

Sommerfeld Enhancement in the Inert Doublet Model

Camilo A. Garcia Cely

Feb 19, 2014

MALT Workshop, Munich



In collaboration with Alejandro Ibarra and Michael Gustafsson

Outline

- Introduction
- Dark Matter as an Inert Scalar
- Gamma-ray spectral features: Annihilation into two photons
- Issues with Unitarity and the Sommerfeld enhancement
- Gamma-ray fluxes and H.E.S.S.
- Conclusions

SM doublet

$$\Phi = \begin{pmatrix} G^+ \\ \frac{v_h + h + iG^0}{\sqrt{2}} \end{pmatrix}$$

Additional doublet

$$\Psi = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H^0 + iA^0) \end{pmatrix}$$

Dark
Matter
Stability

It is possible to
postulate invariance under

$$\Psi \rightarrow -\Psi \quad \Phi \rightarrow \Phi$$

(Z_2 symmetry)

SM doublet

$$\Phi = \begin{pmatrix} G^+ \\ \frac{v_h + h + iG^0}{\sqrt{2}} \end{pmatrix}$$

Additional doublet

$$\Psi = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H^0 + iA^0) \end{pmatrix}$$

Dark
Matter
Stability

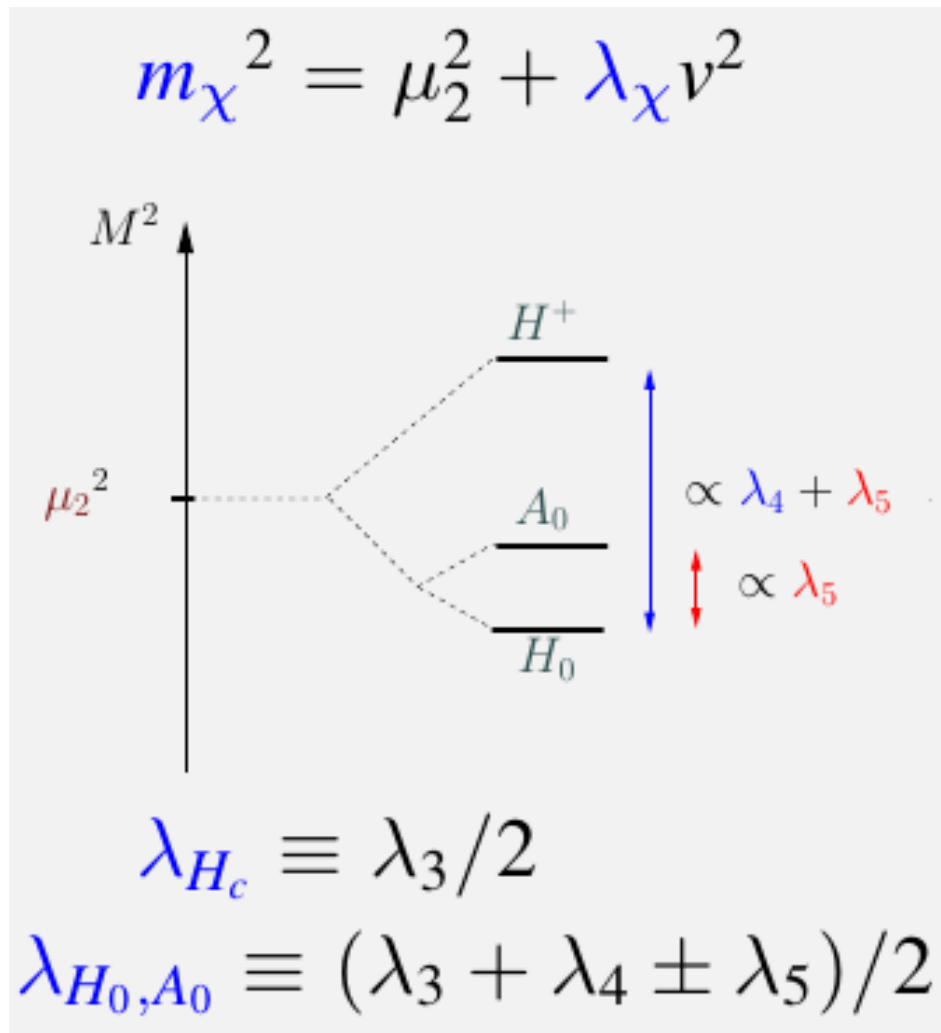
It is possible to
postulate invariance under

$$\Psi \rightarrow -\Psi \quad \Phi \rightarrow \Phi$$

(Z_2 symmetry)

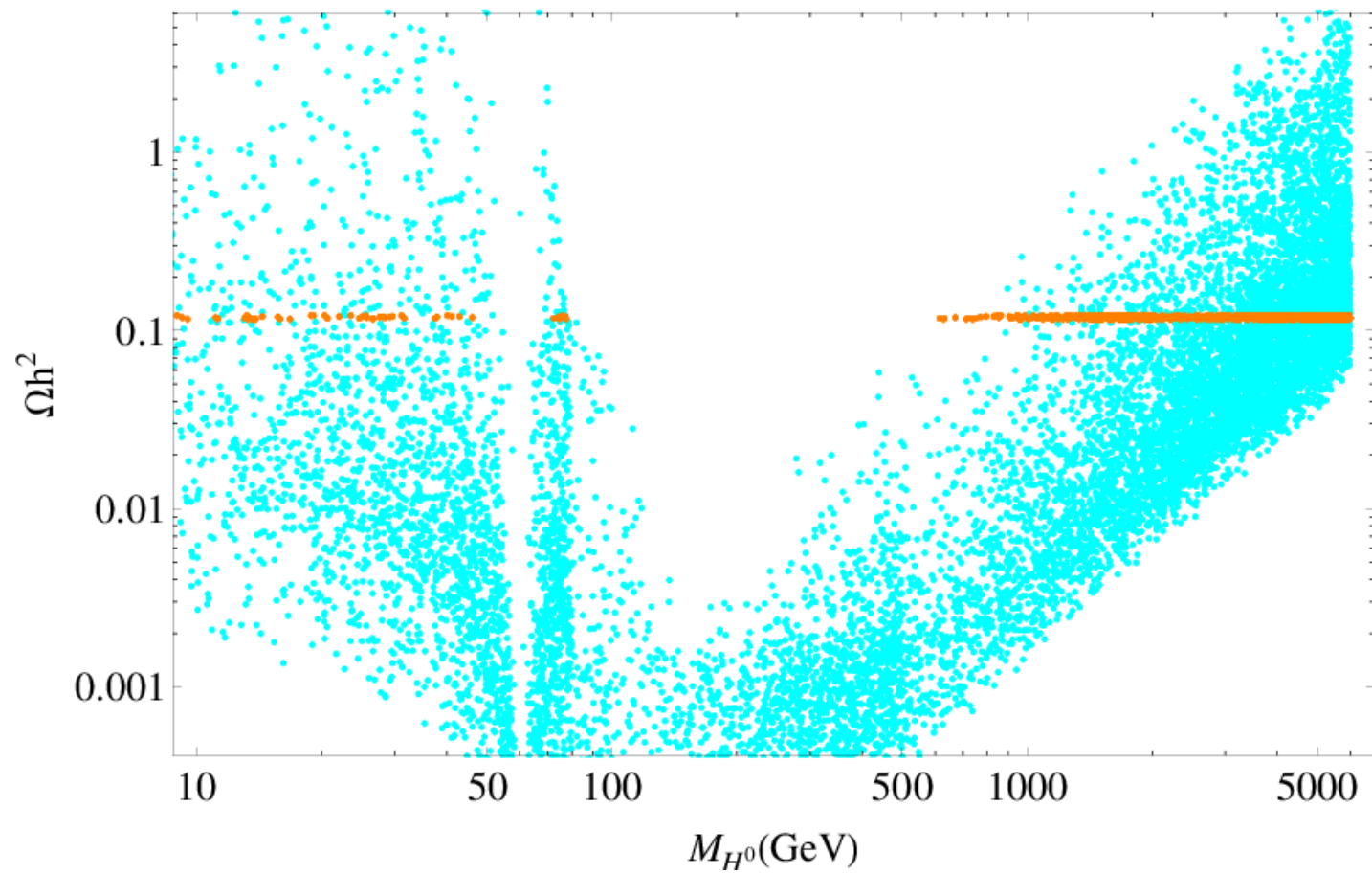
If the lightest particle that is charged under Z_2 is neutral : we have a **dark matter** candidate!!!

$$\mathcal{L}_{\Phi,\Psi} = \mathcal{L}_{\text{Yukawa}} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + (D_\mu \Psi)^\dagger (D^\mu \Psi) - m_1^2 \Phi^\dagger \Phi - m_2^2 \Psi^\dagger \Psi \\ - \lambda_1 (\Phi^\dagger \Phi)^2 - \lambda_2 (\Psi^\dagger \Psi)^2 - \lambda_3 (\Phi^\dagger \Phi) (\Psi^\dagger \Psi) - \lambda_4 (\Phi^\dagger \Psi) (\Psi^\dagger \Phi) - \frac{1}{2} \left(\lambda_5 (\Phi^\dagger \Psi) (\Phi^\dagger \Psi) + \text{h.c.} \right)$$



For a heavy dark matter candidate ($M_{H^0} \gg M_W$) the splitting is relatively small and we expect the particles belonging to the extra doublet to have nearly degenerate masses.

Dark Matter Abundance

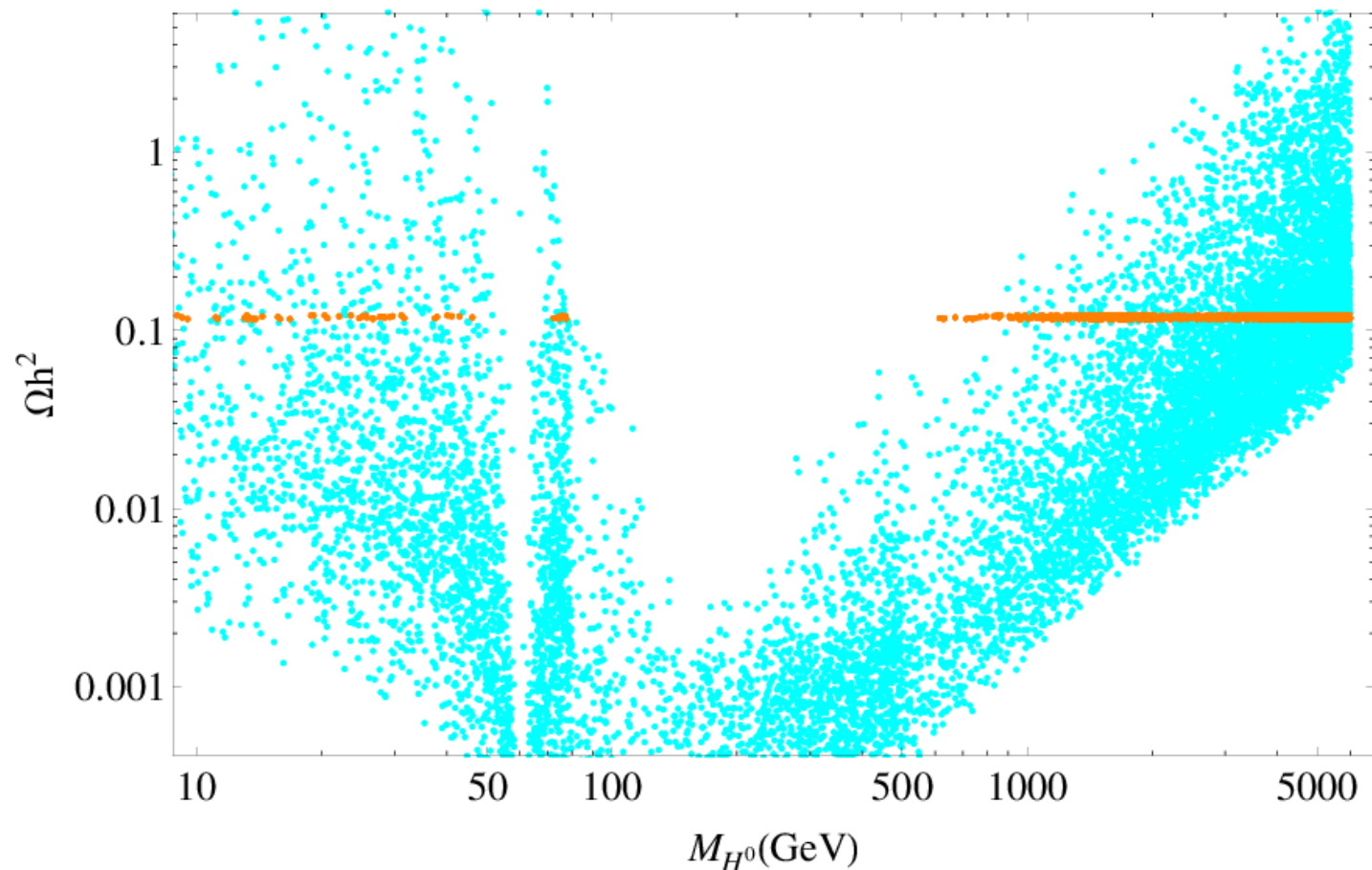


Dark Matter Abundance

$m_{H_0} \lesssim m_W$: GeV range

$$H_0 H_0 \rightarrow h^* \rightarrow \bar{f} f \text{ and } H_0 A_0 \rightarrow Z^* \rightarrow \bar{f} f$$

Barbieri PRD06, LLH JCAP06, Gustafsson PRL07, Cao PRD07, Andreas JCAP08,...



Dark Matter Abundance

$m_{H_0} \lesssim m_W$: GeV range

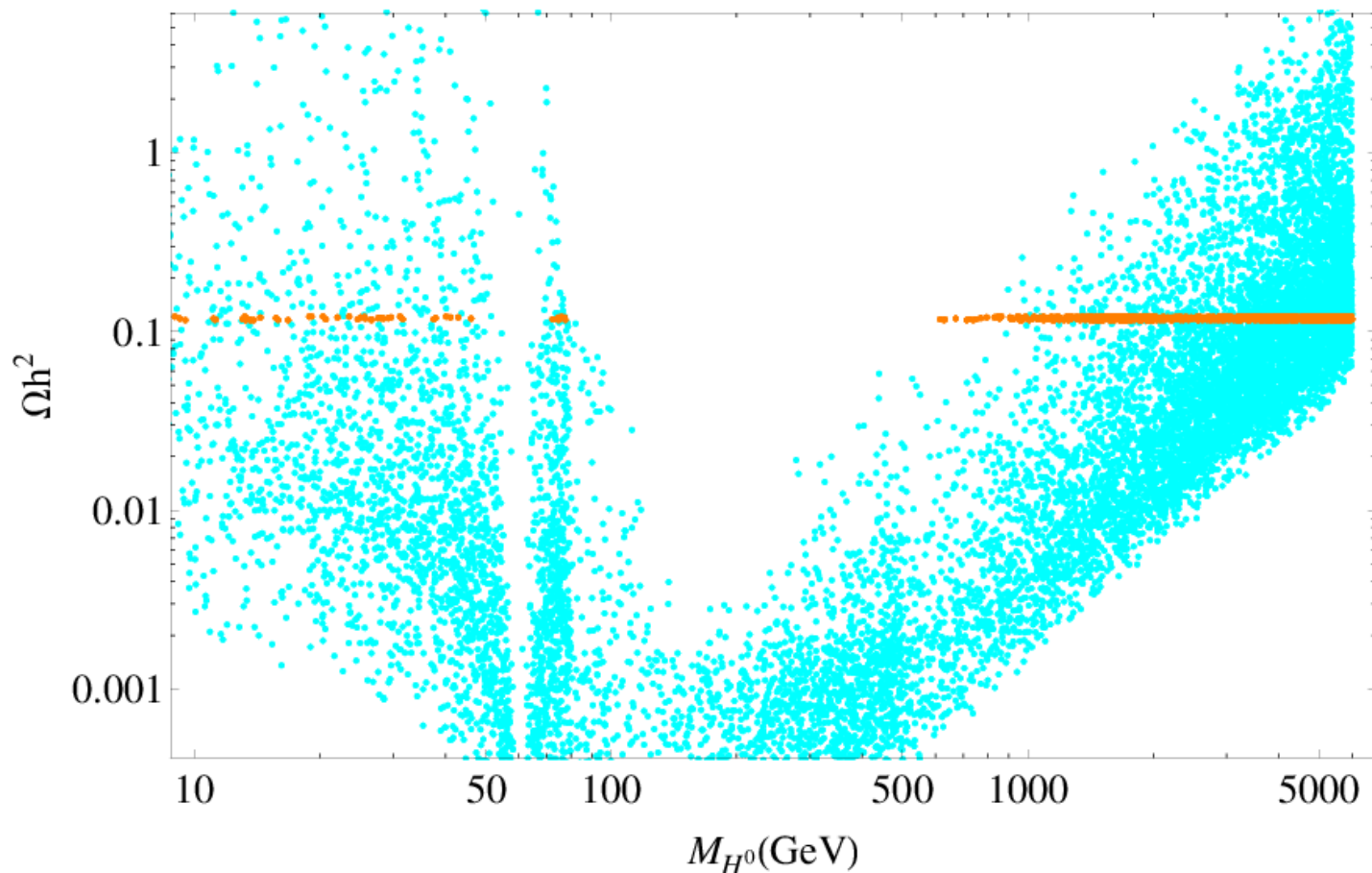
$m_{H_0} \gg m_W$: TeV range

$$H_0 H_0 \rightarrow h^* \rightarrow \bar{f} f \text{ and } H_0 A_0 \rightarrow Z^* \rightarrow \bar{f} f$$

$$H_0 H_0 \rightarrow ZZ, WW, hh$$

Barbieri PRD06, LLH JCAP06, Gustafsson PRL07, Cao PRD07, Andreas JCAP08,...

Cirelli NPB06, Hambye JHEP09



Dark Matter Abundance

$m_{H_0} \lesssim m_W$: GeV range

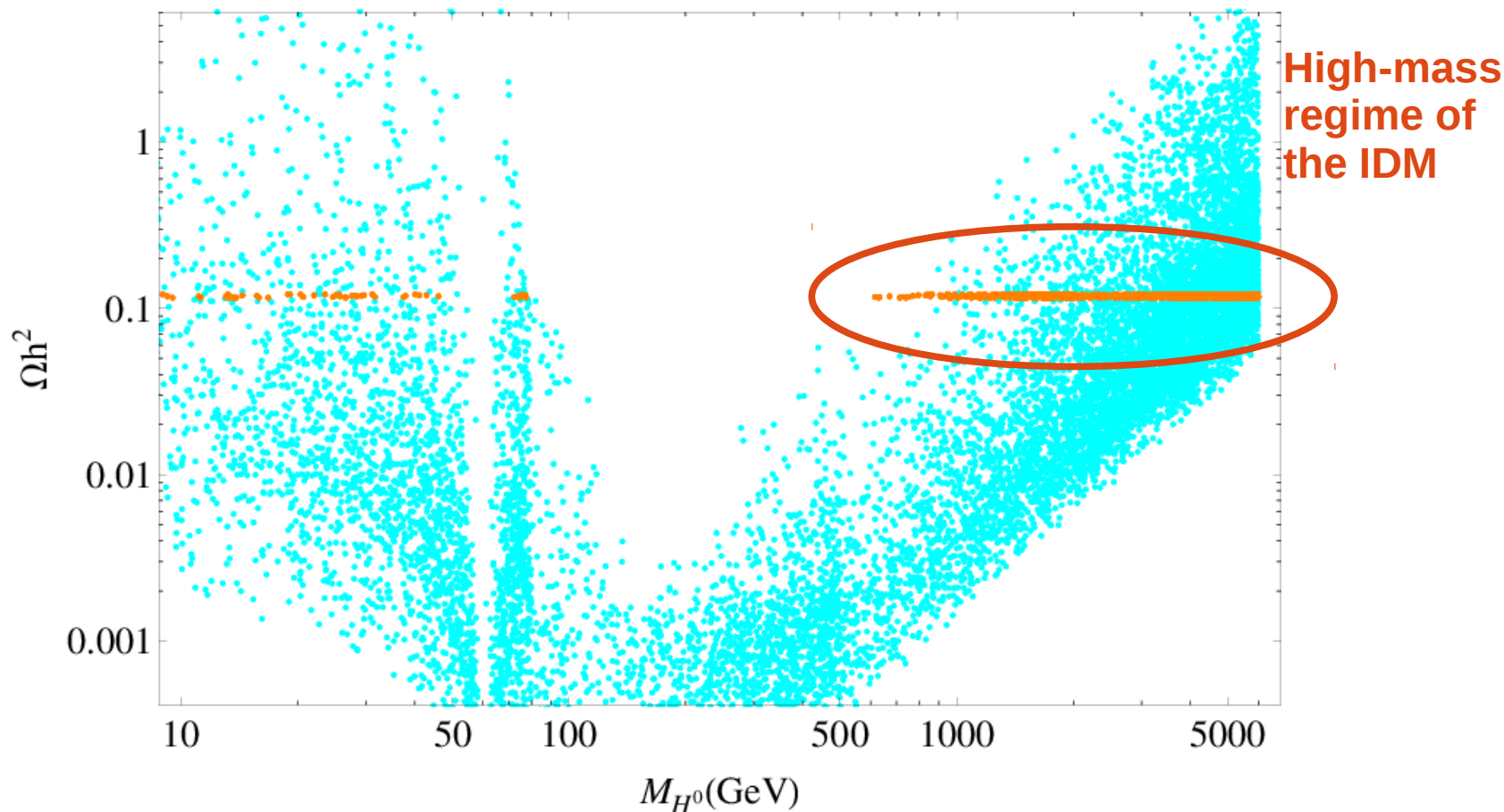
$$H_0 H_0 \rightarrow h^* \rightarrow \bar{f} f \text{ and } H_0 A_0 \rightarrow Z^* \rightarrow \bar{f} f$$

Barbieri PRD06, LLH JCAP06, Gustafsson PRL07, Cao PRD07, Andreas JCAP08,...

$m_{H_0} \gg m_W$: TeV range

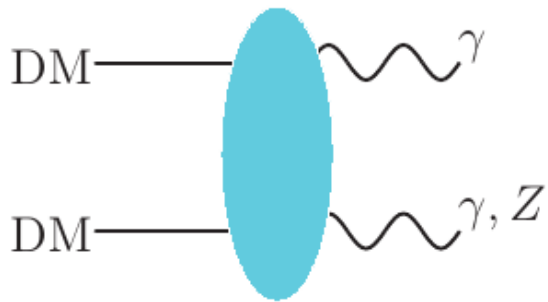
$$H_0 H_0 \rightarrow ZZ, WW, hh$$

Cirelli NPB06, Hambye JHEP09



Gamma-ray spectral features

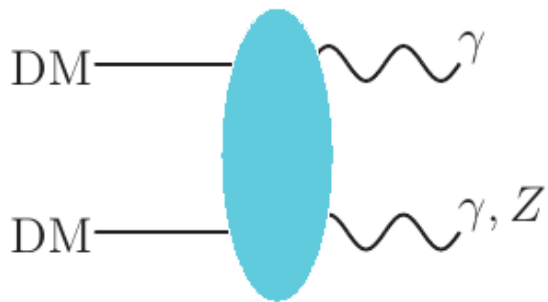
Smoking gun signature for dark matter : no astrophysical process is known to produce a sharp feature in the gamma-ray spectrum



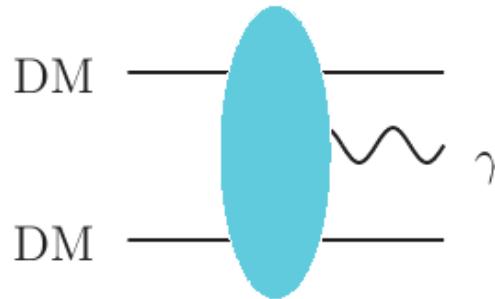
Annihilation into Photons

Gamma-ray spectral features

Smoking gun signature for dark matter : no astrophysical process is known to produce a sharp feature in the gamma-ray spectrum



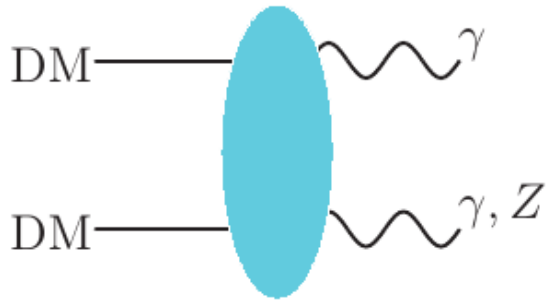
Annihilation into Photons



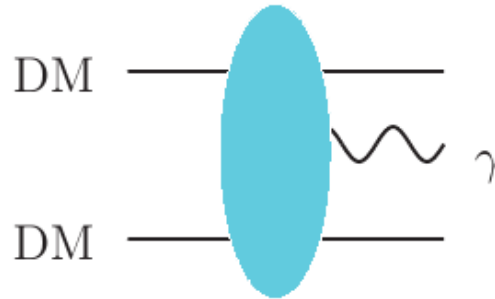
Virtual Internal Bremsstrahlung

Gamma-ray spectral features

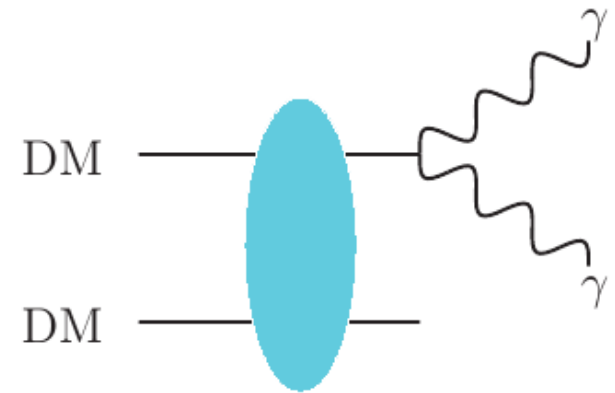
Smoking gun signature for dark matter : no astrophysical process is known to produce a sharp feature in the gamma-ray spectrum



Annihilation into Photons



Virtual Internal Bremsstrahlung

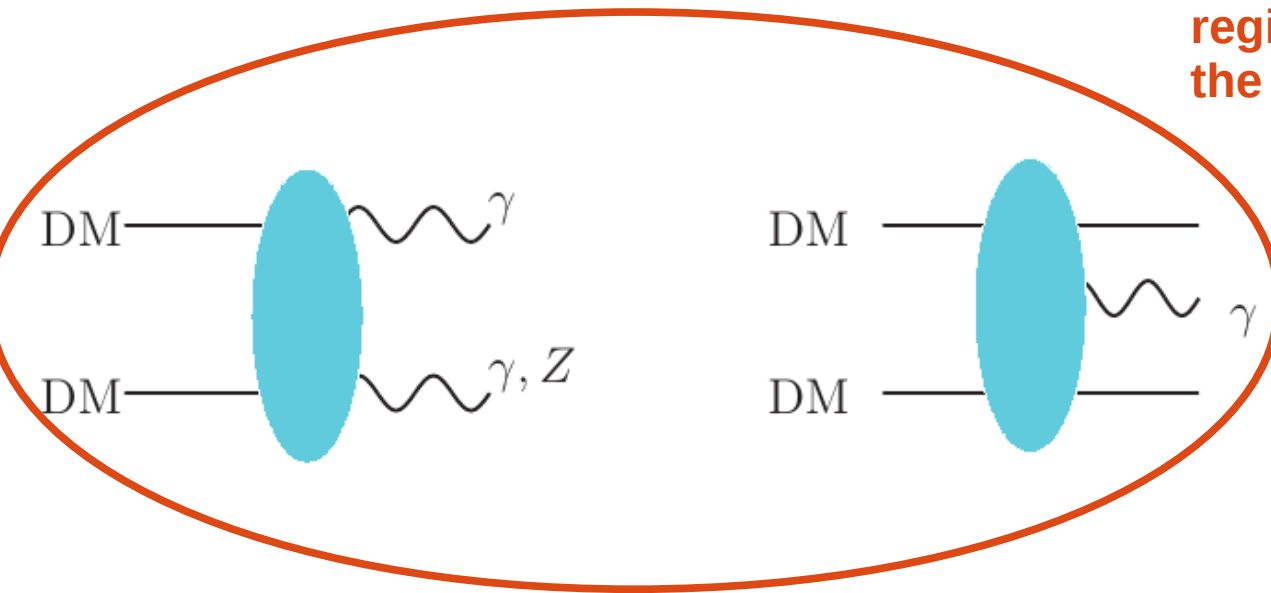


Box-shaped spectra

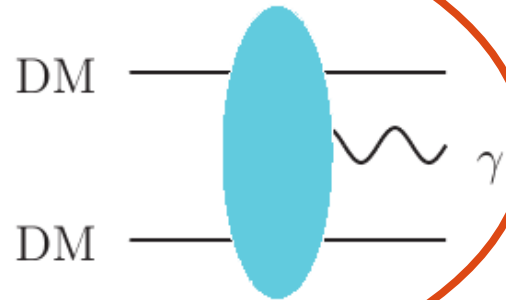
Gamma-ray spectral features

Smoking gun signature for dark matter : no astrophysical process is known to produce a sharp feature in the gamma-ray spectrum

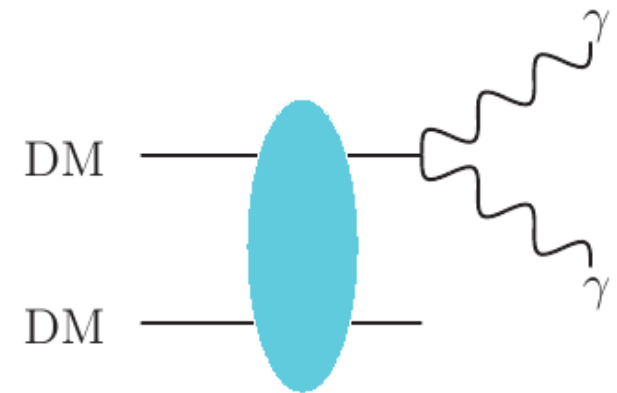
**High-mass
regime of
the IDM**



Annihilation into Photons

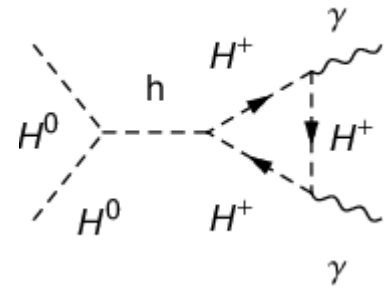
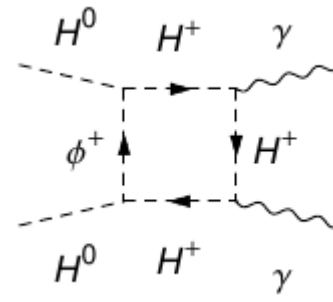
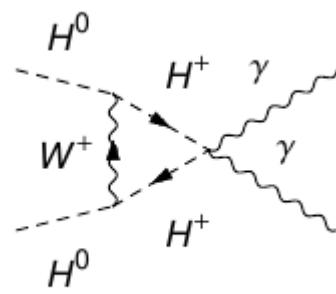


Virtual Internal Bremsstrahlung



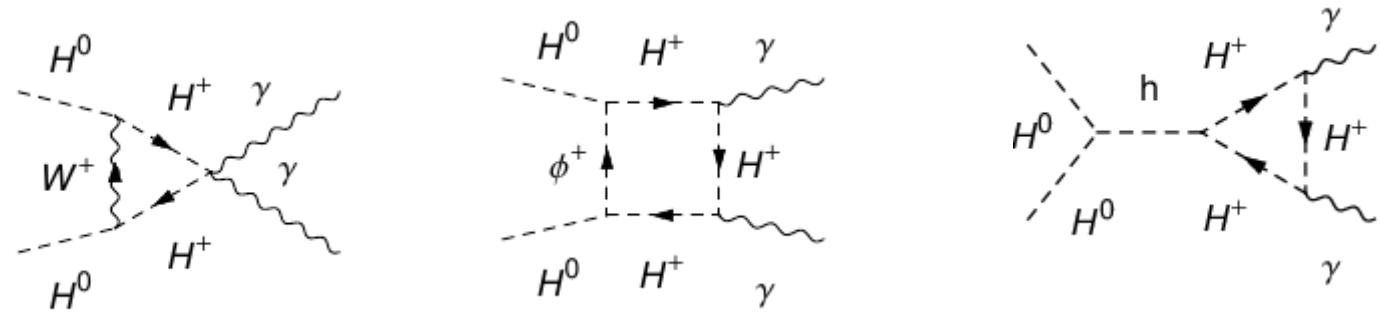
Box-shaped spectra

One-loop annihilation into two photons



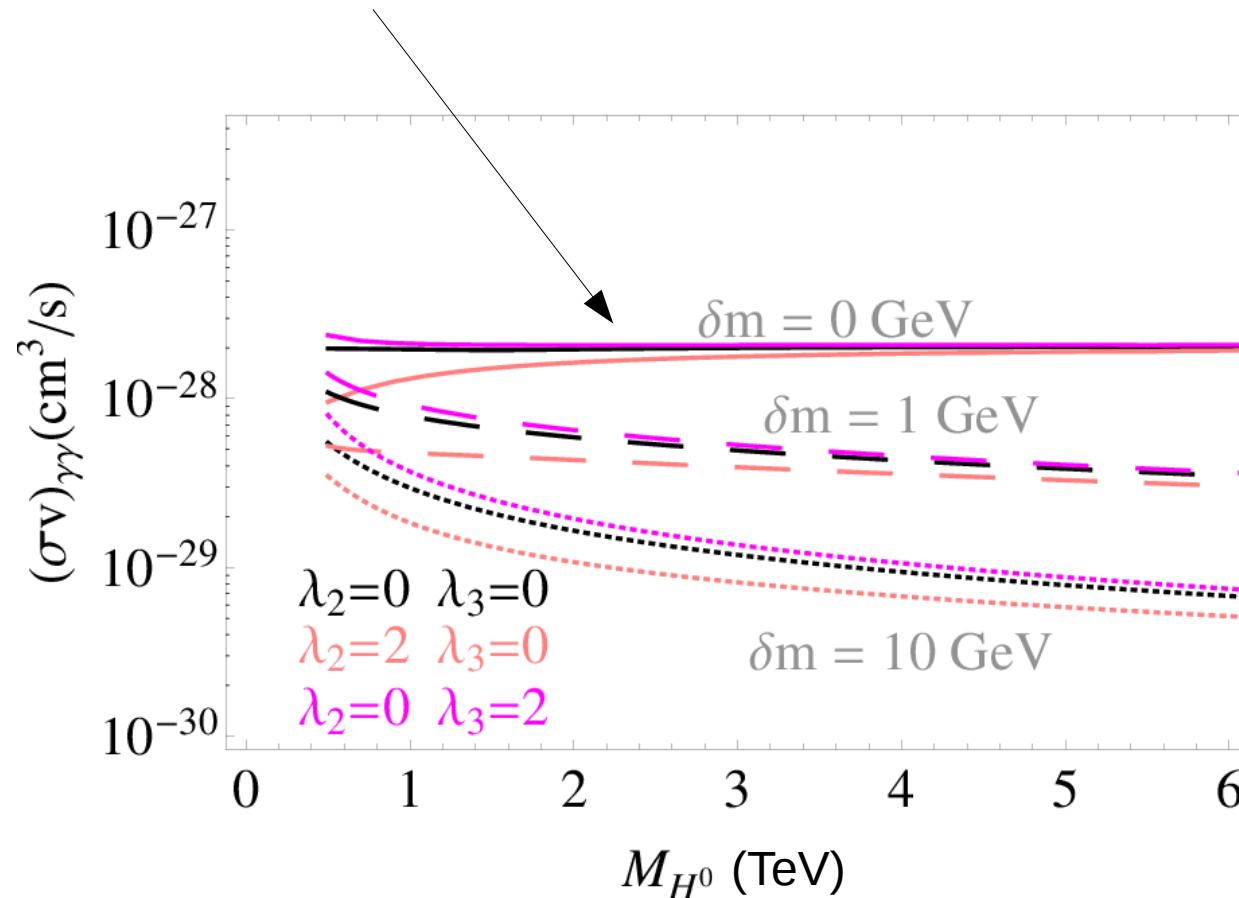
.....and many more

One-loop annihilation into two photons



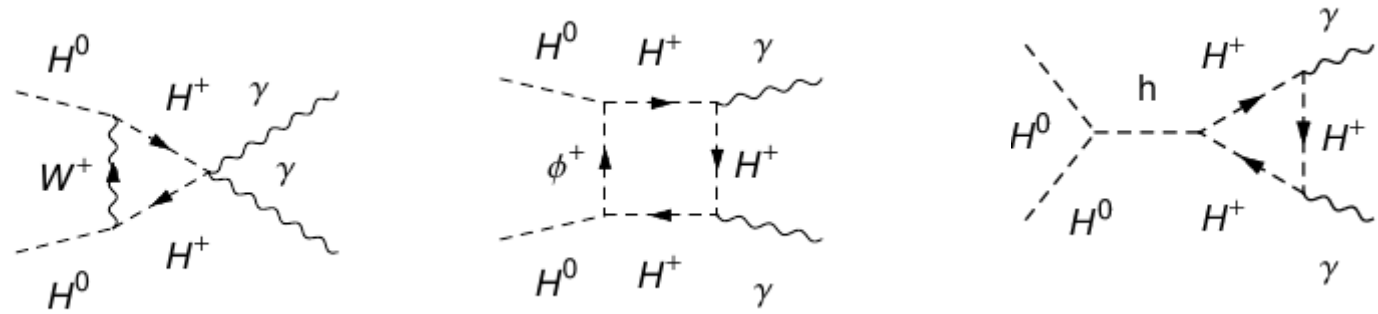
.....and many more

Something is wrong because the cross-section must be bounded from above by the unitarity limit



Preliminary

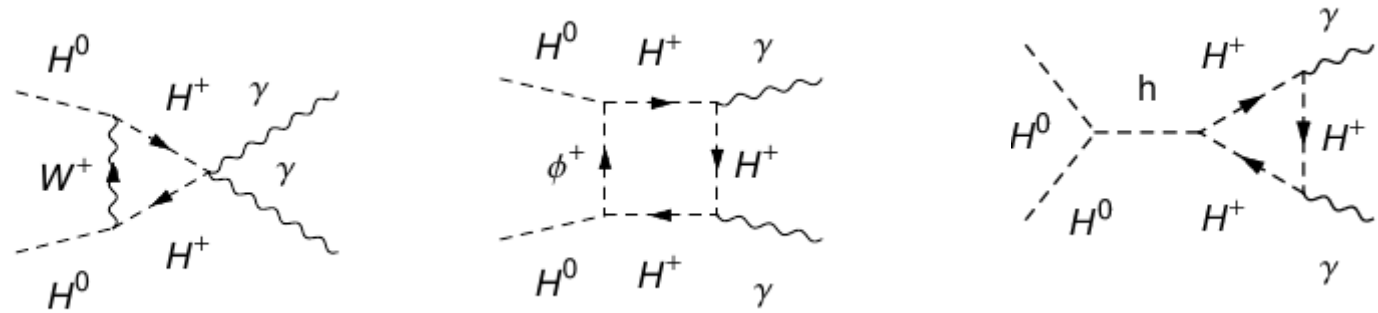
One-loop annihilation into two photons



.....and many more

$$\mathcal{M}(H^0 H^0 \rightarrow \gamma \gamma) \Big|_{s\text{-wave}} = A \left(g^{\mu\nu} - \frac{p_{\gamma_2}^\mu p_{\gamma_1}^\nu}{2M_{H^0}^2} \right) \epsilon_\mu(p_{\gamma_1}) \epsilon_\nu(p_{\gamma_2})$$

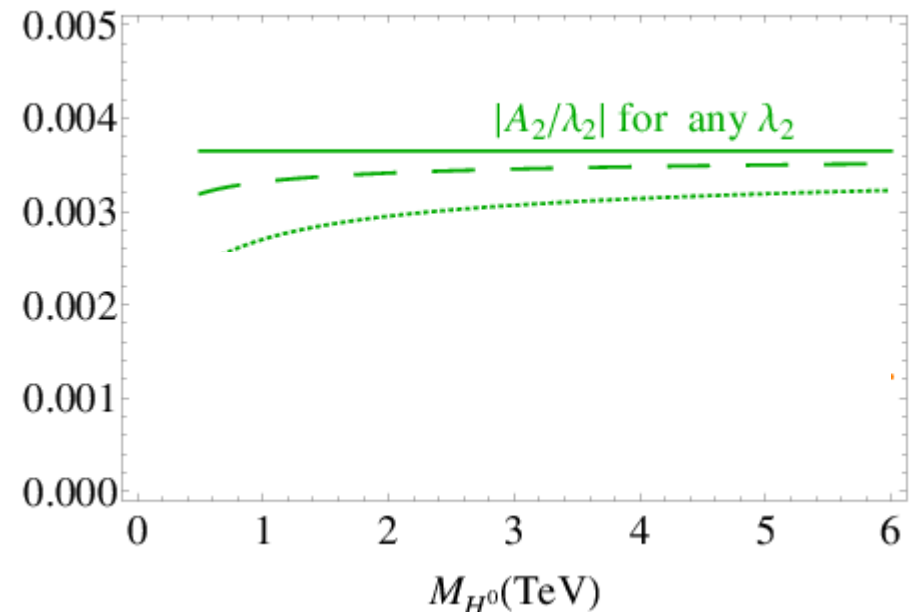
One-loop annihilation into two photons



.....and many more

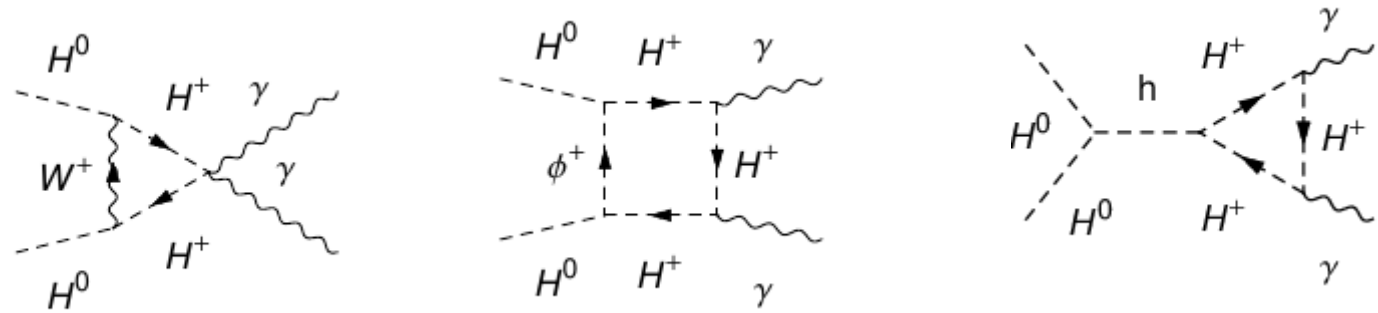
$$+ A_2(M_{H^0}, M_{H^+}, \lambda_2)$$

$$\mathcal{M}(H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = A \left(g^{\mu\nu} - \frac{p_{\gamma_2}^\mu p_{\gamma_1}^\nu}{2M_{H^0}^2} \right) \epsilon_\mu(p_{\gamma_1}) \epsilon_\nu(p_{\gamma_2})$$



Preliminary

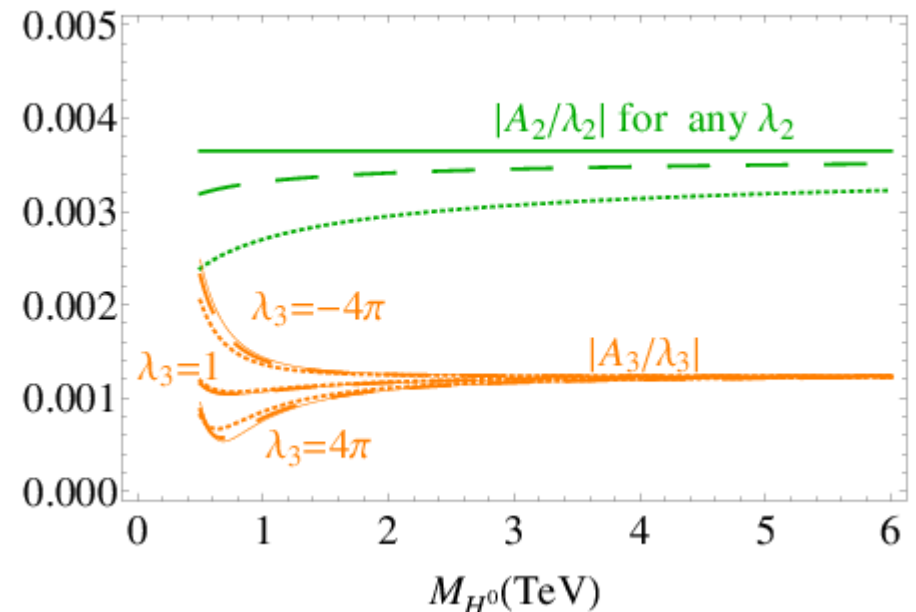
One-loop annihilation into two photons



.....and many more

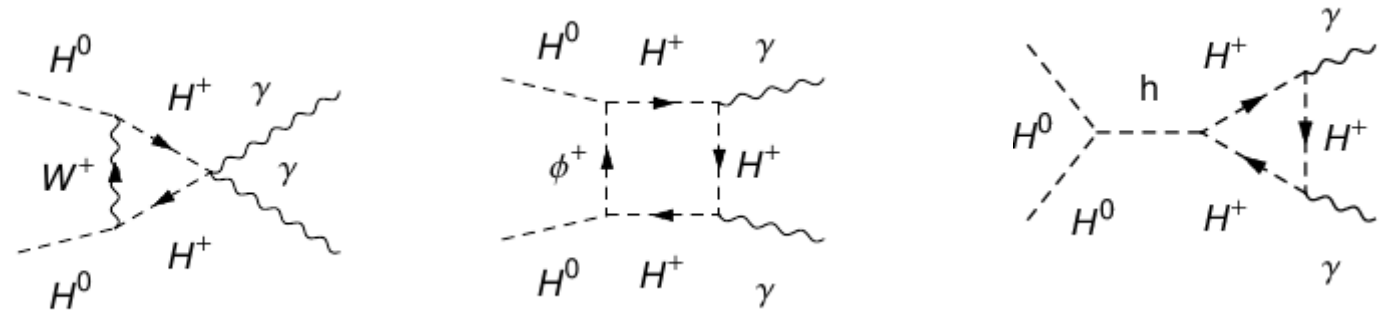
$$+ A_2(M_{H^0}, M_{H^+}, \lambda_2) + A_3(M_{H^0}, M_{H^+}, \lambda_3)$$

$$\mathcal{M}(H^0 H^0 \rightarrow \gamma \gamma) \Big|_{s\text{-wave}} = A \left(g^{\mu\nu} - \frac{p_{\gamma_2}^\mu p_{\gamma_1}^\nu}{2M_{H^0}^2} \right) \epsilon_\mu(p_{\gamma_1}) \epsilon_\nu(p_{\gamma_2})$$



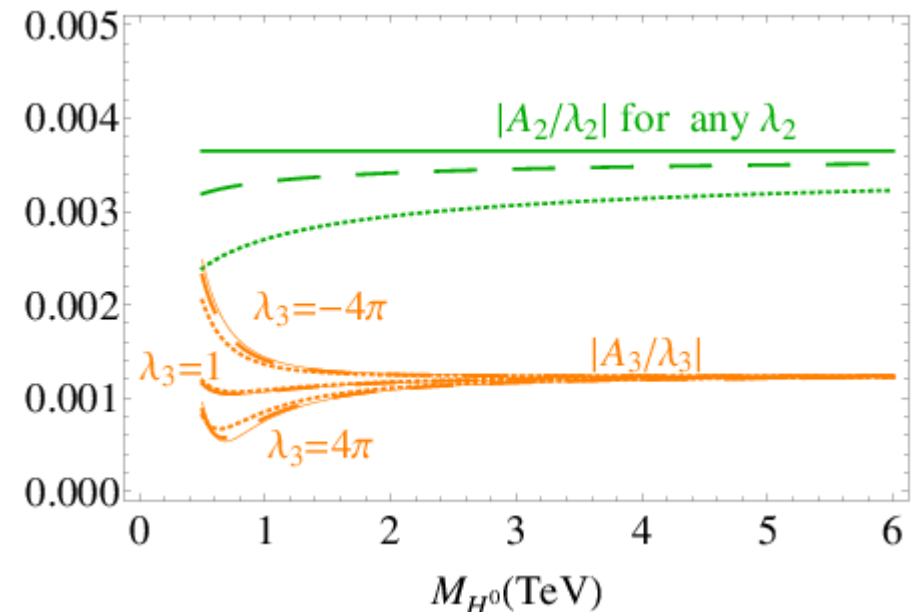
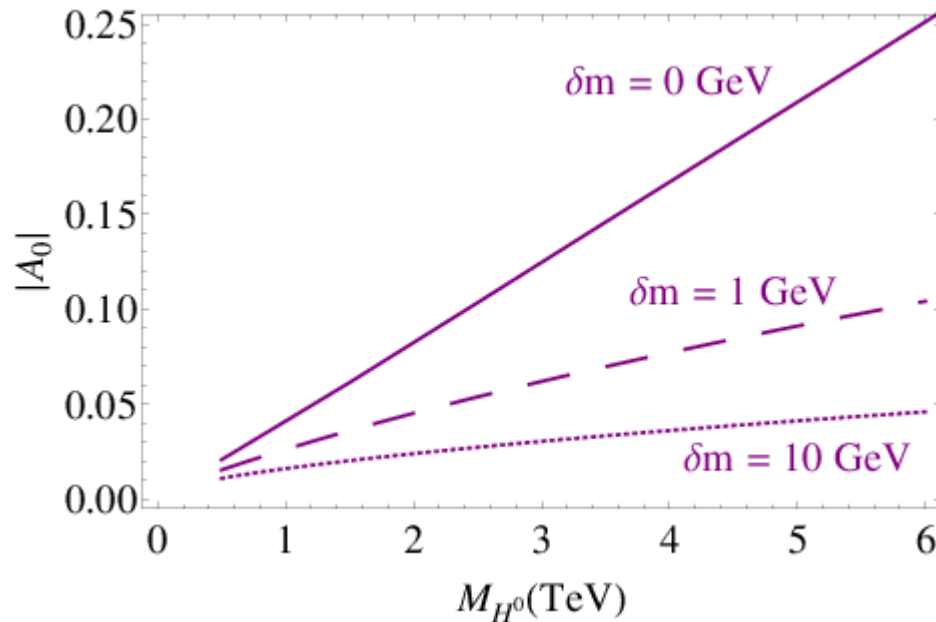
Preliminary

One-loop annihilation into two photons



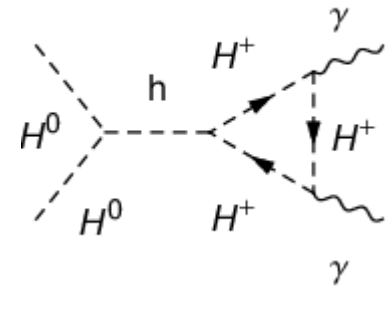
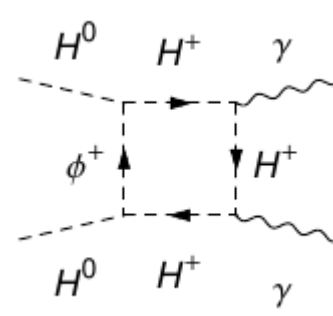
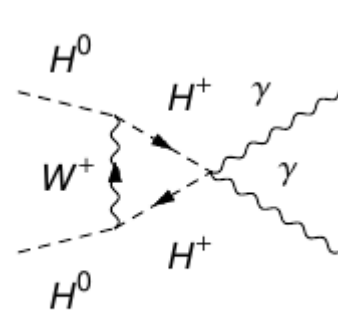
.....and many more

$$\mathcal{M}(H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = \underbrace{A_0(M_{H^0}, M_{H^+}) + A_2(M_{H^0}, M_{H^+}, \lambda_2) + A_3(M_{H^0}, M_{H^+}, \lambda_3)}_A \left(g^{\mu\nu} - \frac{p_{\gamma_2}^\mu p_{\gamma_1}^\nu}{2M_{H^0}^2} \right) \epsilon_\mu(p_{\gamma_1}) \epsilon_\nu(p_{\gamma_2})$$



Preliminary

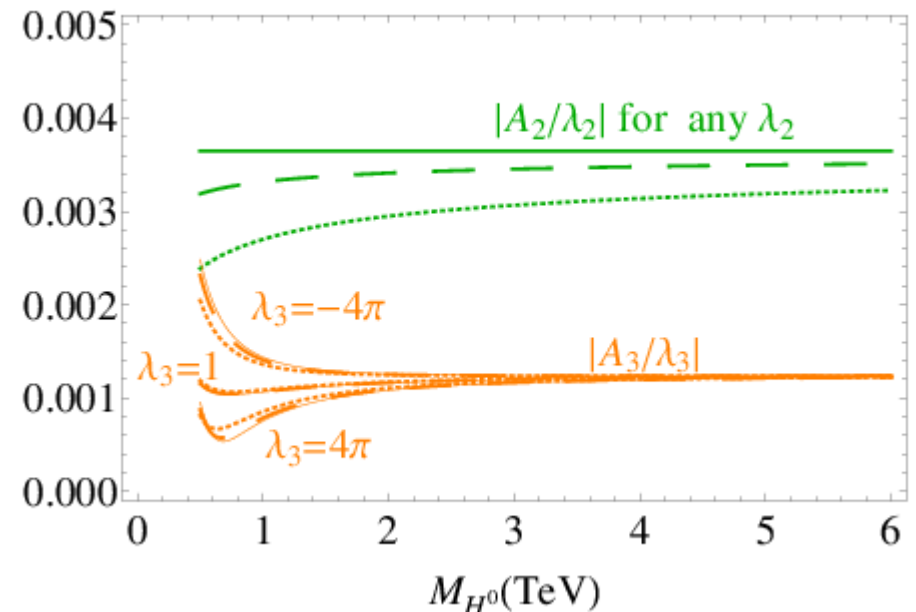
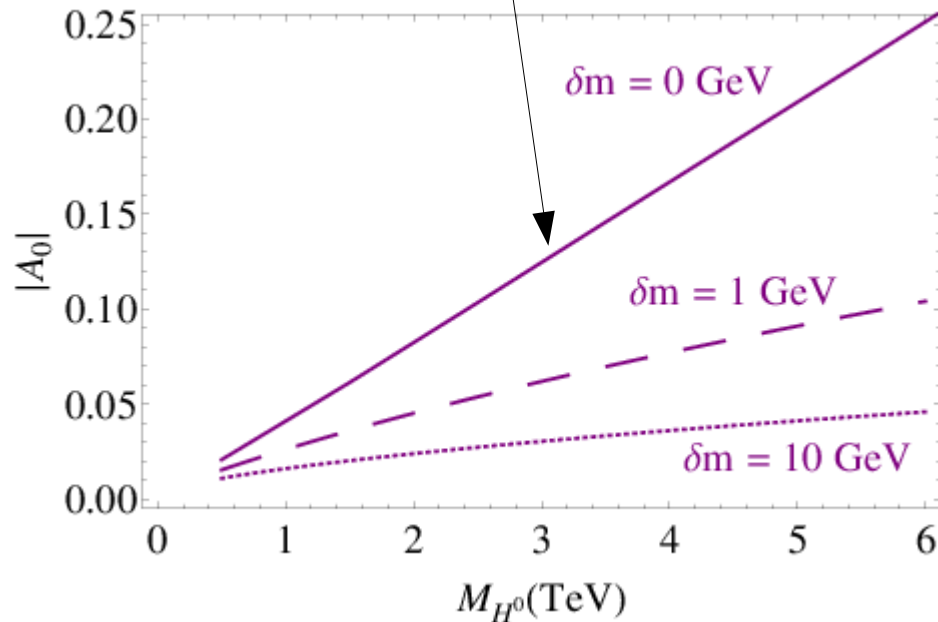
One-loop annihilation into two photons



.....and many more

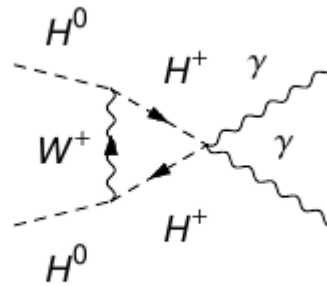
Something is wrong because

$$\sigma v (H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = \frac{|A|^2}{32\pi M_{H^0}^2}$$



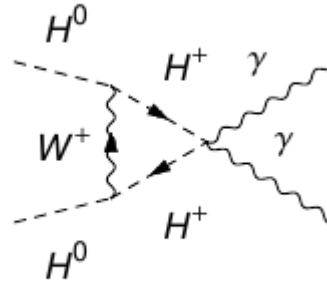
Preliminary

What is wrong?



$$A_0 \supset -\frac{4\alpha\alpha_2 M_{H^0}}{M_W} \int_0^1 dx \frac{1}{\sqrt{1-x}} \arctan \left(\frac{M_{H^0}}{M_W} \frac{x}{\sqrt{1-x}} \right).$$

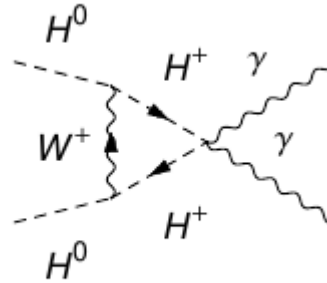
What is wrong?



$$A_0 \supset -\frac{4c\alpha_2 M_{H^0}}{M_W} \int_0^1 dx \frac{1}{\sqrt{1-x}} \arctan \left(\frac{M_{H^0}}{M_W} \frac{x}{\sqrt{1-x}} \right).$$

- For each W boson that is exchanged in the initial state one gets a factor that goes like $\alpha_2 M_{H^0}/M_W$.

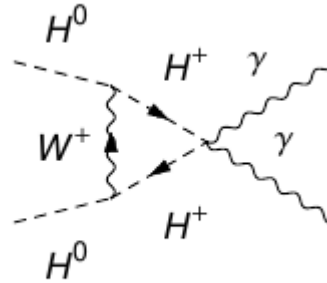
What is wrong?



$$A_0 \supset -\frac{4c\alpha_2 M_{H^0}}{M_W} \int_0^1 dx \frac{1}{\sqrt{1-x}} \arctan \left(\frac{M_{H^0}}{M_W} \frac{x}{\sqrt{1-x}} \right).$$

- For each W boson that is exchanged in the initial state one gets a factor that goes like $\alpha_2 M_{H^0}/M_W$.
- If $M_{H^0} \gtrsim M_W/\alpha_2 \approx 2 \text{ TeV}$, the perturbative calculation breaks down because higher-order loop diagrams become more and more important.

What is wrong?



$$A_0 \supset -\frac{4c\alpha_2 M_{H^0}}{M_W} \int_0^1 dx \frac{1}{\sqrt{1-x}} \arctan \left(\frac{M_{H^0}}{M_W} \frac{x}{\sqrt{1-x}} \right).$$

- For each W boson that is exchanged in the initial state one gets a factor that goes like $\alpha_2 M_{H^0}/M_W$.
- If $M_{H^0} \gtrsim M_W/\alpha_2 \approx 2 \text{ TeV}$, the perturbative calculation breaks down because higher-order loop diagrams become more and more important.
- For these masses and in general in the high-mass regime of the IDM, the one-loop calculation is not reliable until these effects are taken into account.

Sommerfeld Enhancement!

Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that
$$g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$$

Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that

$$g''(r) + M_{H^0} \left(\underbrace{\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r)}_{2\delta m} \right) g(r) = 0$$

Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that

$$g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$$

$2\delta m + V_{\text{Gauge}}(r) + V_{\text{Scalar}}(r)$

Yukawa Potentials

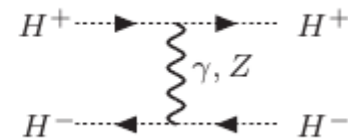
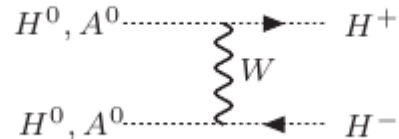
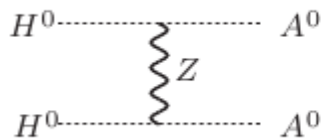
Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that

$$g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$$

$2\delta m + \underbrace{V_{\text{Gauge}}(r) + V_{\text{Scalar}}(r)}_{\text{Yukawa Potentials}}$



Sommerfeld Enhancement

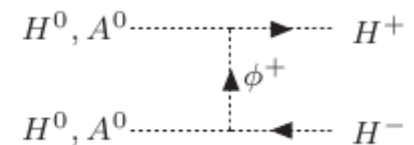
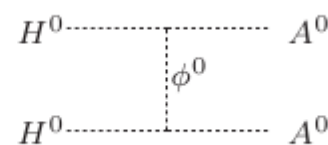
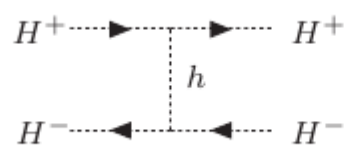
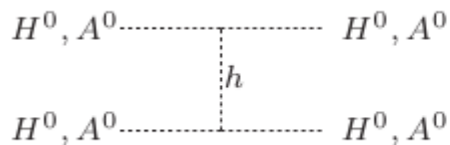
The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that

$$g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$$

$2\delta m + V_{\text{Gauge}}(r) + V_{\text{Scalar}}(r)$

Yukawa Potentials

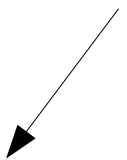


Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that
$$g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$$

Final State



$$\sigma v (H^0 H^0 \rightarrow f) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \cdot \left| \mathcal{M} (H^0 H^0 \rightarrow f) \right|^2$$

Sommerfeld Enhancement

The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that $g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$

$$\sigma v (H^0 H^0 \rightarrow f) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \cdot \left[d_{11} \mathcal{M} (H^0 H^0 \rightarrow f) + \right.$$

2

Enhancement factors

Sommerfeld Enhancement

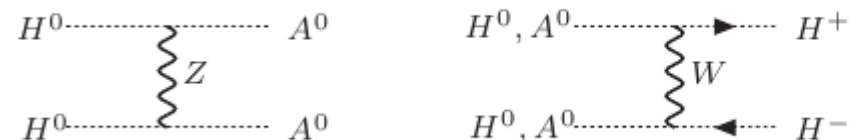
The exchange of W bosons - and in general of any boson- leads to a long range interaction that distorts the wave function of the annihilating particles

Schrödinger equation accounting for that
$$g''(r) + M_{H^0} \left(\frac{1}{4} M_{H^0} v^2 \mathbb{1} - V(r) \right) g(r) = 0$$

Final State

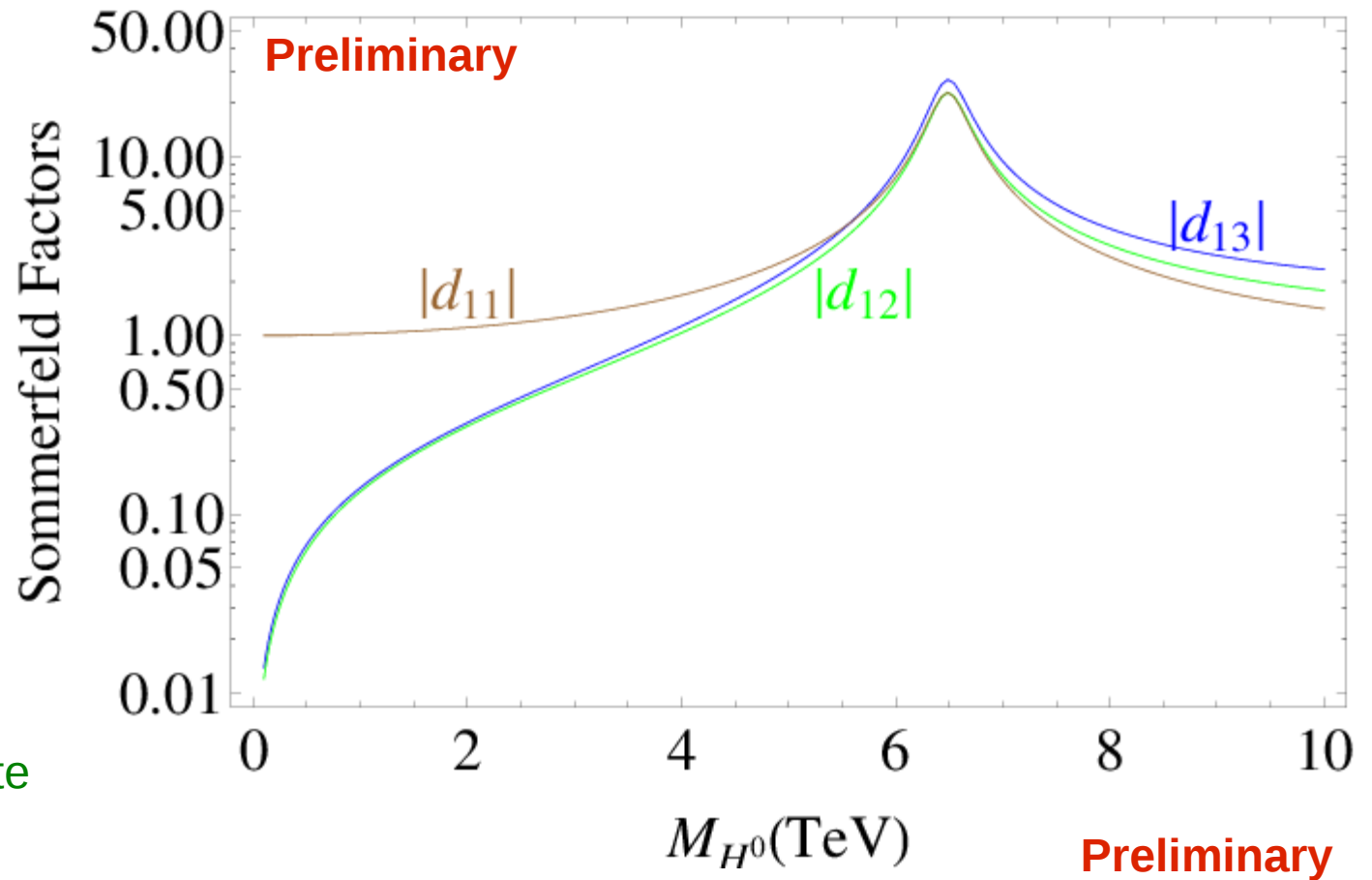
New annihilation channels open-up

$$\sigma v (H^0 H^0 \rightarrow f) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \cdot \left| d_{11} \mathcal{M}(H^0 H^0 \rightarrow f) + d_{12} \mathcal{M}(A^0 A^0 \rightarrow f) + \sqrt{2} d_{13} \mathcal{M}(H^+ H^- \rightarrow f) \right|^2$$



Enhancement factors

Example:
when all the
quartic
couplings
vanish



Final State

$$\sigma v (H^0 H^0 \rightarrow f) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a)$$

$$\cdot \left| d_{11} \mathcal{M} (H^0 H^0 \rightarrow f) + d_{12} \mathcal{M} (A^0 A^0 \rightarrow f) + \sqrt{2} d_{13} \mathcal{M} (H^+ H^- \rightarrow f) \right|^2$$

Including this effect solves the problem with unitarity!!!

Hisano, Matsumoto, Nojiri, Saito PRD05

Including this effect solves the problem with unitarity!!!

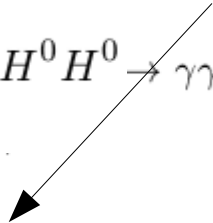
Hisano, Matsumoto, Nojiri, Saito PRD05

$$\sigma v (H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \\ \cdot \left| d_{11} \mathcal{M} (H^0 H^0 \rightarrow \gamma\gamma) + d_{12} \mathcal{M} (A^0 A^0 \rightarrow \gamma\gamma) + \sqrt{2} d_{13} \mathcal{M} (H^+ H^- \rightarrow \gamma\gamma) \right|^2$$

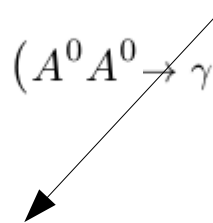
Including this effect solves the problem with unitarity!!!

Hisano, Matsumoto, Nojiri, Saito PRD05

$$\sigma v (H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \cdot \left| d_{11} \mathcal{M}(H^0 H^0 \rightarrow \gamma\gamma) + d_{12} \mathcal{M}(A^0 A^0 \rightarrow \gamma\gamma) + \sqrt{2} d_{13} \mathcal{M}(H^+ H^- \rightarrow \gamma\gamma) \right|^2$$



0

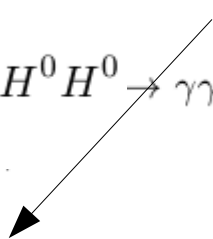


0

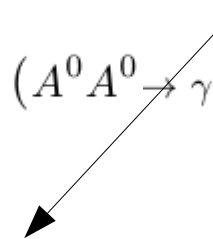
Including this effect solves the problem with unitarity!!!

Hisano, Matsumoto, Nojiri, Saito PRD05

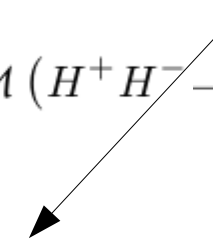
$$\sigma v (H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \cdot \left| d_{11} \mathcal{M}(H^0 H^0 \rightarrow \gamma\gamma) + d_{12} \mathcal{M}(A^0 A^0 \rightarrow \gamma\gamma) + \sqrt{2} d_{13} \mathcal{M}(H^+ H^- \rightarrow \gamma\gamma) \right|^2$$



0



0

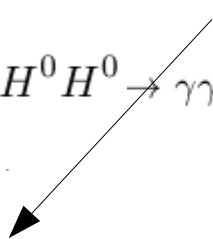


A constant at
tree-level

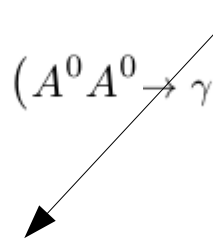
Including this effect solves the problem with unitarity!!!

Hisano, Matsumoto, Nojiri, Saito PRD05

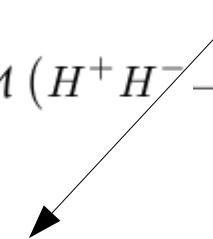
$$\sigma v (H^0 H^0 \rightarrow \gamma\gamma) \Big|_{s\text{-wave}} = \frac{1}{4M_{H^0}^2} \int \left(\prod_{a \in f} \frac{d^3 q_a}{(2\pi)^3 2E_a} \right) (2\pi)^4 \delta^4 (p_{H^0} + p'_{H^0} - \sum_{a \in f} q_a) \cdot \left| d_{11} \mathcal{M}(H^0 H^0 \rightarrow \gamma\gamma) + d_{12} \mathcal{M}(A^0 A^0 \rightarrow \gamma\gamma) + \sqrt{2} d_{13} \mathcal{M}(H^+ H^- \rightarrow \gamma\gamma) \right|^2$$



0

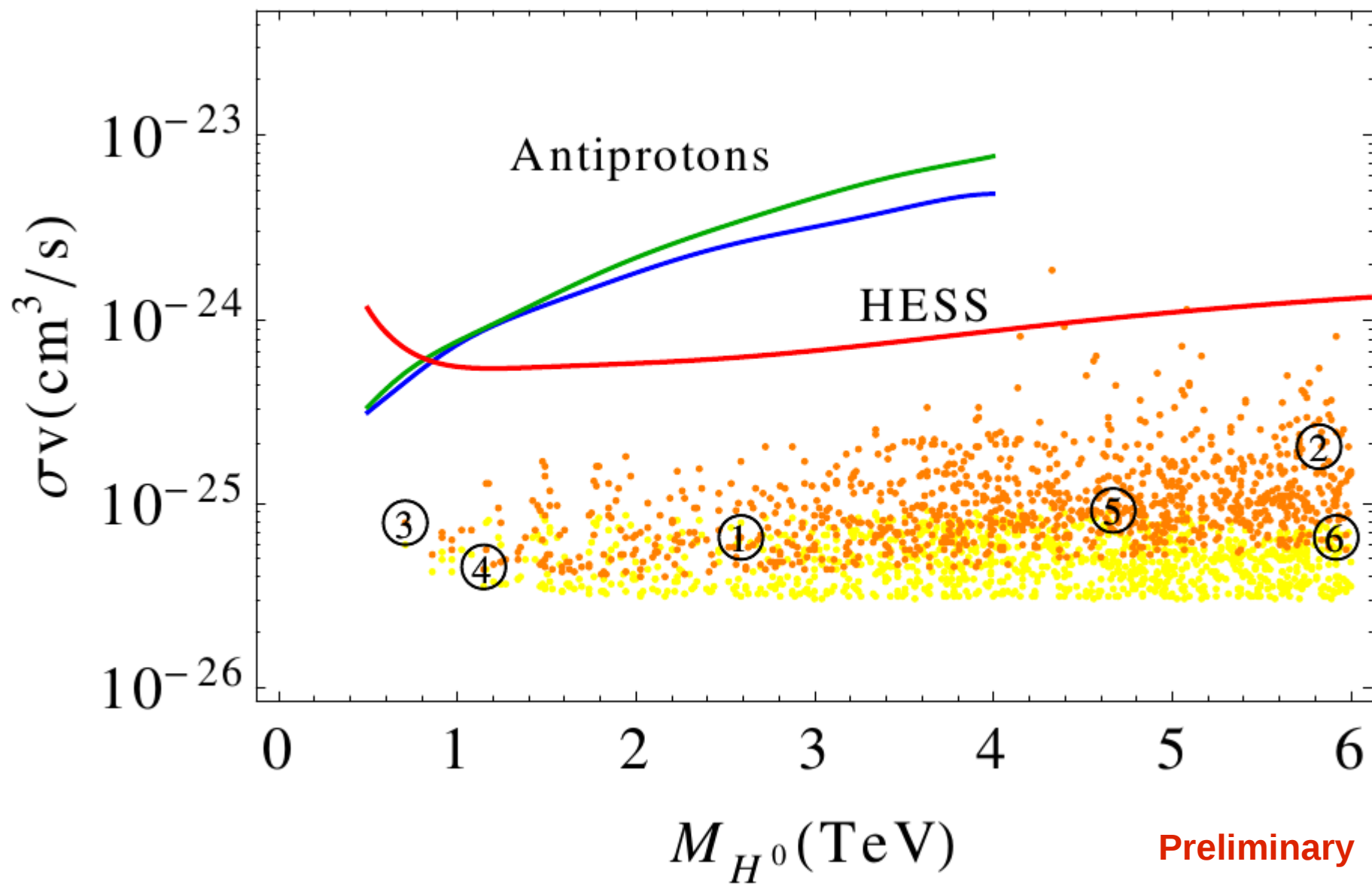


0



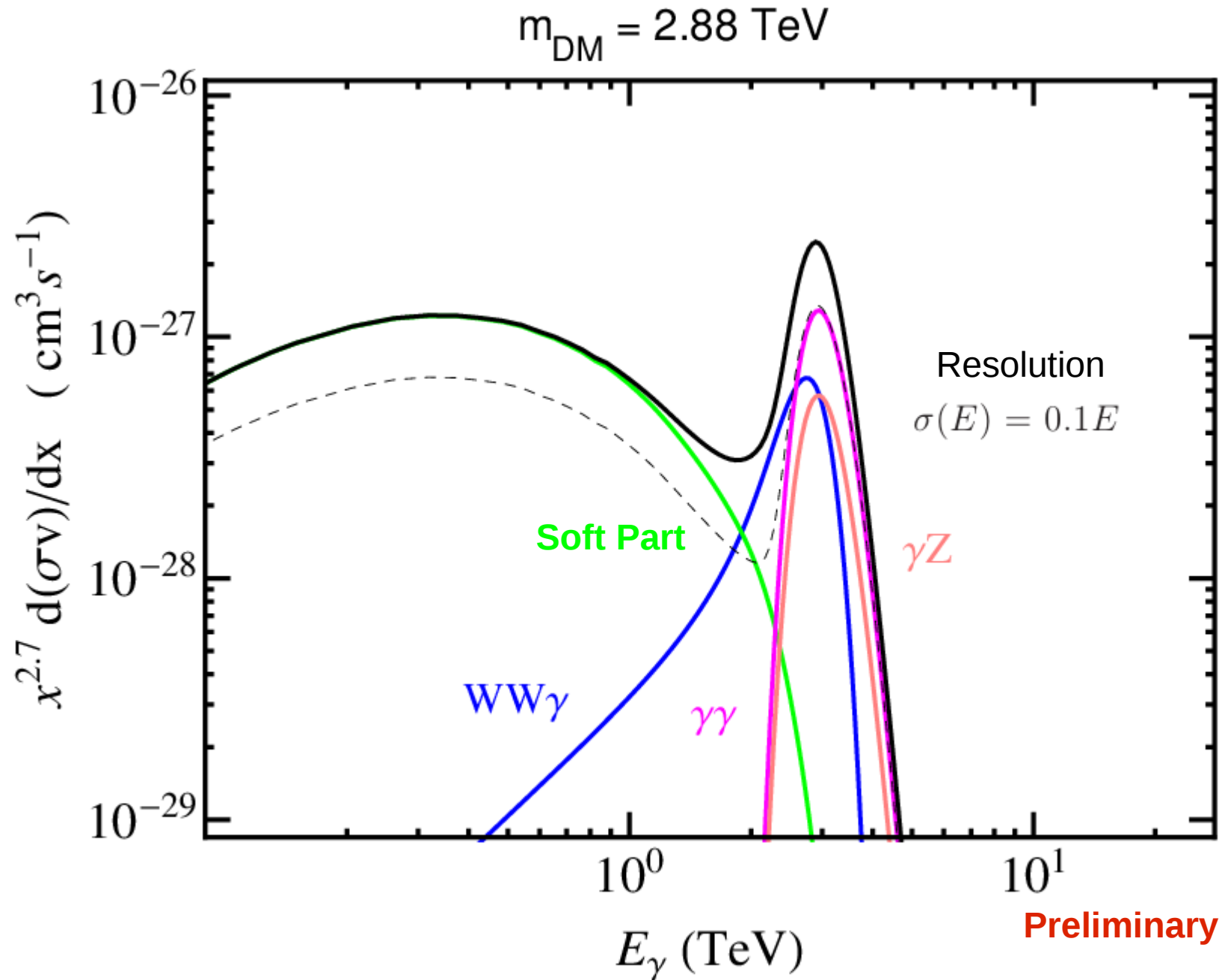
A constant at
tree-level

$$\sigma v (H^0 H^0 \rightarrow \gamma\gamma) = \frac{4\pi\alpha^2 |d_{13}|^2}{M_{H^0}^2}$$



One Benchmark

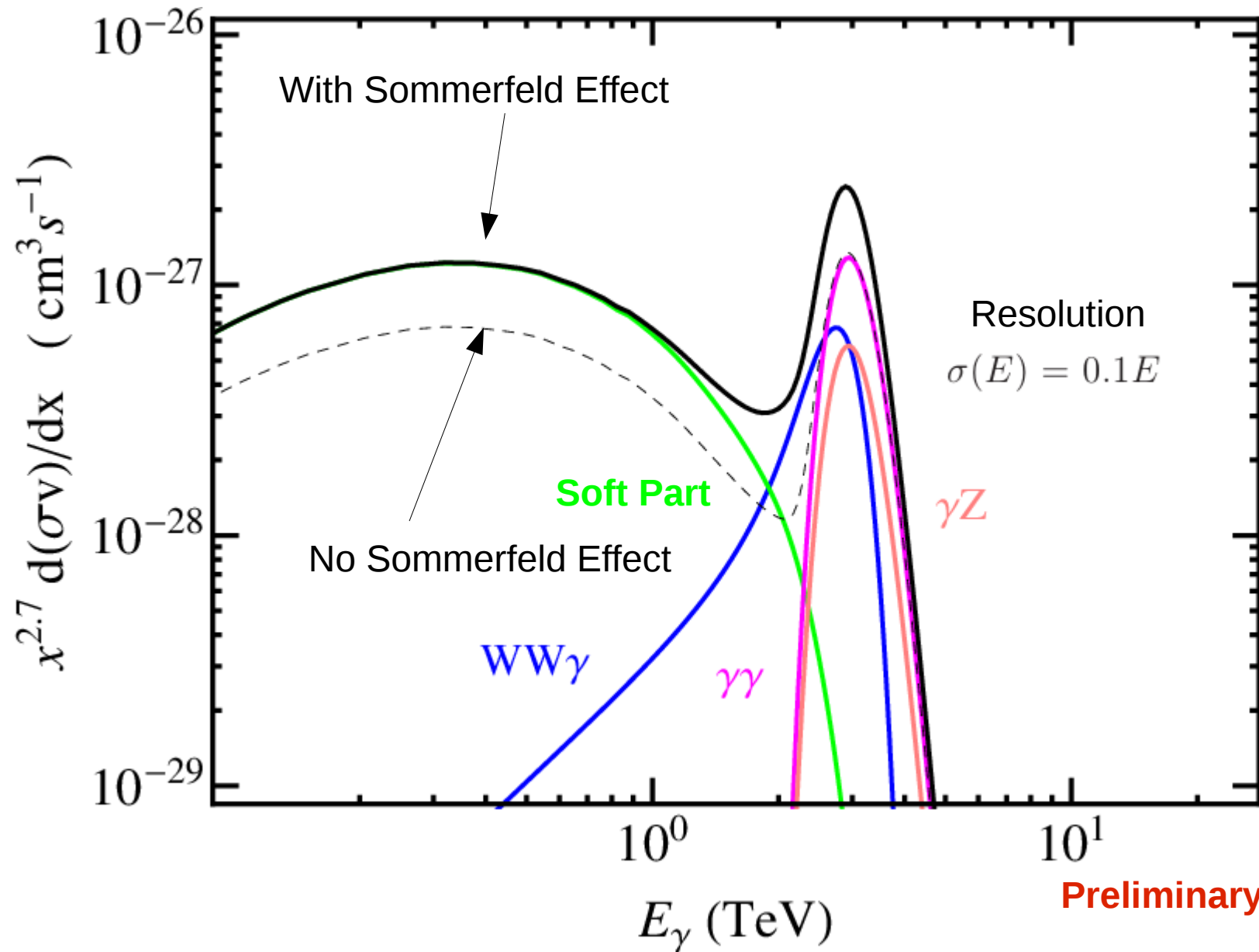
The lines dominate over IB



One Benchmark

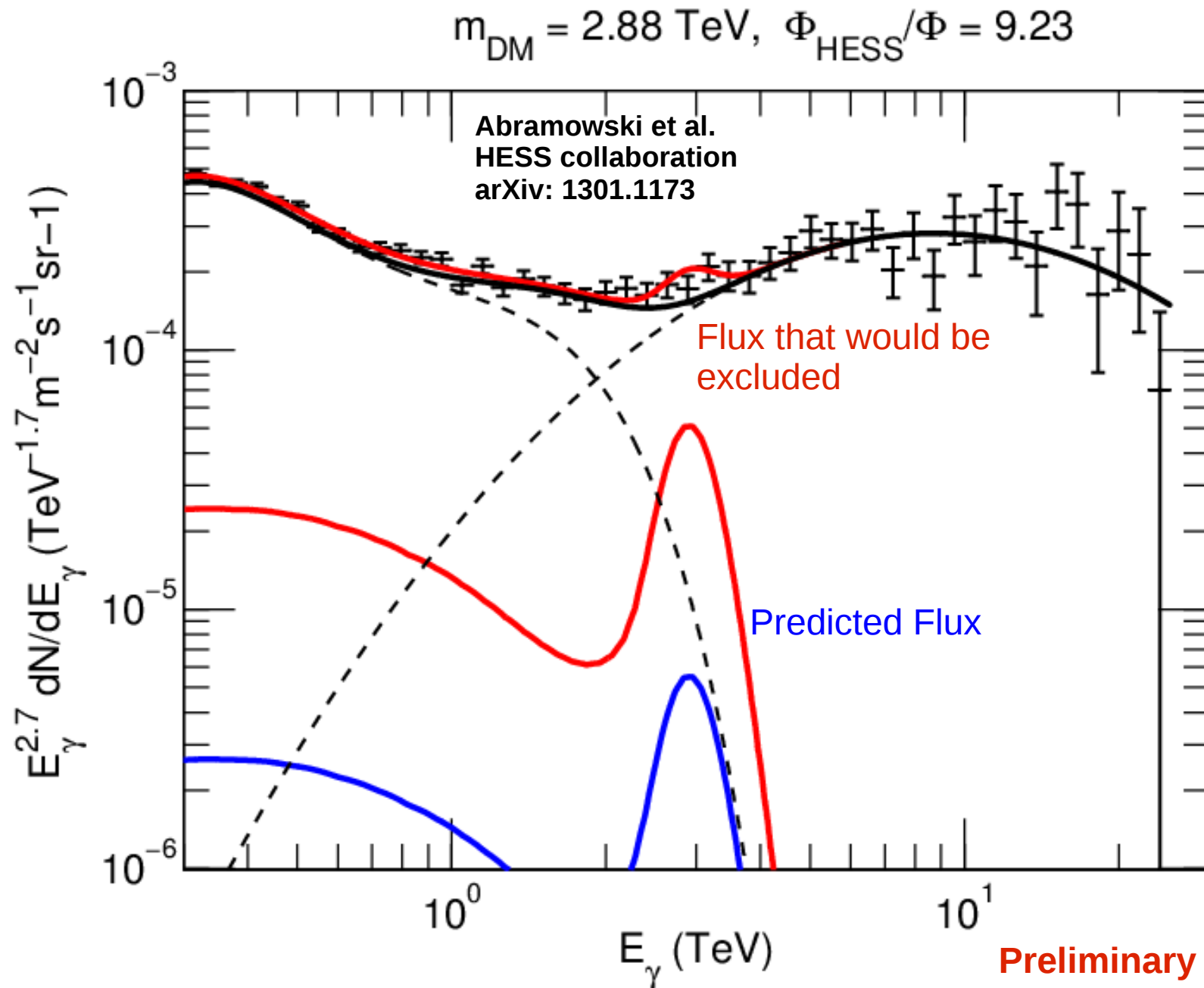
The lines dominate over IB

$$m_{\text{DM}} = 2.88 \text{ TeV}$$



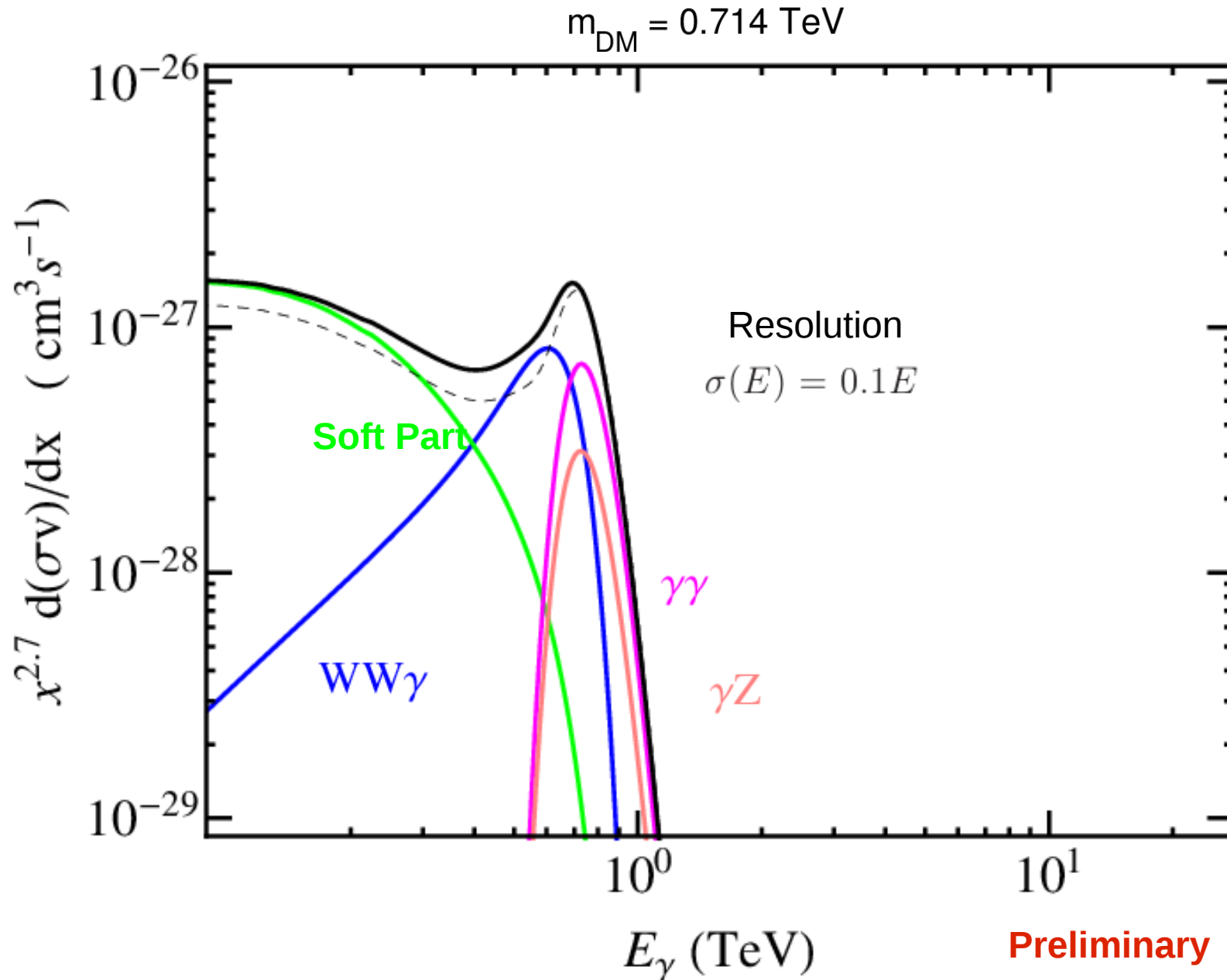
One Benchmark

The lines dominate over IB



Another Benchmark

IB dominates over the lines

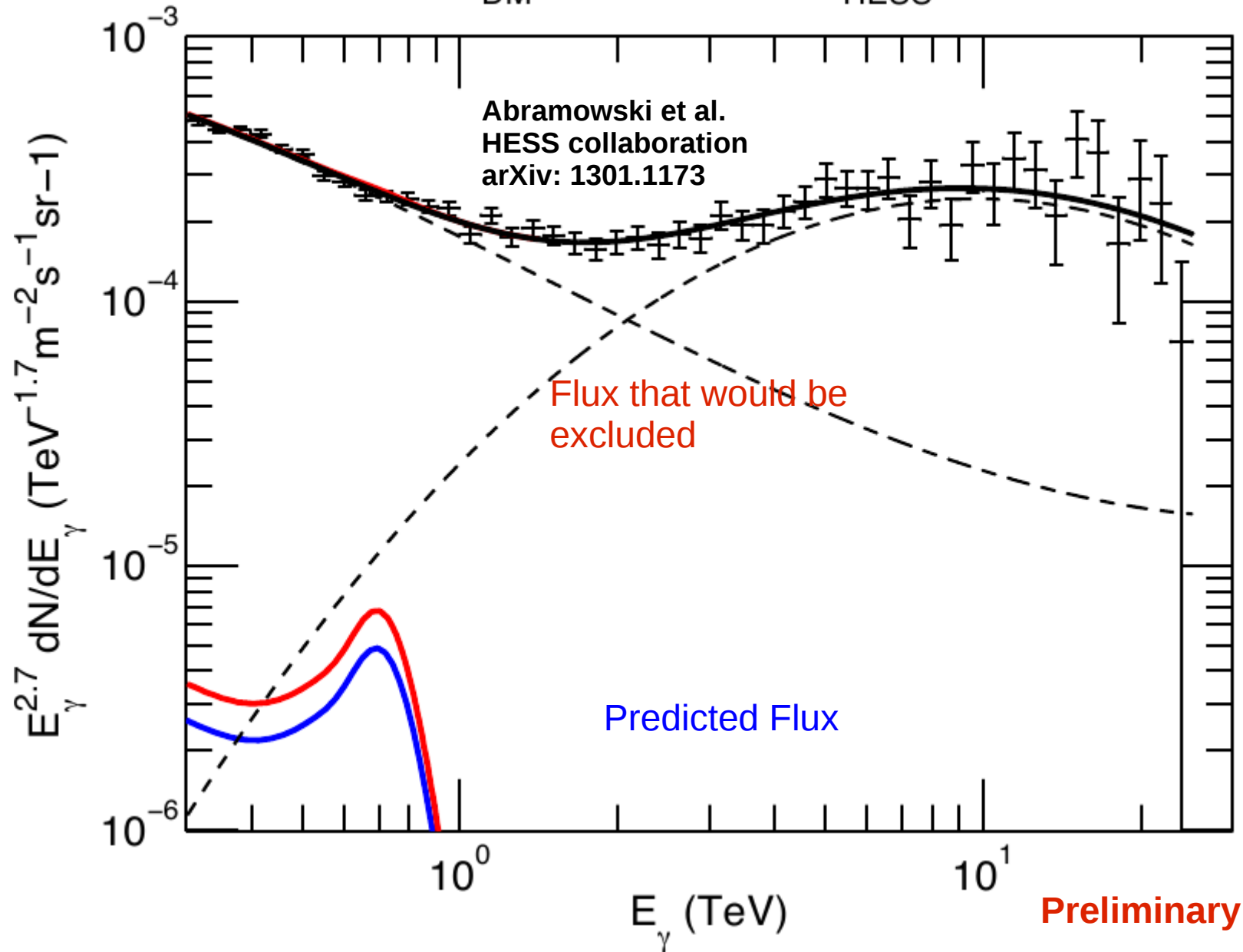


Preliminary

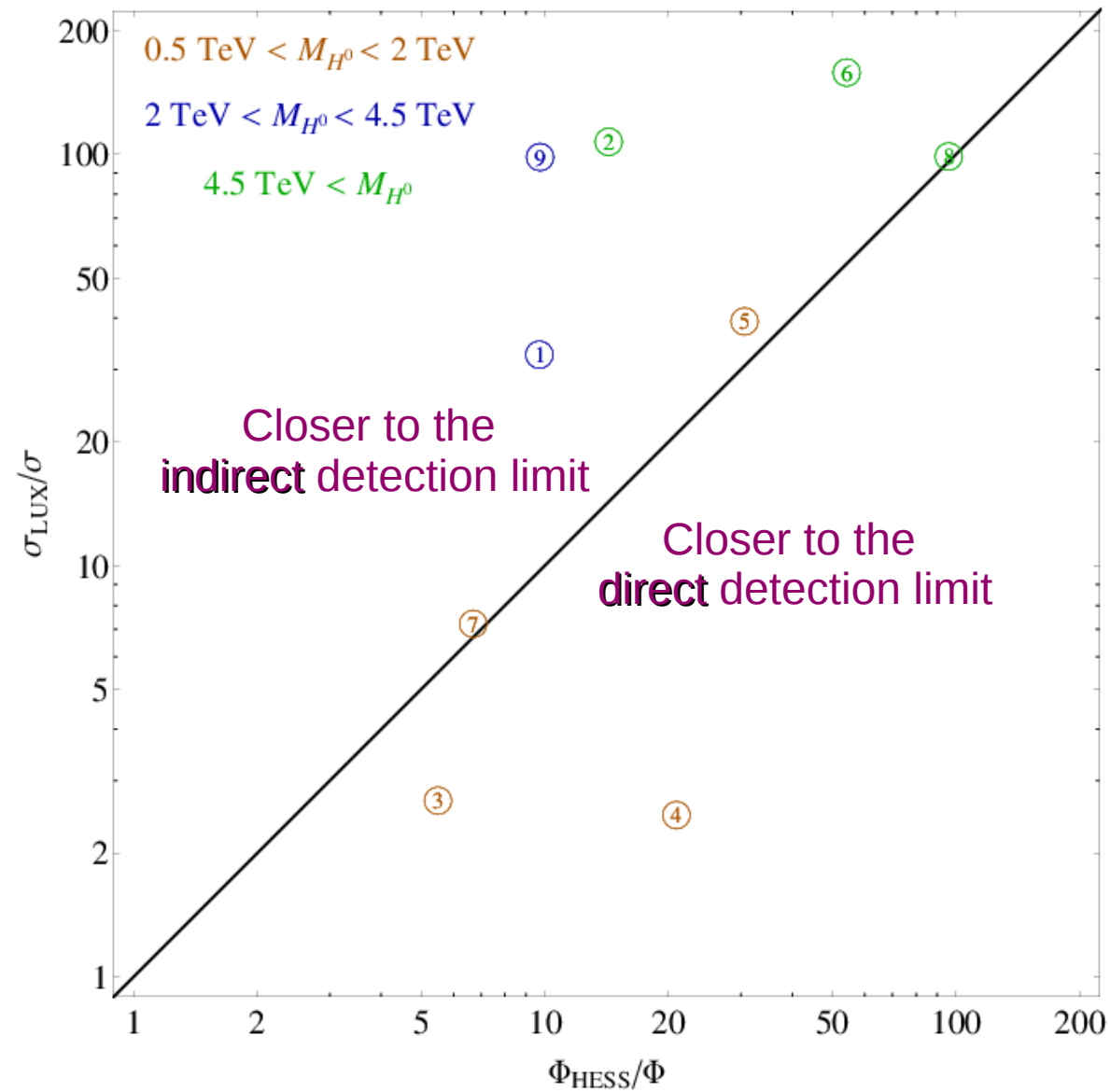
Another Benchmark

IB dominates over the lines

$$m_{\text{DM}} = 0.714 \text{ TeV}, \quad \Phi_{\text{HESS}}/\Phi = 1.38$$

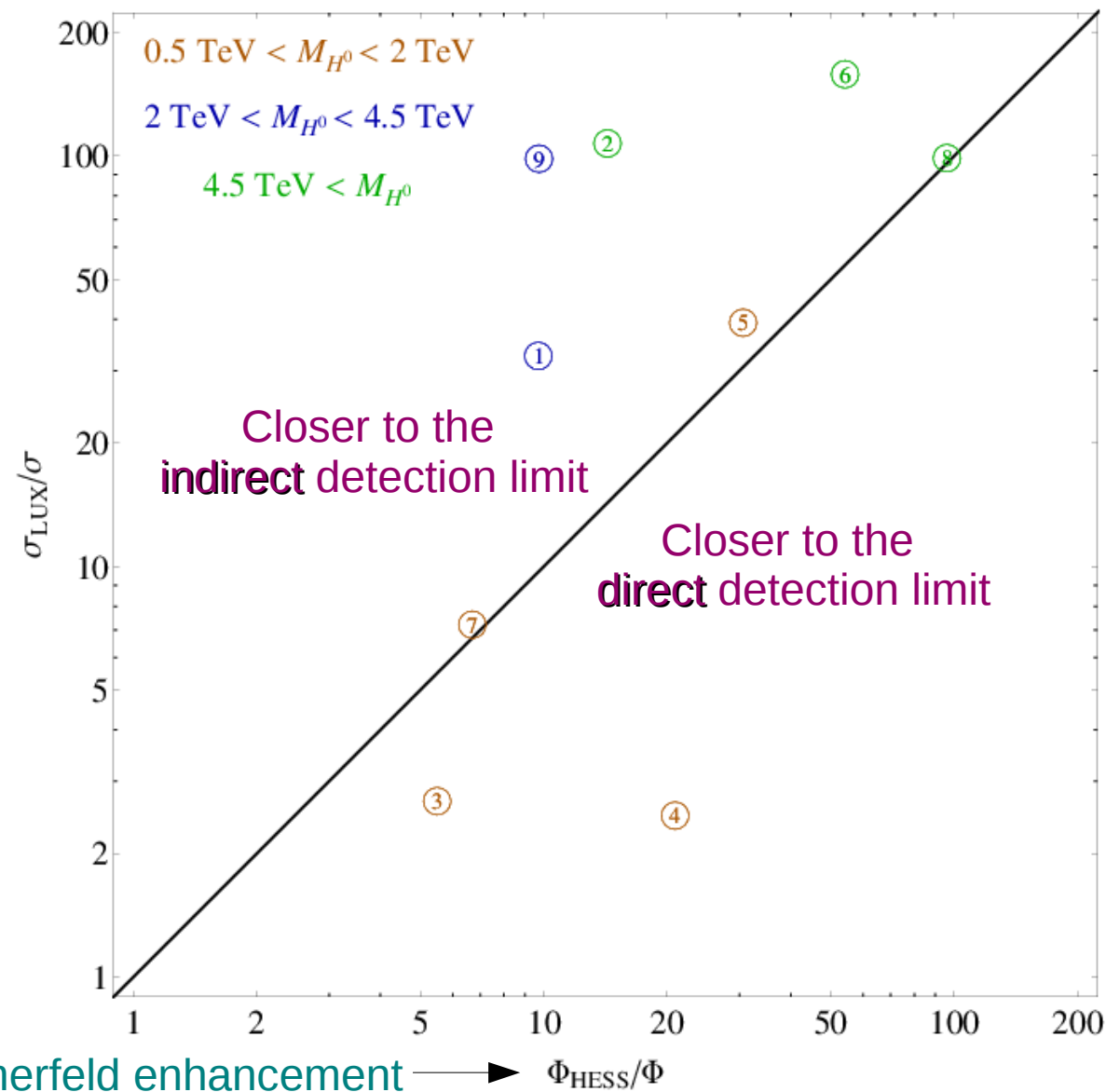


Direct Detection vs. Indirect Detection



Preliminary

Direct Detection vs. Indirect Detection



Preliminary

Conclusions

- In the high mass regime of the inert doublet model, the internal bremsstrahlung process and annihilation into photons generate sharp gamma-ray spectral features.
- The Sommerfeld enhancement has to be taken into account in order to account for perturbative unitarity.
- These spectral features can be searched for with gamma-ray telescopes, and eventually found or excluded in the near future.