡</sup>



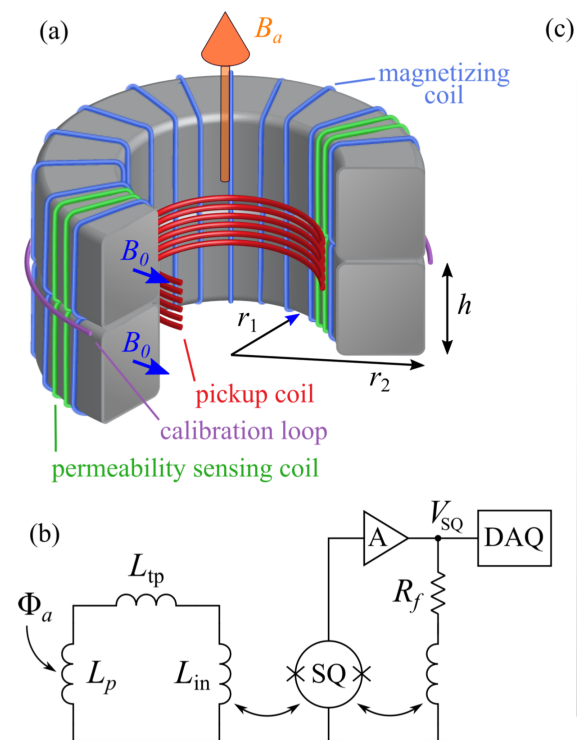
nature physics

SHAFT

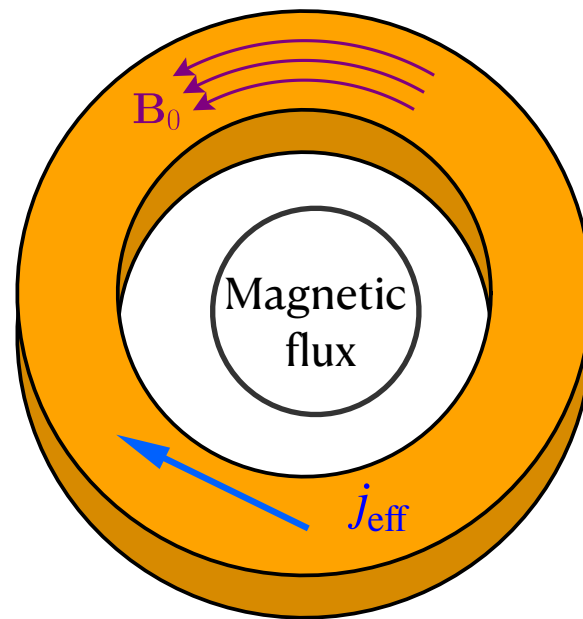
Article | Published: 17 August 2020

## Search for axion-like dark matter with ferromagnets

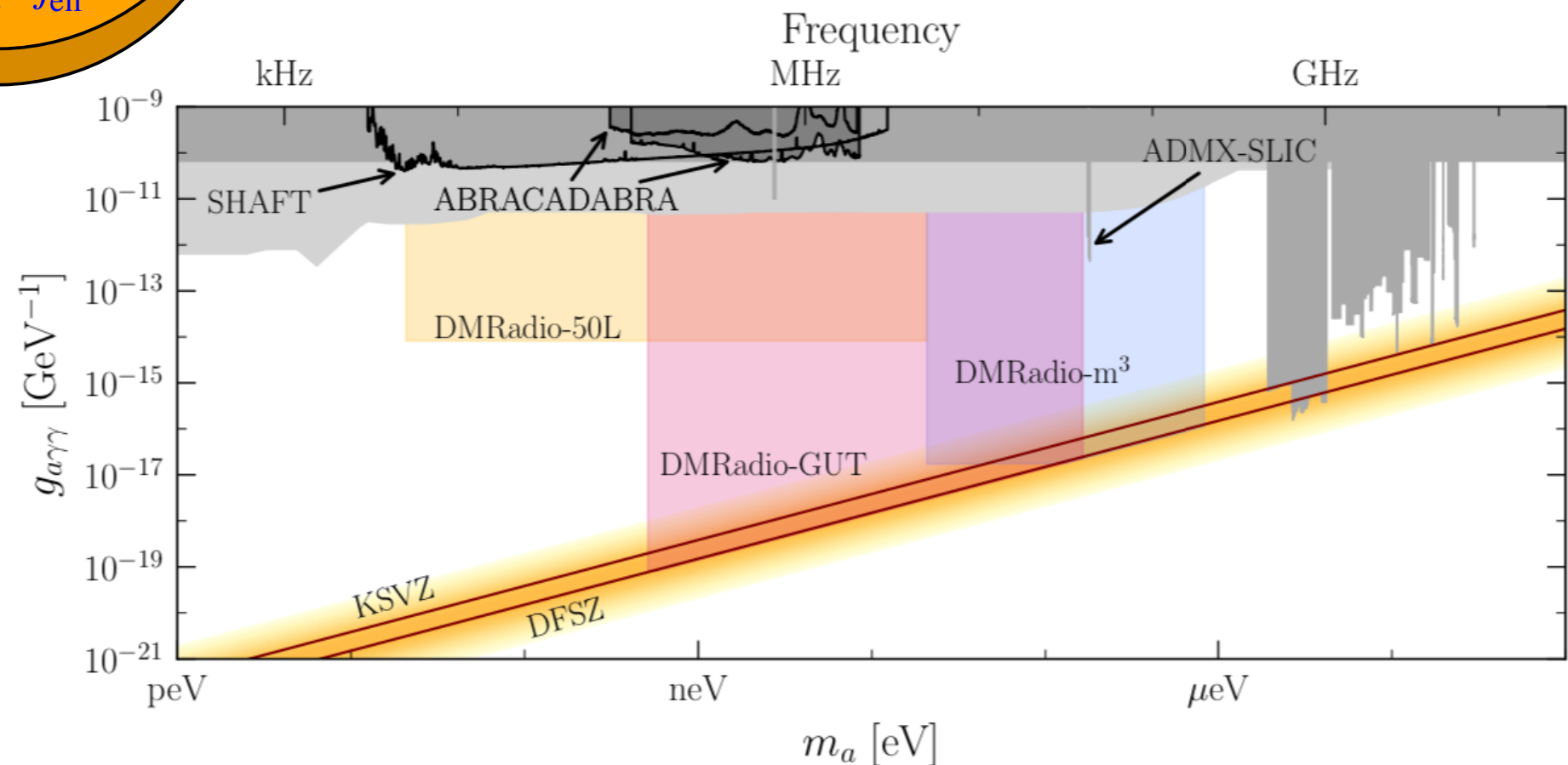
Alexander V. Gramolin, Deniz Aybas, Dorian Johnson, Janos Adam & Alexander O. Sushkov



# Haloscopes based on lumped-element detectors

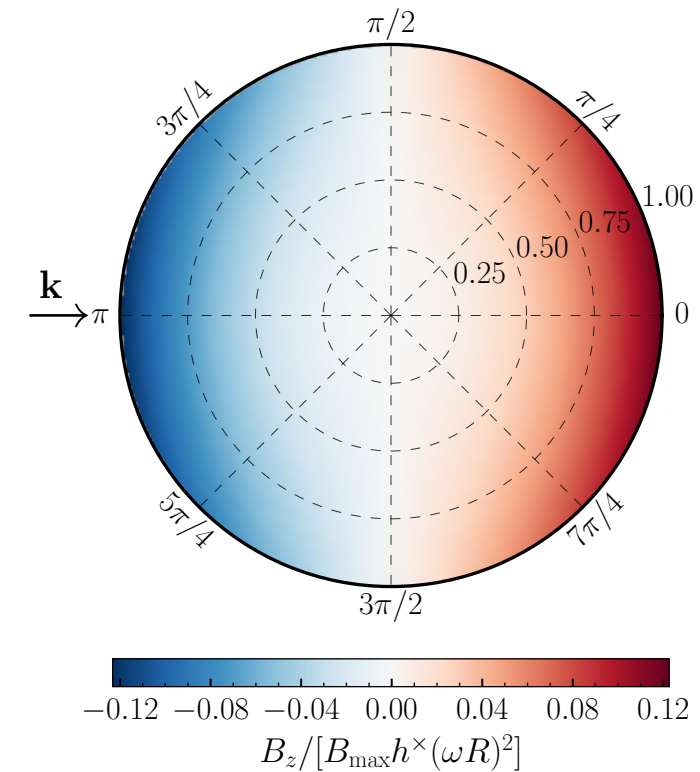
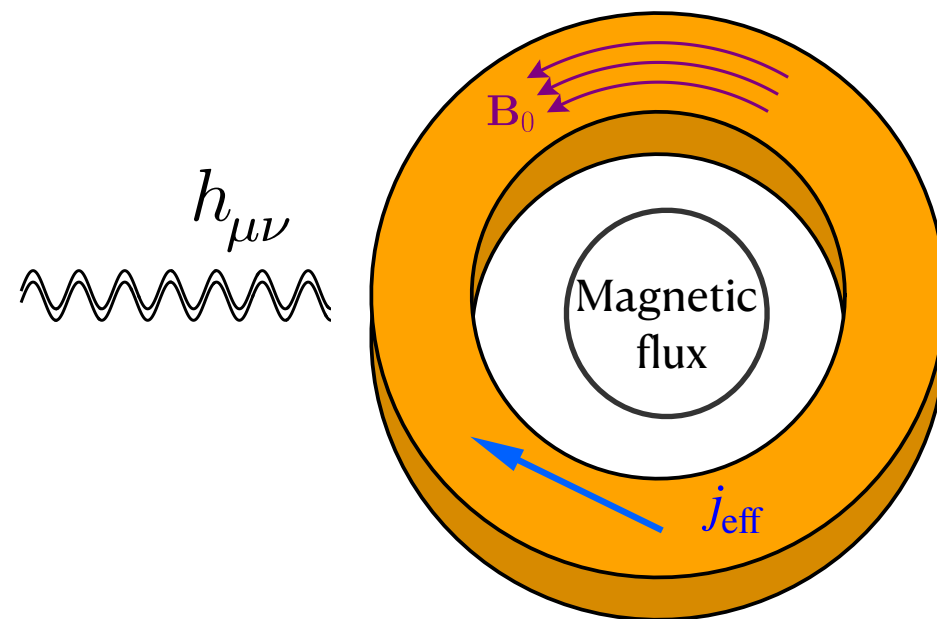


# Proposal for a definitive search for GUT-scale QCD axions

L. Brouwer *et al.* (DMRadio Collaboration)Phys. Rev. D **106**, 112003 – Published 12 December 2022

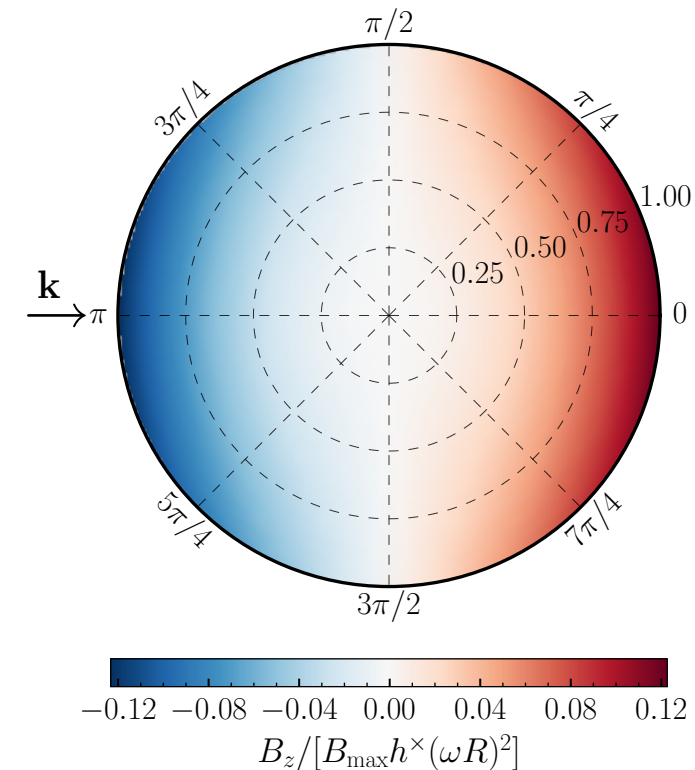
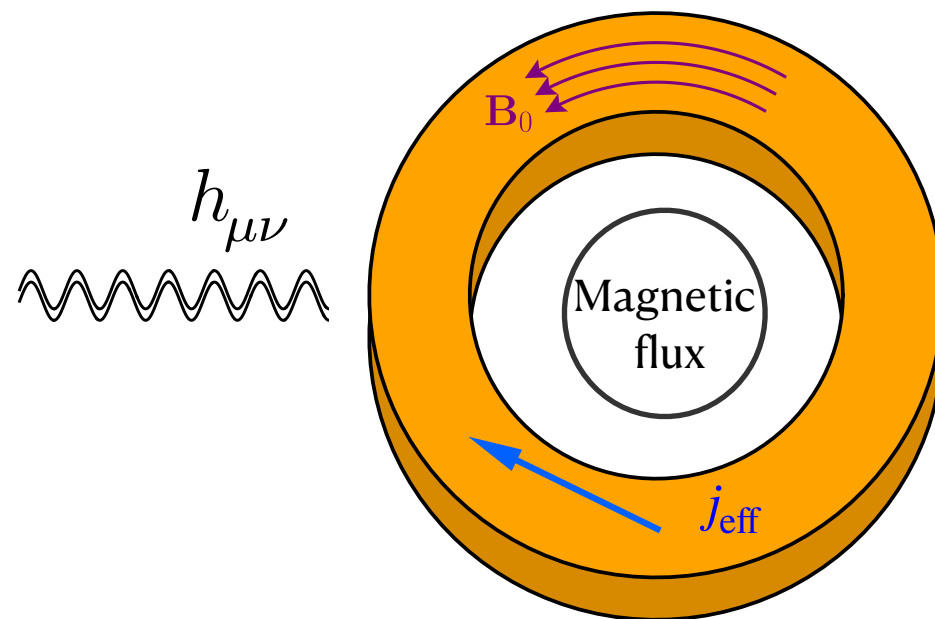
# Toroidal magnetic fields

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd  
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022



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Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd  
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022

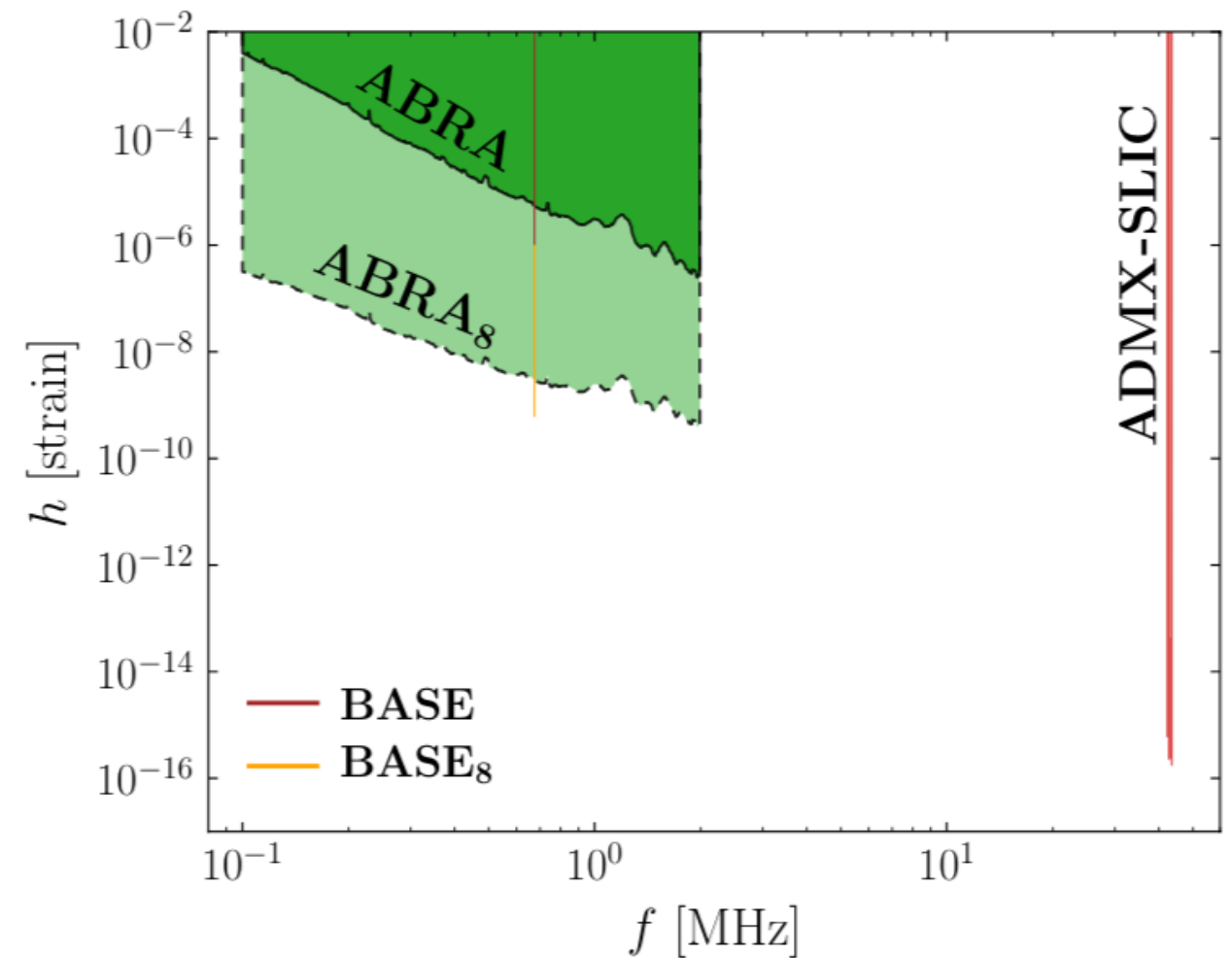
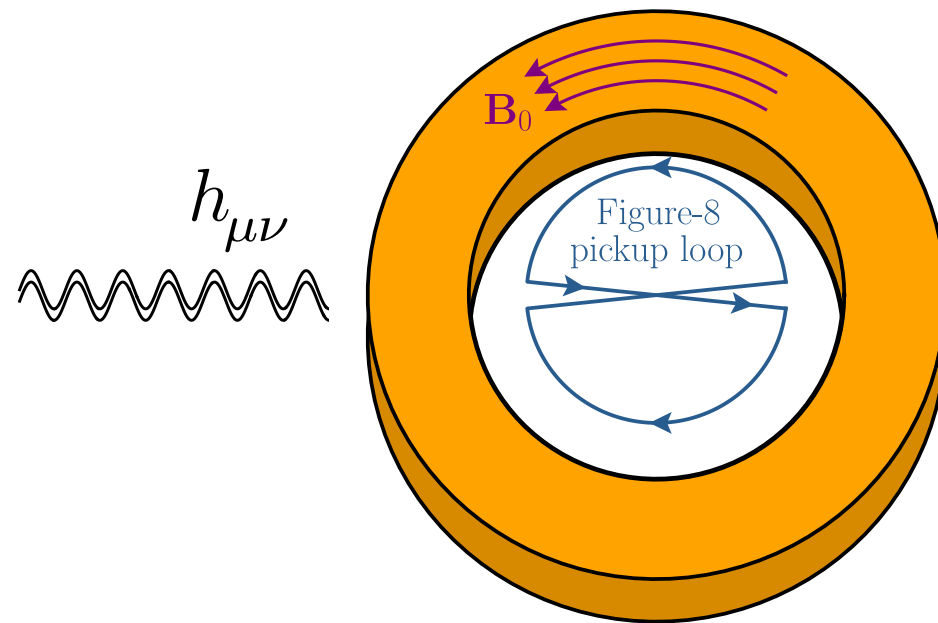


- Only one polarization
- Suppression at small frequencies  $\omega R \ll 1$
- The sensitivity scaling with the volume is faster than for axions

# Toroidal magnetic fields

Valerie Domcke, Camilo Garcia-Cely, and Nicholas L. Rodd  
Phys. Rev. Lett. **129**, 041101 – Published 20 July 2022

Domcke, CGC, Lee, Rodd, 2023





# Solenoidal configurations

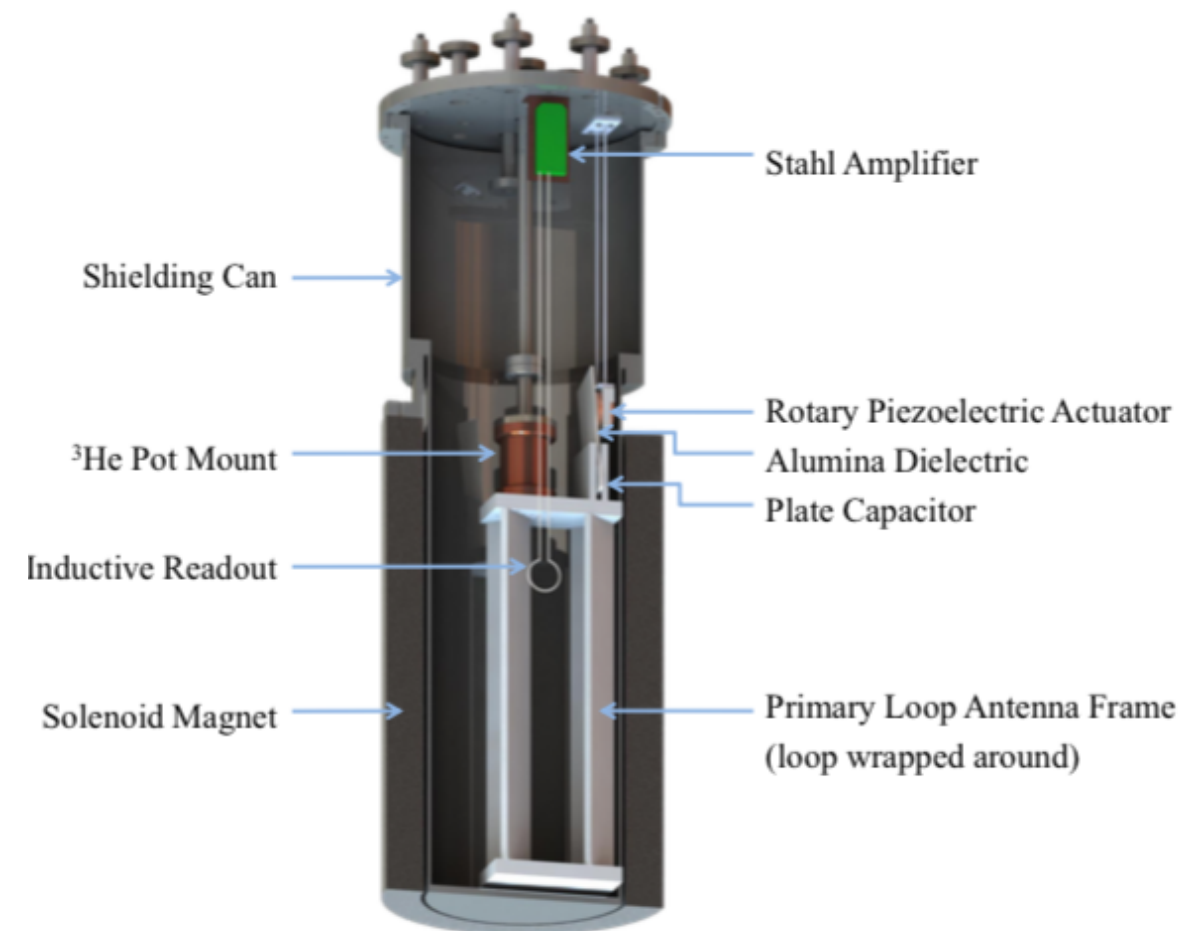
Domcke, CGC, Lee, Rodd, 2023

## ADMX SLIC: Results from a Superconducting LC Circuit Investigating Cold Axions

N. Crisosto, P. Sikivie, N. S. Sullivan, D. B. Tanner, J. Yang, and G. Rybka  
Phys. Rev. Lett. **124**, 241101 – Published 17 June 2020

## Constraints on the Coupling between Axionlike Dark Matter and Photons Using an Antiproton Superconducting Tuned Detection Circuit in a Cryogenic Penning Trap

Jack A. Devlin, Matthias J. Borchert, Stefan Erlewein, Markus Fleck, James A. Harrington, Barbara Latacz, Jan Warncke, Elise Wursten, Matthew A. Bohman, Andreas H. Mooser, Christian Smorra, Markus Wiesinger, Christian Will, Klaus Blaum, Yasuyuki Matsuda, Christian Ospelkaus, Wolfgang Quint, Jochen Walz, Yasunori Yamazaki, and Stefan Ulmer  
Phys. Rev. Lett. **126**, 041301 – Published 25 January 2021



BASE

WISPLC

## Search for dark matter with an $LC$ circuit

Zhongyue Zhang (张钟月), Dieter Horns, and Oindrila Ghosh  
Phys. Rev. D **106**, 023003 – Published 5 July 2022

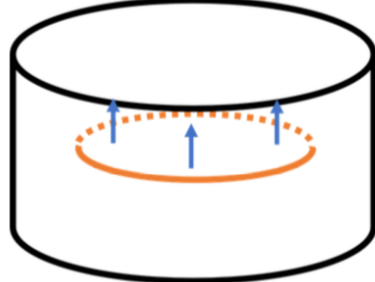
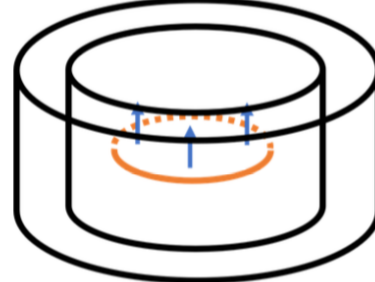
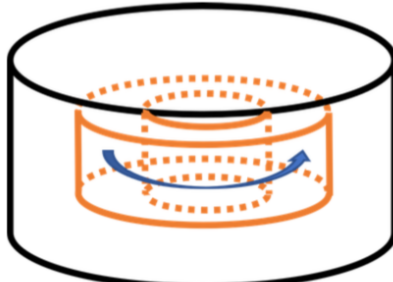
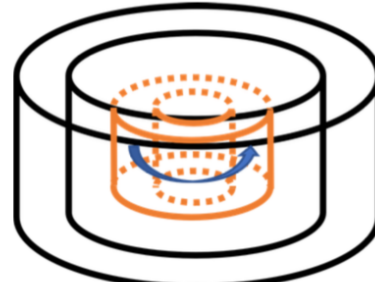
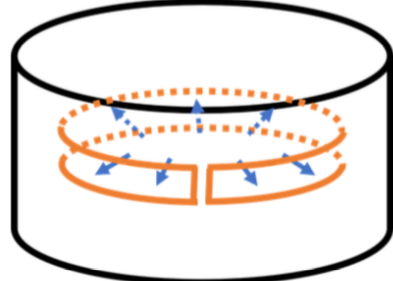
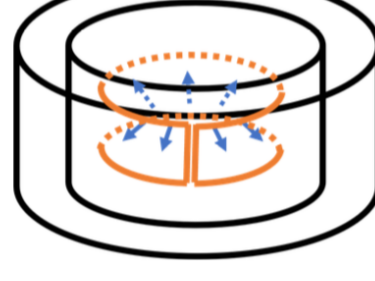


# Beyond toroidal configurations

## Type of external field

Domcke, CGC, Lee, Rodd, 2023

### Pickup loop orientation

	Solenoid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_z$	Toroid: $\mathbf{B}_0 \propto \hat{\mathbf{e}}_\phi$
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_z$	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{e^{-i\omega t}}{48\sqrt{2}} h^+ \omega^2 B_0 s_{\theta_h}^2 \pi r^2 (11r^2 + 14R^2 + 16R^2 \ln \frac{R}{H})$ 	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{48\sqrt{2}} h^\times \omega^3 B_{\max} \pi r^2 a R (a + 2R) s_{\theta_h}^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\phi$	$h^\times, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{96\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 l (12R^2 - 5r^2) s_{\theta_h}^2$ 	$h^+, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^2]$ $\Phi_h = \frac{3e^{-i\omega t}}{4\sqrt{2}} h^+ \omega^2 B_{\max} \frac{\pi r^2 a R l (a + 2R)}{H^2} s_{\theta_h}^2$ 
$\hat{\mathbf{n}}' \propto \hat{\mathbf{e}}_\rho$	$h^+, n \text{ odd} \Rightarrow \mathcal{O}[(\omega L)^3]$ $\Phi_h = \frac{ie^{-i\omega t}}{96\sqrt{2}} h^+ B_0 \omega^3 c_{\theta_h} s_{\theta_h}^2 \times \pi r^2 l (3l^2 - 22(r^2 + 2R^2) - 36R^2 \ln \frac{R}{H})$ 	$h^\times, n \text{ even} \Rightarrow \mathcal{O}[(\omega L)^4]$ $\Phi_h = \frac{e^{-i\omega t}}{32\sqrt{2}} h^\times \omega^4 B_{\max} \pi r^2 a R l (a + 2R) c_{\theta_h} s_{\theta_h}^2$ 

# **Selection rules**

# Selection rules

In the proper detector frame the coordinate system closely matches the intuitive description of an Earth-based laboratory

$$h_{00} = \omega^2 e^{-i\omega t} f(\mathbf{k} \cdot \mathbf{r}) r_m r_n \sum_{A=+, \times} h_A e_{mn}^A(\hat{\mathbf{k}}),$$

$$h_{0i} = \frac{1}{2} \omega^2 e^{-i\omega t} [f(\mathbf{k} \cdot \mathbf{r}) - i f'(\mathbf{k} \cdot \mathbf{r})] [\hat{\mathbf{k}} \cdot \mathbf{r} r_m \delta_{ni} - r_m r_n \hat{k}_i] \sum_{A=+, \times} h_A e_{mn}^A(\hat{\mathbf{k}}),$$

$$h_{ij} = -i \omega^2 e^{-i\omega t} f'(\mathbf{k} \cdot \mathbf{r}) [|\mathbf{r}|^2 \delta_{im} \delta_{jn} + r_m r_n \delta_{ij} - r_n r_j \delta_{im} - r_m r_j \delta_{in}] \sum_{A=+, \times} h_A e_{mn}^A(\hat{\mathbf{k}}),$$

$$f(\xi) = [e^{i\xi} - 1 - i\xi] / \xi^2$$

Domcke, CGC, Rodd, 2023  
Berlin et al, 2023

The  $\omega^2$  dependence is unavoidable

# Selection rules

Domcke, CGC, Lee, Rodd, 2023

Write down the detector response matrix for a wave coming from an arbitrary direction,  
and impose **cylindrical symmetry** for both external magnetic field and loop:

# Selection rules

Domcke, CGC, Lee, Rodd, 2023

Write down the detector response matrix for a wave coming from an arbitrary direction, and impose **cylindrical symmetry** for both external magnetic field and loop:

**Selection Rule 1:** For an instrument with azimuthal symmetry,  $\Phi_h \propto h^+$  at  $\mathcal{O}[(\omega L)^2]$

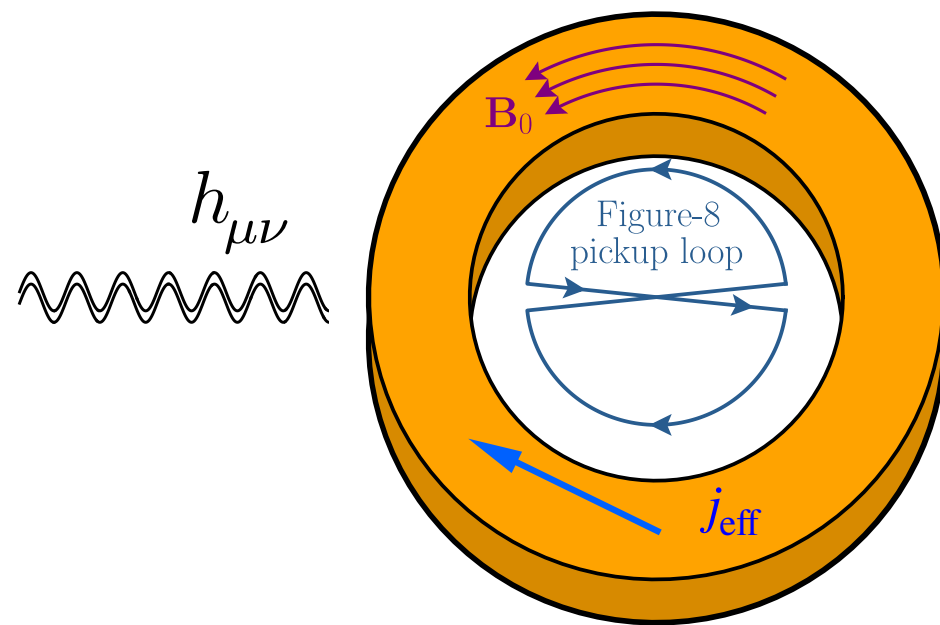
**Selection Rule 2:** For an instrument with azimuthal symmetry, the flux is proportional to either  $h^+$  or  $h^\times$ , but not both. This holds to all orders in  $(\omega L)$ .

**Selection Rule 3:** For an instrument with full cylindrical symmetry,  $\Phi_h$  will contain only even or odd powers of  $\omega$ .

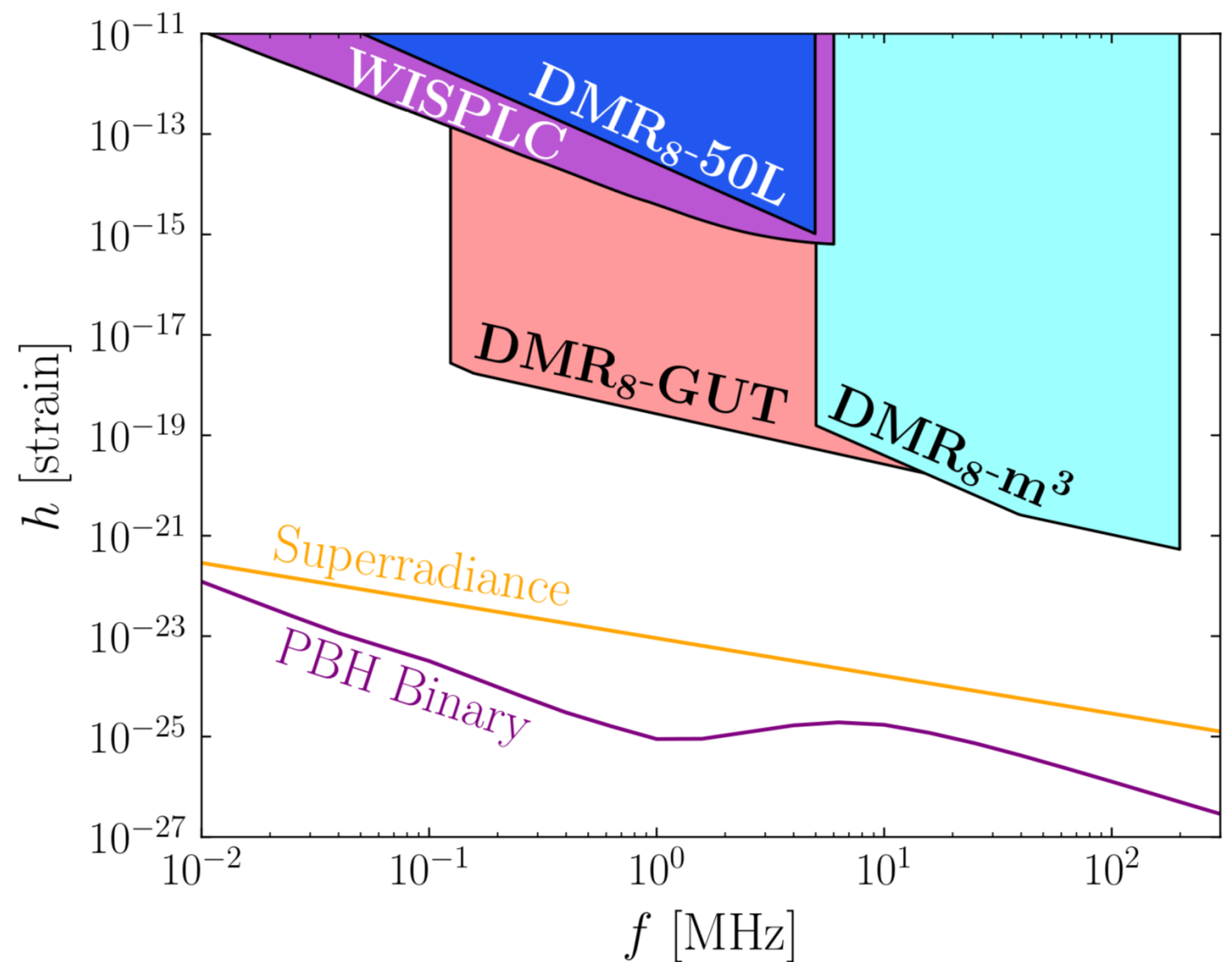
# Gravitational waves in low mass axion haloscopes

Domcke, CGC, Lee, Rodd, 2023

Break cylindrical symmetry



Recast of axion searches to establish GW sensitivity, taking into account the different time scales involved in the signals and detectors.



# Conclusions

The techniques developed for detecting **axion dark matter** could potentially be used to discover new sources of **gravitational waves**.

**Selection rules** in detectors exhibiting cylindrical symmetry enforce cancellations in the flux associated to gravitational waves.

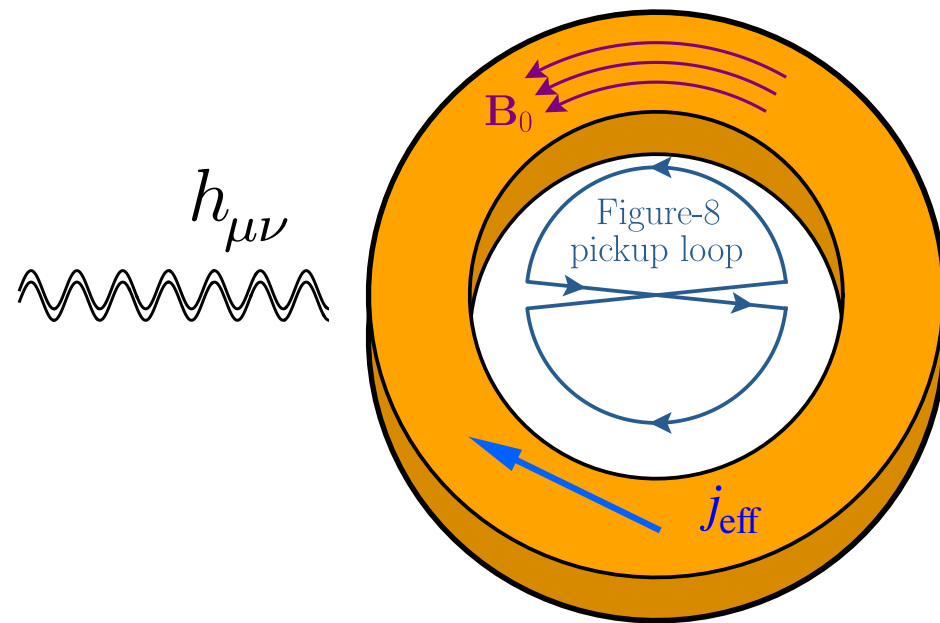
These cancellations can be avoided by **changing the geometry of the pickup loop**. We demonstrate this for different detector geometries, obtaining a parametric increase of sensitivity.

Different experimental proposals have coalesced on a strain sensitivity of  $10^{-22}$  for MHz GWs, **still orders of magnitude away from signals of the early Universe**. Whether we can hope to probe such strain sensitivities remains to be determined.

# Gravitational waves in low mass axion haloscopes

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Break cylindrical symmetry



$$\Phi_h(h^+, h^\times; \phi_h, \theta_h) = \mathcal{R}_c \Phi_a(g_{a\gamma\gamma}),$$

$$\mathcal{R}_c = \left(\frac{T_m}{\tau_h}\right)^{1/4} \left(\frac{Q_a}{Q_h}\right)^{1/4} \begin{cases} 1 & Q_r < Q_a, Q_h, \\ (Q_a/Q_r)^{1/4} & Q_a < Q_r < Q_h, \\ Q_r/Q_h & Q_h < Q_r < Q_a, \\ (Q_a/Q_r)^{1/4} Q_r/Q_h & \text{otherwise.} \end{cases}$$

