Curso de Tercer Ciclo de Fenomenologia Avanzada

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Local: Aula de Juntas del IFIC en Paterna

Horario: Miercoles 17-19 h

Viernes 16-18 h (reservado)

Bibliography

- "Gauge Theories," E. S. Abers and B. W. Lee, Phys. Rept. 9 (1973) 1.
- "Gauge Theory of Elementary Particle Physics", Cheng, T.-P.; Li, L.-F.
- "Field Theory: A Modern Primer" (Frontiers in Physics, Vol 74), P. Ramond
- "Particle Physics and Introduction to Field Theory" (Contemporary Concepts in Physics Series) by T.D. Lee

I will use the same metric conventions of J.J.Sakurai Advanced Quantum Mechanics.

Gauge symmetries in particle physics

consider first the example of electrodynamics, a fermion of mass m and charge Q interacting with the photon

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\psi}[\gamma_\mu \partial_\mu - iQ\gamma_\mu A_\mu + m]\psi$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$				$\tilde{F}_{\mu\nu} = (i/2)\epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$			
0	B_3	$-B_2$	$-iE_1$	0	E_3	$-E_2$	$-iE_1$
	0	B_1	$-iE_2$		0	E_1	$-iB_2$
		0	$-iE_3$			0	$-iB_3$
			0				0

the components of $F_{\mu\nu}$ are just the electric and magnetic fields

$$E_i = iF_{i4} \qquad \epsilon_{ijk} B_k = iF_{ij}$$

with the duality transformation

$$E \to -B \qquad B \to -E$$

the electromagnetic Lagrangean leads to the equations of motion

$$\partial_\mu F_{\mu\nu} = -QJ_\nu \qquad (\gamma_\mu D_\mu + m)\psi = 0$$

and the Bianchi identity $\partial_\mu \tilde{F}_{\mu\nu} = 0$

in vacuo these are just Maxwell's equations

$$
\begin{aligned}
i: &\qquad \dot{E} = \nabla \times B &\qquad E = -\nabla\phi - \dot{A} \\
4: &\qquad \nabla \cdot E = 0 &\qquad B = -\nabla \times A
\end{aligned}
$$

and

$$
\begin{aligned}
i: &\qquad \dot{B} = -\nabla \times E \\
4: &\qquad \nabla \cdot B = 0
\end{aligned}
$$

which defi ne ED
Note that e.m. current conservation

$$\partial_\mu J_\mu - \partial_\mu(\partial_\nu F_{\nu\mu}) \equiv 0$$

is equivalent to local gauge invariance

$$\psi \to e^{i\Lambda}\psi \qquad\qquad A_\mu \to A_\mu + \frac{1}{Q}\partial_\mu \Lambda$$

Covariant derivative

With D_μ defi ned as

$$D_\mu = \partial_\mu - iQA_\mu$$

one can see that $D_\mu \psi$ transformas covariantly

$$\begin{aligned} D_\mu \psi \to D'_\mu \psi' &= [\partial_\mu - iQA'_\mu]\psi' \\ &= [\partial_\mu - iQ(A_\mu + \frac{1}{Q}\partial_\mu \Lambda)]e^{i\Lambda}\psi \\ &= e^{i\Lambda}[\cancel{i}\partial_\mu \Lambda + \partial_\mu - iQA_\mu - \cancel{i}\partial_\mu \Lambda]\psi \\ &= e^{i\Lambda} D_\mu \psi \end{aligned}$$

so that the Lagrangean

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\psi}\gamma_\mu D_\mu \psi - \bar{\psi} m \psi$$

is indeed gauge invariant

Non abelian Yang Mills theories

generalizing from ED which is a U(1) abelian gauge theory one can build a consistent theory based on any gauge group G. For example, G=SU(3) corresponds to QCD under a gauge transf

$$\psi \to U\psi \qquad U^\dagger U = 1 = UU^\dagger \quad , \quad \det U = 1$$

we have $D_\mu \psi = (\partial_\mu - igA_\mu)\psi$; $\psi \in$ "Fundamental" rep

$$\begin{aligned} D_\mu \psi \to & [\partial_\mu - igA'_\mu]U\psi \\ & = [\partial_\mu - ig(UA_\mu U^{-1} + \frac{1}{ig}\partial_\mu U U^{-1})]U\psi \\ & = [\partial_\mu \cancel{U} + U\partial_\mu - igUA_\mu - \partial_\mu \cancel{U}]\psi \\ & = U[\partial_\mu - igA_\mu]\psi \equiv UD_\mu\psi \end{aligned}$$

so that, as in the abelian case, $D_\mu \psi$ transforms covariantly.

Gauge Invariance of the Yang Mills Lagrangean

upgrading the form for the kinetic term expressed in terms of the field strength ("curvature")

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

one can show that under $A_\mu \to U A_\mu U^{-1} + \frac{1}{ig}\partial_\mu U U^{-1}$, $F_{\mu\nu}$ also transforms covariantly, i.e. $F_{\mu\nu} \to U F_{\mu\nu} U^{-1}$ As a result ...

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\, F_{\mu\nu} F_{\mu\nu} - \bar{\psi}(\gamma_\mu D_\mu + m)\psi$$

is invariant under local gauge transformations defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$$D_\mu \psi = (\partial_\mu - igA_\mu)\psi \quad ; \quad \psi \in \text{fund. rep.}$$

Covariant derivative in Adj Repr.

$$D_\mu F_{\mu\nu} = \partial_\mu F_{\mu\nu} - ig[A_\mu, F_{\mu\nu}]$$

Bianchi identity

$$D_\rho \tilde{F}_{\rho\sigma} = 0$$

For constant (x-indep) gauge transf one has that

$$A_{ab} \to U_{aa'} A_{a'b'} U^\dagger_{b'b} \quad ; \quad U = \text{const}$$

transforms as a tensor in the Adj SU(2) Repr. It can be expanded as

$$A_\mu = A_\mu^a t_a \qquad t^a = \frac{1}{2}\tau^a$$

where $t_a = \frac{1}{2}\tau_a = t_a^\dagger$ are the SU(2) generators obeying the group Lie algebra, Tr $t_a t_b = \frac{1}{2}\delta_{ab}$

$$[t_a, t_b] = i\epsilon_{abc} t_c$$

this way one sees that $A_1^2 = \frac{1}{\sqrt{2}} \frac{A_1 - iA_2}{\sqrt{2}}$ and similarly for A_2^1.

Yang Mills Lagrangean in components

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{\psi}\gamma_\mu(\partial_\mu - igA^a_\mu \frac{\tau^a}{2})\psi - \bar{\psi} m \psi$$

where the fi eld tensor is also expanded as $F_{\mu\nu} = F^a_{\mu\nu} t_a$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon_{abc} A^b_\mu A^c_\nu$$

$$D_\mu \psi = (\partial_\mu - \frac{ig}{2} A^a_\mu \tau^a)\psi$$

$$\begin{aligned}(D_\mu F_{\mu\nu})^a = D^{ab}_\mu F^b_{\mu\nu} &= \partial_\mu F^a_{\mu\nu} t_a - i[A^a_\mu t_a, F^b_{\mu\nu} t_b] \\ &= \partial_\mu F^a_{\mu\nu} t_a + \epsilon_{abc} t_c A^a_\mu F^b_{\mu\nu} \\ &= (\delta^{ab}\partial_\mu + g\epsilon_{abc} A^c_\mu) F^b_{\mu\nu}\end{aligned}$$

Yang Mills eqs

Using the Euler-Lagrange equations

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \varphi} = \frac{\delta \mathcal{L}}{\delta \varphi}$$

one derive the YM fi eld eqs

$$(D_\mu F_{\mu\nu})_a = -ig\bar{\psi}\frac{\tau_a}{2}\gamma_\nu \psi$$

$$(\gamma_\mu D_\mu + m)\psi = 0$$

Note that mass terms for gauge fi elds not gauge-invariant. Fermion mass terms OK in ED and CD. As we will see later, a basic feature of the Standard Model is that fermion masses are **not** gauge invariant, since the theory is chiral

Rough structure of the Standard model

We now generalize from SU(2) to an an arbitrary group G defi ned by

$$[t_a, t_b] = iC_{abc} t_c \qquad \text{where C's are structure const.}$$

The gauge group can be a product, each factor with its gauge coupling constant g_i

$$\begin{array}{ccccccc} G = & G_1 & \otimes & G_2 & \otimes & G_3 & \ldots \\ & g_1 & & g_2 & & g_3 & \cdots \end{array}$$

For the Standard model the Gauge group is G=SU(3) $\otimes$ SU(2) $\otimes$ U(1)

- SU(3) describes QCD: 8 gluons
- SU(2) $\otimes$ U(1) : 4 gauge bosons ($W^{\pm}, Z, \gamma$) describing the electroweak part

Fermions may sit in any representation of G, not necessarily irreducible. In the SM we have

$$\left\{ \begin{array}{cc} \begin{pmatrix} \nu \\ e \end{pmatrix}_L & \begin{pmatrix} u \\ d \end{pmatrix}_L \\ e_R & u_R,\ d_R \end{array} \right\}_{i=1,2,3}$$

Note on functional integrals-1

start with 1-dimensional from Gaussian

$$G = \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}ax^2} = \sqrt{\frac{2\pi}{a}}, \quad \mathrm{Re}\, e\, a > 0$$

and go to the multi-dimensional case

$$G = \int_{-\infty}^{\infty} \prod_i dx_i\, e^{-\frac{1}{2}X^T A X}, \quad A = A^T$$

where the matrix A defi nes a quadratic form that be diagonalized as

$$X = RX' \quad \Longrightarrow \quad R^T A R = \Lambda = \text{diag} > 0$$

$$\int_{-}^{+} \prod_i dx_i' \underbrace{\left|\frac{\partial X}{\partial X'}\right|}_{=1} e^{-\frac{1}{2}X'^T R^T A R X'} = \int_{-}^{+} \prod_i dx_i' e^{-x_i' \frac{\Lambda_i}{2} x_i'}$$

$$= \prod_i G_i = \frac{(2\pi)^{\frac{n}{2}}}{\sqrt{\det A}} \tag{1}$$

Note on functional integrals-2

for a general quadratic form

$$Q(x) = \frac{1}{2} X^T A X + b^T X + C$$

peaked at $\bar{x} = -A^{-1} b$ one has

$$\int_{-\infty}^{\infty} dx_1 \cdots dx_n e^{-Q(x)} = e^{-Q(\bar{x})} \frac{(2\pi)^{\frac{n}{2}}}{\sqrt{\det A}}$$

Generalizing to the functional case

$$\int [d\phi]\, e^{-\int \, d^4x \int \, d^4y \, \phi(x) A(x,y) \phi(y)} \propto \frac{1}{\sqrt{\det A}} \quad \text{bosons}$$

$$\int [d\psi]\, [d\bar{\psi}]\, e^{-\int \, d^4x \int \, d^4y \, \bar{\psi}(x) M(x,y) \psi(y)} \propto \det M \quad \text{fermions}$$

Quantization of Gauge Theories

- instead of using the usual canonical procedure

Classical $\mathcal{L}$ $\longrightarrow$ Canonical variables $[\phi, \pi]_{\rm PB} \longrightarrow [\hat{\phi}, \hat{\pi}]_{\pm}$ $\longrightarrow$ Pert. Theory Feynman rules

- we will use Feynman's path integral method which skips the intermediate step
- starts from generating functional in the presence of an external (Schwinger) source J(x)

$$\langle 0|0\rangle^J = e^{iZ[J]} \propto \int [d\phi]\, e^{i\int\, d^4x(\mathcal{L}+J\phi)}$$

- and gives directly the connected n-point Greens functions as (Feynman rules)

$$\left.\frac{\delta^n Z}{\delta J(x_1)\delta J(x_2)\cdots\delta J(x_n)}\right|_{J=0}$$

Free scalar field generating functional

$$\langle 0|0\rangle^J \propto \int [d\phi]\, e^{\frac{i}{2}\int\, d^4x(\phi\Box\phi - m^2\phi^2 + J\phi)}$$

gives, by completing the square,

$$e^{-\frac{1}{2}\, J^T A^{-1} J}$$

so that the propagator is given as

$$A^{-1} \propto \frac{1}{\Box - m^2 + i\epsilon} \to -\frac{i}{(2\pi)^4}\int d^4p \frac{e^{ip\cdot x}}{p^2 + m^2 - i\epsilon}$$

where the iϵ makes the integral well-defined. If we try to do the same for QED, we find that A^{-1} does not exist, since A is a projection

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}(\partial_\mu A_\nu \partial_\mu A_\nu - \partial_\mu A_\nu \partial_\nu A_\mu) \\ &\to -\frac{1}{2}(-A_\nu \Box A_\nu + A_\nu \partial_\mu \partial_\nu A_\mu) = -\frac{1}{2} A_\nu(-\delta_{\mu\nu}\Box + \partial_\mu \partial_\nu) A_\mu \end{aligned}$$

gauge invariance problem

any vector potential can be split as

$$A_\mu = A_\mu^L + A_\mu^T$$

where

$$\begin{aligned} A_\mu^L &= \frac{\partial_\mu \partial_\nu}{\Box} \\ A_\mu^T &= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}\right) A_\nu \end{aligned}$$

so that only the transverse part appears

$$\mathcal{L} \longrightarrow \frac{1}{2} A_\mu \Box \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}\right) A_\nu \equiv \frac{1}{2} A_\nu^T \Box A_\nu^T + \frac{1}{2} A_\nu^L O A_\nu^L$$

$$D_{\mu\nu}^T = -\frac{i}{(2\pi)^4} \int d^4p \frac{\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box}}{p^2} e^{ip\cdot x} \qquad D_{\mu\nu}^L = \frac{1}{0} = \infty$$

Gauge invariance in QED

therefore if one integrates over all gauge potentials

$$\int [dA_\mu]\, e^{i\int d^4x \mathcal{L}} \to \int [dA^T]\, \underbrace{[dA^L][\cdots]}_{\infty}$$

one gets a divergence due to the overcount. One should integrate only over transverse

$$\begin{aligned} \langle 0|0\rangle &= \int [dA^T]\, e^{iS} \\ &= \int [dA_\mu]\, \delta[A^L_\mu]\, e^{iS} \\ &= \int [dA_\mu]\, \delta[F(A)] \left(\det \frac{\delta F}{\delta A^L}\right) e^{iS} \end{aligned}$$

$$\langle 0|0\rangle = \int [dA_\mu] \det\left(\frac{\delta F}{\delta \Lambda}\right) \delta[F(A)] e^{iS^J}$$

where the gauge is specifi ed by the condition $F(A) = 0$

Fadeev-Popov trick in QED

to calculate the generating functional

$$F = \partial_\mu A_\mu - \rho$$

$$\begin{aligned} \delta F &= \frac{1}{Q}\Box\Lambda \\ \frac{\delta F}{\delta\Lambda} &= \frac{1}{Q}\Box \quad \text{indep. A} \quad \Rightarrow \quad \det\left(\frac{\delta F}{\delta\Lambda}\right) \end{aligned}$$

$$\begin{aligned} \langle 0|0\rangle &\propto \int [dA]\,\delta[\partial\cdot A - \rho]e^{iS} \\ &\propto \int [dA]\,[d\rho]\,\delta[\partial\cdot A - \rho]e^{i\int\, d^4x(\mathcal{L} - \frac{\rho^2}{2\alpha})} \\ &= \int [dA]e^{i\int\, d^4x(\mathcal{L} - \frac{1}{2\alpha}(\partial\cdot A)^2)} \end{aligned}$$

now we can invert

$$-\frac{1}{2}\left(\partial_\mu A_\nu \partial_\mu A_\nu - \partial_\mu A_\nu \partial_\nu A_\mu + \frac{1}{\alpha}\partial_\mu A_\mu \partial_\nu A_\nu\right)$$

$$\rightarrow \quad -\frac{1}{2}A_\nu\left(-\delta_{\mu\nu}\Box + \partial_\mu\partial_\nu - \frac{1}{\alpha}\partial_\mu\partial_\nu\right)A_\nu$$

$$D^F_{\mu\nu} \quad \propto \quad \left[\delta_{\mu\nu}\Box - \left(1 - \frac{1}{\alpha}\right)\partial_\mu\partial_\nu\right]^{-1}$$

$$D^F_{\mu\nu} \quad = \quad \frac{1}{\Box}\left(\delta_{\mu\nu} - \frac{1-\alpha}{\Box}\partial_\mu\partial_\nu\right)$$

$$D_{\mu\nu}(p) = -\frac{i}{(2\pi)^4}\int d^4p \frac{1}{p^2 - i\epsilon} e^{ip\cdot x}\left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\alpha)\right]$$

different α choices correspond to different gauges

α	gauge parameter
1	Feynman
0	Landau

Fadeev Popov quantization (non-abelian YM)

As we saw, from

$$\begin{aligned} A'^a_\mu &= U A_\mu U^\dagger - \frac{i}{g}\partial_\mu U U^{-1} \\ U &= e^{i\sum_{a=1}^{\text{dim G}} U^a t_a}, \quad U = U(x) \end{aligned}$$

we get

$$A'^a_\mu = A^a_\mu - C_{abc} u^b A^c_\mu + \frac{1}{g}\partial_\mu u^a \quad \text{a=1} \cdots \text{dim G}$$

Now we choose a covariant gauge specified by

$$F^a = \partial_\mu A^a_\mu - \rho^a = 0$$

and calculate the generating functional with the FP ansatz

$$\langle 0|0\rangle \propto \int [dA^a][d\rho^a]\delta[\partial_\mu A^a_\mu - \rho^a]\text{det}\left(\frac{\delta F^a}{\delta U^b}\right) e^{i\int d^4x\left[\mathcal{L} - \frac{\rho^a\rho^a}{2\alpha}\right]}$$

including the FP determinant and the gauge-fixing term (where the last term in exp)

$$\langle 0|0\rangle \propto \int [dA^a_\mu]\mathrm{det}M\, e^{i\int d^4x\left[\mathcal{L}-\frac{1}{2\alpha}(\partial\cdot A^a)^2\right]}$$

Fadeev-Popov trick

$$\mathrm{det}M = \int [d\phi][d\bar{\phi}]e^{\int_{xy}\bar{\phi}_x M_{x,y}\phi_y} \qquad \text{where } \phi, \bar{\phi} \quad \text{anticommuting}$$

$$\int_{xy} \bar{\phi}^a(x)M^{ab}(x,y)\phi^b(y) = \int d^4x\, d^4y \bar{\phi}^a(x)\left(\frac{\delta F^a(x)}{\delta U^b(y)}\right)\phi^b(y)$$

$$M^{ab}_{x,y} = C_{abc}\partial_{y_\mu}\delta^4(x-y)A^c_\mu(y) + \frac{1}{g}\delta^{ab}\Box\delta^4(x-y)$$

where the FP determinant is

$$\mathrm{det}M \equiv \exp\left(\frac{-i}{g}\right) i\int d^4x d^4y \bar{\phi}^a\Box\phi^a\delta^4(x-y) + g\,\partial_{x_\mu}\bar{\phi}^a_x C_{abc}A^c_{\mu\,y}\phi^b_y\delta^4(x-y)$$

$\Rightarrow e^{i\int d^4x(-\partial_\mu\bar{\phi}^a\partial_\mu\phi^a + g\,C_{abc}\partial_\mu\bar{\phi}^a A^c_\mu\phi^b)}$ non-abelian ghosts couple to gauge fi elds (QCI

Yang Mills generating functional

$$\langle 0|0\rangle^J = \int [dA^a_\mu][d\phi^a][d\bar{\phi}^a] e^{i\int d^4x}(\mathcal{L}_{\text{eff}} + J^a_\mu A^a_\mu) = e^{iZ[J]}$$

$$\mathcal{L}_{\text{TOTAL}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{g-f}} + \mathcal{L}_{\text{ghost}} + \mathcal{L}_{\text{fermions}}$$

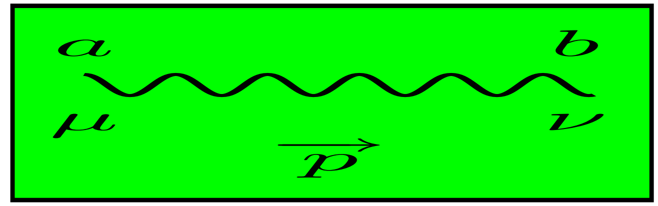
$$\begin{aligned}
\mathcal{L}_{\text{YM}} &= -\frac{1}{2}\text{Tr} F_{\mu\nu}F_{\mu\nu} \\
\mathcal{L}_{\text{g-f}} &= -\frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2 \\
\mathcal{L}_{\text{ghost}} &= -\partial_\mu \bar{\phi}^a \partial_\mu \phi^a + g\, C_{abc} \partial_\mu \bar{\phi}^a A^c_\mu \phi^b \\
\mathcal{L}_{\text{fermions}} &= -\bar{q}(\gamma_\mu D_\mu + m) q
\end{aligned}$$

$$\mathcal{L}_{\text{YM}} = \mathcal{L}_0 + \mathcal{L}' = \begin{cases} \mathcal{L}_0 & \equiv \text{quadratic} \\ \mathcal{L}' & \equiv \text{cubic} + \text{interactions} = \mathcal{L}_3 + \mathcal{L}_4 \end{cases}$$

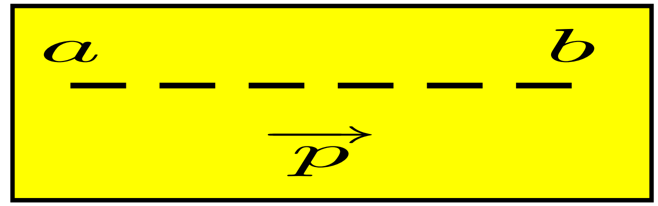
$$\mathcal{L}_3 = -g\, C_{abc} \partial_\mu A^a_\nu A^b_\mu A^c_\nu \qquad \mathcal{L}_4 = -\frac{g^2}{4} C_{abi} C_{cdi} A^a_\mu A^b_\nu A^c_\mu A^d_\nu$$

Non-abelian YM Feynman Rules

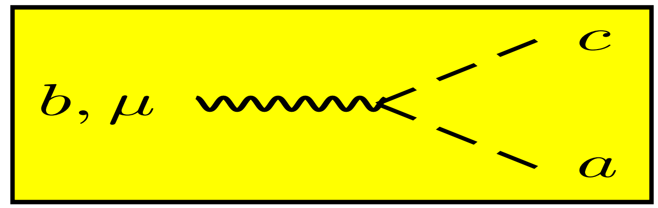
$$\frac{-i}{p^2 - i\epsilon}\delta^{ab}\left[\delta_{\mu\nu} - \frac{(1-\alpha)p_\mu p_\nu}{p^2}\right]$$



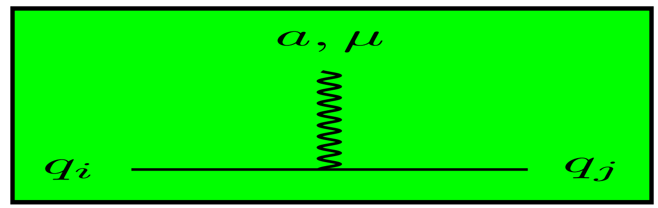
$$-\frac{i}{p^2 - i\epsilon}\delta^{ab}$$



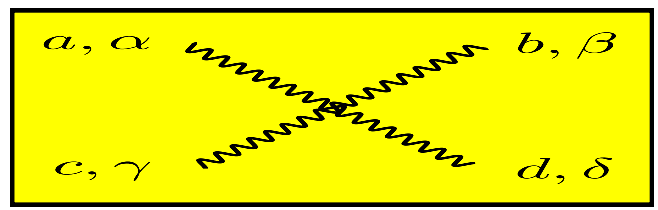
$$-g q_\mu C_{abc}$$



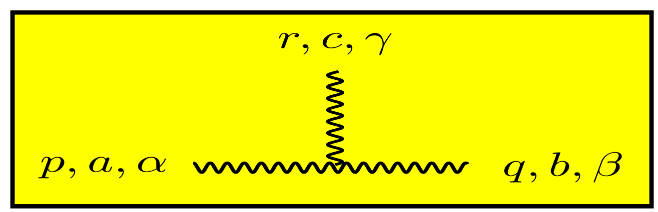
$$-g\gamma_\mu (t^a)_{ij}$$



$$\begin{aligned}
&-ig^2[C_{abf}C_{cdf}(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\beta\gamma}\delta_{\alpha\delta})\\
&+C_{acf}C_{bdf}(\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma})\\
&+C_{adf}C_{bcf}(\delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\gamma}\delta_{\beta\delta})]
\end{aligned}$$



$$g\, C_{abc}[\delta_{\alpha\gamma}(p-r)_\beta + \delta_{\alpha\beta}(q-p)_\gamma + \delta_{\beta\gamma}(r-q)_\alpha]$$

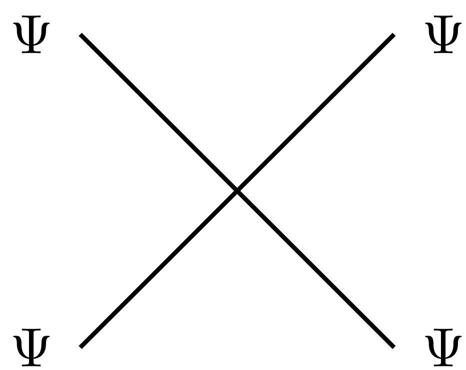


Towards a weak interaction gauge theory

So far we have seen

1. φ^4 scalar field theory
2. unbroken Yan-Mills gayge theory
    - abelian: QED
    - nonabelian: QCD

The basic feature of the weak interaction is that it has short range and is well described by a contact 4-fermion interaction. Although this gives a good phenomenological description of muon and beta decays, $\mu \to e\nu\bar{\nu}$ and $n \to pe\bar{\nu}$, the theory is not renormalizable nor unitary. To develop the weak interaction as a gauge theory in which these processes are described by gauge boson exchange, we need to combine 1 and 2 above. For this to work we need to incorporate mass in the gauge theory, and this is incompatible with gauge invariance. Spontaneous symmetry breaking is the basic ingredient needed to overcome this conflict.



Goldstone model-1

consider the theory of a massive real scalar fi eld

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4$$

this has $\varphi = 0$ as ground sate. Now take 2 scalars

$$\mathcal{L} = -\frac{1}{2}[(\partial_\mu \varphi_1)^2 + \partial_\mu \varphi_2)^2] - V(\varphi_1^2 + \varphi_2^2)$$

with V symmetric under an O(2) rotation,

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \longrightarrow \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

O(2) invariance forbids cubic terms, while renormalizability implies that V is at most of degree 4. Thus there can only be quadratic and quartic terms

Goldstone model-2

An O(2) symmetric of 2 scalar fi elds φ_1, φ_2 can be rewritten as

$$\mathcal{L} = -\partial_\mu \varphi^\dagger \partial_\mu \varphi - V(\varphi^\dagger \varphi)$$

where

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$$

and is symmetric under the U(1)=O(2) transformation $\varphi \to e^{i\alpha}\varphi$. Renormazability and U(1) invariance imply

$$V = -A\varphi^\dagger\varphi + B(\varphi^\dagger\varphi)^2$$

where A, B are constants. For $A < 0$ and $B > 0$ the ground state corresponds to $\varphi_1 = 0 = \varphi_2$. In contrast, if $A, B > 0$ we have (up to irrelevant overall constant)

$$V = B\left[(\varphi^\dagger\varphi)^2 - \frac{A}{B}\varphi^\dagger\varphi + \frac{A^2}{4B^2}\right] = B\left[\varphi^\dagger\varphi - \frac{A}{2B}\right]^2$$

Goldstone model-3

Now one sees that $\varphi_1 = 0 = \varphi_2$ is not the ground state and V is minimized when $\varphi^\dagger \varphi = \frac{A}{2B}$. This gives an infinite degeneracy of vacua, corresponding to any point along the circle define by $\varphi^\dagger \varphi = \frac{A}{2B}$. Let us rewrite the Lagrangean in terms of shifted fields

$$\tilde{\varphi}_a = \varphi_a - \langle \varphi_a \rangle \quad a = 1, 2$$

with small fluctuations around any chosen minimum

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2}(\partial_\mu \tilde{\varphi}_1)^2 - -\frac{1}{2}(\partial_\mu \tilde{\varphi}_2)^2 - \frac{1}{2}\langle V_{,ab}\rangle \tilde{\varphi}_a \tilde{\varphi}_b - \frac{1}{3!}\langle V_{,abc}\rangle \tilde{\varphi}_a \tilde{\varphi}_b \tilde{\varphi}_c \\ &\quad -\frac{1}{4!}\langle V_{,abcd}\rangle \tilde{\varphi}_a \tilde{\varphi}_b \tilde{\varphi}_c \tilde{\varphi}_d \end{aligned}$$

where

$$\begin{aligned} V &= -\frac{A}{2}\varphi_a \varphi_a + \frac{B}{4}(\varphi_a \varphi_a)^2 \\ V_{,a} &= -A\varphi_a + B\varphi_b \varphi_b \varphi_a \quad ; \qquad \langle V_{,a}\rangle = 0 \\ V_{,ab} &= -A\delta_{ab} + 2B\varphi_b \varphi_a + B\varphi_c \varphi_c \delta_{ab}; \quad \langle V_{,ab}\rangle = [B\langle \varphi_c \varphi_c\rangle - A]\delta_{ab} + 2B\langle \varphi_a \varphi_b\rangle \end{aligned}$$

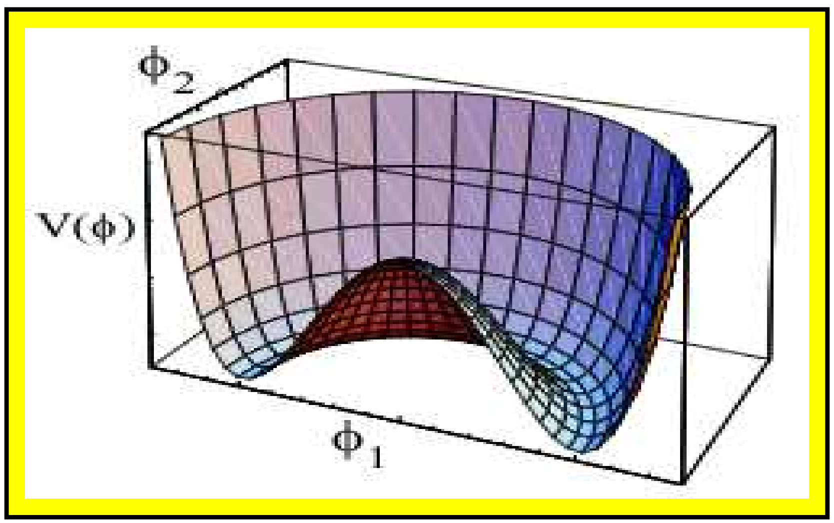
Goldstone theorem

the second derivative of the potential at minimum is the mass-squared matrix of the scalars

$\langle V_{,ab} \rangle$	$\tilde{\varphi}_1$	$\tilde{\varphi}_2$
$\tilde{\varphi}_1$	$2A$	0
$\tilde{\varphi}_2$	0	0

showing that φ_2 is a massless particle and the O(2) symmetry between φ_1 and φ_2 has been spontaneously broken. The appearance of the massless mode (Goldstone boson) is a general consequence of spontaneously breaking a continuous symmetry, O(2) in this case.

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \tilde{\varphi}_1)^2 - \frac{1}{2}(\partial_\mu \tilde{\varphi}_2)^2 - A\tilde{\varphi}_1^2 + 0\varphi_2^2$$



Goldstone theorem in polar coordinates

write scalar fi elds in polar coordinates, as

$$\varphi = \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2) = \frac{1}{\sqrt{2}}\rho e^{i\theta}; \quad \partial_\mu(\rho e^{i\theta}) = (\partial_\mu \rho + i\rho\partial_\mu\theta)e^{i\theta}$$

$$\begin{aligned} -\partial_\mu\varphi^\dagger\partial_\mu\varphi &= -\frac{1}{2}(\partial_\mu\rho - i\rho\partial_\mu\theta)(\partial_\mu\rho + i\rho\partial_\mu\theta) \\ &= -\frac{1}{2}(\partial_\mu\rho)^2 - \frac{1}{2}\rho^2(\partial_\mu\theta)^2 \end{aligned}$$

$$\text{shift} \longrightarrow \begin{cases} \rho = & \langle\rho\rangle + \tilde{\rho} = v + \tilde{\rho} \\ \theta = & \underbrace{\langle\theta\rangle}_{=0} + \tilde{\theta} \end{cases}$$

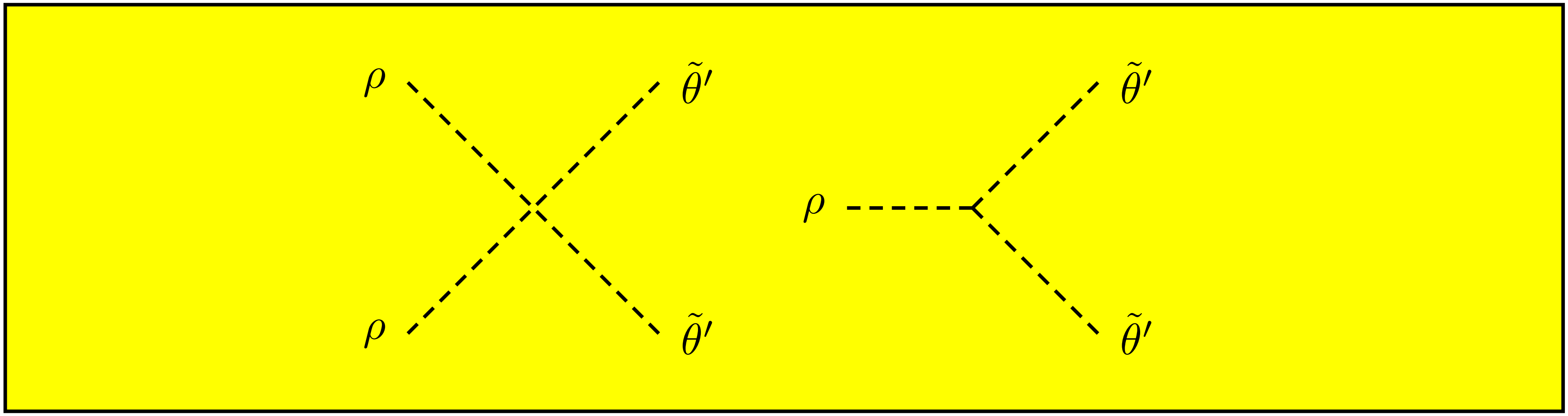
$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\tilde{\rho})^2 - \frac{1}{2}(v+\tilde{\rho})^2(\partial_\mu\tilde{\theta})^2 - V(\rho^2)$$

$$\tilde{\theta}' = v\tilde{\theta}$$

Goldstone theorem in polar coordinates-2

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \tilde{\rho})^2 - \frac{1}{2}(\partial_\mu \tilde{\theta}')^2 - \frac{1}{2}\frac{\tilde{\rho}^2}{v^2}(\partial_\mu \tilde{\theta}')^2 - \frac{\tilde{\rho}}{v}(\partial_\mu \tilde{\theta}')^2 - V(\rho^2)$$

clearly V has no θ-dependence or, equivalently, no mass term for $\tilde{\theta}'$. This is the Goldstone associated to the breaking of U(1) and corresponds to the flat direction in mexican hat. Note that the GB has interactions with ρ. As we will see later in the electroweak theory, since the continuous which breaks spontaneously is the gauge symmetry, the GB will eventually disappear. However, in the case of genuine continuous global symmetries such as lepton number, which break spontanously, say, to generate neutrino masses, Goldstone theorem leads to physical GB, called majoron [the above cubic term then leads to the invisible decay of the Higgs boson, see A. S. Joshipura, J.V., Nucl. Phys. B397 (1993) 105-122]



Goldstone-theorem (non-abelian case)-1

Let T_A be the generators of a Lie group G of structure const. C_{ABC}

$$[T_A, T_B] = iC_{ABC}T_C$$

The T_A are matrices acting on the representation space of the scalar fi elds φ_a's. By assumption, under an infi nitesimal transf in G

$$\delta^A\varphi_a = iu^A T^A_{ab}\varphi_b, \quad \forall\ A, \quad 1 \le A \le \dim G$$

the Lagrangean is invariant

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\varphi_a)^2 - V(\varphi)$$

and in particular the potential V, so that

$$\delta^A V = 0 = iV_{,a}u^A T^A_{ab}\varphi_b, \quad \forall\ u^A$$

differentiate w.r.t. φ_c and setting $\varphi \to \langle\varphi\rangle$ we have $\langle V_{,ac}\rangle\, T^A_{ab}\, \langle\varphi_b\rangle + \underbrace{\langle V_{,a}\rangle}_{=0}\, T^A_{ab}\delta_{bc} = 0$

Goldstone-theorem (non-abelian case)-2

As a result the scalar mass-squared matrix

$$M^2_{ac} \equiv \langle V_{,ac} \rangle \quad \text{has} \quad \psi^A_a = T^A_{ab} \langle \varphi_b \rangle \quad \text{as null vector}$$

$$M^2 \psi^A = 0 \quad \forall \ A$$

this is automatic for those T^A that kill the vacuum, i. e. for the subgroup H of G which is left unbroken since it leaves the vacuum invariant. For the other T^A the scalar mass-squared matrix will have non-trivial null vectors corresponding to the 2 broken generators in G/H. This is the non-abelian version of Goldstone-theorem. Take as an example O(3) broken to O(2)

$$O(3) \xrightarrow{\langle \varphi \rangle \neq 0} O(2)$$

by a nonzero $\langle \varphi \rangle$ of a triplet scalar fi eld $\varphi_a \equiv (\varphi_1, \varphi_2, \varphi_3)$.

Goldstone-theorem O(3) example-1

$$\varphi_a^T \equiv (\varphi_1, \varphi_2, \varphi_3)$$

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \varphi_a)^2 - V(\varphi) \quad G \text{ invariant if } V = V(\varphi^T \varphi)$$

$$\varphi^T \varphi = \sum_{a=1}^{3} \varphi_a \varphi_a \equiv \varphi^T R^T R \varphi \quad \forall \text{ rotation R} \in \ G$$

Let V be minimized for $\langle \varphi \rangle \neq 0$. e. g.

$$V \propto B(\varphi^T \varphi - v^2)^2$$

we can always choose

$$\langle \varphi_a \rangle = \delta_{a3}\, v \rightarrow \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

Goldstone-theorem O(3) example-2

In this case we have

$$M^2_{ac}\epsilon_{a3d} = 0, \quad \forall\, d$$

$$d = 1 \quad \Longrightarrow \quad M^2_{2c} = 0$$

$$d = 2 \quad \Longrightarrow \quad M^2_{1c} = 0$$

thus

M^2	1	2	3
1	0	0	0
2	0	0	0
3	0	0	M^2_{33}

so that M^2_{33} is the only one not forced to vanish. The O(3) generators are repr. by

T_1			T_2			T_3		
0	0	0	0	0	-1	0	1	0
0	0	1	0	0	0	-1	0	0
0	-1	0	1	0	0	0	0	0

$$T^3 \langle \varphi \rangle = 0$$

Goldstone-theorem O(3) example-3

1. $M^2 T^3 \langle\varphi\rangle = M^2 .0 = 0$ corresponding to the surviving symmetry, while
2. $M^2 T^\pm \langle\varphi\rangle = 0$ although $T^\pm \langle\varphi\rangle \neq 0$, corresponding to the 2 broken generators in O(3)/O(2)

Thus there are two Goldstone bosons, the same number as broken symmetry generators. In what follows we will see what happens to these massless scalars (there are none in the PDG) in the presence of gauge fi elds. This so-called Higgs mechanism will form the basis for the construction of a Yang-Mills gauge theory of the weak and electromagnetic interaction

Abelian Higgs mechanism-1

add to Goldstone model a U(1) gauge fi eld

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - (D_\mu\varphi)^\dagger D_\mu\varphi - V(\varphi)$$

where

$$D_\mu\varphi = (\partial_\mu - igA_\mu)\varphi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V = B\left(\varphi^\dagger\varphi - \frac{A}{2B}\right)^2 \equiv \mu^2\varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2 \quad \begin{cases} \lambda = & B > 0 \\ \mu^2 = & -A < 0 \end{cases}$$

this lagrangean has a U(1) gauge invariance

$$\begin{aligned} \varphi &\to e^{i\Lambda(x)}\varphi \\ A_\mu &\to A_\mu + \frac{1}{g}\partial_\mu\Lambda \\ D_\mu\varphi &\to e^{i\Lambda(x)}D_\mu\varphi \\ F_{\mu\nu} &\to F_{\mu\nu} \end{aligned}$$

Abelian Higgs mechanism-2

since

$$\langle \varphi^\dagger \varphi \rangle = \frac{v^2}{2}$$

we write

$$\varphi = \frac{1}{\sqrt{2}}(v+\rho)e^{i\frac{\theta}{v}} = \frac{1}{\sqrt{2}}(v+\rho+i\theta+\cdots)$$

now since $\mathcal{L}$ is gauge invariant we can choose (U-gauge) $\Lambda(x) = \frac{\theta}{v}$

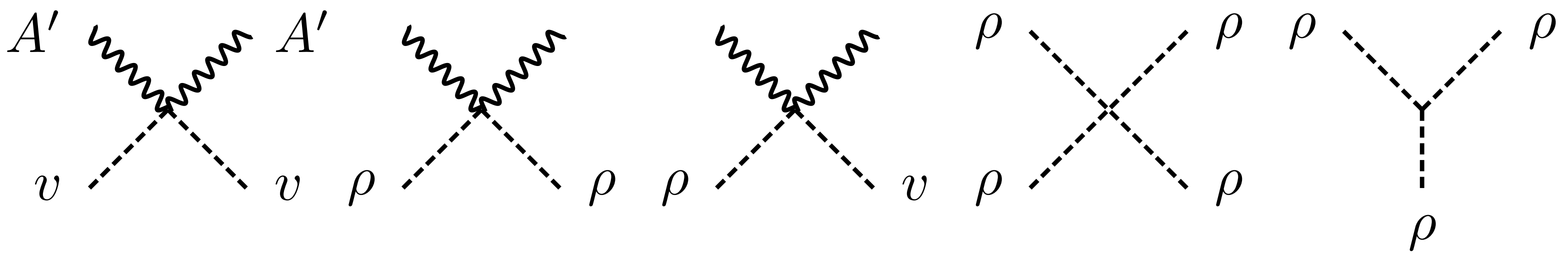
$$\varphi \to e^{-i\frac{\theta}{v}}\varphi = \frac{1}{\sqrt{2}}(v+\rho) \qquad A_\mu \to A_\mu - \frac{1}{gv}\partial_\mu\theta = A'_\mu$$

so that

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F'_{\mu\nu}F'_{\mu\nu} - \frac{1}{2}[\partial_\mu\rho - igA'_\mu(\rho+v)]^\dagger[\partial_\mu\rho - igA'_\mu(\rho+v)] - V(\varphi) \\
&= -\frac{1}{4}F'_{\mu\nu}F'_{\mu\nu} - \frac{1}{2}(\partial_\mu\rho)^2 - \frac{1}{2}(g^2v^2)A'_\mu A'_\mu - \frac{1}{2}g^2\rho^2 A'_\mu A'_\mu - g^2 v\rho A'_\mu A'_\mu \\
&\quad -\frac{B}{4}(\rho^4 + 4\rho^2v^2 + 4\rho^3 v)
\end{aligned}$$

Abelian Higgs mechanism-3

$$m^2(A'_\mu) = g^2 v^2, \qquad m^2(\rho) = B\, v^2$$



- the massless A-vector has been replaced by the massive A' field and the Nambu-Goldston boson has disappeared
- naively we expected
  - 2 transverse degrees of freedom of a massless vector
  - 1 massive scalar
  - 1 massless scalar
- instead we have found
  - 3 components of a massive vector
  - 1 massive scalar

renormalizability and unitarity-1

note that the original description was in the R-gauge where one can check renormalizability (t'Hooft-Veltman) but particle identifi cation can not be made

$$D_\mu \varphi = \frac{1}{\sqrt{2}}(\partial_\mu \rho + \partial_\mu \theta - igA_\mu(v + \rho + i\theta) + \cdots)$$

due to presence of cross terms

$$(D_\mu \varphi)^\dagger D_\mu \varphi = -\frac{1}{2}[(\partial_\mu \rho)^2 + (\partial_\mu \theta)^2 + g^2 v^2 A_\mu^2 - 2gvA_\mu \partial_\mu \theta + \cdots]$$

In contrast in the new description in the U-gauge particle content can be identifi ed (massive vector A'+ scalar ρ) but renormalizability is not manifest, since here we have

$$\Delta_{\mu\nu}^m = -\frac{i}{(2\pi)^4} \frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}}{k^2 + m^2 - i\epsilon} \xrightarrow{k \to \infty} 1$$

in contrast with QED, where the propagator behaves as $\frac{1}{k^2}$,

renormalizability and unitarity-2

using gauge invariance t'Hooft [NPB33 (71) 173] showed that massive gauge propagator can be written in any $\boxed{R_\xi\text{-gauge}}$ as

$$\Delta^{\xi}_{\mu\nu} = -\frac{i}{(2\pi)^4}\left[\frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}}{k^2+m^2-i\epsilon} + \frac{k_\mu k_\nu}{k^2\left(\frac{k^2}{\xi}+m^2\right)}\right]$$

For finite ξ-values we have QED behavior, so that theory is renormalizable

$$\Delta_{\mu\nu} \xrightarrow{k\to\infty} \frac{1}{k^2}$$

As $\xi \to \infty$ one can show that the propagator becomes becomes that of a massive vector boson in the U-gauge

because of gauge invariance theory must be both renormalizable and unitary

Non-abelian Higgs mechanism-1

in non-abelian case we have

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu})^2 - \frac{1}{2}(D_\mu\varphi^\alpha)(D_\mu\varphi^\alpha) - V(\varphi^T\varphi)$$

where a basis is chosen where the generators are chosen to be imaginary and skew

$$-t^* = +t_a = -t_a^T; \quad [t_a, t_b] = if_{abc}t_c$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \quad A_\mu = A_\mu^a t^a, \quad t^a = \frac{\tau^a}{2} \\ D_\mu\varphi &= \partial_\mu\varphi - igt^a A_\mu^a\varphi \qquad \text{or} \qquad \mathrm{D}_\mu\varphi^\alpha = \partial_\mu\varphi^\alpha - \mathrm{igt}^{\mathrm{a}}_{\alpha\beta}\mathrm{A}^{\mathrm{a}}_\mu\varphi^\beta \end{aligned}$$

choose parameters in scalar potential so that minimum occurs for

$$\langle\varphi^\alpha\rangle \equiv \lambda^\alpha \neq 0$$

shift scalars as

Non-abelian Higgs mechanism-2

$$\varphi = \lambda + \tilde{\varphi}$$

and substitute. One can now show that the non-abelian gauge fi elds now get a mass term described by the matrix

$$\boxed{M^2_{ab}(A') \propto \lambda^T t_a t_b \lambda}$$

- thus those generators which anihilate the vacuum correspond to unbroken gauge symmetries and massless intermediate vector boson, like the QED photon
- for each generator broken by the vacuum the corresponding intermediate vector boson becomes massive, as needed to describe the weak interaction
- for example in the O(3) example abobe 2 gauge bosons are eaten by the gauge fi elds in O(3)/O(2) with the O(2) gauge fi eld remaining massless

$$\begin{pmatrix} W^+ \\ W^- \\ \gamma \end{pmatrix}$$

Non-abelian Higgs mechanism-3

this would be a perfect model (Glashow) except that it has no room to account for the weak neutral currents and the existence of a massive Z boson discovered and well-studied at LEP

$$(SU_3^C) \times SU_2 \times U_1$$

so that we have

$$\begin{pmatrix} W^+ \\ W^- \\ Z \\ \gamma \end{pmatrix}$$

The Standard Model

- result of detailed interplay of different fi elds over around 30 years
- we adopt anti-historical approach, present it and derive implications
- The gauge group is a product, each factor with its gauge coupling constant g_i

$$\mathrm{G} = \mathrm{SU}(3) \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1) \qquad \text{with} \qquad \mathrm{g}_3 \qquad \mathrm{g}_2 \qquad \mathrm{g}_1$$

- chiral fermion multiplets
- scalar multiplet
- focus on SU(2) $\otimes$ U(1)

SM lepton assignments

- 3 left-handed, color singlet, SU(2) $\otimes$ U(1) doublet leptons

$$\chi_{AL} = \begin{pmatrix} \chi_{0A} \\ \chi_{-A} \end{pmatrix}_L = \frac{1+\gamma_5}{2}\chi_A \qquad A = 1,2,3$$

- 3 right-handed, color singlet, SU(2) $\otimes$ U(1) singlet leptons

$$\chi_{AR} = \frac{1+\gamma_5}{2}\chi_{AR}$$

A=1,2,3 is the generation index

replication problem: who ordered the muon?

SM quark assignments

- 3 left-handed, SU(3)-color triplet, SU(2) $\otimes$ U(1) doublet quarks

$$\psi_{AL} = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}_{AL} = \frac{1+\gamma_5}{2}\psi_A$$

- 6 right-handed, SU(3)-color triplet, SU(2) $\otimes$ U(1) singlet quarks

$$\begin{aligned} \psi^u_{AR} &= \frac{1-\gamma_5}{2}\psi^u_R \\ \psi^d_{AR} &= \frac{1-\gamma_5}{2}\psi^d_R \end{aligned}$$

we still need to defi ne the U(1) assignments of all multiplets. For this we need to analyse the gauge sector and the symmetry breaking induced by the scalar multiplet. The result is (see next)

$$\boxed{Y = 2(Q - T_3)}$$

SU(2) $\otimes$ U(1) gauge fields

$$\left\{ A_\mu = A_\mu^a \frac{\tau^a}{2} = \frac{1}{2}\begin{pmatrix} A_3 & A_1 - iA_2 \\ \text{h.c.} & -A_3 \end{pmatrix}, B_\mu \right\}$$

$$\begin{aligned} W^\pm &= \frac{A_1 \mp iA_2}{\sqrt{2}} \\ A^3, B &\leftrightarrow Z_\mu, \gamma. \\ \Phi &\to \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix} \end{aligned}$$

the full SM Lagrangean

$$\mathcal{L} = \mathcal{L}_{\text{YM}}^{A} + \mathcal{L}_{\text{S}}^{\varphi,A} + \mathcal{L}_{\text{matter}}^{\psi,\Phi,A}$$

contains

- Yang-Mills gauge fi elds
- scalar fi elds and their gauge couplings (covariant derivative)
- fermions with their gauge (covariant derivative) and Yukawa couplings

Symmetry structure-1

the YM Lagrangean is

$$\mathcal{L}_{\rm YM} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{2}{\rm Tr}(F_{\mu\nu}F_{\mu\nu})$$

$$F_{\mu\nu} = \frac{\tau^a}{2}F^a_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\nu, A_\mu]$$

The scalar-gauge Lagrangean can be written as

$$\mathcal{L}_S = -D_\mu\varphi^\dagger D_\mu\varphi - V(\varphi)$$

where

$$\begin{aligned} V(\varphi) &= a(\varphi^\dagger\varphi - v^2/2)^2, \quad a, v^2 > 0 \\ &\equiv \mu^2\varphi^\dagger\varphi + 2(\varphi^\dagger\varphi)^2 + \text{const} \end{aligned}$$

$$D_\mu\varphi = \partial_\mu\varphi - igA_\mu\varphi + \frac{ig'}{2}B_\mu\varphi$$

where g and g' are the SU(2) and U(1) gauge couplings

Symmetry structure-2

the scalar potential can be chosen as

$$V = a\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^2 \equiv \underbrace{-av^2}_{\mu^2<0}\varphi^\dagger\varphi + a(\varphi^\dagger\varphi)^2 + \text{const}$$

is invariant under

$$\begin{aligned} \varphi &\rightarrow e^{i\alpha(x)}\varphi \\ \varphi &\rightarrow U(x)\varphi \end{aligned}$$

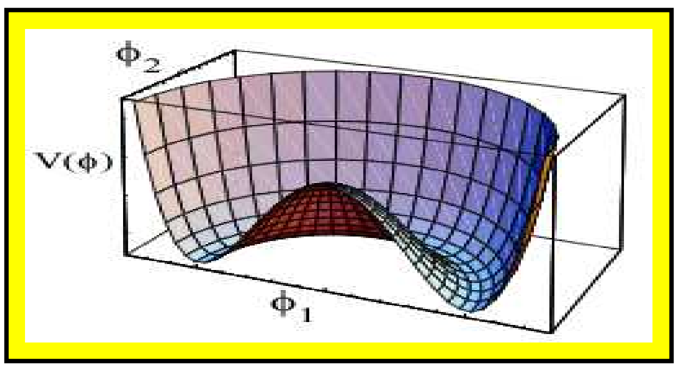
where

$$U^\dagger U = 1 \qquad \det U = 1$$

For

$$a, v^2 > 0 \longrightarrow \langle\varphi^\dagger\varphi\rangle = \frac{v^2}{2} \neq 0$$

as a stable minimum



Symmetry structure-3

under a gauge transformation it is allways possible to bring

$$\varphi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ (\tilde{\varphi} + v)/\sqrt{2} \end{pmatrix}$$

so that

$$\begin{aligned} D_\mu \varphi &= \begin{pmatrix} 0 \\ \partial_\mu \tilde{\varphi}/\sqrt{2} \end{pmatrix} - \frac{ig}{2} \begin{pmatrix} A^3_\mu & \sqrt{2} W^+_\mu \\ \sqrt{2} W^+_\mu & -A^3_\mu \end{pmatrix} \begin{pmatrix} 0 \\ (\tilde{\varphi} + v)/\sqrt{2} \end{pmatrix} \\ &\quad - \frac{ig'}{2} \begin{pmatrix} 0 \\ (\tilde{\varphi} + v)/\sqrt{2} \end{pmatrix} B_\mu \\ &= \begin{pmatrix} \frac{-ig}{2} \frac{\tilde{\varphi}+v}{\sqrt{2}} \sqrt{2} W^+_\mu \\ \frac{\partial_\mu \tilde{\varphi}}{\sqrt{2}} + i \frac{\tilde{\varphi}+v}{2\sqrt{2}} (g A^3_\mu + g' B_\mu) \end{pmatrix} \end{aligned}$$

where

$$\sqrt{2} W^{\pm} = (A^1_\mu \mp A^2_\mu),$$

Symmetry structure-4

$$\sqrt{2}W^{\pm} = A^1_\mu \mp A^2_\mu, \quad \varphi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ (\tilde{\varphi}+v)/\sqrt{2} \end{pmatrix}$$

$$\begin{aligned} -(D_\mu\varphi)^\dagger D_\mu\varphi &= -\frac{g^2}{4}(\tilde{\varphi}+v)^2 W^+_\mu W^-_\mu - \frac{1}{2}(\partial_\mu\tilde{\varphi})^2 \\ &- \frac{(\tilde{\varphi}+v)^2}{8}\underbrace{\left(\frac{gA^3_\mu + g'B_\mu}{\sqrt{g^2+g'^2}}\right)^2}_{Z^2_\mu}\underbrace{\frac{(g^2+g'^2)}{g^2}}_{\frac{1}{\cos^2\theta_W}}g^2 \end{aligned}$$

$$\begin{aligned} \frac{g^2v^2}{4} &= m^2_W \\ \frac{(g^2+g'^2)v^2}{4} &= m^2_Z \end{aligned}$$

$$\boxed{\frac{m_W}{m_Z} = \frac{g}{\sqrt{g^2+g'^2}} = \cos\theta_W}$$

$$e = g\,\sin\theta_W = g'\,\cos\theta_W$$

Symmetry structure-5

$$\tau_3 \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ -v \end{pmatrix}$$

$$Y \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$(\tau_3 + Y)|0\rangle$$

$$\tau_3 + 1 = 2I_3 + Y \equiv 2Q$$

$$\boxed{Q = I_3 + \frac{Y}{2}}$$

$$\mathcal{L}^{\text{gauge}}_{\text{mass}} = -m_W^2 W^+_\mu W^-_\mu - \frac{m_Z^2}{2} Z^2_\mu + 0\, A^2_\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix}$$

The Standard SU(2) $\otimes$ U(1) Model

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$e_R \qquad \mu_R \qquad \tau_R$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$u_R \qquad c_R \qquad t_R$$

$$d_R \qquad s_R \qquad b_R$$

$$\varphi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}$$

χ_L	2	-1	1
χ_R	1	-2	1
ψ_L	2	1/3	3
ψ_R^u	1	4/3	3
ψ_R^d	1	-2/3	3

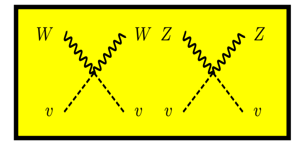
gauge-scalar sector			
W_μ, Z_μ	3	0	1
A_μ	1	0	1
g_μ	1	0	8
φ	2	1	1

$$\boxed{Y = 2(Q - T_3)}$$

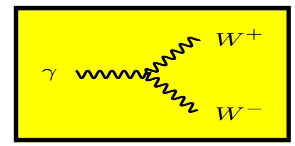
The Standard SU(2) $\otimes$ U(1) Model Lagrangean

the SM Lagrangean can be rewritten in terms of the mass-eigenstate gauge fields as follows as a sum of seven terms $\boxed{m_W = m_Z \cos\theta_W}$ $\boxed{\frac{\sin\theta_W}{\cos\theta_W} = \frac{g'}{g} = \tan\theta_W}$ $\boxed{e = g\sin\theta_W = g'\cos\theta_W}$

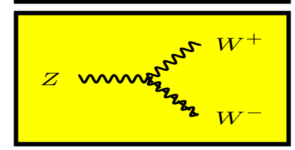
$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2$$



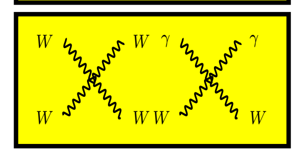
$$-m_W^2 W_\mu^+ W_\mu^- - \frac{m_Z^2}{2} Z_\mu^2$$



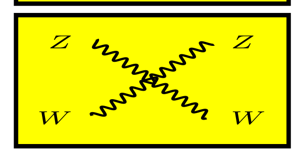
$$ie[A_\mu(W_\nu^+ \overleftrightarrow{\partial}_\mu W_\nu^- + W_\nu^+ \overleftrightarrow{\partial}_\mu W_\mu^- - \partial_\nu W_\mu^+ W_\nu^-) + (\partial_\mu A_\nu - \partial_\nu A_\mu) W_\mu^+ W_\nu^-]$$



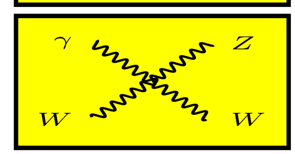
$$igc_W[Z_\mu(W_\nu^+ \overleftrightarrow{\partial}_\mu W_\nu^- + W_\nu^+ \overleftrightarrow{\partial}_\mu W_\mu^- - \partial_\nu W_\mu^+ W_\nu^+) + (\partial_\mu Z_\nu - \partial_\nu Z_\mu) W_\mu^+ W_\nu^-]$$



$$-\frac{g^2}{2}(W_\mu^+ W_\mu^-)^2 + \frac{g^2}{2}(W_\mu^+ W_\nu^-)^2 + e^2(A_\mu A_\nu W_\mu^+ W_\nu^- - A_\mu A_\mu W_\nu^+ W_\mu^-)$$



$$g^2 c_W^2 (Z_\mu Z_\nu W_\mu^+ W_\nu^- - Z_\mu^2 W_\nu^+ W_\nu^-)$$



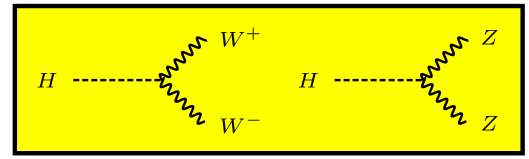
$$egc_W[A_\mu Z_\nu(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\nu W_\nu^+ W_\nu^-]$$

Higgs boson piece

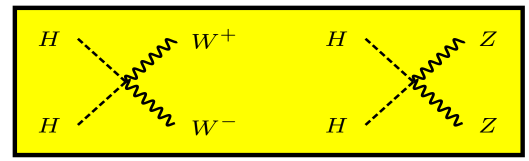
In addition the we have the Higgs interactions with e-w gauge bosons $\boxed{m_H^2 = 8av^2}$

$$\mathcal{L}_{\tilde{\varphi}-\text{gauge}} = \mathcal{L}^{(3)}_{\tilde{\varphi}-\text{gauge}} + \mathcal{L}^{(4)}_{\tilde{\varphi}-\text{gauge}}$$

$$\mathcal{L}^{(3)}_{\tilde{\varphi}-\text{gauge}} = -\frac{g m_W}{2}\tilde{\varphi}\left(W^+_\mu W^-_\nu + \frac{1}{2\cos^2\theta_W} Z^2_\mu\right)$$

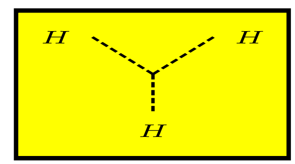


$$\mathcal{L}^{(4)}_{\tilde{\varphi}-\text{gauge}} = -\frac{g^2}{4}\tilde{\varphi}^2\left(W^+_\mu W^-_\nu + \frac{1}{2\cos^2\theta_W} Z^2_\mu\right)$$

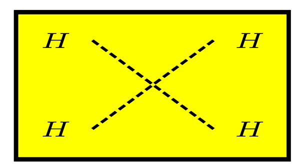


$$\mathcal{L}_{\tilde{\varphi}} = -\frac{1}{2}(\partial_\mu\tilde{\varphi})^2 - \frac{1}{2}m_H^2\tilde{\varphi}^2 + \mathcal{L}^{(3)}_{\tilde{\varphi}} + \mathcal{L}^{(4)}_{\tilde{\varphi}}$$

$$\mathcal{L}^{(3)}_{\tilde{\varphi}} = -\frac{g}{4}\frac{m_H^2}{m_W}\tilde{\varphi}^3$$



$$\mathcal{L}^{(4)}_{\tilde{\varphi}} = -\frac{g^2}{32}\frac{m_H^2}{m_W^2}\tilde{\varphi}^4$$



note the Higgs mass is undetermined, if heavy, Higgs becomes more strongly coupled

Yukawa couplings and fermion masses

we should also add to the SM Lagrangean couplings of fermions to the scalar doublet, since these are allowed by consistent with gauge invariance and renormalizability

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\sum_{AB} h^e_{AB}\bar{\chi}^A_R\varphi^\dagger\chi^B_L + \text{h.c.} \\ &\quad -\sum_{AB}\left[h^d_{AB}\bar{\psi}^A_{dR}\varphi^\dagger\psi^B_L + h^u_{AB}\bar{\psi}^A_L(i\tau_2\varphi^*)\psi^B_{uR}\right] + \text{h.c.}\end{aligned}$$

Note that $i\tau_2\varphi^*$ transforms as φ under SU(2). These Yukawa terms introduce many new arbitrary parameters

$$\begin{array}{cccc|c} \lambda & g & g' & v & \\ \theta_W & m_H & m_W & m_Z & h^{d,u,e}_{AB} \end{array}$$

some are spurious, others directly given by experiment

rewrite $\mathcal{L}_{\text{Yukawa}}$ in matrix form

$$\boxed{\mathcal{L}_{\text{Yukawa}} = -\bar{\chi}_R \varphi^\dagger h^e \chi_L - \bar{\psi}_R^d \varphi^\dagger h^d \psi_L - \bar{\psi}_R^u (i\tau_2 \varphi^*) h^u \psi_L + \text{h.c.}}$$

where h^e, h^u, h^d are arbitrary, non-hermitian matrices. In the U-gauge we have

$$\varphi^\dagger = \left(0, \frac{v+H}{\sqrt{2}}\right)$$

thus the lepton piece becomes

$$\begin{aligned} & \bar{\chi}_R \left(0, \frac{v+H}{\sqrt{2}}\right) h^e \begin{pmatrix} \chi_L^0 \\ \chi_L^- \end{pmatrix} \\ = \quad & \bar{\chi}_R^- h^e \chi_L^- \frac{(v+H)}{\sqrt{2}} + \text{h.c.} = \bar{E}_R \Omega_R^\dagger h^e \Omega_L E_L \frac{(v+H)}{\sqrt{2}} + \text{h.c.} \end{aligned}$$

After SSB these Yukawa terms will generate the lepton masses as

$$h^e \quad \longrightarrow \quad m_e, m_\mu, m_\tau$$

general matrix diagonalization theorem

(Dirac) fermion Yukawa couplings are generally specified by arbitrary square matrices. Any such matrix M may be decomposed in polar form as

$$\underbrace{M}_{2n^2} = \underbrace{UK}_{n^2+n^2} \quad \text{where } K = K^\dagger \quad \text{and} \quad U^\dagger U = UU^\dagger = 1$$

one can always find a unitary matrix V so that

$$\begin{aligned} K &= VDV^\dagger \qquad && D \text{ real diag} > 0 \\ & && V^\dagger V = VV^\dagger = 1 \\ M &= UVDV^\dagger && \end{aligned}$$

$$\Rightarrow W^\dagger M V = D; \quad \text{where} \quad W = UV$$

thus any square matrix can be bi-diagonalized, if $M = M^\dagger$ then $V = W$. The diagonalizing matrices may be found by solving the eigenvalue equations:

$$\begin{aligned} MM^\dagger &= WDV^\dagger VDW^\dagger = WD^2W^\dagger \\ M^\dagger M &= VDW^\dagger WDV^\dagger = VD^2V^\dagger \end{aligned}$$

similarly for the quarks

$$\mathcal{L}_{\text{Yuw}} = -\sum_i \bar{u}_i \left[m(u_i) u_i \left(1 + \frac{H}{v}\right) + \bar{d}_i m(d_i) d_i \left(1 + \frac{H}{v}\right)\right]$$

where

$$m(u) = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad m(d) = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

so we finally have

$$\frac{v}{\sqrt{2}} \Omega_R^{e\dagger} h^e \Omega_L^e = m(e) \text{ real diagonal}$$

$$\frac{v}{\sqrt{2}} \Omega_R^{u\dagger} h^u \Omega_L^u = m(u) \text{ real diagonal}$$

$$\frac{v}{\sqrt{2}} \Omega_R^{d\dagger} h^d \Omega_L^d = m(d) \text{ real diagonal}$$

note that, in SM, $m_{\nu_i} \equiv 0$

fermion masses

- not gauge invariant, but arise from SSB.
- not predicted by theory, Yukawas are fi tted parameters
  experiment gives

$$\begin{array}{lll} m_e \approx 0.511 \text{ MeV}, & m_u = 1.5 \div 4.5 \text{ MeV}, & m_d = 5 \div 8.5 \text{ MeV} \\ m_\mu \approx 105.66 \text{ MeV}, & m_c = 1 \div 1.4 \text{ GeV}, & m_s = 80 \div 155 \text{ MeV} \\ m_\tau \approx 1.777 \text{ GeV}, & m_t = 174.3 \pm 5.1 \text{ GeV}, & m_b = 4.0 \div 4.5 \text{ GeV} \end{array}$$

Higgs boson couplings

- prop to mass, all Yukawa terms proportional to 1+H/v
- Higgs boson couplings diagonal in mass eigenstate basis
  thus no Higgs-mediated FCNC

fermion gauge interactions

all gauge interactions determined by the fermion covariant derivatives, whose SU(2) piece we already know, and whose U(1) piece is given by the table of hypercharge assignments

$$
\begin{aligned}
D_\mu \chi_L &= \partial_\mu \chi_L - igA_\mu \chi_L - i\frac{g'}{2} B_\mu \chi_L \\
D_\mu \chi_R &= \partial_\mu \chi_R \qquad\qquad\qquad - ig' B_\mu \chi_R \\
D_\mu \psi_L &= \partial_\mu \psi_L - igA_\mu \psi_L - i\frac{g'}{6} B_\mu \psi_L \\
D_\mu \psi_{uR} &= \partial_\mu \psi_{uR} \qquad\qquad\qquad + \frac{2ig'}{3} B_\mu \psi_{uR} \\
D_\mu \psi_{dR} &= \partial_\mu \psi_{dR} \qquad\qquad\qquad - \frac{ig'}{3} B_\mu \psi_{dR}
\end{aligned}
$$

so that

$$
\begin{aligned}
\mathcal{L}_{\text{matter}} &= -\sum_A (\bar\chi_{AL}\gamma_\mu D_\mu \chi_{AL} + \bar\chi_{AR}\gamma_\mu D_\mu \chi_{AR}) \\
&\quad - \sum_A (\bar\psi_{AL}\gamma_\mu D_\mu \psi_{AL} + \bar\psi_{AR}\gamma_\mu D_\mu \psi_{AR}) + \mathcal{L}_{\text{Yukawa}}
\end{aligned}
$$

understanding hypercharge assignments

we already saw that the generator $\tau_3 + Y$ annihilates the vacuum $|0\rangle \equiv \begin{pmatrix} 0 \\ v \end{pmatrix}$, i. e.

$$\tau_3 \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ -v \end{pmatrix}$$

$$Y \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\tau_3 + 1 = 2I_3 + Y \equiv 2Q$$

this allows us to identify the electric charge as $\boxed{Q = I_3 + \frac{Y}{2}}$ using

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}$$

and $\boxed{e = g \sin \theta_W = g' \cos \theta_W}$ one can understand the U(1) piece on previous page

CC and NC weak interactions

the general structure of the gauge interactions of the fermions is very simple form in the original (weak-eigenstate) basis, namely

$$\begin{array}{ll} \text{weak CC} & \bar{E}\gamma_\mu N W_\mu + \bar{D}\gamma_\mu U W_\mu + \text{h.c.} \\ \text{weak NC} & \bar{\psi}_L \gamma_\mu \psi_L Z_\mu \qquad \bar{\psi}_R \gamma_\mu \psi_R Z_\mu \\ \text{em current (QED) has L+R form} & \bar{\psi}\gamma_\mu \psi A_\mu \end{array}$$

where the original SU(2) doublet fermions are denoted $\chi = (N, E)$ and $\psi = (U, D)$, and similarly for the SU(2) singlets, with N, E, U, D being 3-dimensional vectors in generation space

$$\begin{array}{lll} E & e_L \mu_L \tau_L & e_R \mu_R \tau_R \\ N & \nu_{eL} \nu_{\mu L} \nu_{\tau L} & \\ U & u_L c_L t_L & u_R c_R t_R \\ D & d_L s_L b_L & d_R s_R b_R \end{array}$$

in contrast to kinetic terms and the QED couplings which are form-invariant (parity conservation) the weak couplings must be rewritten in mass-eigenstate basis

CC weak interactions-1

the next step is to the gauge interactions of the fermions in the mass-eigenstate basis. For the leptons we have

$$
\begin{aligned}
D_\mu \chi_L &= \ldots - \frac{ig}{\sqrt{2}} \begin{pmatrix} & W_\mu^+ \\ & \end{pmatrix} \begin{pmatrix} \chi_0 \\ \chi_- \end{pmatrix}_L + \cdots \\
&\to \frac{-ig}{\sqrt{2}} W_\mu^+ \chi_{-L}
\end{aligned}
$$

thus we have $\quad -\chi_L \gamma_\mu D_\mu \chi_L \to \mathcal{L}_{\mathrm{CC}}^{\ell} = \dfrac{ig}{\sqrt{2}} W_\mu^+ \bar{\chi}_{0L} \gamma_\mu \chi_{-L} + \text{h.c.}$

where
$$
\begin{aligned}
\chi_{0L} &= \Omega_L^\nu N_L \\
\chi_{-L} &= \Omega_L^e E_L
\end{aligned}
$$

thus $\quad \mathcal{L}_{\mathrm{CC}}^{\mathrm{lep}} = \dfrac{ig}{\sqrt{2}} W_\mu^+ \bar{N}_L \gamma_\mu \underbrace{\Omega_L^{\nu\dagger} \Omega_L^e}_{1} E_L + \text{h.c.}$

since neutrinos can be redefined in an arbitrary way

CC weak interactions-2

$$\mathcal{L}_{\rm CC}^{q} = \frac{ig}{\sqrt{2}} W_\mu^+ \bar{U}_L \gamma_\mu D_L + \text{h.c.}$$

$$\mathcal{L}_{\rm CC}^{q} = \frac{ig}{\sqrt{2}} W_\mu^+ \bar{u}_L \gamma_\mu \underbrace{\Omega_L^{u\dagger} \Omega_L^d}_{V} d_L + \text{h.c.}$$

note that quark phases can be redefi ned keeping mass terms invariant, so as to simplify the form of V, leaving only $n^2 - n - (n-1) = n^2 - 2n + 1 = (n-1)^2$, thus 9-3-2=4 physical KM parameters in V

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \longrightarrow \begin{pmatrix} e^{i\theta_u} & & \\ & e^{i\theta_c} & \\ & & e^{i\theta_t} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \longrightarrow \begin{pmatrix} e^{i\theta_d} & & \\ & e^{i\theta_s} & \\ & & e^{i\theta_b} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

CC weak interactions-3

V can be written in many parametrizations, such as the original one of M. Kobayashi, T. Maskawa, Prog.Theor.Phys.49:652-657,1973

$$V_{\text{CKM}} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} \end{pmatrix}$$

$$\begin{array}{ccc|c} u & c & t & \\ \hline c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} & d \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta_{13}} & c_{13}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} & s \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} & b \end{array}$$

which is essentially the same as Schechter & Valle, Phys.Rev.D22 (1980) 2227, with the convenient ordering of Wolfenstein Phys.Rev.Lett. 51 (1983) 1945, who also introduced a nice truncated form

Only one CP violating phase responsible for violation seen in K system

CC weak interactions-4

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} \sim 1 & V_{us} \sim \lambda & V_{ub} \sim \lambda^3 \\ V_{cd} \sim \lambda & V_{cs} \sim 1 & V_{cb} \sim \lambda^2 \\ V_{td} \sim \lambda^3 & V_{ts} \sim \lambda^2 & V_{tb} \sim 1 \end{pmatrix}$$

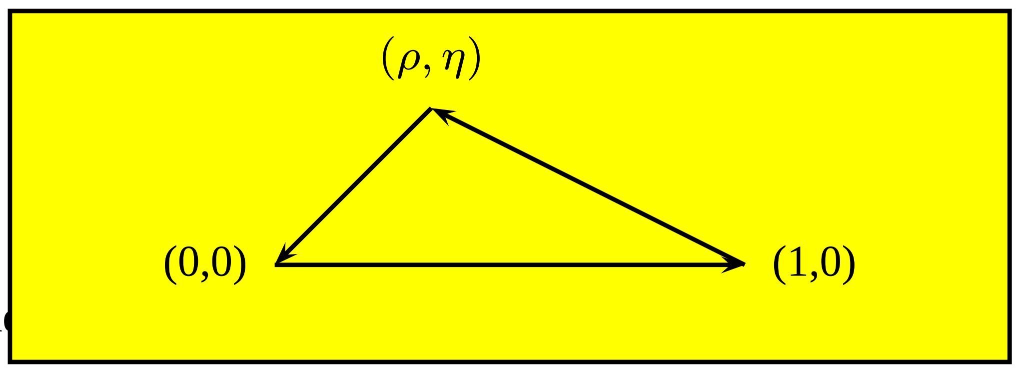
$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \rho e^{i\varphi} \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho e^{-i\varphi}) & -A\lambda^2 & 1 \end{pmatrix}$$

where $\lambda \sim \theta_c$ is small. This form shows explicitly that CP violation is small

unitarity triangle

$$V_{ud}\bar{V}_{ub} + V_{cd}\bar{V}_{cb} + V_{td}\bar{V}_{tb} = 0$$

$$A\lambda^3(\rho + i\eta) - A\lambda^3 + A\lambda^3 - A\lambda^3(\rho + i\eta) = 0$$



CC weak interactions-5

from various weak measurements and unitarity we have (PDG) (see Pich's lectures)

- $|V_{ud}| \approx 0.9741 \div 0.9756$
- $|V_{us}| \approx 0.219 \div 0.226$
- $|V_{ub}| \approx 0.0025 \div 0.0048$
- $|V_{ub}| \approx 0.0025 \div 0.0048$
- $|V_{cd}| \approx 0.219 \div 0.226$
- $|V_{cs}| \approx 0.9732 \div 0.9748$
- $|V_{cb}| \approx 0.038 \div 0.044$
- $|V_{td}| \approx 0.004 \div 0.014$
- $|V_{ts}| \approx 0.037 \div 0.044$
- $|V_{tb}| \approx 0.9990 \div 0.9993$

fix parameters from muon decay

$$\mathcal{L}_{\rm CC} = \frac{ig}{\sqrt{2}} W^+_\mu \bar{N}_L \gamma_\mu E_L + \bar{u}\gamma_\mu V D_L + \text{h.c.}$$

$$\mathrm{amp} = -\mathrm{i}\frac{\mathrm{g}^2}{8\mathrm{M}_\mathrm{W}^2}[\bar{\mathrm{u}}_1\gamma_\mu(1+\gamma_5)\mathrm{v}_2][\bar{\mathrm{u}}'\gamma_\alpha(1+\gamma_5)\mathrm{u}]$$

$$\Gamma_{\mu^- \to e^- \nu_e \nu_\mu} = \frac{1}{192\pi^3} G_F^2 m_\mu^5$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

$$G_F \approx 1.16 \times 10^{-5}\ \text{GeV}^{-2}$$

link between Fermi phenomenological model and the gauge theory
One can now easily calculate all leptonic weak processes, also semi-leptonic weak processes, like beta decyas. Non-leptonic weak processes are hard (QCD effective lagrangeans)

neutral currents in SM-1

$$
\begin{aligned}
D_\mu &= \cdots - igA^3_\mu T_3^\psi \psi + \frac{ig'}{2} B_\mu Y_W^\psi \psi \\
&= \cdots [-ig(c_W Z_\mu - s_W A_\mu) T_3^\psi + \frac{ig'}{2}(s_W Z_\mu - c_W A_\mu) Y_W^\psi] \psi \\
&= \cdots [i \underbrace{g s_w}_{e} A_\mu \underbrace{(T_3 + \frac{1}{2} Y_W)_\psi}_{Q_\psi} - \frac{ig'}{2} Z_\mu \underbrace{(T_3 - s_W^2 Q)_\psi}_{\text{neutral current (NC)}}] \psi
\end{aligned}
$$

$$
\begin{aligned}
J_\mu^{\text{em}} &= i \sum_{\text{all}} \bar{\psi} \gamma_\mu Q_\psi \psi \\
J_\mu^Z &= i \sum_{\text{all}} \bar{\psi} \gamma_\mu (T_3 - xQ) \psi
\end{aligned}
$$

$\boxed{g' c_W = e = g s_W}$ and $\boxed{x \equiv \sin^2 \theta_W \equiv s_W^2}$ with no FCNC, explaining rareness of $K \to \mu^+ \mu^-$

neutral currents in SM-2

ψ	T_3	Q	$T_3 - xQ$
ν_L	$1/2$	0	$1/2$
e_L	$-1/2$	-1	$-1/2 + x$
u_L	$1/2$	$2/3$	$1/2 - 2/3x$
d_L	$-1/2$	$-1/3$	$-1/2 + 1/3x$
e_R	0	-1	x
u_R	0	$2/3$	$-2/3x$
d_R	0	$-1/3$	$1/3x$

$$Z_\mu \sum \bar{\psi}\gamma_\mu (T_3 - xQ)_\psi \psi \Rightarrow$$

$$\bar{\psi}_L(\cdots)_{\psi_L}\psi_L + \bar{\psi}_R(\cdots)_{\psi_R}\psi_R$$

$$()_L\frac{(1+\gamma_5)}{2} + ()_R\frac{(1-\gamma_5)}{2}$$

$$\underbrace{\frac{[()_L + ()_R]}{2}}_{v_\psi} + \gamma_5 \underbrace{\frac{[()_L - ()_R]}{2}}_{\gamma_5 a_\psi}$$