Curso de Tercer Ciclo de Fenomenologia Avanzada

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Local: Aula de Juntas del IFIC en Paterna

Horario: Miercoles 17-19 h

Viernes 16-18 h (reservado)

NEUTRINOS

Basic refs

[Schechter, Valle, PRD22 (1980) 2227, D23 (1981) 1666]

[Schechter, Valle, PRD24 (1981) 1883, D25 (1982) 283]

Bibliography

- "Massive Neutrinos in Physics and Astrophysics" (World Scientifi c Lecture Notes in Physics, Vol. 41), R.N. Mohapatra and P. B. Pal
- "Physics of Neutrinos" by M Fukugita, T Yanagida, Springer Verlag (2003)
- "Gauge Theories And The Physics Of Neutrino Mass," J. W. F. Valle, Prog. Part. Nucl. Phys. 26 (1991) 91.
- "Neutrino properties before and after KamLAND", S. Pakvasa and JV HEP-PH 0301061
- "Status of three-neutrino oscillations after the SNO-salt data", M. Maltoni, T. Schwetz, M.A. Tortola, JV, Phys.Rev. D68 (2003) 113010

Our goal is to review gauge theoretic formulation of nu-mass, the corresponding extensions of the standard electroweak theory, the current experimental status and implications for the future. These include the possibility of new signatures in laboratory experiments, both at high energy accelerators, as well as in nonaccelerator and nuclear physics experiments. Some of the implications in cosmology and astrophysics are also discussed.

history

- Since its proposal as a fundamental ingredient in the description of β decay by Pauli and Fermi, we have learned a lot about the neutrino. First we now know for certain that there are at least 3 types ν_e (1956), ν_μ (1962) and ν_τ (2000).
- Although the possibility of neutrinos having mass was present since the birth of the neutrino idea, it was abandoned in the 50's because of the great success of the V-A hypothesis of Lee and Yang, in accounting for the observed parity violation in the weak interaction (Wu et al). This was then taken as an indication that neutrinos are massless.
- In the 80's, however, there was a tremendous amount of activity in neutrino physics, devoted mainly to the issue of nonzero masses and other novel properties of neutrinos, absent in the standard model, partly due to theoretical ideas such as grand unification.
- Gauge theoretic formulation of neutrino mass and oscillations with the characterization of the charged and neutral current weak interactions in gauge theories of massive neutrinos.
- Other important landmarks were the birth of extra solar system neutrino astronomy with the detection of neutrinos from SN87a and the formulation of the MSW effect
- The ultimate confirmation of the neutrino mass hypothesis only came recently, with the conclusive atmospheric neutrino data and the subsequent resolution of the solar neutrino problem which came by combining these data with reactor data
- However many crucial issues in neutrino physics are still open

open issues

- What is the origin of neutrino mass? Why are neutrino masses so small ? Is it a remnant from unifi cation ? Or does it follow from weak-scale physics ? Is this related to the origin of parity violation in the weak interaction ?
- Are neutrinos Dirac or Majorana particles? Does neutrinoless double beta decay take place?
- how to explain the pattern of neutrino mixing?
- Is CP violated in the lepton sector?
- Do lepton flavour violating processes, *e.g.*, $\mu \to e + \gamma$, $\mu \to 3e$, $\tau \to 3\mu$ *etc.* , occur and at what rates? $\mu \to e$ conversion oin nuclei, etc Does lepton flavour violation take place in the domain of high energy processes?
- are there other properties of the neutrinos, such as decays and electromagnetic interactions ? what is their effect in astrophysics?
- Following the recent discoveries which have led to the Nobel prize in physics in 2002, and because of its key role in science,

  neutrinos have become a major focus in astro, particle and nuclear physics

Dirac and Majorana Masses-1

The first basic kinematical concept is that of a *Majorana fermion* and how it contrasts with the more familiar one of a *Dirac fermion*. A massive Majorana fermion has just half of the number of degrees of freedom of a conventional massive spin $1/2$ Dirac fermion and corresponds to the lowest representation of the Lorentz group. The basic Lagrangean describing such particle is

$$L_M = -i\rho^\dagger \sigma_\mu \partial_\mu \rho - \frac{m}{2}\rho^T \sigma_2 \rho + H.C.$$

given in terms of a 2-component spinor ρ. Here I use the 2×2 σ matrices, with σ_i being the usual Pauli matrices and $\sigma_4 = -i\,\mathbf{I}$, $\mathbf{I}$ being the identity matrix. As before, I also use Pauli's metric convention for Minkowski coordinates, where the dot product of two four vectors is $a.b \equiv a_\mu b_\mu \equiv \vec{a} \cdot \vec{b} + a_4 b_4$, where $a_4 = ia_0$. Under a Lorentz transformation, $x \to \Lambda x$, the spinor ρ transforms as $\rho \to S(\Lambda)\rho(\Lambda^{-1}x)$ where S obeys

$$S^\dagger \sigma_\mu S = \Lambda_{\mu\nu}\sigma_\nu$$

The kinetic term is clearly invariant. Similarly, the mass term is invariant, as a result of unimodular property $det\ S = 1$. However, it is not invariant under a phase transformation

$$\boxed{\rho \to e^{i\alpha}\rho}$$

Dirac and Majorana Masses-2

The resulting equation of motion is

$$-i\sigma_\mu \partial_\mu \rho = m\sigma_2 \rho^*$$

As a result of the conjugation and Clifford properties of the σ-matrices, one can verify that each component of the spinor ρ obeys the Klein-Gordon wave-equation. In order to display clearly the relationship between our theory, and the usual theory of a massive spin $1/2$ Dirac fermion, defined by the familiar Lagrangean

$$L_D = -\bar{\Psi}\gamma_\mu \partial_\mu \Psi - m\,\bar{\Psi}\Psi,$$

we construct the solutions to the Majorana equation in terms of those of the Dirac eqn, which are well known. For this we can use any representation of the Dirac algebra $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\,\delta_{\mu\nu}$. To develop the weak interaction theory, however, it is convenient to use the *chiral* representation, in which γ_5 is diagonal,

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix} \quad \gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad \boxed{\gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}}$$

Dirac and Majorana Masses-3

In this representation the charge conjugation matrix C obeying

$$C^T = -C$$
$$C^\dagger = C^{-1}$$
$$C^{-1}\,\gamma_\mu\,C = -\,\gamma_\mu^T$$

is simply given by

$$C = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

A Dirac spinor may then be written as

$$\boxed{\Psi_D = \begin{pmatrix} \chi \\ \sigma_2\,\phi^* \end{pmatrix}}$$

so that the corresponding charge-conjugate spinor $\Psi_D^c = C\,\bar{\Psi}_D^T$ is the same as Ψ_D but exchanging ϕ and χ, *i.e.*,

$$\Psi_D^c = \begin{pmatrix} \phi \\ \sigma_2\,\chi^* \end{pmatrix}$$

Dirac and Majorana Masses-4

A 4-component spinor is said to be Majorana or self-conjugate if $\Psi = C\bar{\Psi}^T$ which amounts, to setting $\boxed{\chi = \phi}$. We can rewrite the D-Lagragean as follows

$$L_D = -i \sum_{\alpha=1}^{2} \rho_\alpha^\dagger \sigma_\mu \partial_\mu \rho_\alpha - \frac{m}{2} \sum_{\alpha=1}^{2} \rho_\alpha^T \sigma_2 \rho_\alpha + H.C.$$

where

$$\chi = \frac{1}{\sqrt{2}}(\rho_2 + i\rho_1)$$
$$\phi = \frac{1}{\sqrt{2}}(\rho_2 - i\rho_1)$$

are the left handed components of Ψ_D and of the charge-conjugate field Ψ_D^c, respectively. This way the Dirac fermion is shown to be equivalent to two Majorana fermions of equal mass.

Dirac and Majorana Masses-5

As a result of this mass degeneracy, the D-theory is invariant under a continuous rotation symmetry between ρ_1 and ρ_2

$$\rho_1 \longrightarrow c\rho_1 + s\rho_2$$
$$\rho_2 \longrightarrow -s\rho_1 + c\rho_2$$

This corresponds to the phase symmetry

$$\Psi_D \longrightarrow e^{i\alpha}\Psi_D$$

associated to *fermion number conservation* in the Dirac theory.

Note that the mass term in the M-theory Lagrangean vanishes unless ρ and ρ^* are anti-commuting, so we consider the Majorana fermion, right from the start, as a second $quantized$ fi eld.

quantum theory of Majorana Masses-1

In order to obtain solutions of the M-theory we start from the usual Fourier expansion for the Dirac spinor,

$$\Psi_D = (2\pi)^{-3/2} \int d^3k \sum_{r=1}^{2} (\frac{m}{E})^{1/2} [e^{ik.x} a_r(k) u_r(k) + e^{-ik.x} b_r^{\dagger}(k) v_r(k)]$$

where $u = C\,\bar{v}^T$ and $E(k) = (\vec{k}^2 + m^2)^{1/2}$ is the mass-shell condition. From our decomposition we then derive the corresponding expansion for one of our 2-component Majorana spinors.
For example, for $\Psi_M = \rho_2$ we find

$$\Psi_M = (2\pi)^{-3/2} \int d^3k \sum_{r=1}^{2} (\frac{m}{E})^{1/2} [e^{ik.x} A_r(k) u_{Lr}(k) + e^{-ik.x} A_r^{\dagger}(k) v_{Lr}(k)]$$

with a similar expression for ρ_1. Here L denotes left-handed chiral projection, and the operators A are defined are $A = (a + b)/\sqrt{2}$, for each value of r and k, from where it follows that they obey canonical anticommutation rules. This gives the

expansion for massive 2-component M-spinors

quantum theory of Majorana Masses-2

Note that

- the massive Majorana field operator is given in terms of the chiral projections of the *ordinary* massive Dirac wave functions u and v.
- the $same$ creation and annihilation operators appear in Ψ_M showing explicitly that there are only $half$ the number of degrees of freedom in the Majorana field.

This gives a consistent Fock-space particle interpretation of the Majorana theory. For example, the energy-momentum is given as

$$P_\mu = \int d^3k \sum_{r=1}^{2} k_\mu A_r^\dagger(k) A_r(k) \ ,$$

apart from the zero point energy.
Another important concept of the Majorana theory are the 2 types of propagators (L conserving and violating) that follow from Lorentz invariance

$$\langle 0 \mid \rho(x)\ \rho^*(y) \mid 0\rangle = i\sigma_\mu \partial_\mu \Delta_F(x-y;m)$$

$$\boxed{\langle 0 \mid \rho(x)\ \rho(y) \mid 0\rangle = m\, \sigma_2\, \Delta_F(x-y;m)}$$

quantum theory of Majorana Masses-3

In the above $\Delta_F(x-y;m)$ is the usual Feynman function. The first is the "normal" propagator that intervenes in total lepton number conserving ($\mid \Delta L \mid = 0$) processes, while the second describes the virtual propagation of Majorana neutrinos in $\mid \Delta L \mid = 2$ processes such as neutrinoless double-beta decay.

It is instructive to consider the massless limit of the Majorana theory. For this purpose we define helicity eigenstate wavefunctions by

$$\vec{\sigma} \cdot \vec{k}\, u_L^{\pm}(k) = \pm \mid \vec{k} \mid u_L^{\pm}(k)$$
$$\vec{\sigma} \cdot \vec{k}\, v_L^{\pm}(k) = \mp \mid \vec{k} \mid v_L^{\pm}(k)$$

The old 2-component massless neutrino theory is recovered from this by noting that, out of the 4 linearly independent wave functions $u_L^{\pm}(k)$ and $v_L^{\pm}(k)$, *only two* survive as the mass approaches zero, namely, $u_L^{-}(k)$ and $v_L^{+}(k)$ [Schechter, Valle, PRD24 (1981) 1883 D25 (1982) 283].

In summary, we have a perfectly consistent Lorentz invariant quantum field theory

quantum theory of Majorana Masses-4

It can easily be generalized for a system of an arbitrary number of Majorana neutrinos. In this case the most general Lagrangean allowed by Lorentz invariance is of the type

$$L_M = -i \sum_{\alpha=1}^{n} \rho_\alpha^\dagger \sigma_\mu \partial_\mu \rho_\alpha - \frac{1}{2} \sum_{\alpha,\beta=1}^{n} M_{\alpha\beta} \rho_\alpha^T \sigma_2 \rho_\beta + H.C.$$

where the sum runs over α and β. By Fermi statistics the mass coeffi cients $M_{\alpha\beta}$ must form a symmetric matrix, in general complex. This matrix can always be diagonalized by a complex $n \times n$ unitary matrix U [SV, D22]

$$M_{diag} = U^T M U \ .$$

When M is taken to be real (CP conserving) its diagonalizing matrix U may be chosen to be orthogonal and, in general, the mass eigenvalues can have different signs. These may be assembled as a signature matrix

$$\eta = diag(+, +, ..., -, -, ..)$$

quantum theory of Majorana Masses-5

- note that when we decomposed the D-spinor we chose to get rid of these signs by introducing appropriate factors of i in the wave functions. This is perfectly consistent, as long as one only has the free theory.
- In the presence of interactions, such as the charged currents, these signs are physical, and theories characterized by different signature matrices *differ in an essential way*. For example, there are two inequivalent models containing 2 Majorana neutrinos: one characterized by $\eta = diag(+,+)$ and another by $\eta = diag(+,-)$. A $Dirac$ neutrino belongs to the second class. For example, the condition for CP invariance is different for these two cases. As emphasized by Wolfenstein, these signs play an important role in the discussion of neutrinoless double beta decay
- It should be apparent from the above analysis that there is no reason, in general, to expect a conserved fermion number symmetry to arise in a gauge theory where the basic building blocks are 2-component massive electrically neutral fermions, such as neutrinos or the supersymmetric $inos$.
- D-M confusion theorem

  Majorana and Dirac neutrinos can only be distinguished in the standard $SU(2) \otimes U(1)$ electroweak theory, to the extent that neutrinos are massive

Electroweak interactions of massive neutrinos

In the standard model there are no neutrino masses, since there are no gauge invariant Yukawa couplings that can generate them. As a result one can redefi ne neutrinos at will through an arbitrary unitary transformation, thus eliminating altogether mixing as well as the possibility of CP violation from the lepton sector.

The structure of the leptonic charged current will depend on whether there are isosinglet neutrinos, and how many, and upon whether neutrinos are Dirac or Majorana particles. We will classify the possibilities according to Dirac versus Majorana, analysing, in each case, the various possible assumptions regarding the number, m, of isosinglet leptons.

For the time being I will just analyse, in a model independent way, the generic structure that the $\mathrm{SU(2)} \otimes \mathrm{U(1)}$ charged and neutral currents take, when expressed in terms of physical mass-eigenstate neutrinos, in analogy with what we did for quarks.

Later on I will discuss various models of neutrino mass

The Leptonic Charged Current-Dirac case-1

Neutrinos may be given a mass by coupling new fermions beyond those in table, such as right handed neutrinos, in such a way that total lepton number is preserved. The new leptons will be denoted by ν_i^c, $1 \leq i \leq m$. From the point of view of the standard model one can add any number m of isosinglets since, being completely inert under the gauge group, they do not upset the renormalizability of the theory. There is, in this case a new gauge invariant Yukawa interaction, similar to that of the up quarks,

$$h_{Dia} \nu_i^c \ell_a \tau_2 \phi^* .$$

assuming $n = m$ this generates a $Dirac$ mass for the neutrinos.

in this case we find that the structure of lepton mixing is identical with that describing quarks, so the same parametrization applies. It can be given as

$$K = O_{23} \Gamma_\delta O_{13} \Gamma_\delta^\dagger O_{12}$$

where O_{ij} is the orthogonal rotation matrix in the ij-plane which depends on the mixing angle θ_{ij}, and $\Gamma_\delta = \mathrm{diag}(1, 1, \mathrm{e}^{\mathrm{i}\delta_{\mathrm{CP}}})$, δ_{CP} being the Dirac-type CP-violating phase.

The Leptonic Charged Current-Dirac case-2

one may write the (3,3) case explicitely as

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix},$$

This parameterization is a variant of the form introduced in Eqs. (2.5) and (2.10) of PRD22 (1980) 2227

However, as already discussed, in gauge theories of massive neutrinos there is in general no reason for having an equal number os isosinglets and isodoublet neutrinos

The Leptonic Charged Current-Dirac case-3

- we now consider the case where the number m of isosinglets is smaller than that of isodoublet neutrinos, m < n.
- A model of the (n, m) type, with $m \neq 1$ and conserved total lepton number has n(n-1)/2 - (n-m)(n-m-1)/2 mixing angles θ_{ij} and 1+ n(m-1)-m(m+1)/2 CP violating phases ϕ_{ij}.
- For the (3,1) case where a single right handed neutrino is added to the 3 generations of isodoublets, 2 neutrinos remain massless. As a result, the structure of lepton mixing is simpler because one can perform arbitrary rotations (not only rephasings) in the degenerate sector of the 2 massless neutrinos and this way eliminate unphysical parameters, so that the mixing matrix can always be brought to the form

$$K = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix},$$

- the above matrix provides a good approximation to the structure of lepton mixing indicated by current experiments

The Leptonic Charged Current-Dirac case-4

Finally, when the number of isosinglets is larger than that of isodoublets, $m > n$, new conceptual possibilities arise for the physics of leptons. For example, if $m = 2n$, the masslessness of the observed neutrinos may be enforced by imposing the conservation of total lepton number, yet allowing for the remarkable possibility of lepton flavour and CP nonconservation despite neutrinos being strictly massless.

The currents will have a very specifi c form, due to the L symmetry

for the 3,6 model they were given in Prog. Part. Nucl. Phys. 26 (1991) 91 and refes therein.

gauge theories suggest that neutrinos are Majorana

However, as already discussed, in gauge theories of massive neutrinos there is in general no reason for the emergence of a conserved total lepton number symmetry.

Indeed, in unified gauge theories, such as $SO(10)$, one often finds that lepton number is violated, either by an effective left-handed Majorana neutrino mass that follows from the exchange of some heavy neutral fermions such as right handed neutrinos

in broken R parity supersymmetry this effective left-handed Majorana neutrino mass that follows from the exchange of neutral supersymmetric fermions, called neutralinos

in either case neutrinos are then Majorana particles and their mass term is of the form given above

Majorana neutrino mixing-1

The mass matrix of Majorana neutrinos, given is manifestly not invariant under rephasings of the neutrino fi elds. [Schechter, V, PRD22 (1980) 2227, D23 (1980) 1666]

As a result, there are additional sources of CP violation in the currents of gauge theories with Majorana neutrinos. For example, CP can be violated in a theory with just two generations of Majorana neutrinos. The charged current of a (2,0) model may be parametrized as

$$\begin{pmatrix} c_{12} & e^{i\phi_{12}} s_{12} \\ -e^{-i\phi_{12}} s_{12} & c_{12} \end{pmatrix}$$

where ϕ_{12} is the Majorana phase.

although irrekevant for neutrino oscillations, this phase enters in processes that violate L-number

For (n,0) case these Majorana phases make up a total of $(n-1)$ additional phases over and above those characterizing Dirac neutrino mixing. As fi rst noted in PRD23 (1980) 1666, these are genuine physical parameters, and may play a role in $\mid \Delta L \mid = 2$ processes, such as neutrinoless double beta decay

Majorana Neutrino mixing-2

An important subtlety arises regarding the conditions for CP conservation in gauge theories of massive Majorana neutrinos. Unlike the case of Dirac fermions, where CP invariance implies that the mixing matrix should be real, in the Majorana case the condition reads [Schechter, Valle, PRD24 (1981) 1883, D25 (1982) 283].

$$K^* = K\eta$$

where $\eta = diag(+,+,...,-,-,..)$ is the signature matrix describing the relative signs of the neutrino mass eigenvalues that follow from diagonalizing the mass matrix with real matrices. These important signs determine the CP properties of the neutrinos.

in contrast to the Dirac case the value $\phi_{12} = \pi/2$ (just as $\phi_{12} = 0$) is CP conserving.

In the most general situation where there are $m \neq 0$ two-component SU(2) $\otimes$ U(1) singlet leptons (RH neutrinos) present in the theory, there is in general no reason to forbid a gauge and Lorentz invariant Majorana mass term of the type

$$M_{Rij} \nu_i^c \nu_j^c$$

which breaks total lepton number symmmetry.

Majorana Neutrino mixing-3

As a result, in full generality structure of the weak currents in gauge theories with Majorana neutrinos is substantially more complex than that of quarks for 2 reasons:

- the weak charged current is characterized by rectangular matrix K.
- there are Majorana phases

For example, the (3,3) seesaw model has 12 angles and 12 phases

in the description of neutrino oscillations we will neglect all these complexities

- rectangularity. This is justifi ed when isosinglets are suffi ciently heavy
- Majorana phases, justifi ed because they do not appear in “standard” neutrino oscillations

In what follows we will develop in two directions

- the phenomenological the description of neutrino oscillations (set of three lectures on neutrino oscillations by Akhmedov) and
- some examples of models of neutrino mass (lectures by myself and Hirsch)