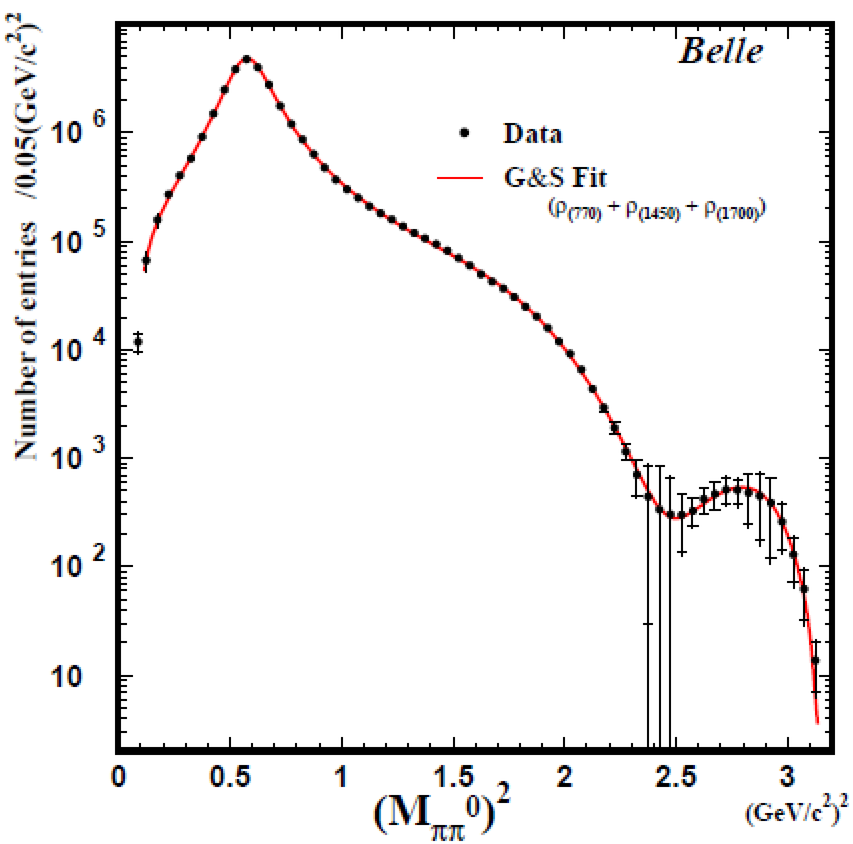
DISPERSIVE REPRESENTATION OF THE $\pi\pi$ VFF

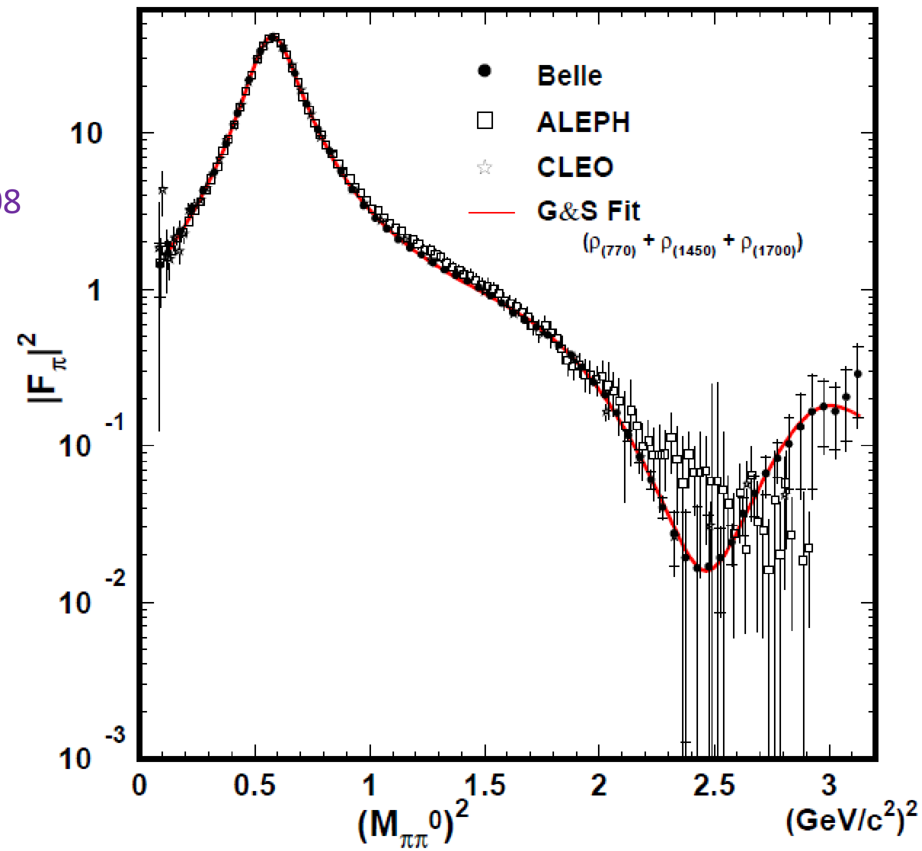
Pablo Roig (IF-UNAM)

Work done in collaboration with Daniel Gómez Dumm (La Plata, Argentina)

CINVESTAV, 12/11/13 & IF/ICN-UNAM 13/11/13



Belle '08



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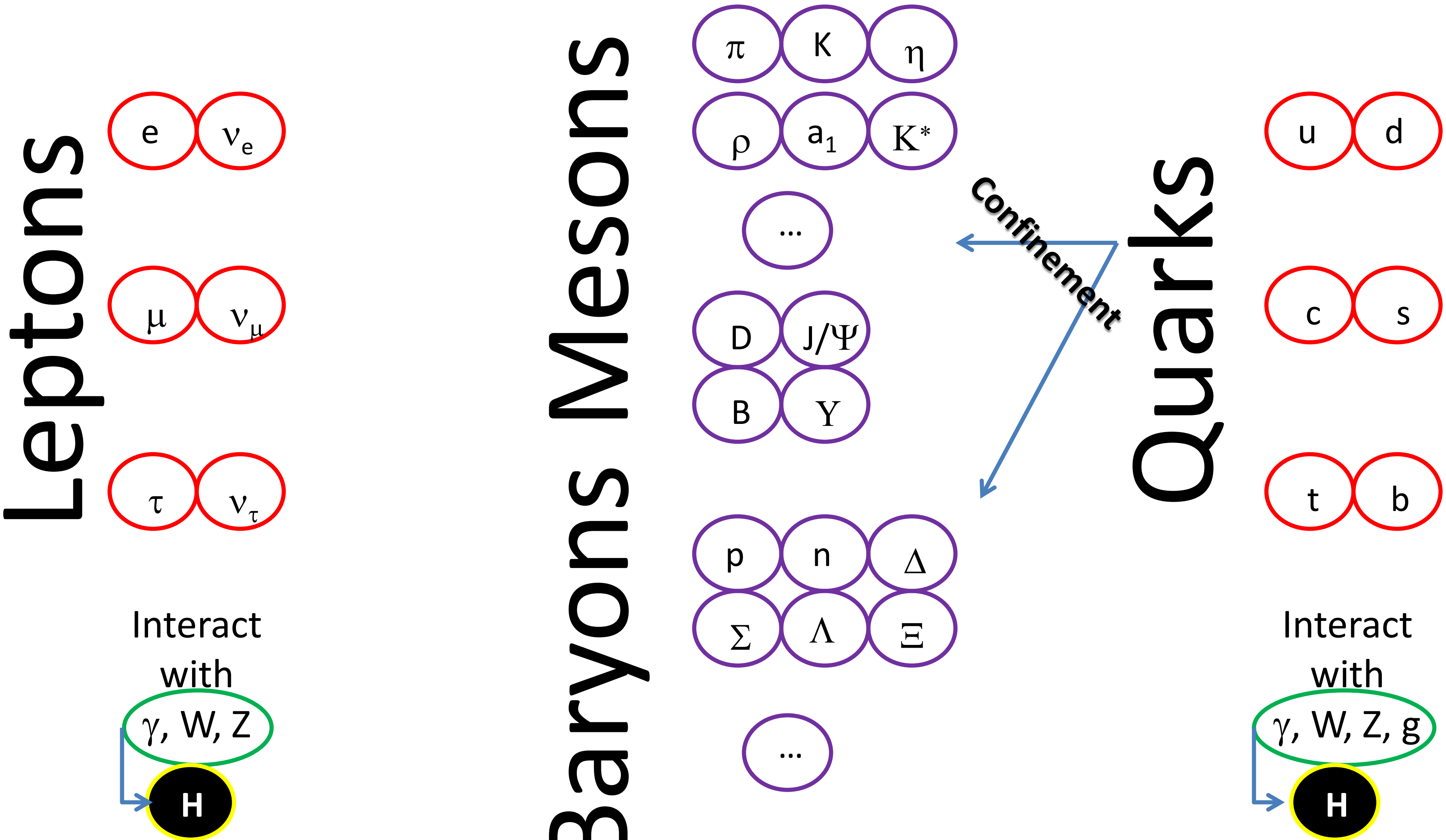
- Introduction
- Theoretical setting
- The vector form factor of $\pi^-\pi^0$ and fits to data
- Conclusions

INTRODUCTION



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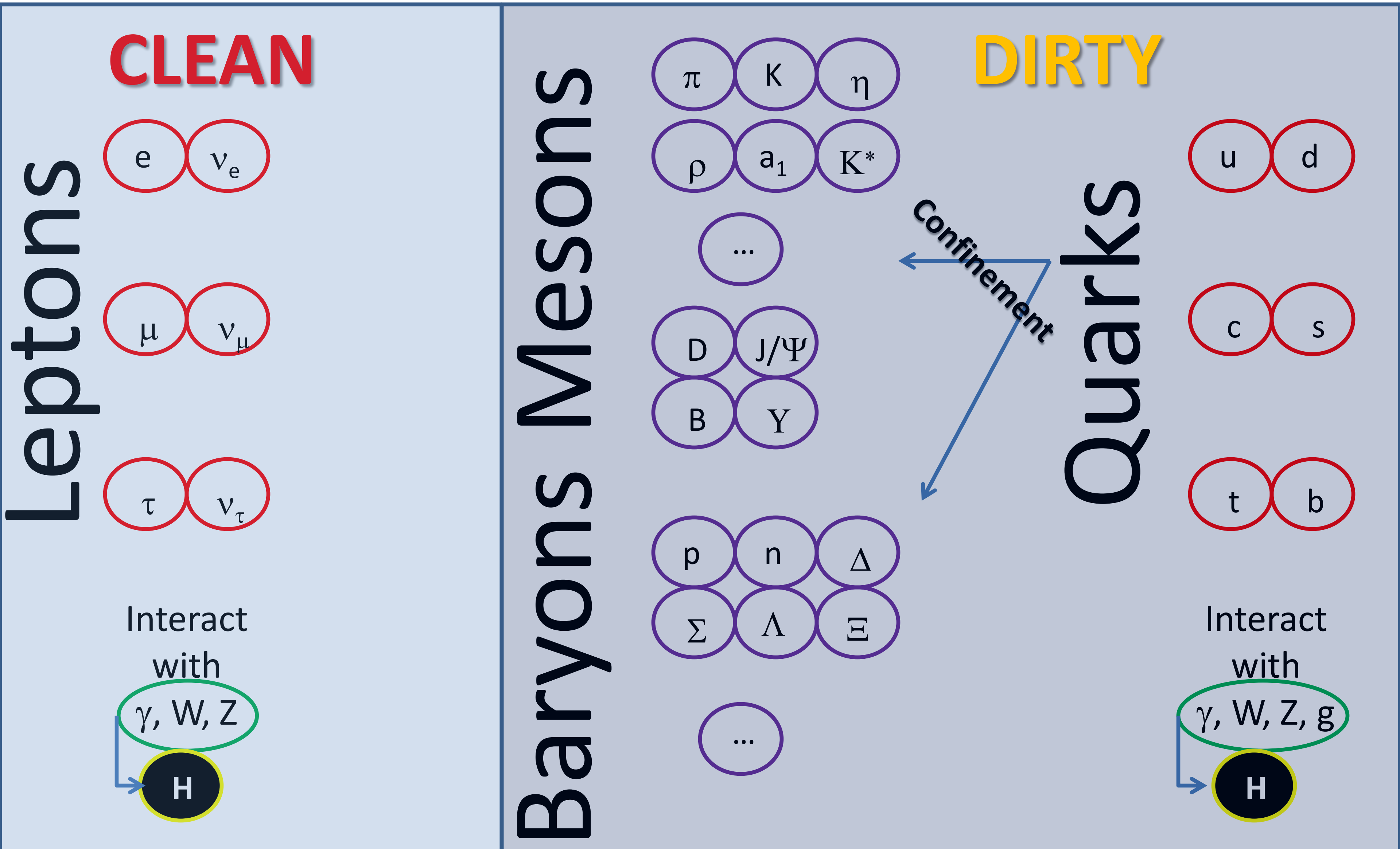

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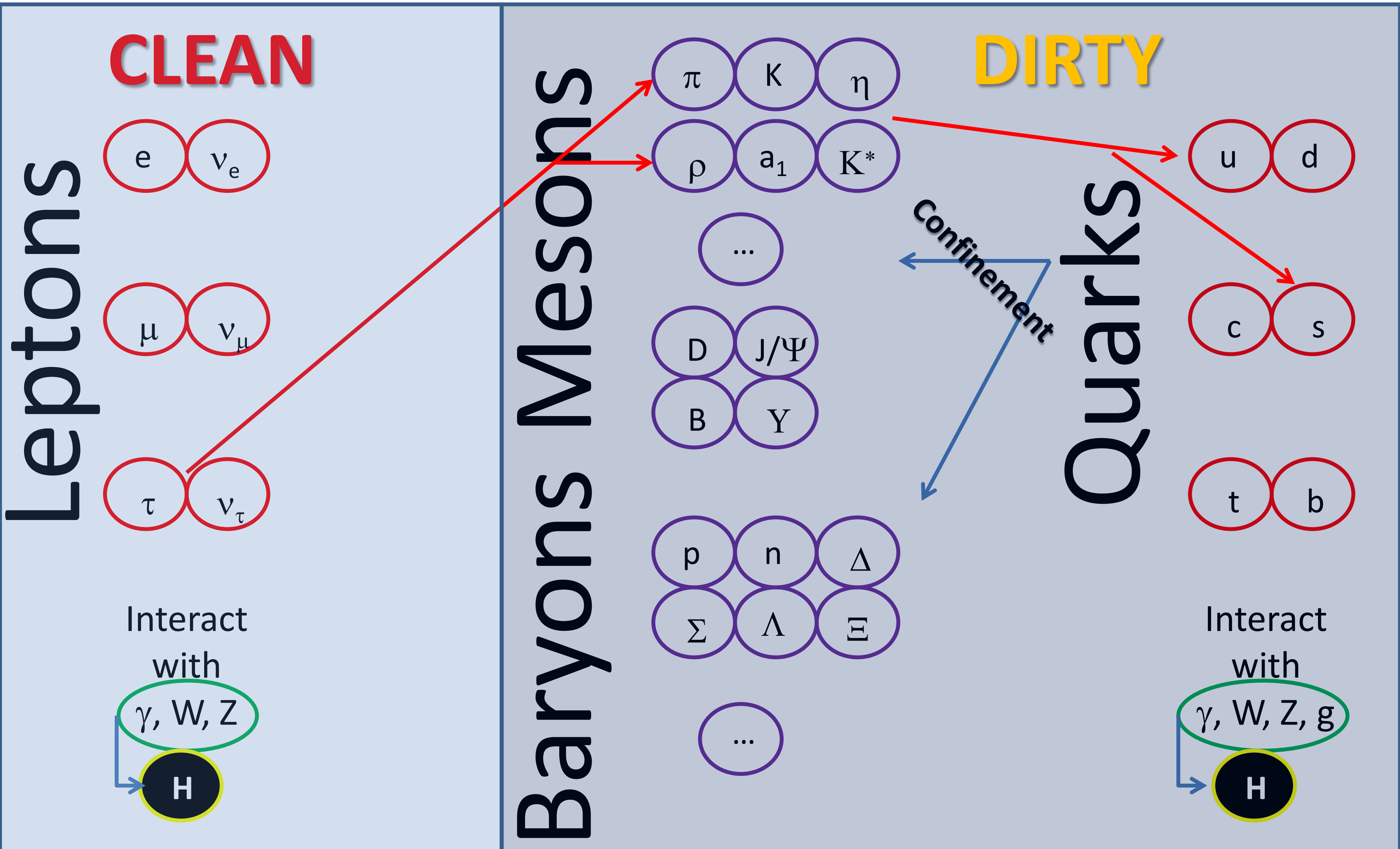

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Davier, Hocker, Zhang ‘05; Pich, ‘13

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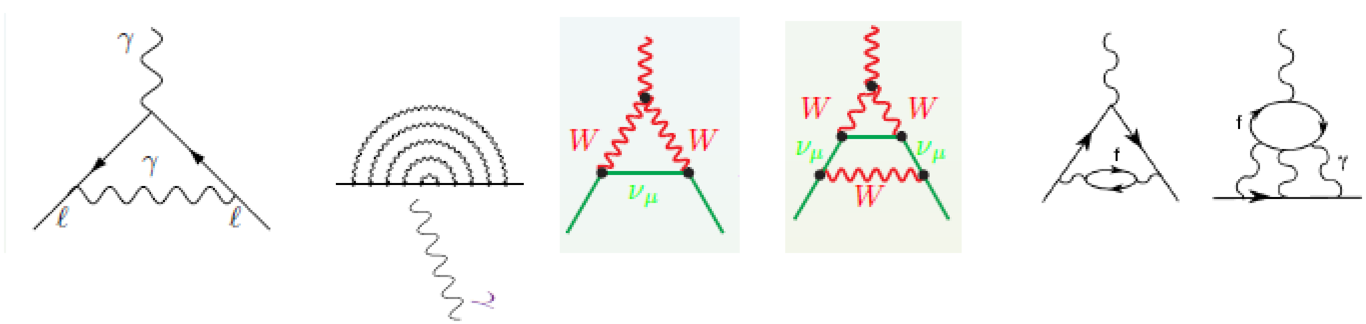

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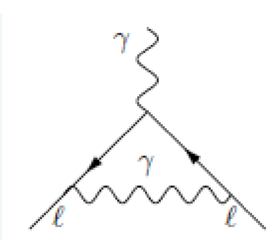

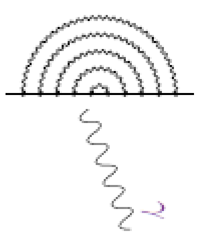

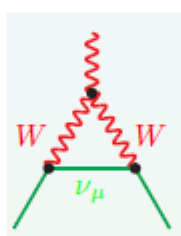

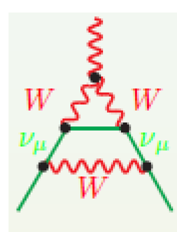

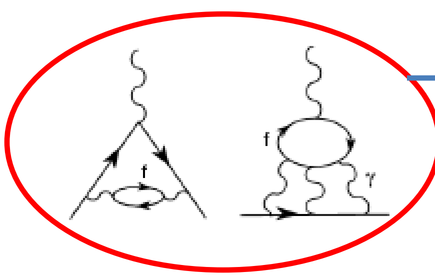

QCD not applicable for quark loops at **low E**!!

Dispersive representation of $\pi^-\pi^0$ VFF

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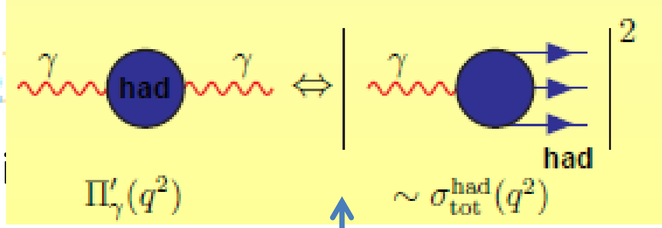

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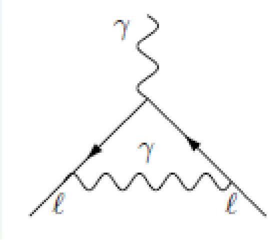

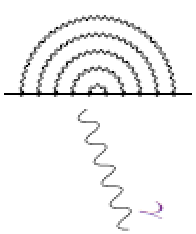

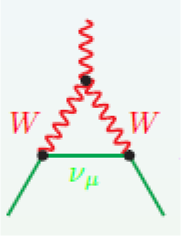

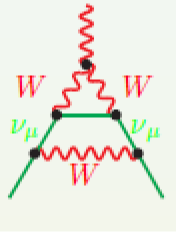

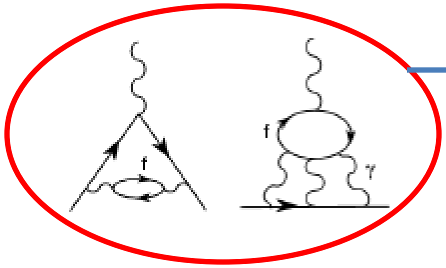

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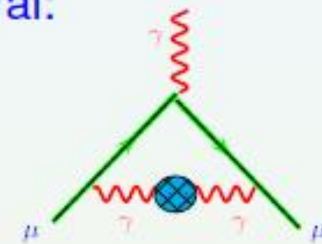
The Hadronic Vacuum Polarization Contribution

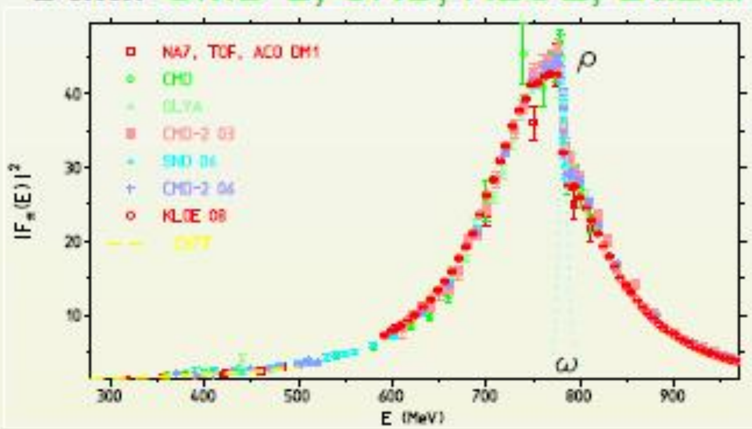
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \mathrm{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:



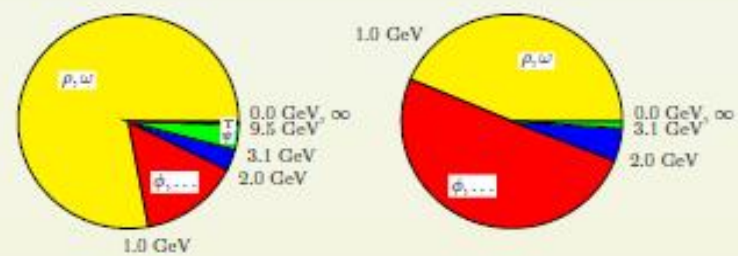
$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} ds \frac{R_\gamma^{\rm data}(s)\,\hat{K}(s)}{s^2} + \int_{E^2_{\rm cut}}^{\infty} ds \frac{R_\gamma^{\rm pQCD}(s)\,\hat{K}(s)}{s^2} \right)$$

- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

Data: CMD-2, SND, KLOE, BaBar

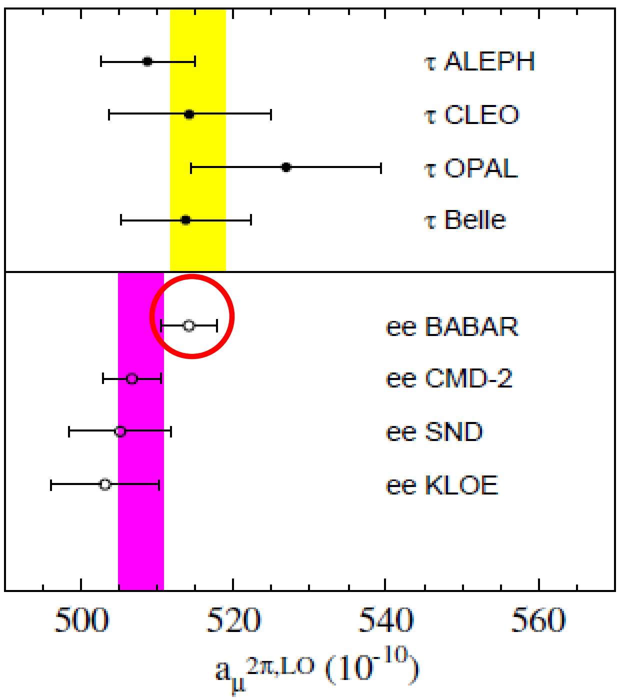


$$a_\mu^{\rm had(1)} = (690.7 \pm 4.7)[695.5 \pm 4.1]\ 10^{-10}$$
e^+e^-–data based [incl. BaBar MD09]



INTRODUCTION

BaBar 1205.2228 hep-ex



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en including LO isospin corrections Cirigliano, Ecker, Neufeld ‘01

stand the hadronization of QCD currents at low energies (**chiral**

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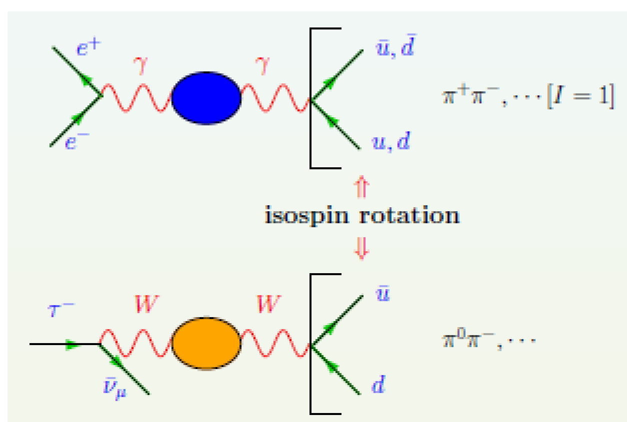
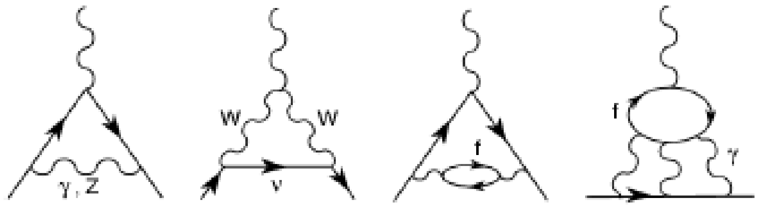
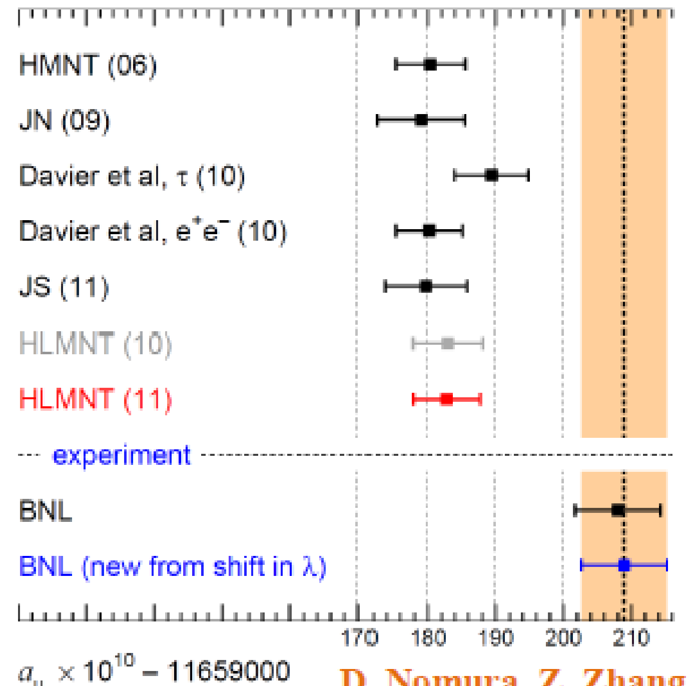


μ Anomalous Magnetic Moment



$$a_\mu^{\exp} = (11\,659\,208.9 \pm 6.3)\cdot 10^{-10}$$

BNL-E821



D. Nomura, Z. Zhang

$10^{10}\cdot a_\mu^{th} = 11\,658\,471.895 \pm 0.008$	QED		**Aoyama-Hayakawa-Kinoshita-Nio**
$+\ 15.4 \pm 0.2$	EW		**Czarnecki et al**, Knecht et al
$+\ 696.4 \pm 4.6$	hvp	$(701.5 \pm 4.7)_\tau$, $(692.4 \pm 4.1)_{e^+e^-}$	**Davier et al**, Hagiwara et al, Jegerlehner-Nyffeler
$-\ 9.8 \pm 0.1$	hvp NLO		**Krause**, Hagiwara et al
$+\ 10.5 \pm 2.6$	light-by-light		**de Rafael-Prades-Vainshtein**, Melnikov-Vainshtein, Knecht et al, Bijnens et al, Hayakawa et al, Nyffeler

$= 11\,659\,184.4 \pm 5.3$ **$(11\,659\,189.5 \pm 5.4)_\tau$, $(11\,659\,180.4 \pm 4.9)_{e^+e^-}$**

$a_\mu^{\exp} - a_\mu^{th} = 3.0\,\sigma$ $2.3\,\sigma$ $3.6\,\sigma$

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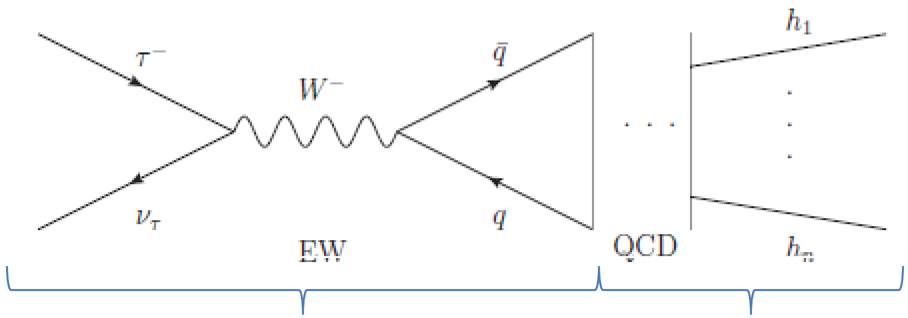
From the high-E perspective the $\pi^-\pi^0$ **and** $\pi^-\pi^-\pi^+$ channels are **essential** to follow the **spin** in the **Higgs**(-like) **di-tau channels** at LHC (New hadronic currents in TAUOLA Shekhovtsova, PR *et. al.* '12, '13)

THEORETICAL SETTING



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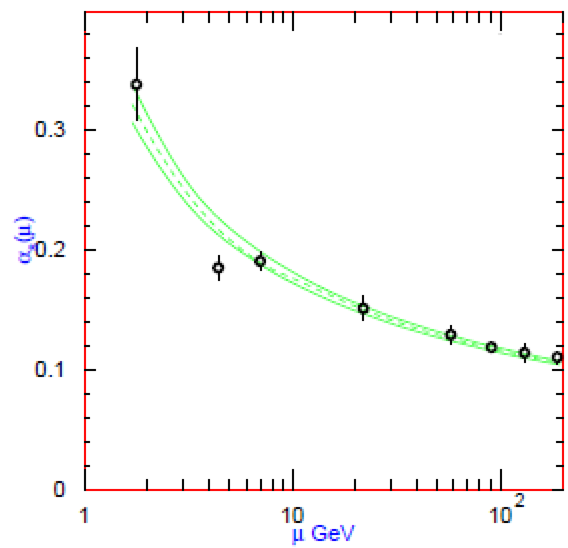
Problem

Hadronization in $2m_\pi \leq E \leq M_\tau$



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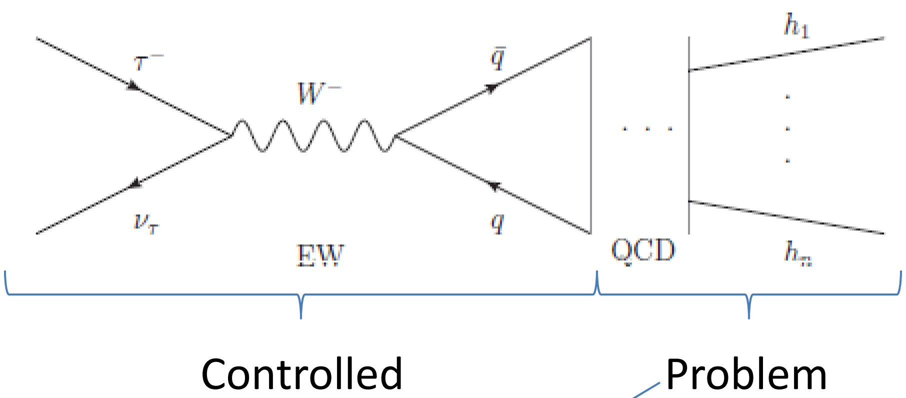

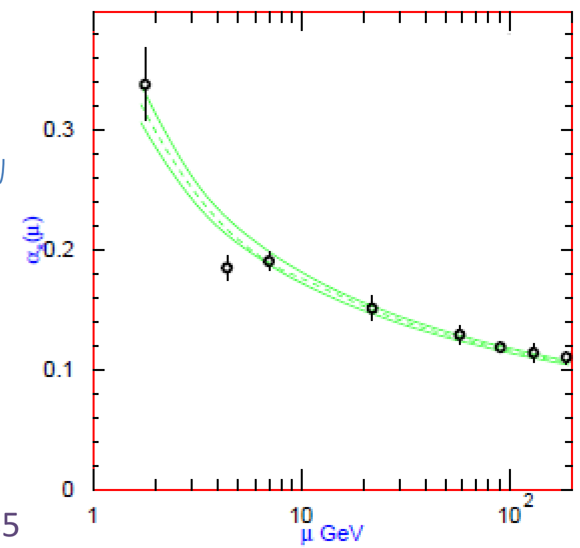

Hadronization in $2m_\pi \leq E \leq M_\tau$

→ At very low energies, **χPT** is the **EFT** of **QCD** Gasser, Leutwyler '85

Only light quarks (u, d, s). In the massless limit $G \equiv SU(n_f)_L \otimes SU(n_f)_R$

Spontaneous Symmetry Breaking $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$

Approximately massless pGbs $\pi^\pm, \pi^0, \eta, K^\pm, K^0, \bar{K}^0$

Small masses ($m<M_\rho,M_{a_1}$) by explicit Symmetry Breaking

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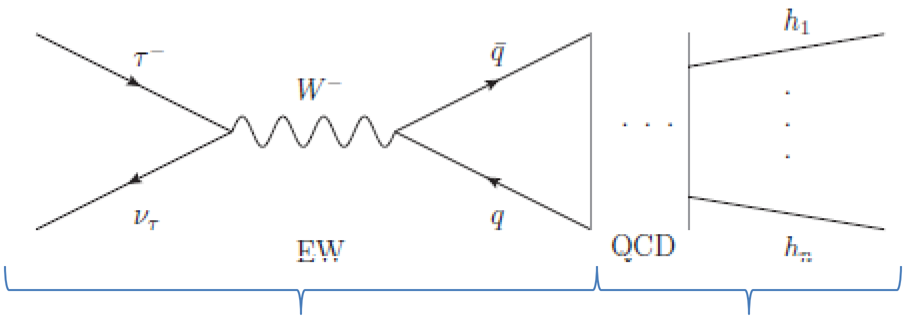

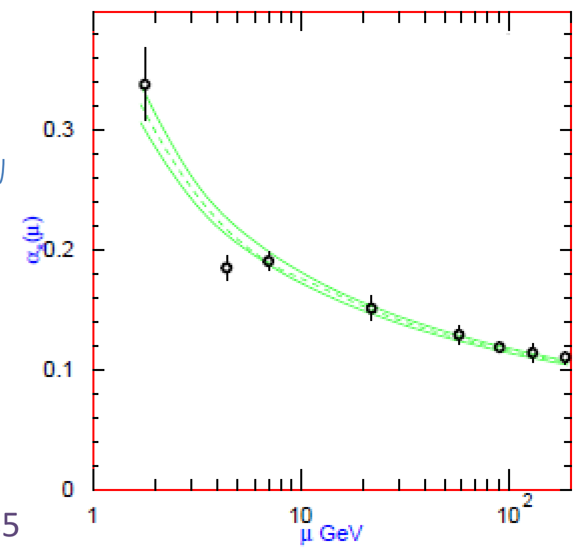

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→ At very low energies, **χPT** is the **EFT** of **QCD** Gasser, Leutwyler '85

Only light quarks (u, d, s). In the massless limit $G \equiv SU(n_f)_L \otimes SU(n_f)_R$

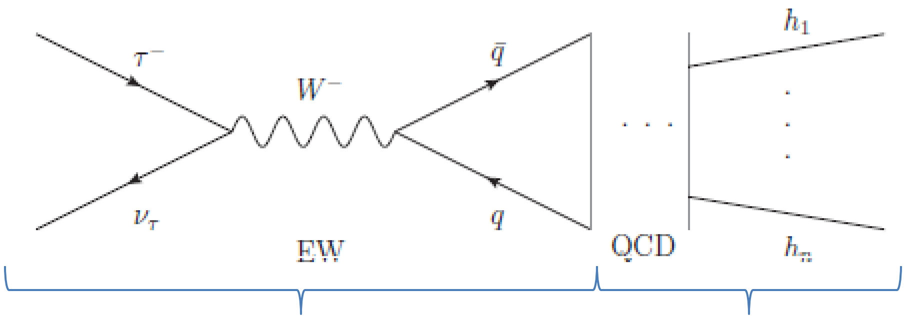
Spontaneous Symmetry Breaking $SU(3)_L \otimes SU(3)_R \to SU(3)_V$

Approximately massless pGbs $\pi^\pm, \pi^0, \eta, K^\pm, K^0, \bar{K}^0$

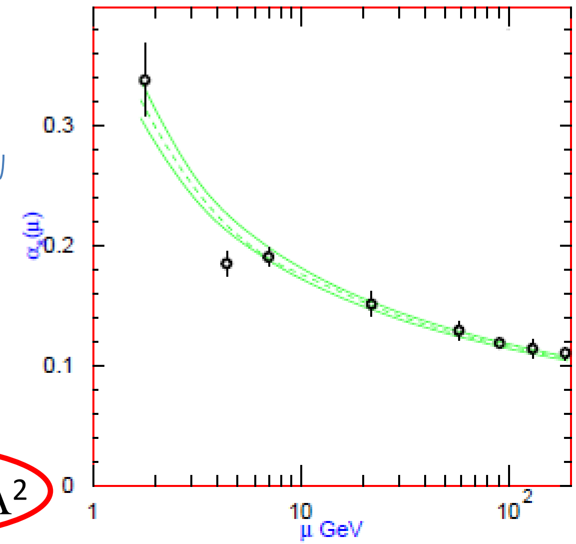
EFT: $(p^2,m^2)/\Lambda^2$

Small masses ($m<M_\rho,M_{a_1}$) by explicit Symmetry Breaking

THEORETICAL SETTING



Controlled Problem



Hadronization in $2m_\pi \leq E \leq M_\tau$

→ At very low energies, **χPT** is the **EFT** of **QCD** EFT: $(p^2,m^2)/\Lambda^2$

→ At larger energies the expansion breaks down and new dofs are needed

1/Nc 't Hooft '74 ↔ ρ, K^*, a_1, a_0,...

RχT Ecker et. al. '89

Simple description at LO

Preserves χPT results at low energies

THEORETICAL SETTING

Portolés ’10

Gasser, Leutwyler, ‘84, ‘85

- **QCD** has a well-defined expansion at low-energies that allows to build an **EFT**: χ**PT**.

Bijnens, Colangelo, Ecker ‘99

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• The energy of the hadronic system in semileptonic tau decays is not small enough through all phase space to allow for the low-energy expansion done by χ**PT**. Colangelo, Finkemeier, Urech '96

• Alternative expansion parameter to extend χ**PT** to higher energies: $1/N_c$. 't Hooft '74, Witten '79

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• LO in $1/N_c$-expansion → ∞ **number** of **stable** resonances. At NLO **tree level** exchanges.

• However: - We **cut** the ∞ **spectrum** of states (Nature).

Gómez-Dumm, Pich, Portolés '00

- QFT-based off-shell **width** for resonances within **RχT** (NLO).

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Ecker, Gasser, Pich, De Rafael '89 Ecker, Gasser, Leutwyler, Pich, De Rafael '89

• Finally, QCD **high-energy** behaviour imposed to the **Green functions** or **form factors**.

Ruiz-Femenía, Pich, Portolés '03 Cirigliano, Ecker,Eidemüller, Pich, Portolés '04

Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés '05, '06 Kampf, Novotny '11

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

Different approaches to deal with the diverse energy regimes

- For $E<M_\rho \to \chi$PT up to $O(p^6)$ Gasser, Leutwyler'85, Bijnens, Colangelo, Talavera '98, Bijnens, Talavera'02

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Guerrero, Pich '97

- For $M_\rho \leq E \leq 1$ GeV $\to$ Match χPT results to VMD using an Omnés solution for dispersion relation.

Omnés solution for dispersion relation Pich, Portolés '01

Unitarization approach Trocóniz, Yndúrain '01, Oller, Oser, Palomar '01

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- 1 GeV $\leq E \leq$ 2 GeV $\rightarrow$ Include ρ' through Schwinger-Dyson-like resummation.

Sanz-Cillero, Pich '03

Tower of resonances based on dual QCD

Domínguez '01, Bruch, Khodjamiriam, Kuhn '05

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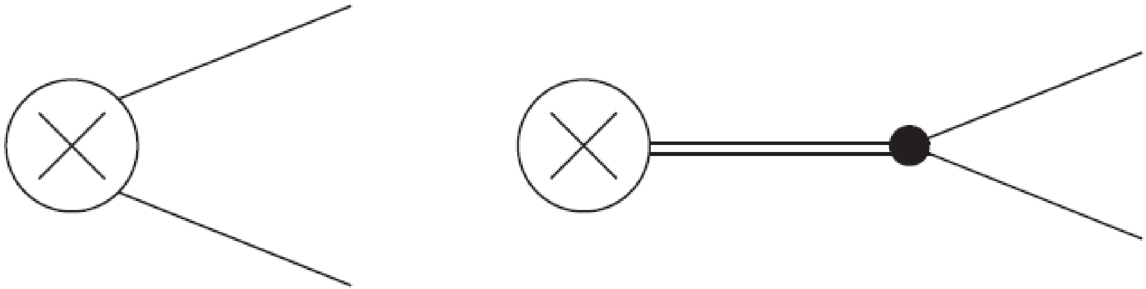

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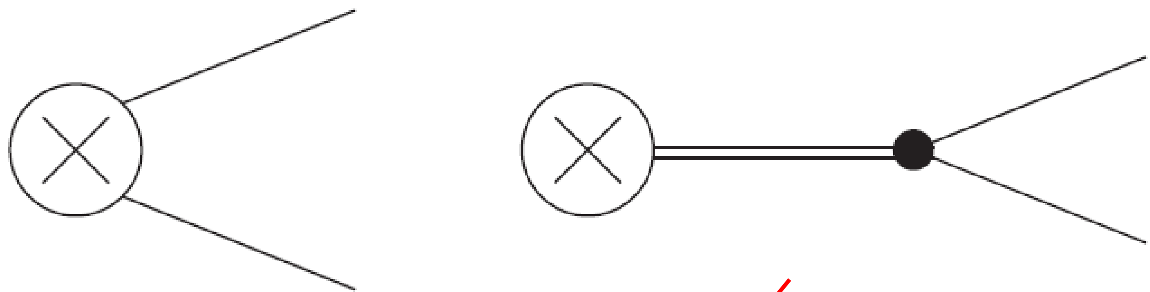
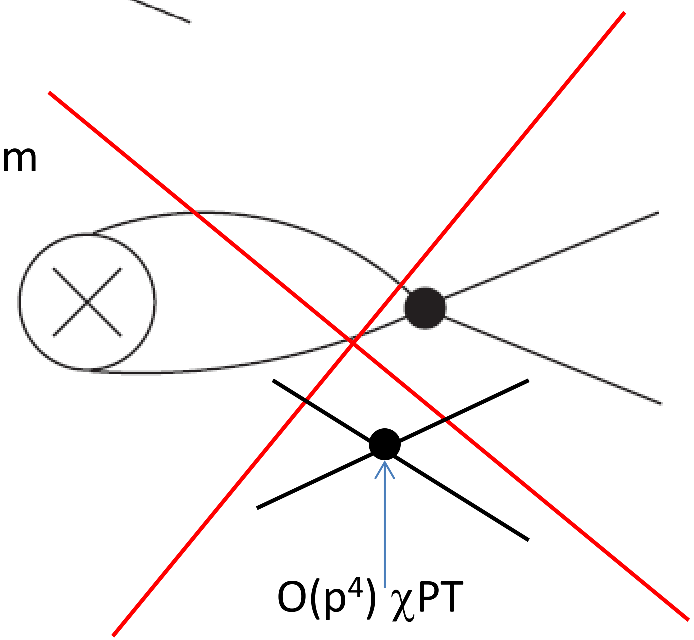


Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.* '89



To avoid double counting

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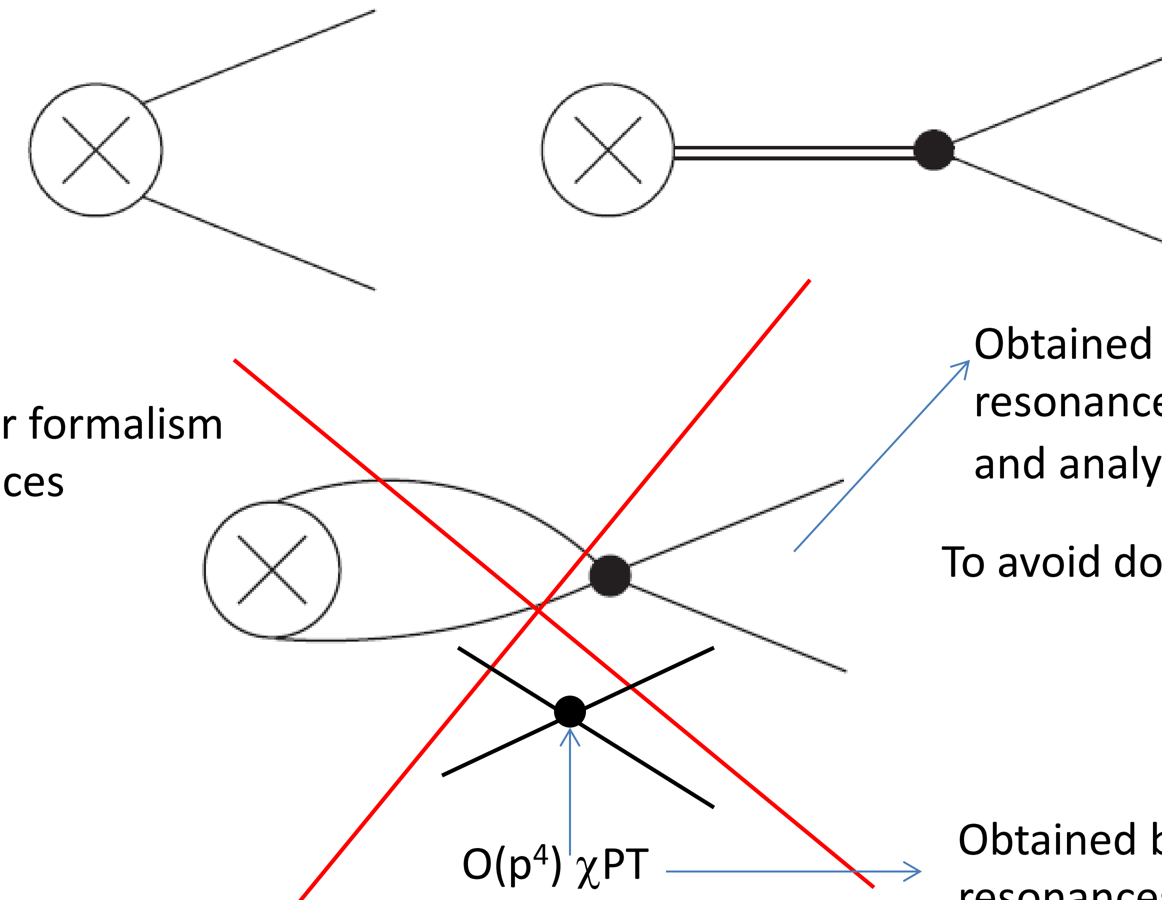

Obtained via the resonances width and analyticity

To avoid double counting

Obtained by integrating resonances out

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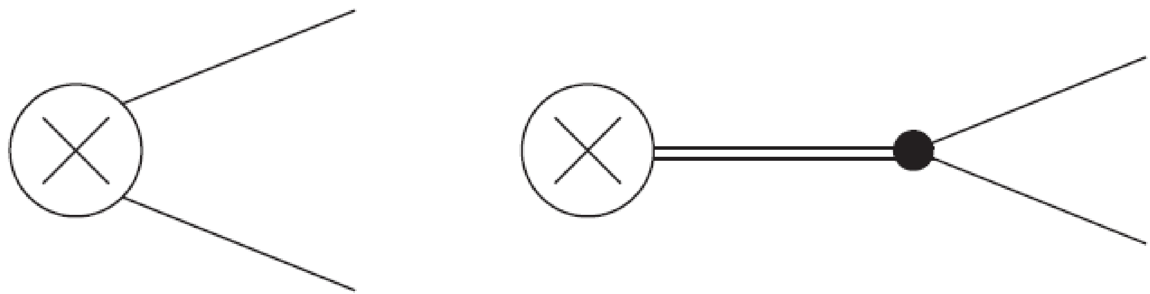

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$f_\pm^{\mu\nu} = u\,F_L^{\mu\nu}\,u^\dagger \pm u^\dagger\,F_R^{\mu\nu}\,u$$

$$u_\mu = i\,\{u^\dagger(\partial_\mu - ir_\mu)\,u - u\,(\partial_\mu - il_\mu)\,u^\dagger\}$$

$$u(\varphi) = \exp\left\{i\frac{\Phi}{\sqrt{2}\,F}\right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}}\sum_{a=1}^{8}\lambda_a\,\varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

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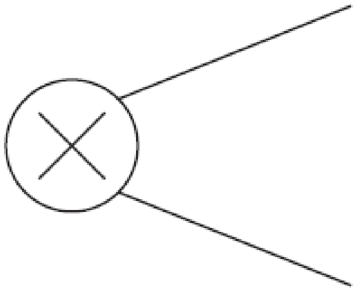

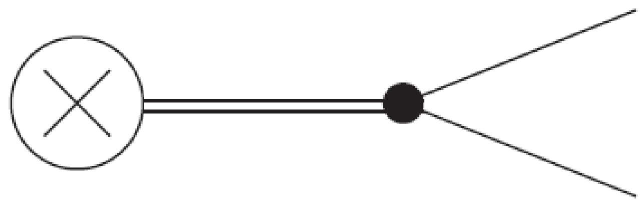

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Short-distance constraints

$$F(s)^V = 1 + \frac{F_V G_V}{f_\pi^2}\frac{s}{M_\rho^2 - s}$$

F(s) $\rightarrow$0, for s$\rightarrow\infty$ $\Rightarrow F_V\, G_V = F^2$

$$\Downarrow$$

$$F(s)^{\rm VMD} = \frac{M_\rho^2}{M_\rho^2 - s}$$

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Guerrero, Pich '97

Gasser, Leutwyler '85

$$F(s)^{\mathrm{ChPT}}_{O(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2}s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

$O(p^6)$ Gasser, Meissner '91; Bijnens, Colangelo, Talavera '98,; Bijnens, Talavera '02

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

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ChPT+VMD

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ChPT+VMD Guerrero, Pich '97

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Unitarity+Analiticity Omnés, '58

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$O(p^2)$ result for $\delta^1{}_1(s)$

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Guerrero, Pich '97

$$\Gamma_\rho(s) = \frac{M_\rho\, s}{96\pi f_\pi^2}\left\{\theta(s-4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s-4m_K^2)\,\sigma_K^3\right\}$$

Gómez-Dumm, Pich, Portolés '00

$$= -\frac{M_\rho\, s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

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Match χPT results to VMD using an Omnés solution for dispersion relation

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- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)
- Analiticity and unitarity constraints (NNLO)

Idea: Follow the approach of Boito, Escribano, Jamin '08 preserving analiticity and unitarity exactly using a dispersive representation of the VFF while retaining (some of) these nice properties

Also using a dispersive representation: Pich, Portolés '02

Hanhart '12

Celis, Cirigliano and Passemar '13

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$$F_V(s) = \exp\left\{\alpha_1 s+\frac{\alpha_2}{2}s^2+\frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

The dispersive representation is matched to

$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2 \left[1 + \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s}$$
$$- \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\,[1 + s\, C_{\rho'} A_\pi(s)] - s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\,[1 + s\, C_{\rho''} A_\pi(s)] - s}$$

$$C_R = \frac{\Gamma_R}{\pi M_R^3 \sigma_\pi^3(M_R^2)} \qquad \Gamma_R(s) = \Gamma_R \frac{s}{M_R^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_R^2)}\, \theta(s - 4m_\pi^2)$$

at higher energies

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

The dispersive representation is matched to

$$F_V^{\pi}(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2 \left[1 + \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\,[1 + s\, C_{\rho'} A_\pi(s)] - s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\,[1 + s\, C_{\rho''} A_\pi(s)] - s}$$

$$C_R = \frac{\Gamma_R}{\pi M_R^3\, \sigma_\pi^3(M_R^2)} \qquad \Gamma_R(s) = \Gamma_R \frac{s}{M_R^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_R^2)}\, \theta(s - 4m_\pi^2)$$

at higher energies

Possible improvement: Above the onset of inelasticities ($s \geq 4m_K^2$) the elastic approximation shall be replaced by a coupled channel ($\pi^-\pi^0$, $\pi^-\eta$, $\pi^-\eta'$) formalism.

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

The dispersive representation is matched to

$$F_V^\pi(s) = \frac{M_\rho^2 + (\alpha' e^{i\phi'} + \alpha'' e^{i\phi''})\, s}{M_\rho^2 \left[1 + \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2} A_K(s)\right)\right] - s} - \frac{\alpha' e^{i\phi'}\, s}{M_{\rho'}^2\,[1 + s\, C_{\rho'} A_\pi(s)] - s} - \frac{\alpha'' e^{i\phi''}\, s}{M_{\rho''}^2\,[1 + s\, C_{\rho''} A_\pi(s)] - s}$$

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at higher energies

Possible improvement: Above the onset of inelasticities ($s \geq 4m_K^2$) the elastic approximation shall be replaced by a coupled channel ($\pi^-\pi^0$, $\pi^-\eta$, $\pi^-\eta'$) formalism.

→ See Hanhart '12; Celis, Cirigliano, Passemar '13 for other approaches

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$
\begin{aligned}
F_V(s) &= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s} \\
&= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}
\end{aligned}
$$

$\longleftrightarrow$?

$$
\begin{aligned}
F_V(s) &= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s} \\
&\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}
\end{aligned}
$$

$$
\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}
$$

$$
F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \, \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}
$$

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

?

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But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$\longleftrightarrow$ **?**

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
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$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

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Cirigliano, Ecker, Neufeld '01

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Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^{-}\pi^{0}$ AND FITS TO DATA

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s} = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$\longleftrightarrow$?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s} \sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to the AMMM, but not to fit the VFF

Belle '08 from Flores-Báez et. al. '08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

$$F_V(s) = \exp\left\{\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3(s'-s-i\epsilon)}\right\}$$

Dispersive representation of $\pi^{-}\pi^{0}$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

?

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Fit I SU(2)

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

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Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
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?

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$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit II ~~SU(2)~~ **SU(2) kin**

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to the AMMM, but not to fit the VFF

Belle '08 from Flores-Báez et. al. '08

$$\tan\delta_1^1(s) = \frac{\Im m F_V(s)}{\Re e F_V(s)}$$

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Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_\pi(s)+A_K(s)/2\right)\right]-s}$$
$$= \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s)+\Re e A_K(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s}$$
$$\sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit III

SU(2) LO

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld '01

Factor used by Belle to obtain the had LO contribution to the AMMM, but not to fit the VFF

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Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

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?

$$F_V(s) = \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(A_{\pi^-\pi^0}(s)+A_{K^-K^0}(s)/2\right)\right]-s} \sim \frac{M_\rho^2}{M_\rho^2\left[1+\frac{s}{96\pi^2F_\pi^2}\left(\Re e A_{\pi^-\pi^0}(s)+\Re e A_{K^-K^0}(s)/2\right)\right]-s-iM_\rho\Gamma_\rho(s)}$$

Fit I **SU(2)**

Fit II **~~SU(2) kin~~**

Fit III **~~SU(2) LO~~**

$$|F_V^-(s)|^2 \to |F_V^-(s)|^2 G_{\rm EM}(s)$$

But these are not the complete LO SU(2) corrections

Cirigliano, Ecker, Neufeld ‘01

Factor used by Belle to obtain the had LO contribution to the AMMM, but not to fit the VFF

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Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

Fit III ~~SU(2)~~ LO

$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{ds} = \frac{G_F^2\, m_\tau^3}{384\,\pi^3} S_{EW} |V_{ud}|^2 \left(1-\frac{s}{M_\tau^2}\right)^2 \left(1+\frac{2s}{M_\tau^2}\right)$$

$$\lambda^{3/2}\left(1, \frac{m_{\pi^0}^2}{s}, \frac{m_{\pi^+}^2}{s}\right) |f_+(s)|^2 G_{EM}(s)\,,$$

$S_{EW} \neq 1$, $m_{\pi^0}^2 \neq m_{\pi^+}^2$, $G_{EM} \neq 1$

$$f_+(s) = F_V^{\pi^+}(s) + f_{\rm local}^{\rm elm}$$

Chiral LECs

$$f_{\rm local}^{\rm elm} = \frac{\alpha}{4\pi}\left(-\frac{3}{2} - \frac{1}{2}\log\frac{M_\tau^2}{\mu^2} - \log\frac{m_\pi^2}{\mu^2} + 2\log\frac{M_\tau^2}{M_\rho^2} - X(\mu)\right)$$

Cirigliano, Ecker, Neufeld '01

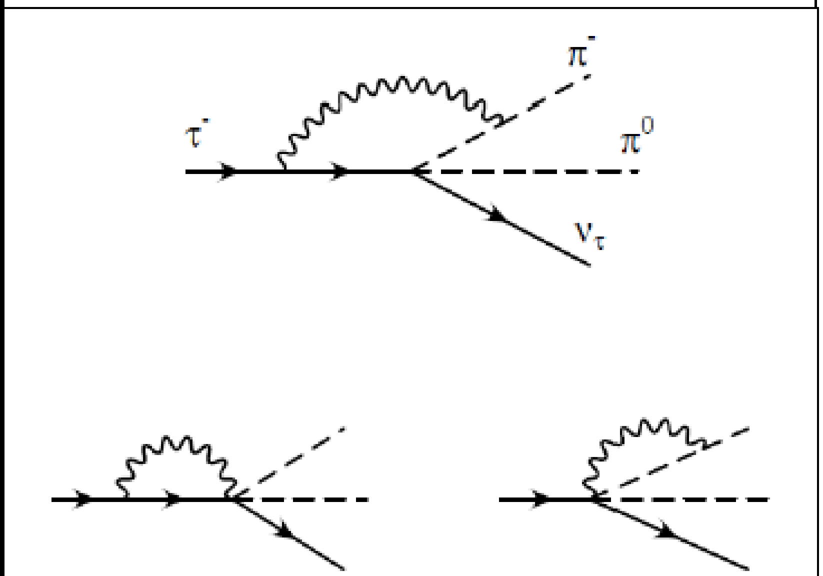
$A_\pi(s)$ and $A_K(s) \longrightarrow A_{\pi^-\pi^0}(s)$ and $A_{K^-K^0}(s)$

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

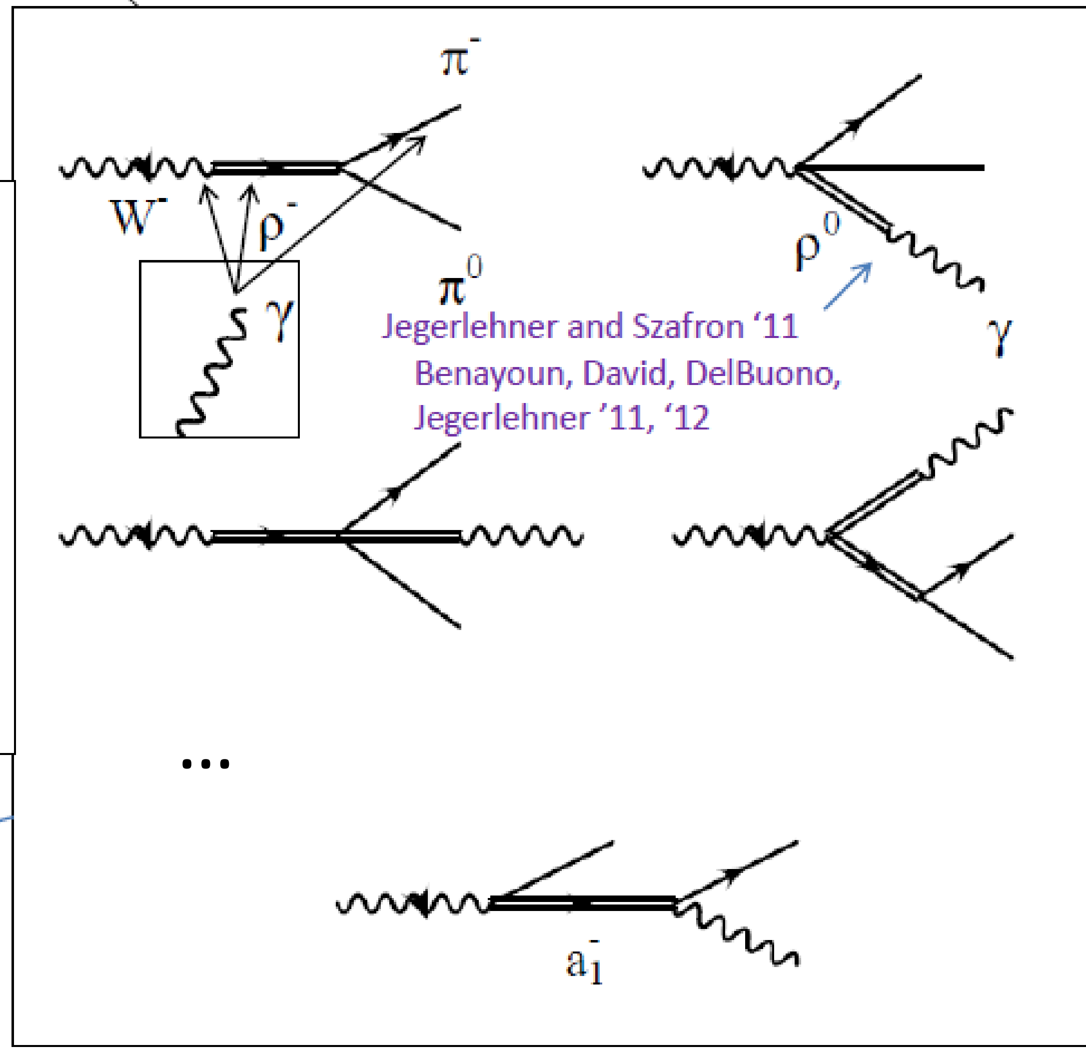
Fit III

SU(2) LO

Photon loop diagrams



Diagrams with external photon



Cirigliano, Ecker, Neufeld '01

Flores-Tlalpa et. al. '05

$G_{EM}(s)$

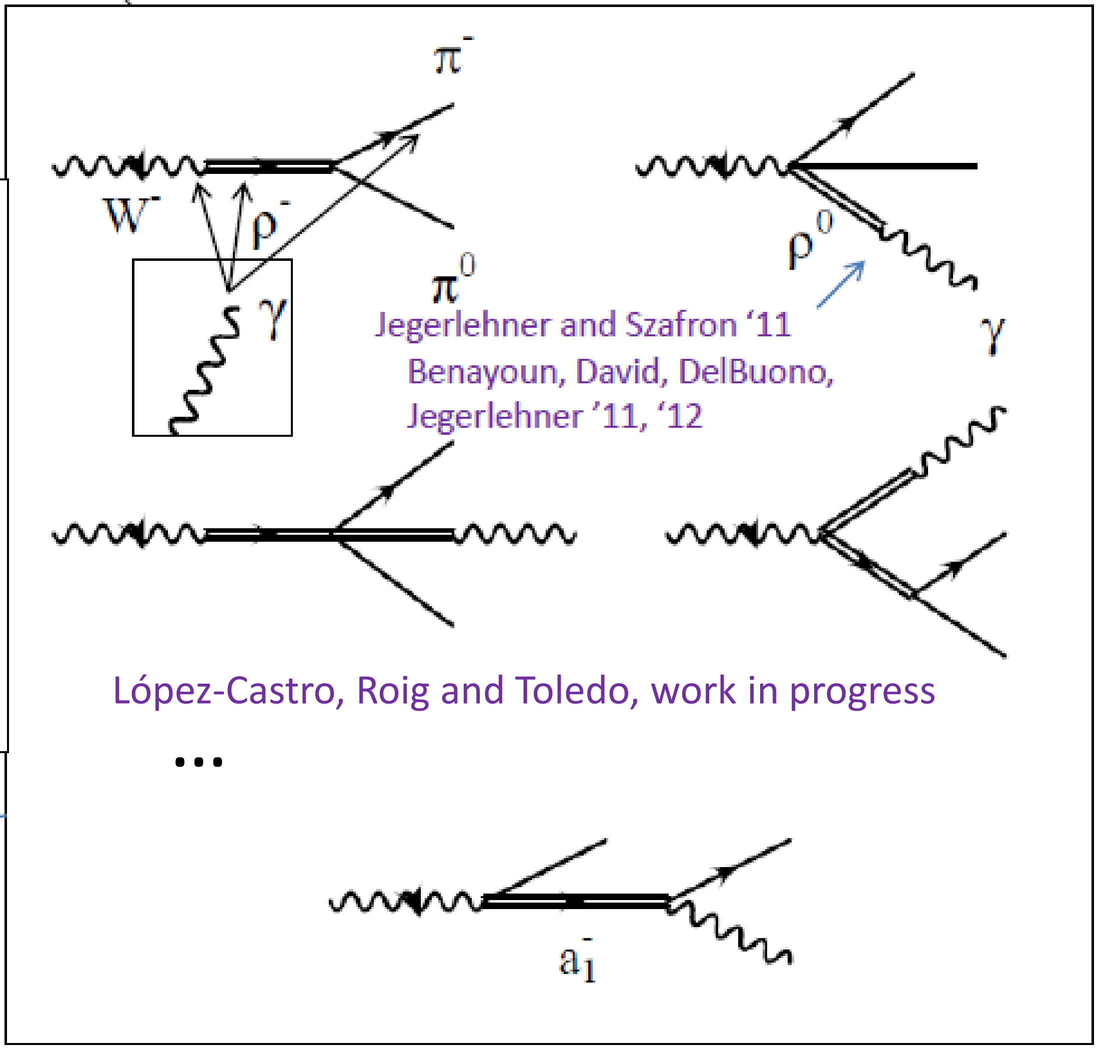
Dispersive representation of $\pi^-\pi^0$ VFF

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

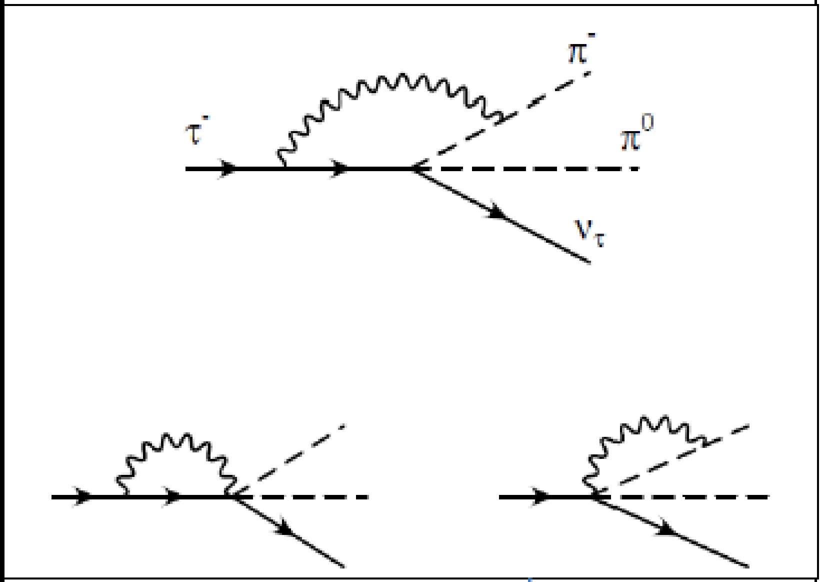
Fit III

~~SU(2) LO~~

Diagrams with external photon



Photon loop diagrams



Cirigliano, Ecker, Neufeld '01

Flores-Tlalpa et. al. '05

$G_{EM}(s)$

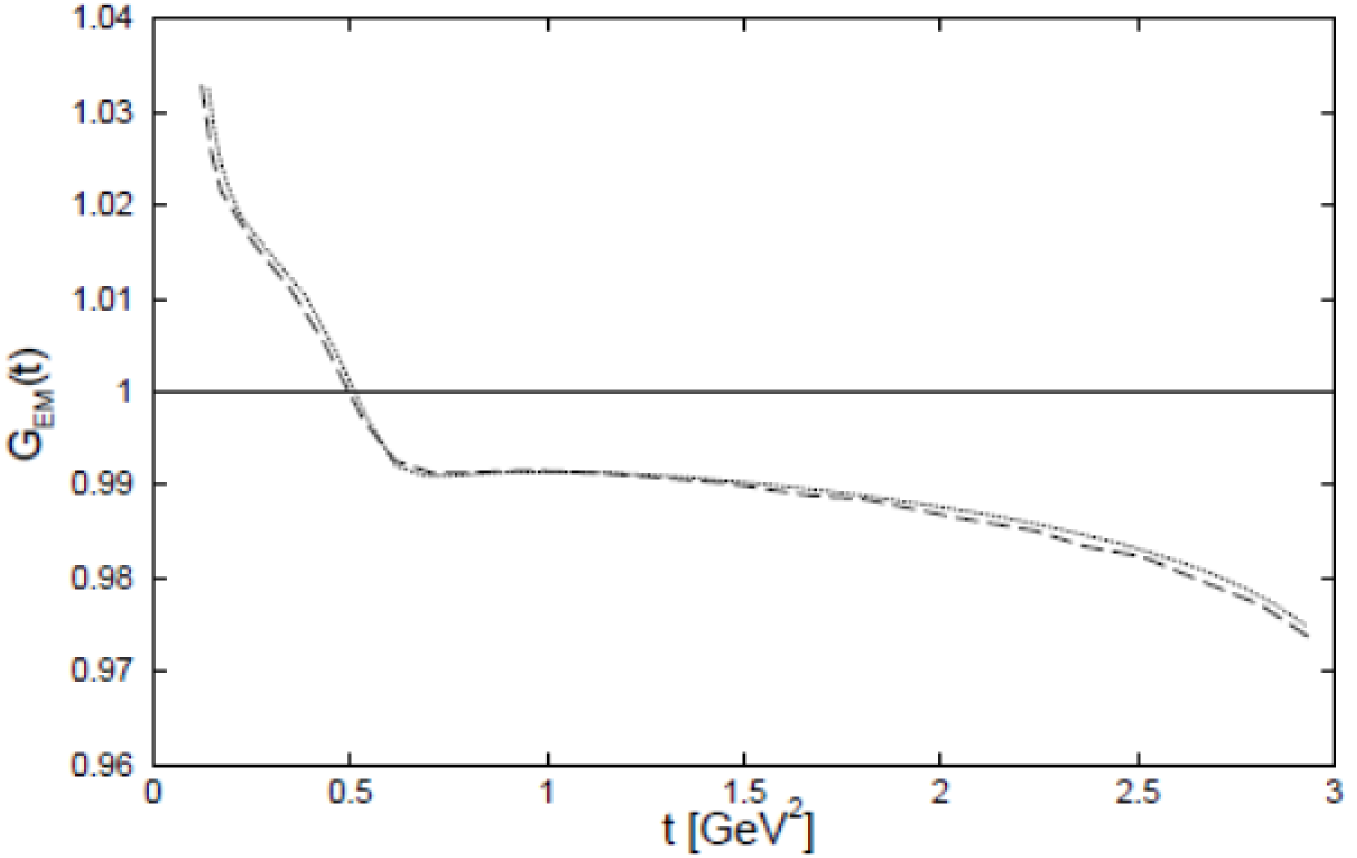
VFF OF $\pi^-\pi^0$ AND FITS TO DATA

Fit III ~~SU(2)~~ LO

$G_{EM}(s)$ has been calculated by Cirigliano, Ecker, Neufeld '01 (Chiral Perturbation Theory) '02 (including resonances) and Flores-Tlalpa et. al. '05.

Cirigliano, Ecker, Neufeld '01 → dashed-dotted

Flores-Tlalpa et. al. '05 → long-dashed



VFF OF $\pi^-\pi^0$ AND FITS TO DATA

Fit III SU(2) LO

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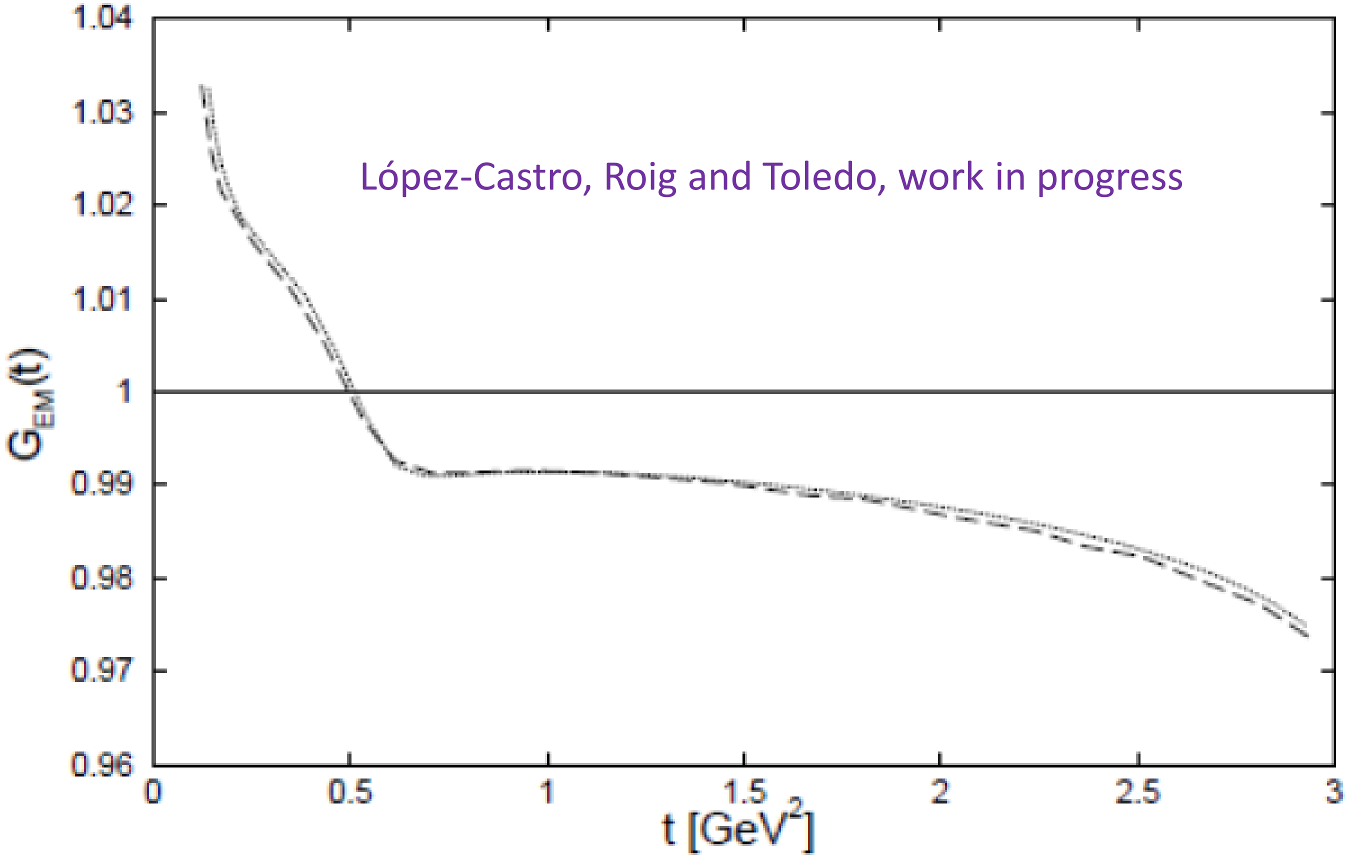
→ dashed-dotted

→ long-dashed



VFF OF $\pi^-\pi^0$ AND FITS TO DATA

	Fit value (I)	Fit value (II)	Fit value (III)
M_ρ [GeV]	0.8430(5)(17)	0.8427(5)(14)	0.8426(5)(20)
F_π [GeV]	0.0901(2)(5)	0.0902(2)(4)	0.0906(2)(4)
α_1 [GeV^{-2}]	1.87(1)(3)	1.87(1)(3)	1.81(1)(2)
α_2 [GeV^{-4}]	4.29(1)(7)	4.31(1)(7)	4.40(1)(6)
χ^2/dof	1.37	1.37	1.55
$\Gamma_\rho(M_\rho^2)$ [GeV]	0.206(1)(3)	0.206(1)(3)	0.204(1)(3)

Table 1: Results of our fits. The first and second numbers in brackets correspond to the statistic and theoretical systematic errors, respectively. $\Gamma_\rho(M_\rho^2)$ is obtained using the fitted values of M_ρ and F_π and is given only for reference.

$$\sqrt{s_{\text{pole}}} = M_\rho^{\text{pole}} - \frac{i}{2}\Gamma_\rho^{\text{pole}} \longrightarrow M_\rho^{\text{pole}} = (748.2 \pm 0.8)\,\text{MeV}\,, \quad \Gamma_\rho^{\text{pole}} = (153.0 \pm 0.7)\,\text{MeV} \quad \text{(Fit III)}$$

The χ^2 is similar in all cases and the inclusion of isospin breaking corrections does not improve the quality of the fits.

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

	Fit value (I)	Fit value (II)	Fit value (III)
M_ρ [GeV]	0.8430(5)(17)	0.8427(5)(14)	0.8426(5)(20)
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?

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VFF OF $\pi^-\pi^0$ AND FITS TO DATA

$$\sqrt{s_{\rm pole}} = M_\rho^{\rm pole} - \frac{i}{2}\Gamma_\rho^{\rm pole} \rightarrow M_\rho^{\rm pole} = (748.2 \pm 0.8)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (153.0 \pm 0.7)\,{\rm MeV} \quad ({\rm Fit\ III})$$

?

The complex variable s in the dispersive VFF is not in the same Riemann sheet where the pole is.

We use one-pole Padé approximants to expand the VFF around the pole at s_0.

(Sanz-Cillero, Masjuan '12)

pole $\rightarrow z_p = s_0 + a_N/a_{N+1}$

$$P_1^N(s;s_0) = \sum_{k=0}^{N-1} a_K (s-s_0)^k + \frac{a_N (s-s_0)^N}{1 - \frac{a_{N+1}}{a_N}(s-s_0)}$$

The de Montessus de Ballore Th. ensures that the Padé pole converges to the original pole.

$$M_\rho^{\rm pole} = (759 \pm 2)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (146 \pm 6)\,{\rm MeV} \quad ({\rm Fit\ I})\,;$$
$$M_\rho^{\rm pole} = (760 \pm 2)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (147 \pm 6)\,{\rm MeV} \quad ({\rm Fit\ III})\,.$$

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

	Fit value (I)	Fit value (II)	Fit value (III)
M_ρ [GeV]	0.8430(5)(17)	0.8427(5)(14)	0.8426(5)(20)
F_π [GeV]	0.0901(2)(5)	0.0902(2)(4)	0.0906(2)(4)
α_1 [GeV^{-2}]	1.87(1)(3)	1.87(1)(3)	1.81(1)(2)
α_2 [GeV^{-4}]	4.29(1)(7)	4.31(1)(7)	4.40(1)(6)
χ^2/dof	1.37	1.37	1.55
$\Gamma_\rho(M_\rho^2)$ [GeV]	0.206(1)(3)	0.206(1)(3)	0.204(1)(3)

Table 1: Results of our fits. The first and second numbers in brackets correspond to the statistic and theoretical systematic errors, respectively. $\Gamma_\rho(M_\rho^2)$ is obtained using the fitted values of M_ρ and F_π and is given only for reference.

$$\sqrt{s_{\rm pole}} = M_\rho^{\rm pole} - \frac{i}{2}\Gamma_\rho^{\rm pole} \xrightarrow{\text{Padés}} \begin{array}{ll} M_\rho^{\rm pole} = (759 \pm 2)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (146 \pm 6)\,{\rm MeV} & \text{(Fit I)}\,; \\ M_\rho^{\rm pole} = (760 \pm 2)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (147 \pm 6)\,{\rm MeV} & \text{(Fit III)}\,. \end{array}$$

Belle could also have determined the pole values of their GS (Gounaris-Sakurai '67) fit

We obtain $M_\rho^{\rm pole} = (760.9 \pm 0.6)\,{\rm MeV}\,, \quad \Gamma_\rho^{\rm pole} = (142.2 \pm 1.6)\,{\rm MeV}\,.$

VFF OF $\pi^- \pi^0$ AND FITS TO DATA

Reference	$M_\rho^{\rm pole}$	$\Gamma_\rho^{\rm pole}$	Data	Analysis
Sanz-Cillero *et al.* [35]	$764.1^{+4.8}_{-3.7}$	$148.2^{+2.5}_{-6.2}$	τ & e^+e^-	DSE
Ananthanarayan *et al.* [69]	762.5 ± 2	142 ± 7	$\pi\pi \to \pi\pi$	RE
Feuillat *et al.* [70]	758.3 ± 5.4	145.1 ± 6.3	τ & e^+e^-	*S*MA
Peláez [71]	754 ± 18	148 ± 20	$\pi\pi \to \pi\pi$	UχPT
Zhou *et al.* [72]	763.0 ± 0.2	139.0 ± 0.5	$\pi\pi \to \pi\pi$	χU
Masjuan *et al.* [64]	763.7 ± 1.2	144 ± 3	τ	RA
Results from our fit I	759 ± 2	146 ± 6	τ	DR
Results from our fit III	760 ± 2	147 ± 6	τ	DR
Results from GS model	760.9 ± 0.6	142.2 ± 1.6	τ	GS

Table 2: Comparison between different results for the pole mass and width of the $\rho(770)$ meson (values are in MeV). Abbreviations for the type of analysis carried out are DSE: Dyson-Schwinger equations; RE: Roy equations; *S*MA: *S* matrix approach; UχPT: Unitarized Chiral Perturbation Theory; χU: Chiral unitarization; RA: Rational approximants; DR: Dispersive representation; GS: Gounaris-Sakurai parametrization.

We also determined the peak or visible mass: $\sqrt{s_{\pi/2}} = (775.0 \pm 0.2)\,\mathrm{MeV}$.

in agreement with ACGL: $\sqrt{s_{\pi/2}} = (774 \pm 3)\ \mathrm{MeV}$

VFF OF $\pi^-\pi^0$ AND FITS TO DATA

	Fit value (I)	Fit value (II)	Fit value (III)
M_ρ [GeV]	0.8430(5)(17)	0.8427(5)(14)	0.8426(5)(20)
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Table 1: Results of our fits. The first and second numbers in brackets correspond to the statistic and theoretical systematic errors, respectively. $\Gamma_\rho(M_\rho^2)$ is obtained using the fitted values of M_ρ and F_π and is given only for reference.

The χ^2 is similar in all cases and the inclusion of isospin breaking corrections does not improve the quality of the fits.

Small contribution (EM)

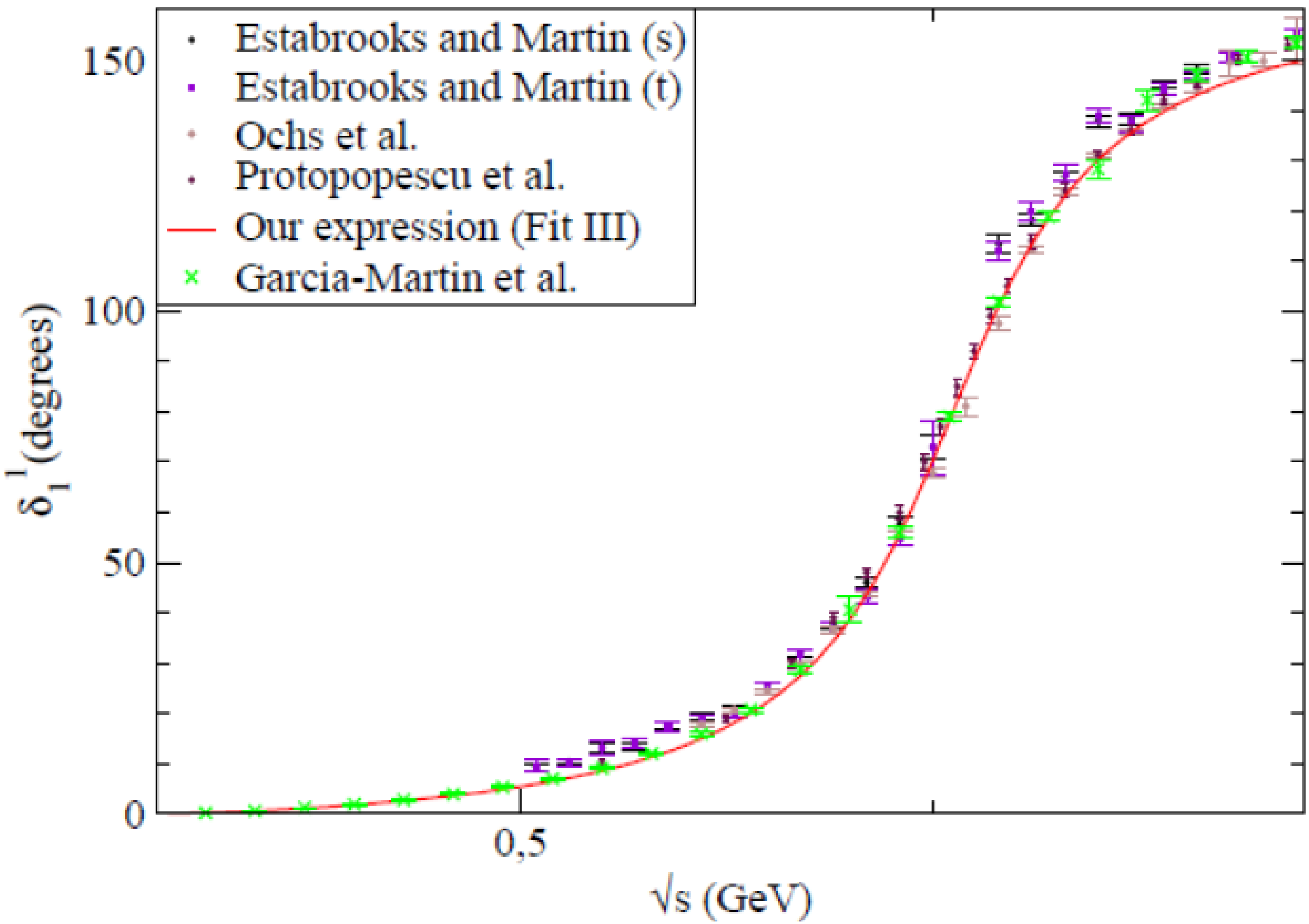
This is the key: However, the error is of the size of the discrepancy with the SM/10 (FF)

t>1 GeV^2 contributions are negligible

$t_{\rm max}$	$S_{\rm EW}$	KIN	EM	FF	$\Delta a_\mu^{\rm vacpol}$ (total)
1	- 95	- 75	- 11	$61 \pm 26 \pm 3$	- 119
2	- 97	- 75	- 10	$61 \pm 26 \pm 3$	- 120
3	- 97	- 75	- 10	$61 \pm 26 \pm 3$	- 120

In units of 10^{-11}

VFF OF $\pi^-\pi^0$ AND FITS TO DATA



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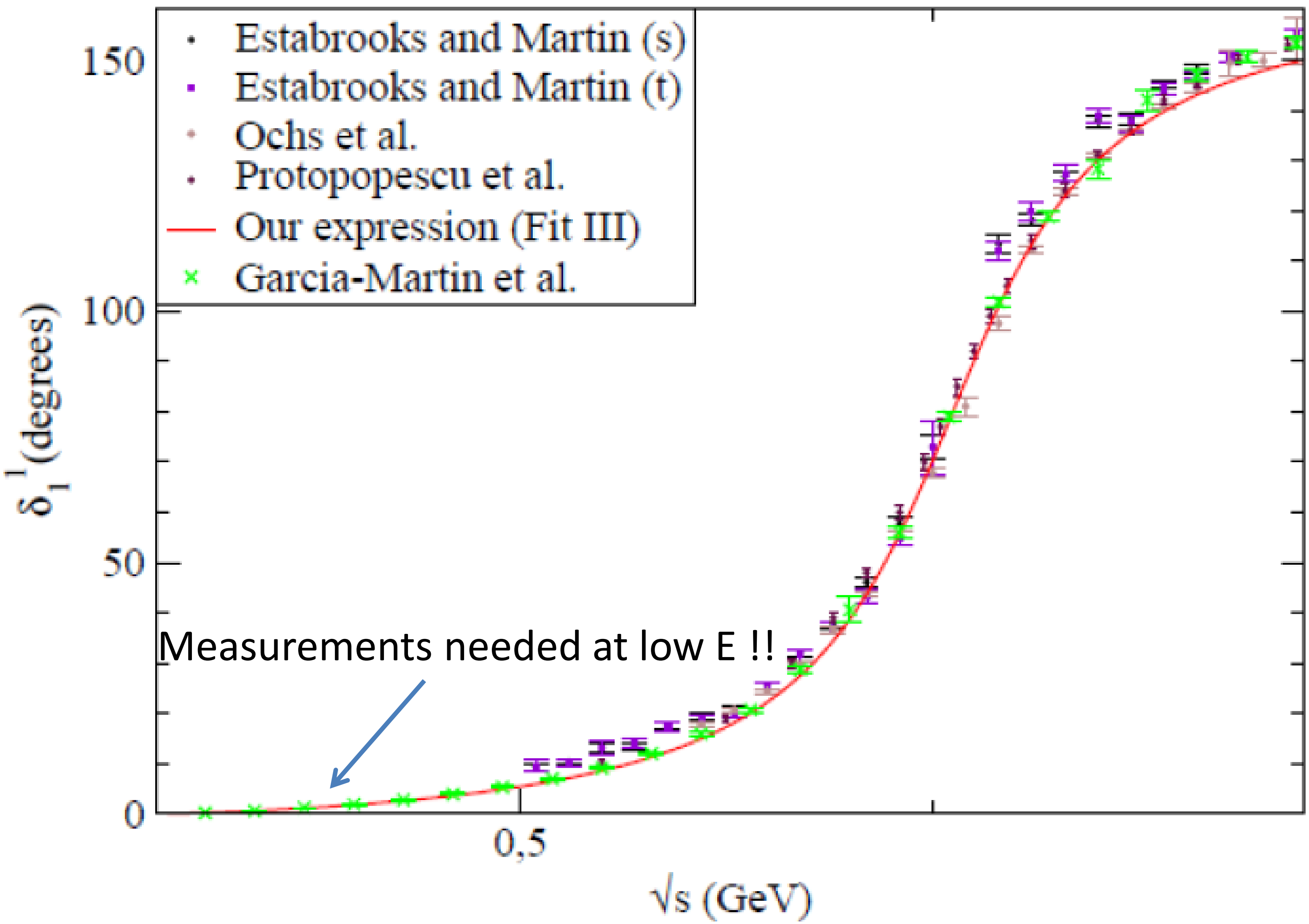

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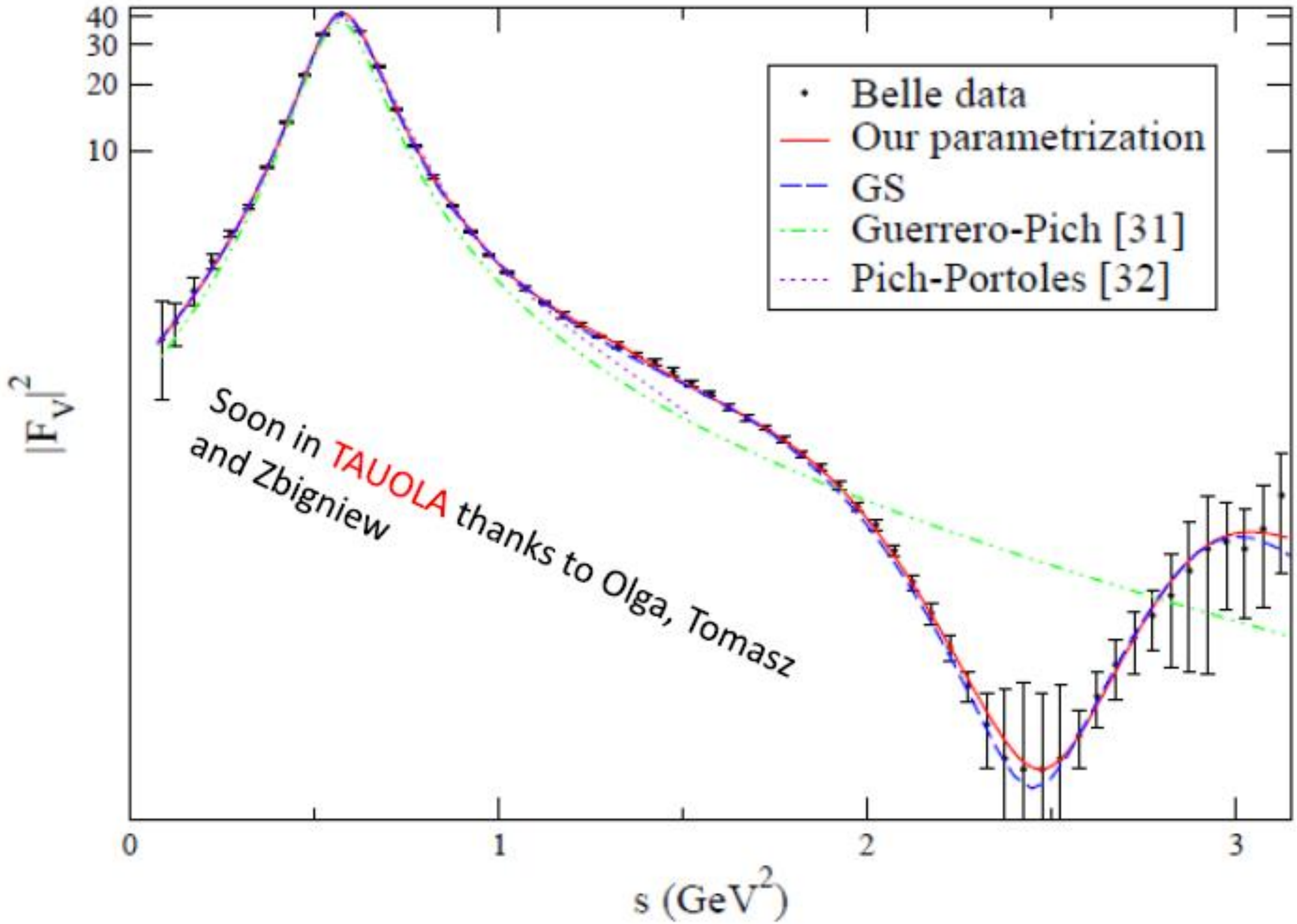

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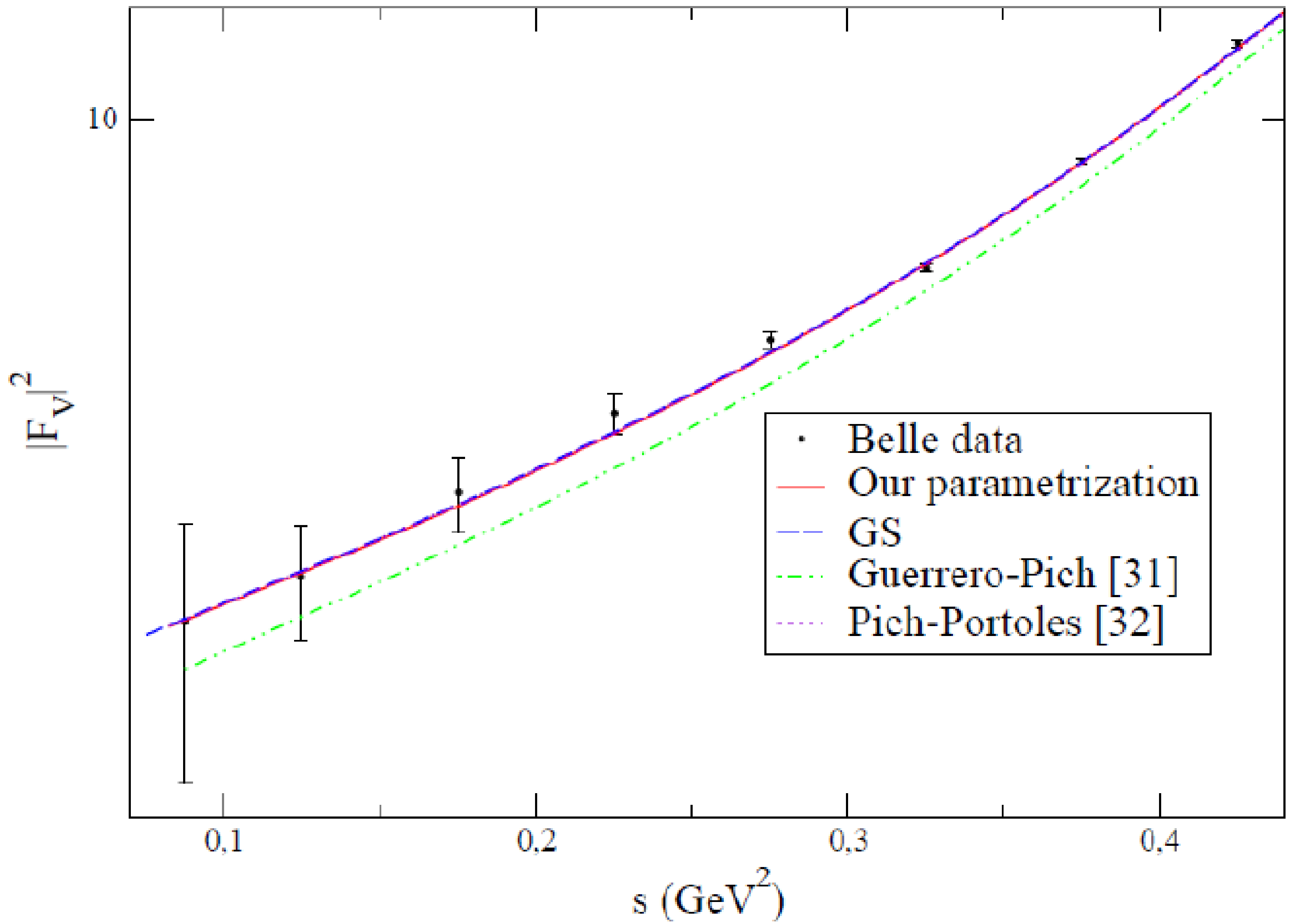

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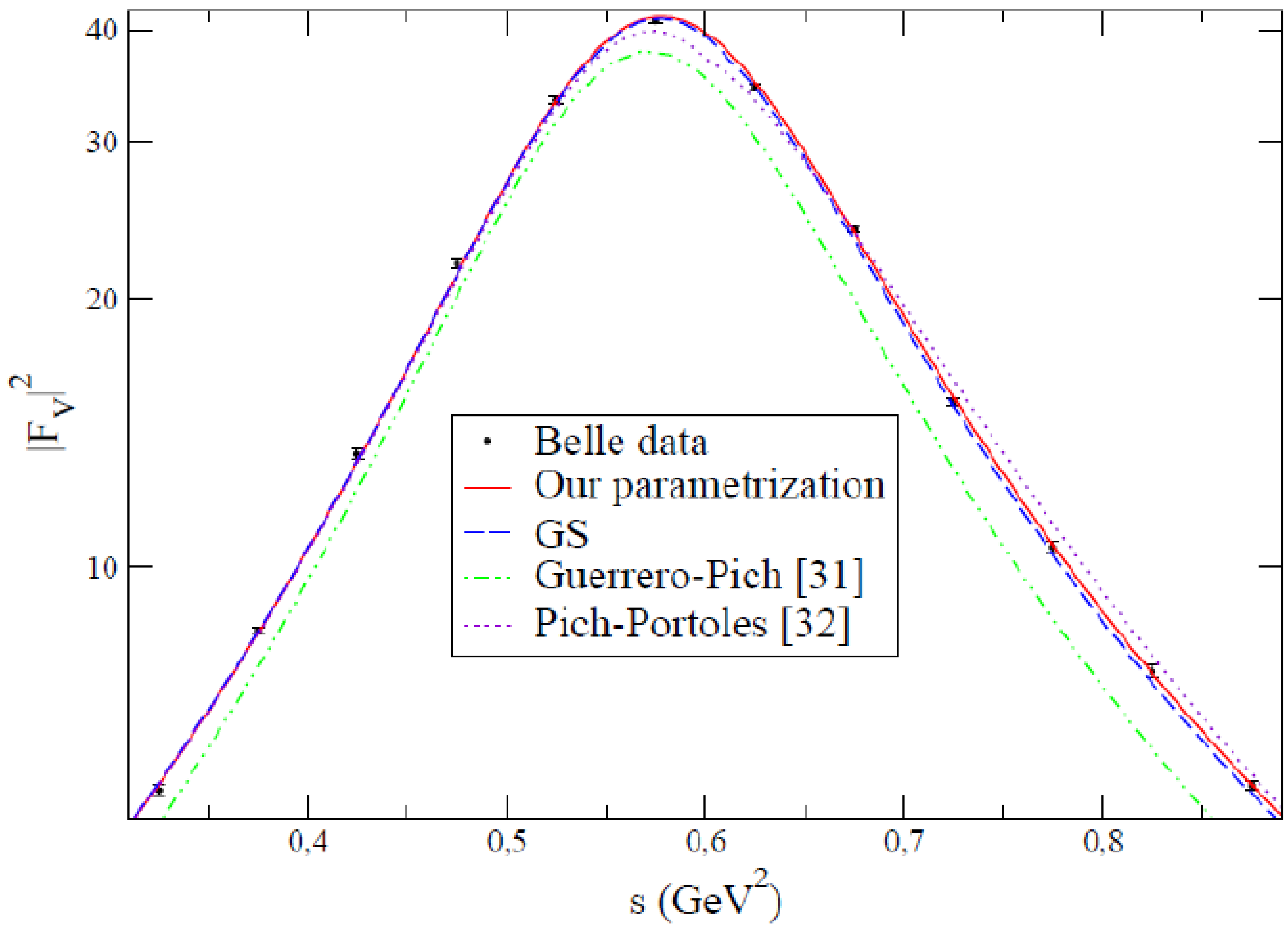

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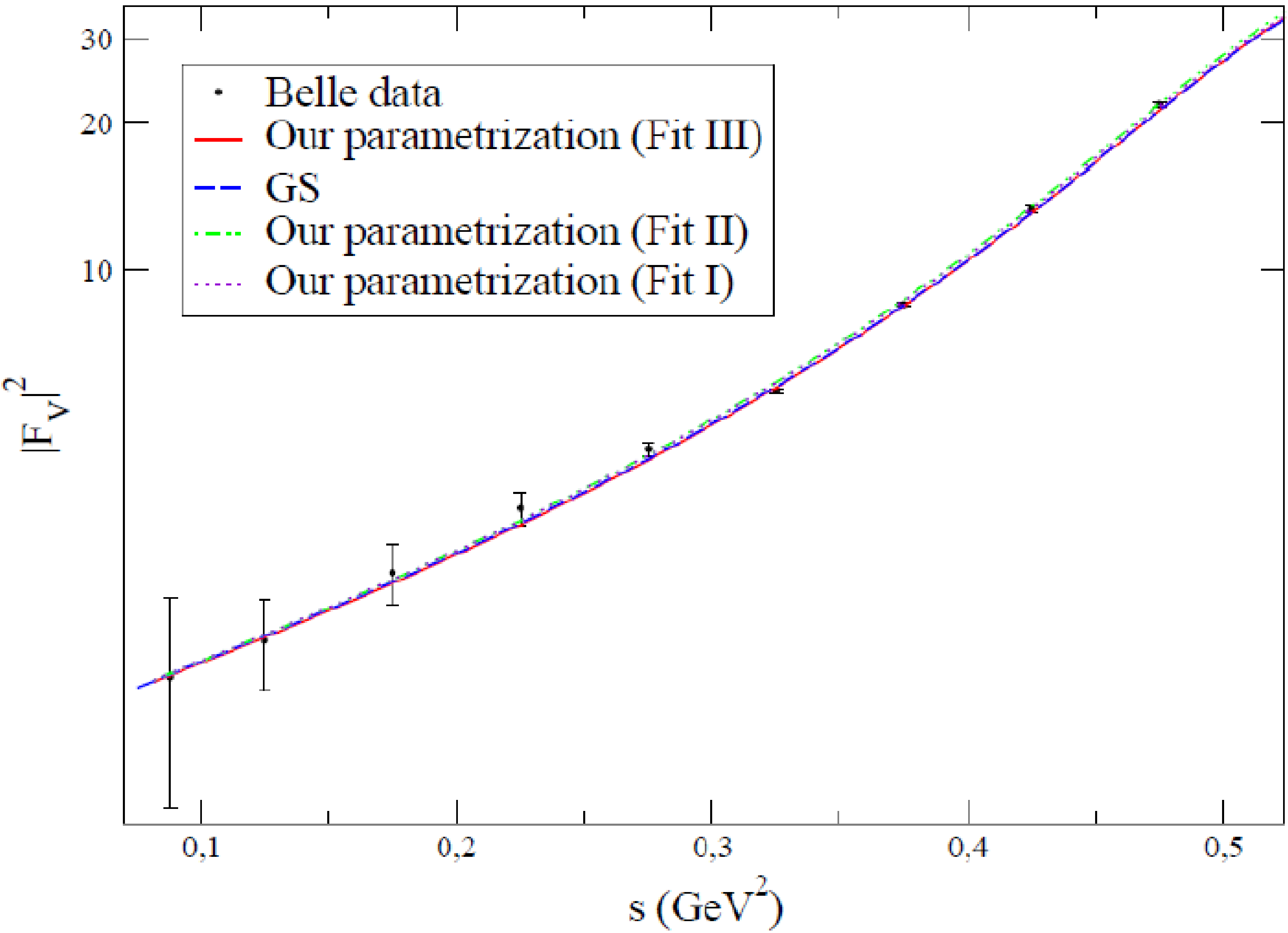


VFF OF $\pi^-\pi^0$ AND FITS TO DATA

Low-E expansion

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi\, s + c_V^\pi\, s^2 + d_V^\pi\, s^3 + \ldots$$

$$\langle r^2\rangle_V^\pi = 6\,\alpha_1\,, \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right)$$

$$\alpha_k = \frac{k!}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}s'\, \frac{\delta_1^1(s')}{s'^{k+1}}$$

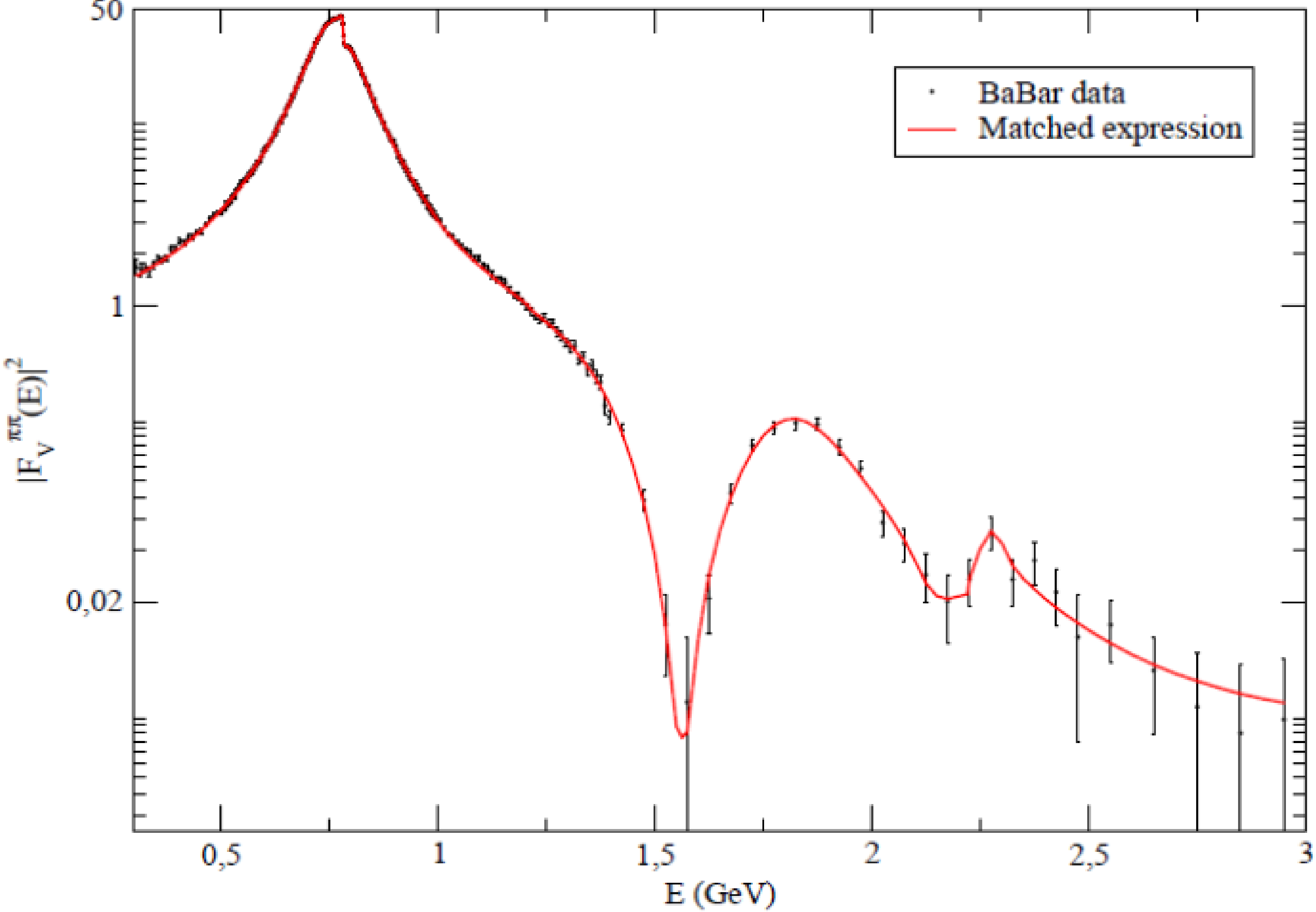
$$\langle r^2\rangle_V^\pi = 10.86 \pm 0.14\ \mathrm{GeV}^{-2}\,, \quad c_V^\pi = 3.84 \pm 0.03\ \mathrm{GeV}^{-4}$$

$$d_V^\pi = \frac{1}{6}\left(\alpha_3 + 3\alpha_1\alpha_2 + \alpha_1^3\right) = 9.84 \pm 0.05\ \mathrm{GeV}^{-6}$$

In good agreement with the literature with higher precision

$e^+e^- \to \pi^+\pi^-$ AND FITS TO DATA

Analogous method is applied (Gómez-Dumm, Jamin and Roig, work in progress)



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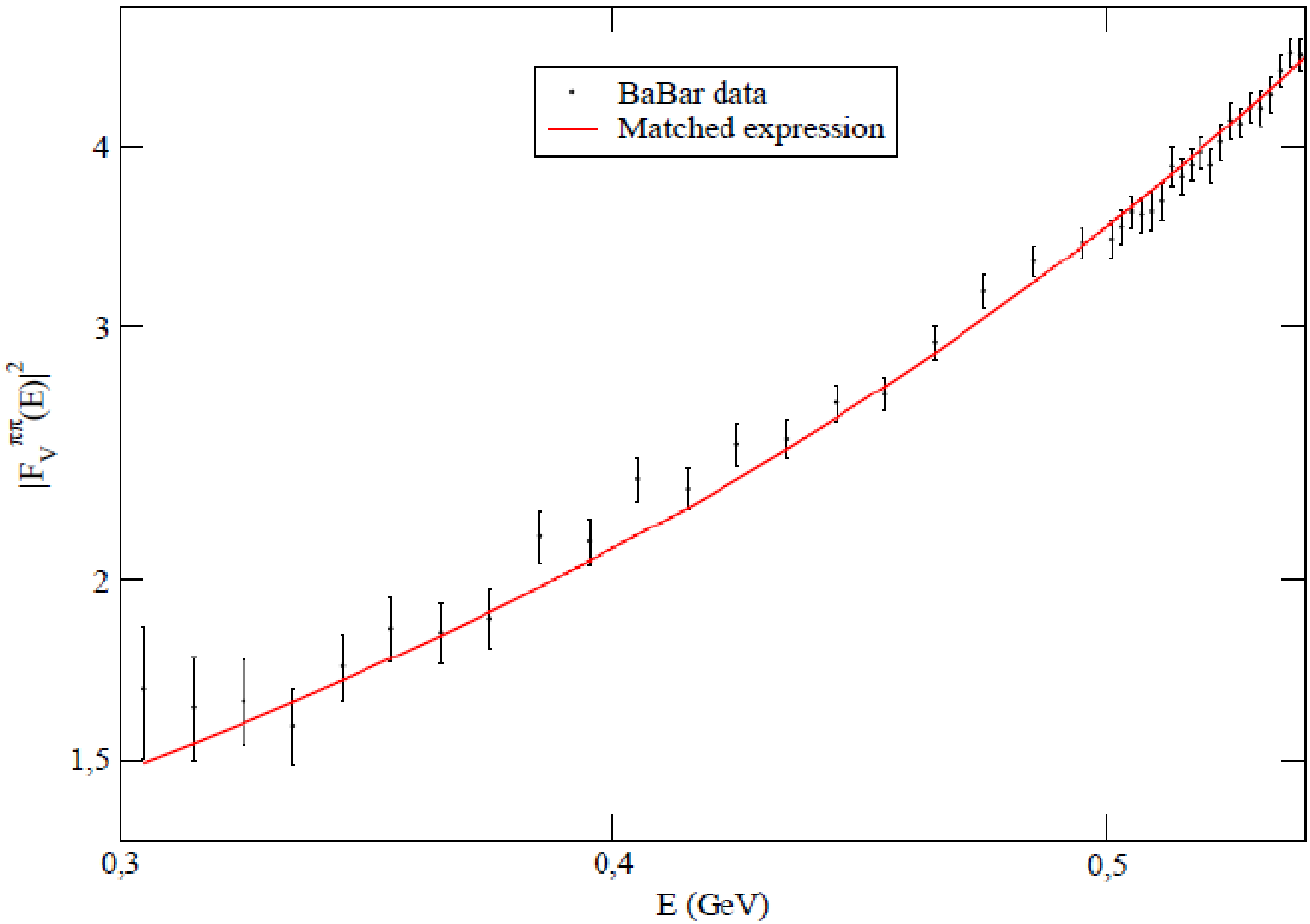

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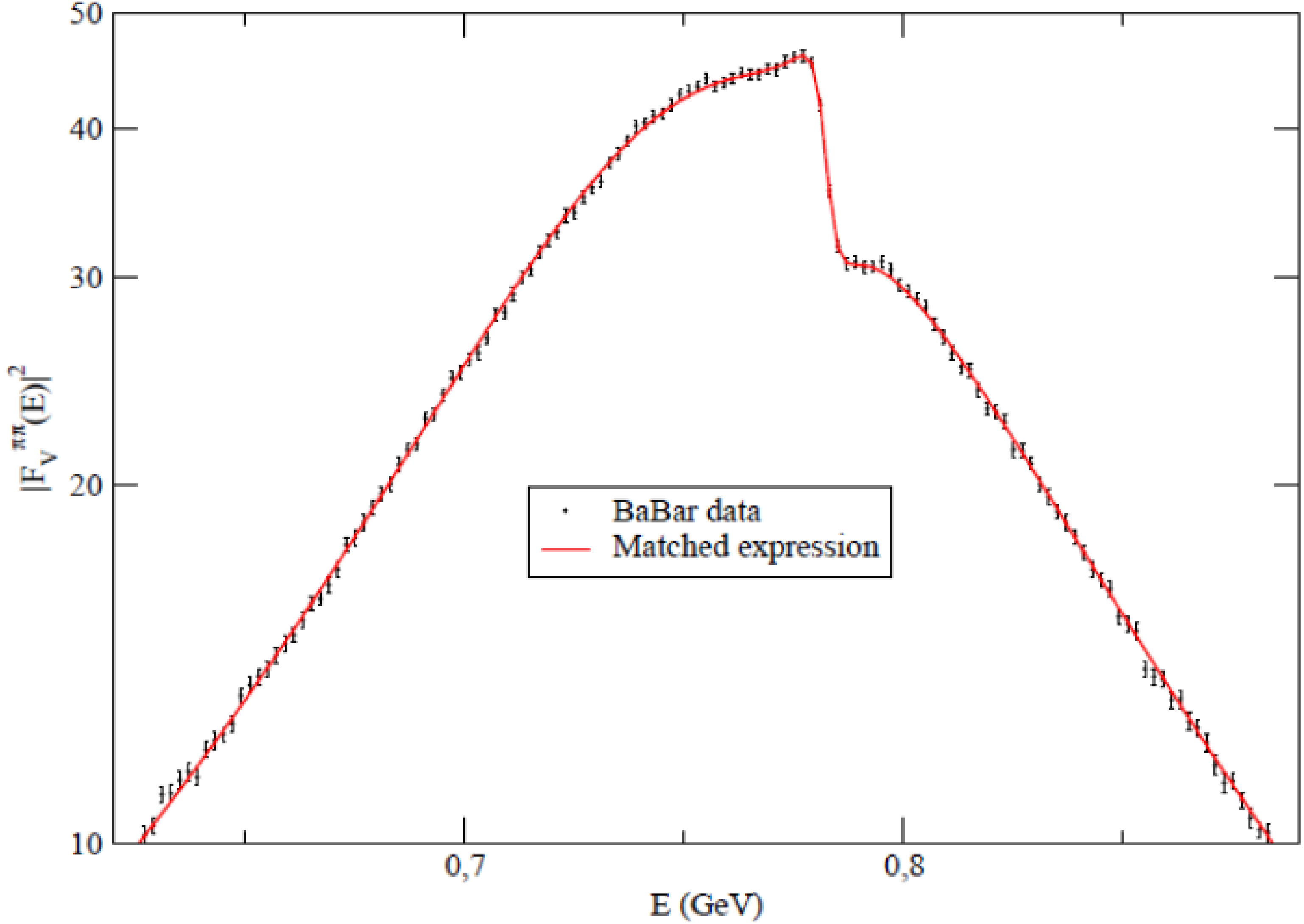

CONCLUSIONS

• We have elaborated a **dispersive description** of the $\pi^-\pi^0$ **VFF** which preserves **analiticity and unitarity** exactly and reproduces χ**PT** up to $O(p^4)$ with leading $O(p^6)$ contributions (soon in **TAUOLA**).

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(Gómez-Dumm, Jamin and Roig, work in progress)

•Our framework is also able to provide **good fits to the low-energy $\sigma(e^+e^- \to \pi^+\pi^-)$**. Once the issue of SU(2) breaking will be completely understood this will allow us to evaluate $a_\mu^{\pi\pi}$ **both from e^+e^- and τ decays consistently**.

(López-Castro, Roig and Toledo, work in progress)

SKIPPED SLIDES

DISPERSIVE REPRESENTATION OF THE $\pi\pi$ VECTOR FORM FACTOR

$$\frac{d\Gamma(\tau^- \to \pi^0\pi^-\nu_\tau)}{dt} = \frac{\Gamma_e^{(0)} S_{\rm EW}|V_{ud}|^2}{2m_\tau^2}\beta_{\pi^0\pi^-}(t)\,(1-\frac{t}{m_\tau^2})^2\left\{|f_+(t)|^2\left[(1+\frac{2t}{m_\tau^2})\beta^2_{\pi^0\pi^-}(t) + \frac{3\Delta_\pi^2}{t^2}\right] + 3|f_-(t)|^2 - 6{\rm Re}\,[f_+^*(t)f_-(t)]\frac{\Delta_\pi}{t}\right\} G_{\rm EM}(t) \quad (4)$$

It **vanishes** even including isospin breaking corrections to first order (Cirigliano, Ecker, Neufeld '01)

$$\Gamma_e^{(0)} = \frac{G_F^2 m_\tau^5}{192\pi^3}\ ,\ \Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2\ ,\ \beta_{\pi^0\pi^-}(t) = \lambda^{1/2}(1, M_{\pi^0}^2/t, M_{\pi^+}^2/t)$$

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$$\frac{d\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau)}{ds} = \frac{G_F^2\, m_\tau^3}{384\,\pi^3} S_{EW}\,|V_{ud}|^2\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right) \lambda^{3/2}\left(1, \frac{m^2_{\pi^0}}{s}, \frac{m^2_{\pi^+}}{s}\right)|f_+(s)|^2\, G_{EM}(s)\ ,$$

Only one relevant form factor: Vector Form Factor

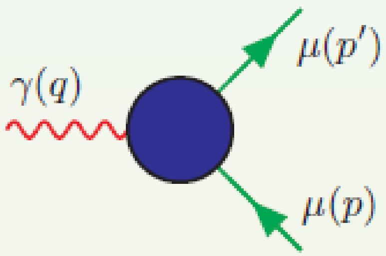
Introduction

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s} \;\; ; \quad g_\mu = 2\,(1 + a_\mu)$$

Dirac: $g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots$ muon anomaly

Electromagnetic Lepton Vertex

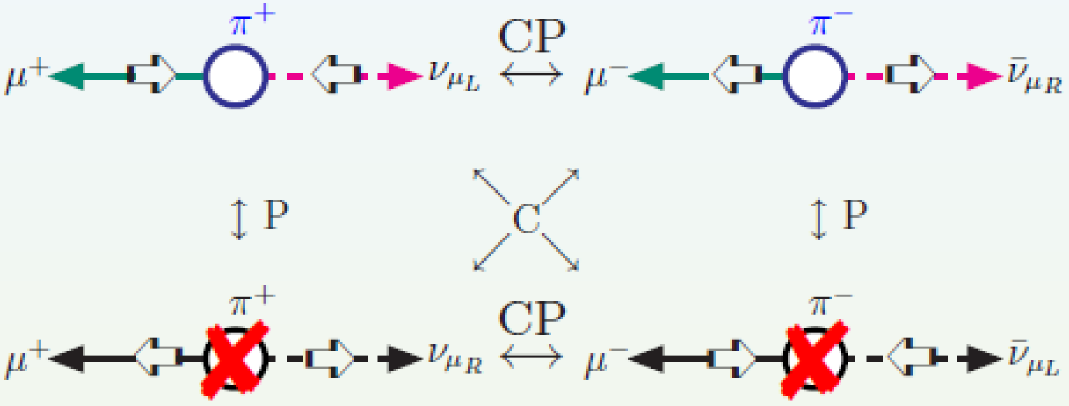


$$= (-\mathrm{i}e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \mathrm{i}\,\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2)\right] u(p)$$

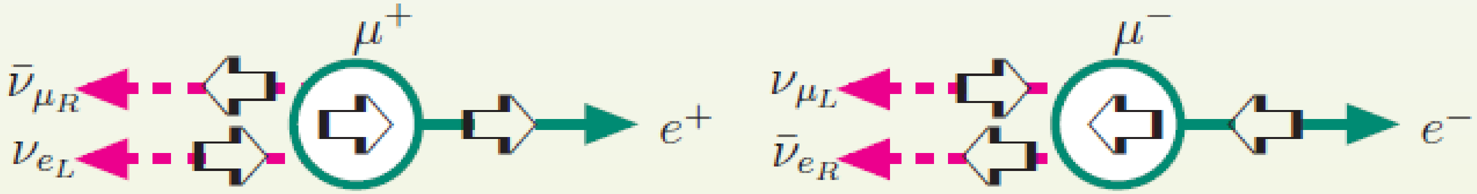
$$F_1(0) = 1 \;\; ; \quad F_2(0) = a_\mu$$

a_μ responsible for the Larmor precession

Production:



Decay:



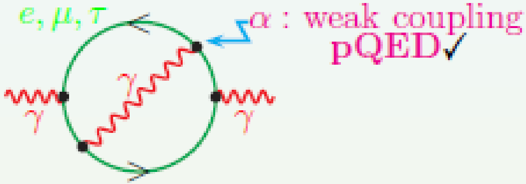
The electrons are thus emitted in the direction of the muon spin, i.e. measuring the direction of the electron momentum provides the direction of the muon spin.

❒ Hadronic stuff: the limitation to theory

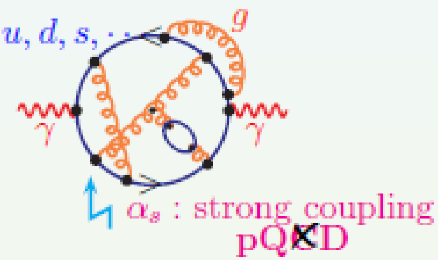
General problem in electroweak precision physics:

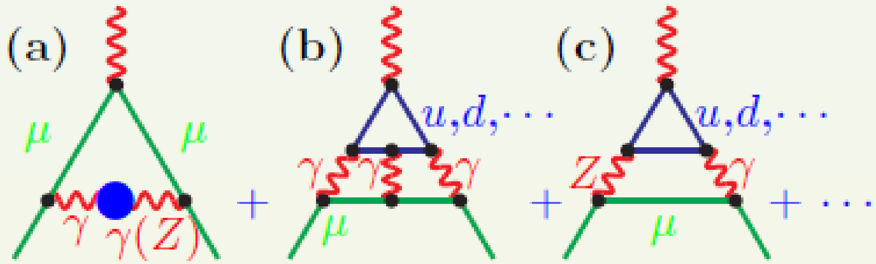
contributions from hadrons (quark loops) at low energy scales

Leptons



Quarks





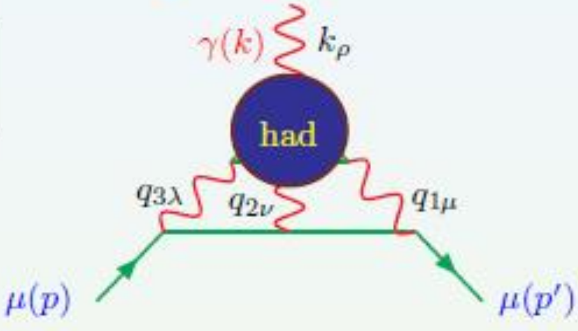
(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$ — Light quark loops

(b) Hadronic light-by-light scattering $O(\alpha^3)$ — ↓

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$ — Hadronic "blobs"

The Hadronic Light-by-Light Scattering Contribution

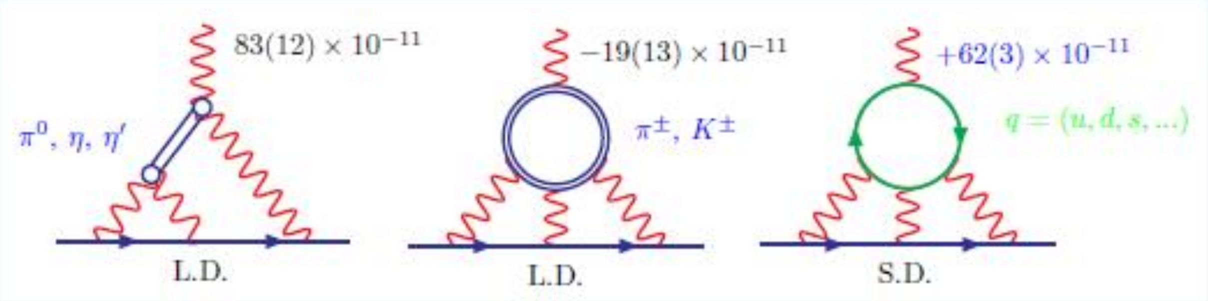
Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



Key object full rank-four hadronic vacuum polarization tensor

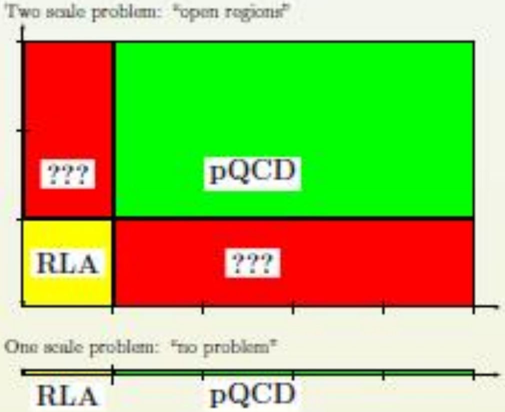
$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{e}^{\mathrm{i}\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\, 0\,|\,T\{j_\mu(x_1)\,j_\nu(x_2)\,j_\lambda(x_3)\,j_\rho(0)\}\,|\,0\,\rangle\;.$$

- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- 32 can contribute to $g-2$



LD contribution requires low energy effective hadronic models : simplest case $\pi^0\gamma\gamma$ vertex

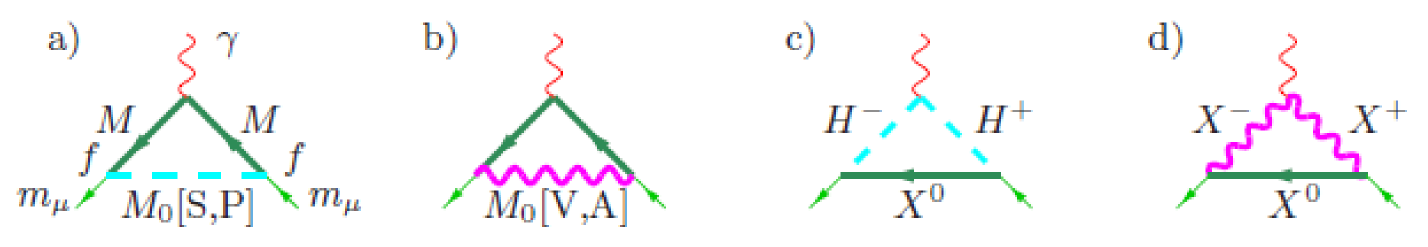
Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



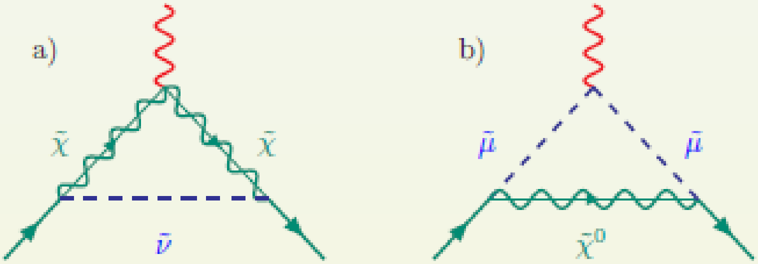
???

– Data, OPE,
– QCD factorization,
– Brodsky-Lepage approach

Most natural New Physics contributions: (examples)

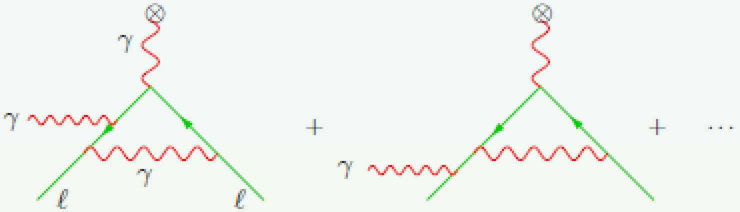


neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector



Leading SUSY contributions to $g-2$ in supersymmetric extension of the SM.

- Could it be a missing SM contribution?
  Could have been missing some electromagnetic radiation effects.
  Do we measure what we calculate i.e. $F_2(0)$?



Does real radiation not affect $g-2$ measurement? Could yield IR finite correction to helicity flip amplitude?

The role of QCD in high precision physics: α from a_e

Example: the electron $g-2$: $a_e^{\rm exp} = 0.001\,159\,652\,180\,73(28)$ Gabrielse et al. 2008

$\alpha^{-1}(a_e) = 137.0359991657(331)(68)(46)(24)[0.25\ \mathrm{ppb}]$ Aoyama et al 2012

Most precise non-a_e determination $\alpha^{-1}(\mathrm{Rb11}) = 137.035999037(91)[0.66\ \mathrm{ppb}]$
yields (QED-test!) $a_e^{\rm exp} - a_e^{\rm the} = -1.13(0.82) \times 10^{-12}$
Total hadronic contribution:

$$a_e^{\rm had} = a_e^{(4)}(\mathrm{vap,\ had}) + a_e^{(6)}(\mathrm{vap,\ had}) + a_e(\mathrm{LbL,\ had})$$
$$a_e^{\rm had} = (1.834 \pm 0.014^{1} - 0.219 \mp 0.002 + 0.037 \pm 0.005) \times 10^{-12}$$

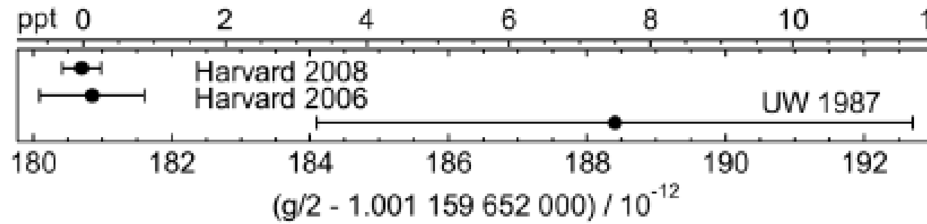
- $a_e^{\rm had} = 1.652(13) \times 10^{-12}$ relevant now, save only due to the fact the high energy tail of the dispersion integral is controlled by QCD (AF).

[1] First precise determination $a_e^{\rm had} = 1.884(41) \times 10^{-12}$ S. Eidelman, F.J. 1995 illustrates progress since

Electron Anomalous Magnetic Moment

Hanneke-Fogwell-Gabrielse '08

$$a_e = 0.001\,159\,652\,180\,73\ \ (28)$$



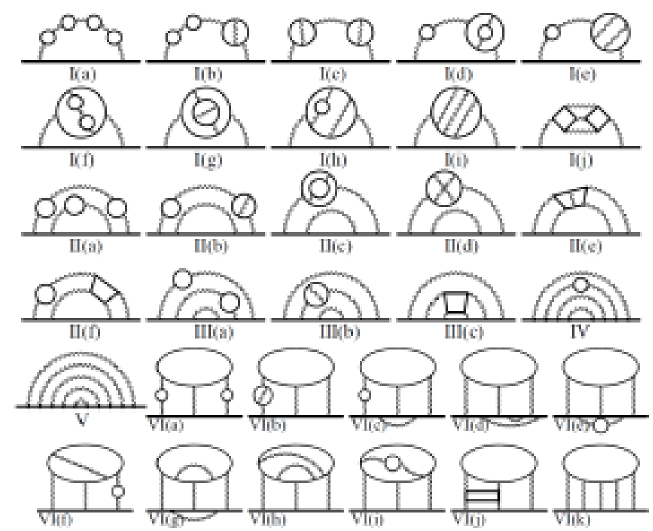
$$a_e^{\rm QED} = \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n a_e^{(2n)} \qquad a_e^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_e/m_\mu) + A_2^{(2n)}(m_e/m_\tau) + A_3^{(2n)}(m_e/m_\mu, m_e/m_\tau)$$

Aoyama-Hayakawa-Kinoshita-Nio

$$A_1^{(8)} = -1.9106\ (20) \quad , \quad A_2^{(8)}(m_e/m_\mu) = 9.222\ (66)\cdot 10^{-4}$$

$$A_2^{(8)}(m_e/m_\tau) = 8.24\ (12)\cdot 10^{-6} \quad , \quad A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.465\ (18)\cdot 10^{-7}$$

$$A_1^{(10)} = 9.16\ (58) \quad , \quad A_2^{(10)}(m_e/m_\mu) = -3.82\ (39)\cdot 10^{-3}$$



$$a_e^{\rm QCD} = 1.685\ (33)\times 10^{-12} \quad , \quad a_e^{\rm Weak} = 0.0297\ (5)\times 10^{-12}$$

$$\alpha^{-1}_{\rm Rb2010} = 137.035\,999\,049\ (90) \ \Rightarrow\ a_e^{\rm th} = 0.001\,159\,652\,181\,78\ \ (6)_{\rm 8th}(4)_{\rm 10th}(3)_{\rm had}(77)_\alpha$$

$$a_e \ \Rightarrow\ \alpha^{-1}_{a_e} = 137.035\,999\,173\,6\ (68)_{\rm 8th}(46)_{\rm 10th}(26)_{\rm had}(331)_{\rm exp} \qquad [0.25\ \rm ppb]$$

Best determination of fine structure constant:

$$a_e^{\rm QED}(\alpha) = \sum_{n=1}^{N} C_n\,(\alpha/\pi)^n \;\;;\;\; N = 5$$

$a_e^{\rm exp} = a_e^{\rm QED}(\alpha) + a_e^{\rm had} \;\Rightarrow$ result $\alpha(a_e)$ given above

At present precision weak contributions $a_e^{\rm weak} = 0.03043 \times 10^{-12}$ not relevant.

- so QCD plays role in the determination of the most fundamental constant $\alpha_{\rm em}$ but even more in the running $\alpha_{\rm em}(s) = \alpha_{\rm em}(0)/(1 - \Delta\alpha(s))$

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027498 \pm 0.000135\;[0.027510 \pm 0.000218],$$
$$\alpha^{-1}(M_Z^2) = 128.962 \pm 0.018\;[128.961 \pm 0.030]\;,$$

evaluated in Adler function approach [in braces standard evaluation].

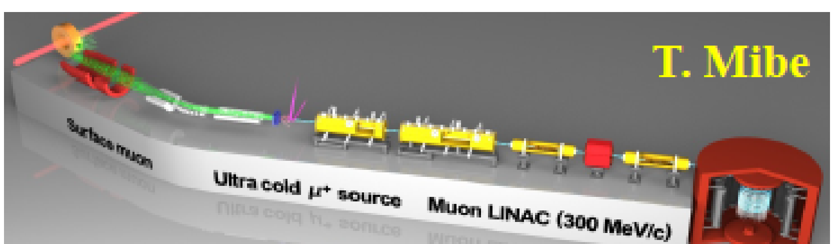
Future Challenge

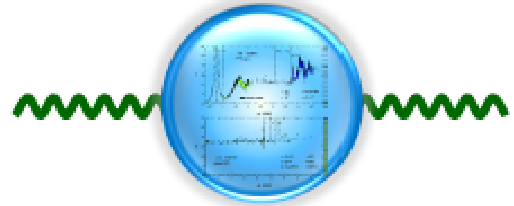
New Muon g-2 Experiment at FNAL

Goal: $\Delta a_\mu = 1.6\ 10^{-10}$ (0.14 ppm)

Alternative proposal at J-PARC

Ultra-slow μ, E=0, 0.1 ppm

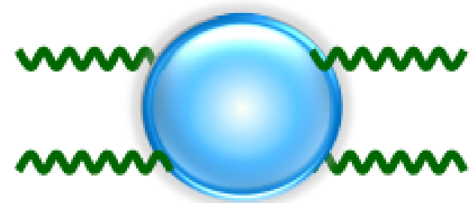




- Improved data
- Radiative return
- Isospin breaking
- Improved theoretical tools

  (Bodenstein et al, …)
- Lattice simulations

  (Blum)
- Light-by-Light (de Rafael)



τ Anomalous Magnetic Moment

Difficult to measure! $a_\tau^{\exp} = (-0.018 \pm 0.017)$ **DELPHI**

$-0.007 < a_\tau^{\text{New Phys}} < 0.005$ González-Springer, Santamaria, Vidal '00 (LEP/SLD data)

Eidelman, Passera

$$
\begin{aligned}
10^8 \cdot \; a_\tau^{\text{th}} = &\; 117\,324 \pm 2 && \text{QED} \\
&+ 47.4 \pm 0.5 && \text{EW} \\
&+ 337.5 \pm 3.7 && \text{hvp} \\
&+ 7.6 \pm 0.2 && \text{hvp NLO} \\
&+ 5 \pm 3 && \text{light-by-light} \\
&= \mathbf{117\,721 \pm 5} &&
\end{aligned}
$$

Enhanced sensitivity to new physics: $\left(m_\tau/m_\mu\right)^2 = 283$

	Electron	Muon	Tau
a^{EW}/a^{HAD}	1/56	1/45	1/7
$a^{EW}/\delta a^{HAD}$	1.6	3	10

M. Passera

The tau g-2 is essentially unknown. We are studying the possibility to measure it at B factories via its radiative leptonic decays.