Track fitting, vertex fitting and detector alignment. Excercises

3 de mayo de 2013

$\boxed{\textbf{Excercise 1}}$

What is the maximum transverse momentum that can be measured with a 1-m-radius cylindrical tracking system embedded in a uniform magnetic field of 1 Tesla along its axis and 20 μm spatial resolution ?

$\boxed{\textit{Solution}}$

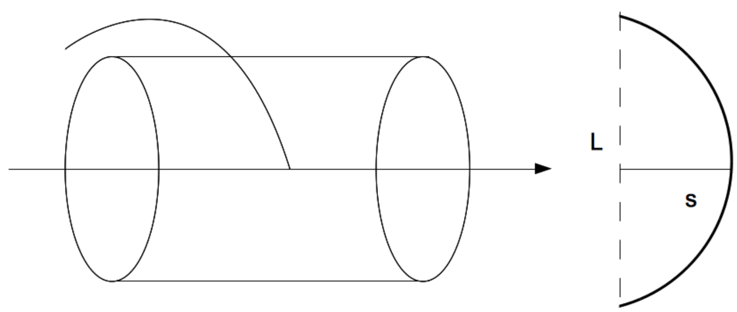
The transverse momentum of the particle can be expressed as:

$$p_{\mathrm{T}} = 0{,}3\ B(\mathrm{T})\ \rho(\mathrm{m}) = 0{,}3\ B \left(\frac{L^2}{8s} + \frac{s}{2} \right)$$

where L is the size of the tracking system while ρ and s are the curvature radius and the sagitta of the track respectively.

Usually as $L \gg s$ the above expression can be approximated by:

$$p_{\mathrm{T}} \approx 0,3\ B\ \frac{L^2}{8s}$$

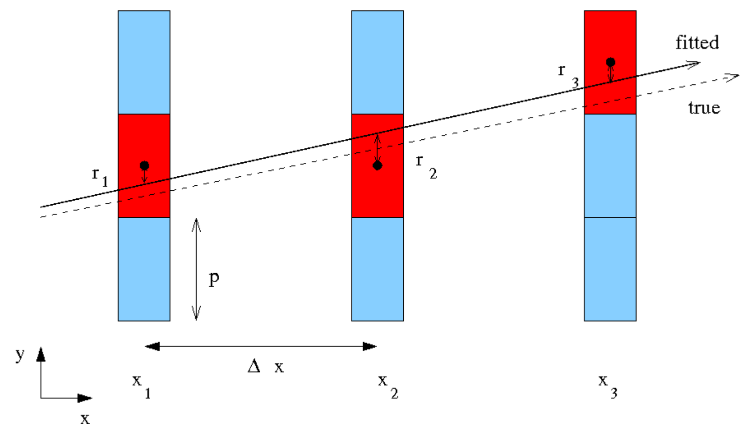


Now, $B = 1$ T, $L = 1$ m and $s = 20$ μm. Then:

$$p_{\mathrm{T}}^{\max} \approx\ 1,9\ \mathrm{TeV}$$

Excercise 2

Let's consider a tracking system built using 3 equidistant identical planes. The incident particles fly along a straight path. Given the three measurements (one in each plane), compute the residuals that minimize the track χ^2.



Hint: the track can be paramterized as: $y = y_0 + x \tan\phi$, so the track parameters set is given by:

$$\mathbf{t} = (y_0, \tan\phi)$$

Solution

The residuals of the track in each plane can be derived from the χ^2 minimization condition:

$$\chi^2 = \mathbf{r}^T V^{-1} \mathbf{r} \qquad \frac{d\chi^2}{d\mathbf{t}} = 0 \quad \rightarrow \quad \left(\frac{d\mathbf{r}}{d\mathbf{t}}\right)^T V^{-1}\mathbf{r} = 0$$

Let be $\mathbf{m}$ the vector of the measurements (with m_i the measurement in plane i), $\mathbf{e}$ the vector of extrapolated points in each plane:

$$\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} \quad \mathbf{e} = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} y_0 + x_1 \tan\phi \\ y_0 + x_2 \tan\phi \\ y_0 + x_3 \tan\phi \end{pmatrix}$$

Therefore the residuals vector, $\mathbf{r}$:

$$\mathbf{r} = \mathbf{m} - \mathbf{e} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} m_1 - (y_0 + x_1 \tan\phi) \\ m_2 - (y_0 + x_2 \tan\phi) \\ m_3 - (y_0 + x_3 \tan\phi) \end{pmatrix}$$

One needs to compute the derivative of the residuals with respect to the track parameters:

$$\frac{d\mathbf{r}}{d\mathbf{t}} = \begin{pmatrix} \frac{dr_1}{dt_1} & \frac{dr_1}{dt_2} \\ \frac{dr_2}{dt_1} & \frac{dr_2}{dt_2} \\ \frac{dr_3}{dt_1} & \frac{dr_3}{dt_2} \end{pmatrix} = \begin{pmatrix} \frac{dr_1}{dy_0} & \frac{dr_1}{d\tan\phi} \\ \frac{dr_2}{dy_0} & \frac{dr_2}{d\tan\phi} \\ \frac{dr_3}{dy_0} & \frac{dr_3}{d\tan\phi} \end{pmatrix} = \begin{pmatrix} -1 & -x_1 \\ -1 & -x_2 \\ -1 & -x_3 \end{pmatrix}$$

Now the covariance matrix is diagonal as all the planes are identical, therefore they have the same resolution σ:

$$V = \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{pmatrix} \quad \rightarrow \quad V^{-1} = \frac{1}{\sigma^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

With all this, the χ^2 minimization can be written as:

$$\begin{pmatrix} -1 & -x_1 \\ -1 & -x_2 \\ -1 & -x_3 \end{pmatrix}^T \frac{1}{\sigma^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Now, for the sake of simplicity, one can set the first plane at the origin ($x_1 = 0$). The other two planes are at $x_2 = \Delta x$ and $x_3 = 2\Delta x$. The equation system becomes:

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & \Delta x & 2\Delta x \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

which has 2 equations and 3 unknowns (r_1, r_2 and r_3).

$$\left.\begin{aligned} r_1 + r_2 + r_3 &= 0 \\ r_2 + 2r_3 &= 0 \end{aligned}\right\}$$

Therefore:

$$r_1 = r_3$$

$$r_2 = -2r_1$$