Physics of the tau lepton

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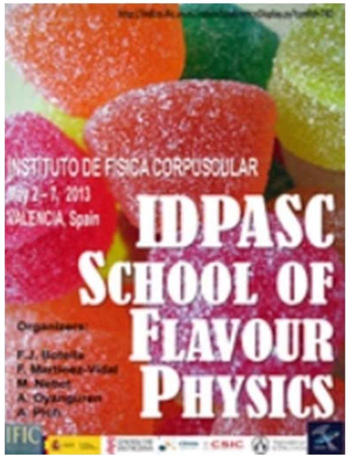


Leptons

$$L_\ell = \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L , \quad \ell^-_R$$

$$\ell = e, \mu, \tau$$

$$N_{(\nu_\ell, \ell^-)} = +1$$
$$N_{(\overline{\nu}_\ell, \ell^+)} = -1$$

$$\Delta N_\ell = 0$$

$$\mathrm{Br}(\mu^+ \to e^+ \gamma) < 5.7 \times 10^{-13}$$

90%CL, MEG [1]

Discovery of the tau lepton

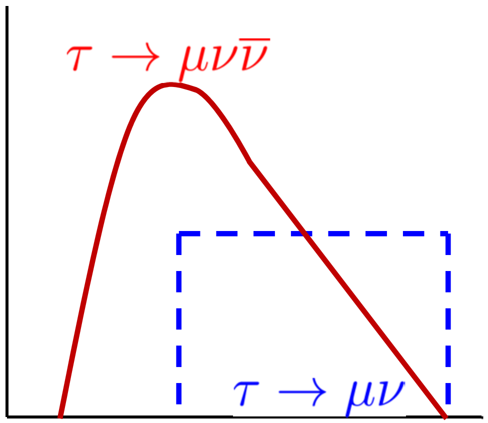
SLAC-LBL (SLAC,1975), PLUTO (DESY,1976)

"anomalous" e μ events → $e^+e^- \to \mu^\mp e^\pm$

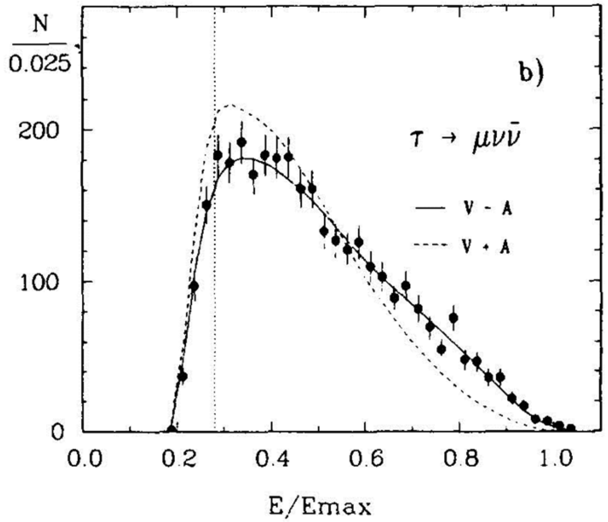
$$e^+e^- \to \tau^+\tau^-$$

$\tau^+ \to \mu^+\nu_\mu\overline{\nu}_\tau$

$\tau^- \to e^-\overline{\nu}_e\nu_\tau$



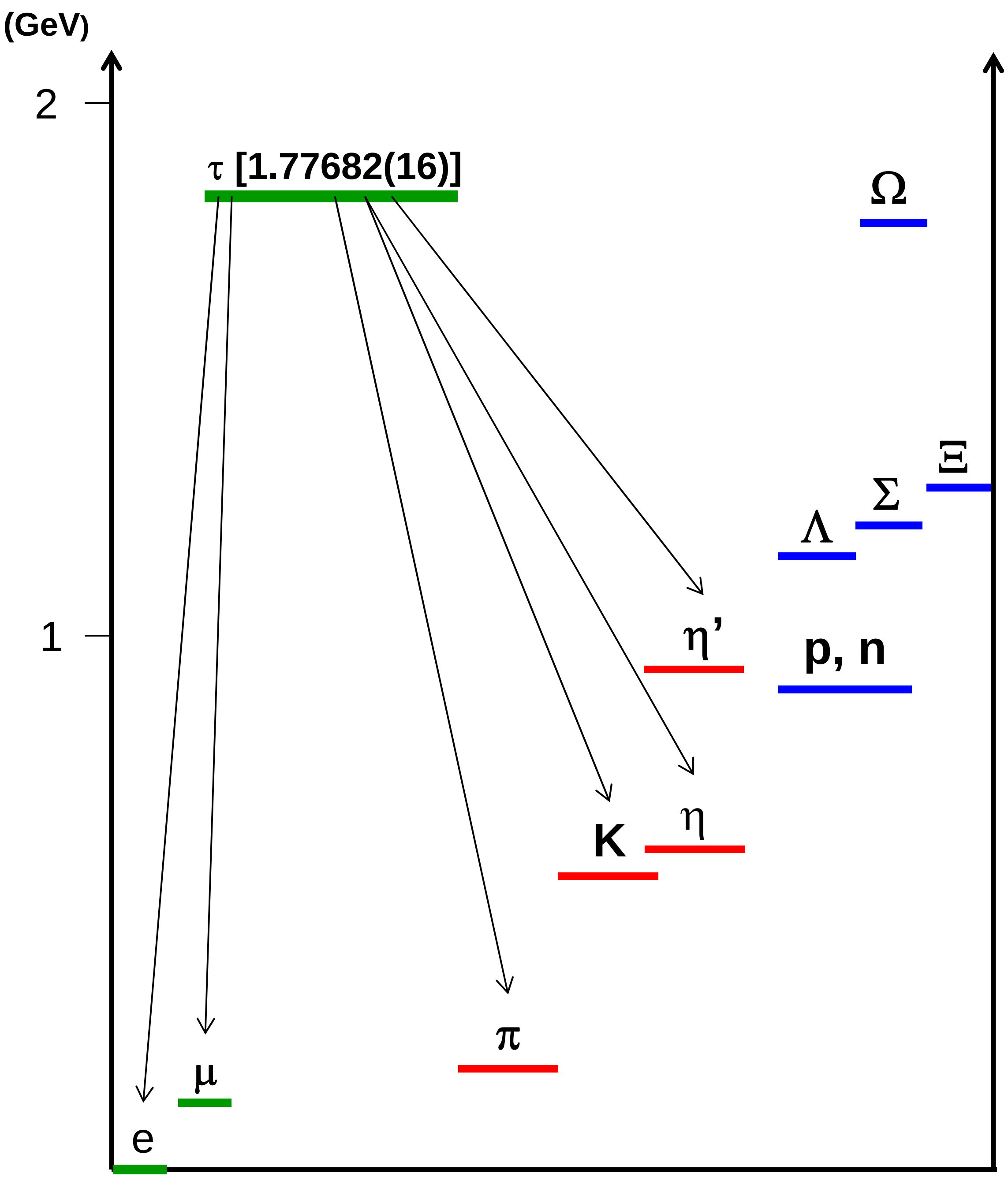
Momentum of e or μ →

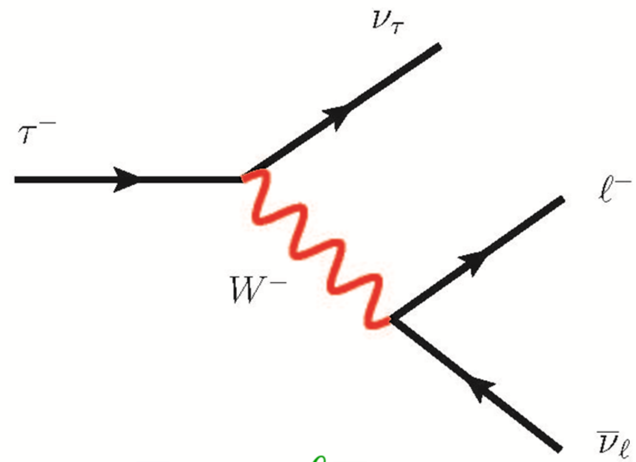


[2]

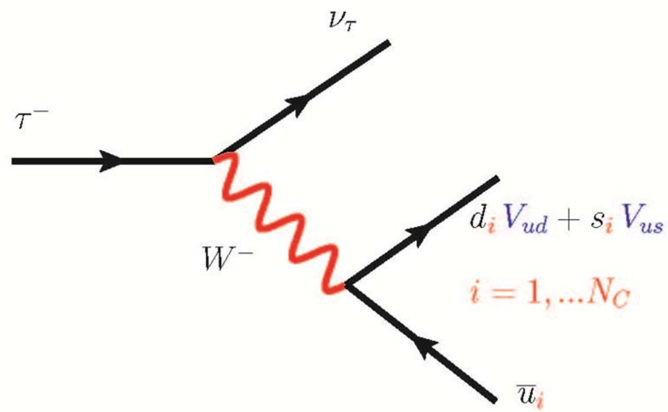
ν_τ → DONuT (Direct Observation of Nu Tau), 2000

Decay spectrum



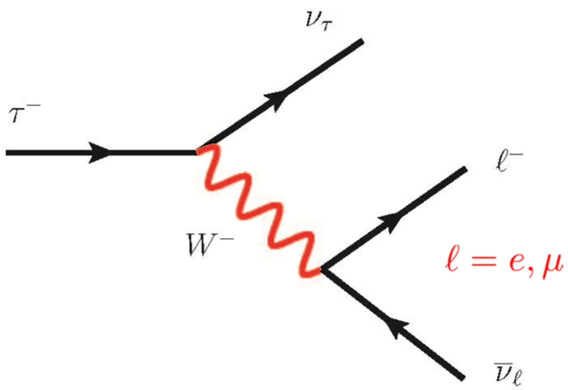


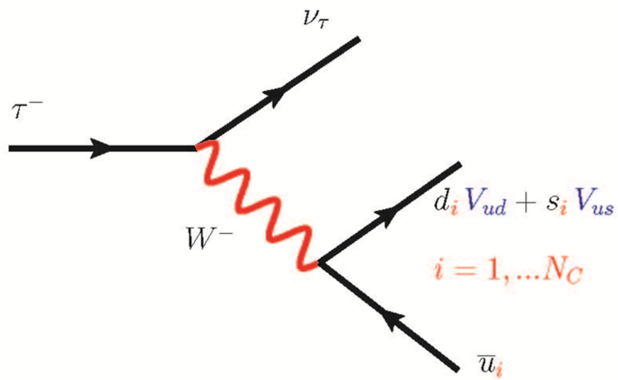
$$\tau^- \to \ell^- \overline{\nu}_\ell \nu_\tau$$



$$\tau^- \to (P, PP, ...)^- \nu_\tau$$

$P \equiv$ Pseudoscalar meson





Process	Estimate	Experiment
$\mathrm{B}_e \equiv \mathrm{Br}(\tau \to e\overline{\nu}\nu)$ $\mathrm{B}_\mu \equiv \mathrm{Br}(\tau \to \mu\overline{\nu}\nu)$	$\dfrac{1}{2 + N_C\,(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2)} \simeq 20\,\%$	$(17.83 \pm 0.04)\%$ $(17.41 \pm 0.04)\%$
$\mathrm{Br}(\tau \to \text{non-strange hadrons})$	$\dfrac{N_C \lvert V_{ud}\rvert^2}{2 + N_C\,(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2)} \simeq 58\,\%$	$(62 \pm 4)\%$
$\mathrm{Br}(\tau \to \text{strange hadrons})$	$\dfrac{N_C \lvert V_{us}\rvert^2}{2 + N_C\,(\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2)} \simeq 2\,\%$	$(2.6 \pm 0.7)\%$

Outline

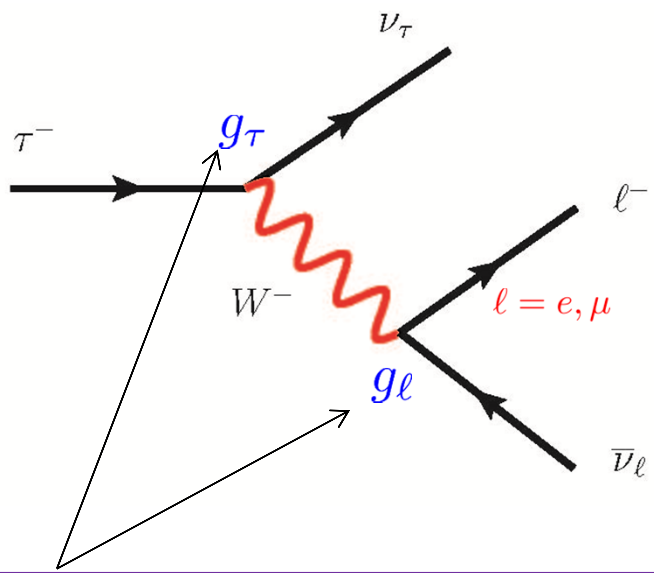
- Leptonic decays
- Hadron decays

  I. Inclusive tau decays: $\alpha_S(M_\tau)$ and $|V_{us}|$

  II. Exclusive tau decays: Hadronization of QCD currents

  E. g. $\tau \to \pi\pi\nu_\tau\,,\ \pi\pi\pi\nu_\tau$

1. Lepton decays



$$\Gamma\left(\tau \to \ell\,\overline{\nu}_\ell\,\nu_\tau\right) = \frac{G_F^2 M_\tau^5}{192\,\pi^3}\, f\left(M_\ell^2/M_\tau^2\right) r_{\rm EW}$$

$$f(x) = 1 - 8\,x + 8\,x^3 - x^4 - 12\,x^2 \ln x$$

$$f\left(\frac{M_e{}^2}{M_\tau^2}\right) = 0.999999, \qquad f\left(\frac{M_\mu{}^2}{M_\tau^2}\right) = 0.972559$$

$$r_{\rm EW} \underset{[3]}{=} \left(1 + \frac{3}{5}\frac{M_\tau^2}{M_W^2}\right)\left[1 + \frac{\alpha(M_\tau)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right] = 0.9960$$

Charged current universality $g_\tau = g_\mu = g_e$

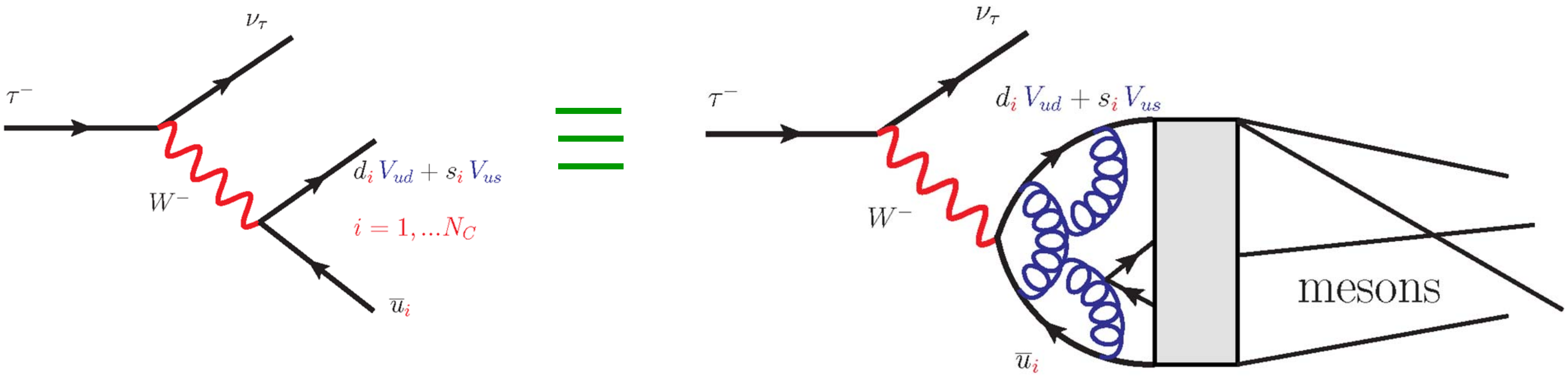
$\mathrm{Br}\left(\mu^- \to e^- \overline{\nu}_e \nu_\mu\right) \simeq 100\,\%$

	$\lvert g_\tau / g_e \rvert$
$B_{\tau\to\mu}\, \tau_\mu/\tau_\tau$	$1.0024(21)$
$B_{W\to\tau}/B_{W\to e}$	$1.023(11)$

	$\lvert g_\tau / g_\mu \rvert$
$B_{\tau\to e}\, \tau_\mu/\tau_\tau$	$1.0006(21)$
$B_{W\to\tau}/B_{W\to\mu}$	$1.032(12)$

	$\lvert g_\mu / g_e \rvert$
$B_{\tau\to\mu}/B_{\tau\to e}$	$1.0018(14)$
$B_{W\to\mu}/B_{W\to e}$	$0.991(9)$

2. Hadron decays



$$\mathcal{M}\,(\tau \to \nu_\tau \mathrm{H}) = \frac{G_F}{\sqrt{2}}\,V_{\mathrm{CKM}}\,\overline{u}_{\nu_\tau}\,\gamma^\mu\,(1-\gamma_5)\,u_\tau\,\langle H|\,(V_\mu - A_\mu)\,e^{i\,\mathrm{L_{QCD}}}\,|\Omega_{\mathrm{H}}\rangle$$

form factors

$$\langle H|\,(V_\mu - A_\mu)\,e^{i\,\mathrm{L_{QCD}}}\,|\Omega_{\mathrm{H}}\rangle = \sum_i (\text{Lorentz structure})^i{}_\mu\, \underbrace{F_i(Q^2, s, \ldots)}_{\text{form factors}}$$

$$d\Gamma\,(\tau \to \nu_\tau H) = \frac{G_F^2}{4\,M_\tau}\,|V_{\mathrm{CKM}}|^2\,L_{\mu\nu}\,H^{\mu\nu}\,d\mathrm{PS}$$

$$\begin{cases} L_{\mu\nu}H^{\mu\nu} \overset{[4]}{=} \sum_X L_X W_X \\ W_X \equiv \textbf{structure functions} \end{cases}$$

What can we get?

1. Inclusive decays: full hadron spectra. Precision physics.

$$\tau^- \to \nu_\tau \, (\bar{u}d, \bar{u}s)$$

➡ Study of Standard Model parameters : $\alpha_S(M_\tau)$, $|V_{us}|$, m_S

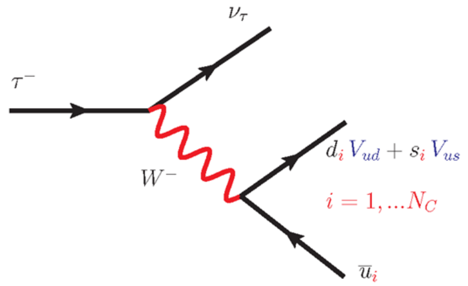
2. Exclusive decays: specific hadron spectrum. Approximate physics

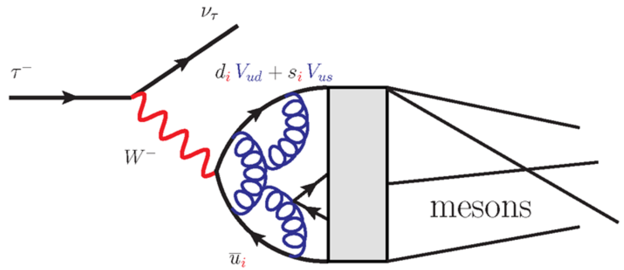
$$\tau^- \to \nu_\tau \, (PP, PPP, \ldots)$$

P = pseudoscalar meson

➡ Study of form factors, resonance parameters (M_R, Γ_R), hadronization of QCD currents.

2.1 Inclusive hadron decays

$\Big|$  $\Big|^2$

$\equiv \sum_{\text{mesons}}$ $\Big|$  $\Big|^2$

$e^+e^- \longrightarrow \text{hadrons}$

$$V^i_\mu = \bar{q}\,\gamma_\mu \frac{\lambda^i}{2} q, \qquad q = (u, d, s)^T$$

$$\sigma_{e^+e^-\to\text{had}}(q^2) = \frac{e^4}{2q^6} L^{\mu\nu} \sum_h (2\pi)^4 \delta^4(p_h - q) \langle \Omega_h |\, J_\mu(0)\, | h \rangle \langle h |\, J_\nu(0)\, | \Omega_h \rangle$$

$$J_\mu = V^3_\mu + \frac{1}{\sqrt{3}} V^8_\mu = \frac{2}{3}\,\bar{u}\,\gamma_\mu\, u - \frac{1}{3}\,\bar{d}\,\gamma_\mu\, d - \frac{1}{3}\,\bar{s}\,\gamma_\mu\, s$$

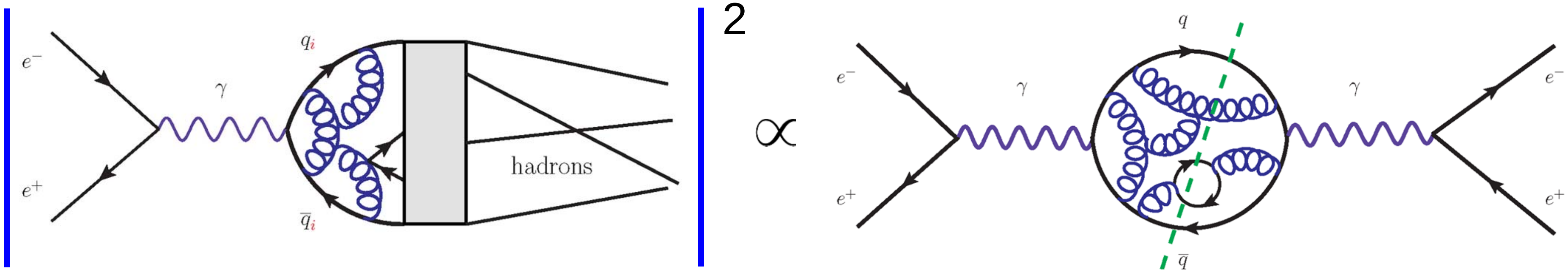
$$\sum_h (2\pi)^4 \delta^4(p_h - q) \langle \Omega_h | J_\mu(0) | h \rangle \langle h | J_\nu(0) | \Omega_h \rangle = \int d^4x\, e^{iqx}\, \langle \Omega_h | J_\mu(x) J_\nu(0) | \Omega_h \rangle$$

$$= \int d^4x\, e^{iqx}\, \langle \Omega_h |\, [J_\mu(x), J_\nu(0)]\, | \Omega_h \rangle$$

$$\int d^4x\, e^{iqx} \langle \Omega_h | \left[J_\mu(x), J_\nu(0) \right] | \Omega_h \rangle \overset{[5]}{=} 2\,\mathrm{Im} \left[i \int d^4x\, e^{iqx} \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle \right]$$

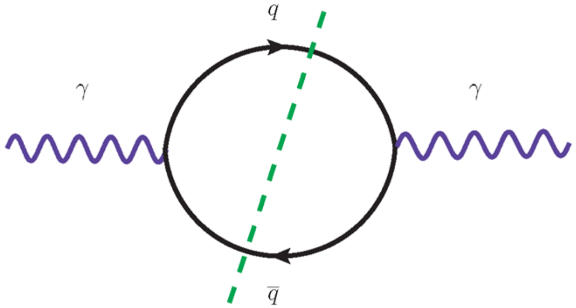
$$i \int d^4x\, e^{iqx} \langle \Omega_h | T J_\mu(x) J_\nu(0) | \Omega_h \rangle = \left(q_\mu q_\nu - q^2 g_{\mu\nu} \right) \Pi_V(q^2)$$

$$\sigma_{e^+e^- \to \mathrm{had}}(q^2) = \frac{16\pi^2\alpha^2}{q^2} \,\mathrm{Im}\, \Pi_V(q^2)$$



$$\sigma_{e^+e^- \to \mu^+\mu^-} = \frac{4\pi\alpha^2}{3\,q^2} \qquad\qquad R(q^2) = \frac{\sigma_{e^+e^- \to \mathrm{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} = 12\,\pi\,\mathrm{Im}\,\Pi_V(q^2)$$

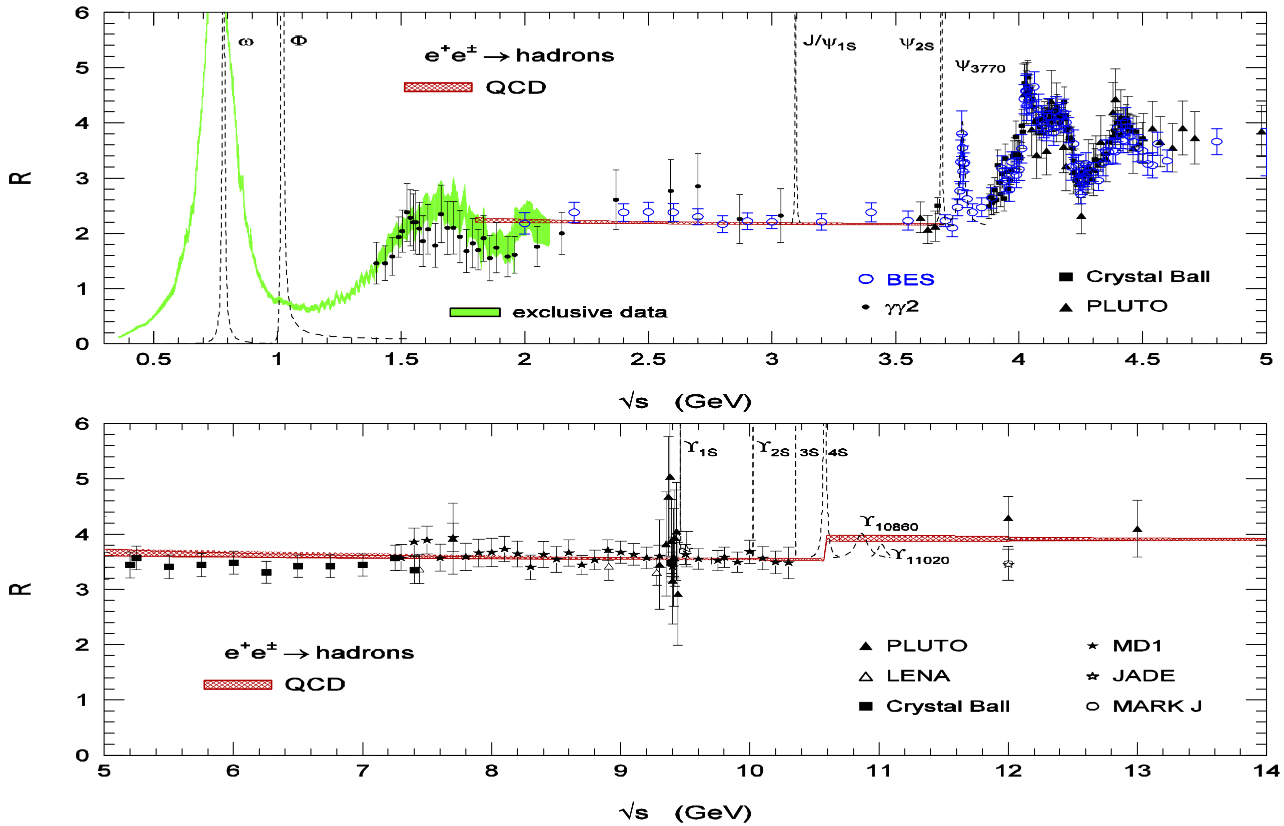
$$\mathrm{Im}\,\Pi_V(q^2) \; = $$



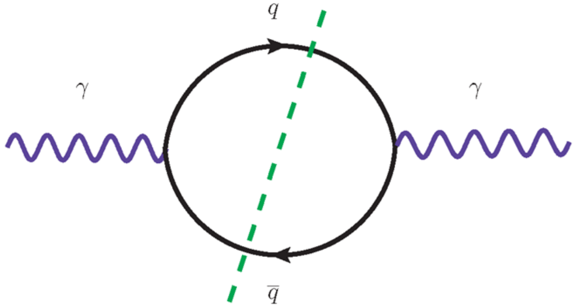
$$= \frac{N_C}{12\,\pi} \sum_i Q_i^2$$

$$R(q^2) \;=\; \frac{\sigma_{e^+e^-\to\text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} \;=\; 12\,\pi\,\mathrm{Im}\,\Pi_V(q^2) \;=\; N_C \sum_i Q_i^2 \;=\; 2,\ \frac{10}{3},\ \frac{11}{3}$$

$$N_F \;=\; 3,\quad 4,\quad 5$$



$$\mathrm{Im}\,\Pi_V(q^2) \; = $$

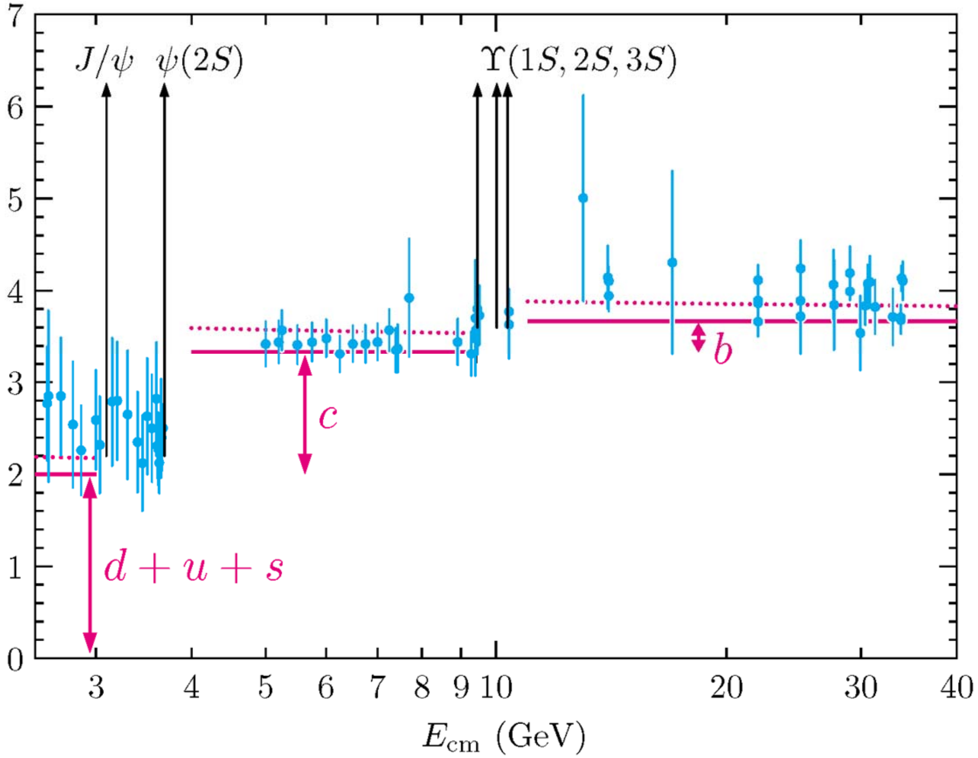


$$= \frac{N_C}{12\,\pi} \sum_i Q_i^2$$

$$R(q^2) = \frac{\sigma_{e^+e^-\to\mathrm{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = 12\,\pi\,\mathrm{Im}\,\Pi_V(q^2) = N_C \sum_i Q_i^2 = 2,\ \frac{10}{3},\ \frac{11}{3}$$

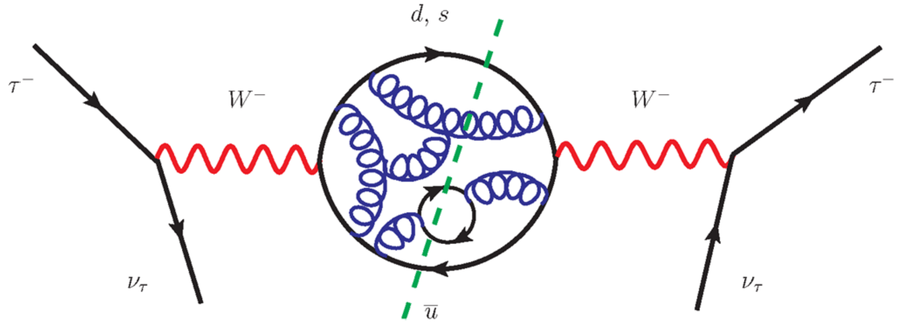
$$N_F = 3,\quad 4,\quad 5$$

$R = \sigma(\mathrm{hadrons})/\sigma(\mu^+\mu^-)$

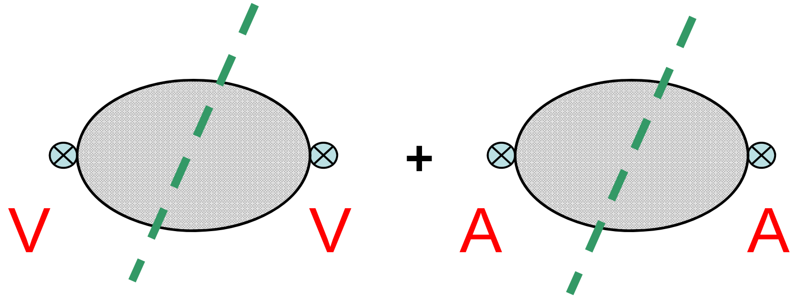


$$\tau \longrightarrow \nu_\tau \text{ mesons}$$

$$\Gamma(\tau \longrightarrow \nu_\tau \text{ mesons}) \quad \propto$$



$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau \text{ mesons})}{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \quad \propto$$



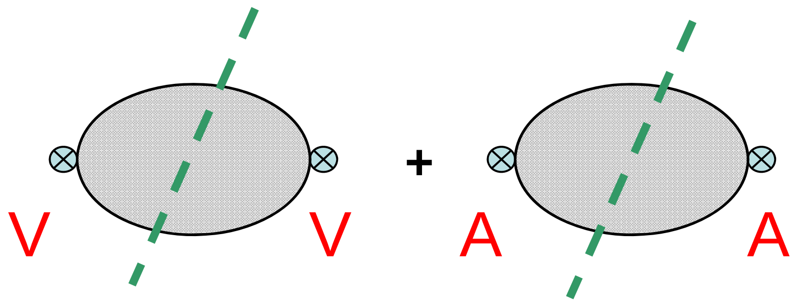
$$R_\tau = \underbrace{R_{\tau,V} + R_{\tau,A}}_{S=0} + \underbrace{R_{\tau,S}}_{S=1} \simeq \frac{N_C}{2}|V_{ud}|^2 + \frac{N_C}{2}|V_{ud}|^2 + N_C|V_{us}|^2$$

$$R_\tau \simeq N_C$$

$$R_\tau^{\exp} = \frac{\sum_i \Gamma(\tau \to \nu_\tau h_i)}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} \overset{[6]}{=} 3.628 \pm 0.009$$

$$R_\tau^{\exp} = \frac{1 - \mathrm{B}_e - \mathrm{B}_\mu}{\mathrm{B}_e} = 3.632 \pm 0.011$$

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau \,\mathrm{mesons}\right)}{\Gamma\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)} \propto$$



$$\Pi^{\mu\nu}_{ij,V}(q) \equiv i \int d^4x \, e^{iqx} \, \langle \Omega_h | \, T V^\mu_{ij}(x) V^\nu_{ij}(0)^\dagger \, | \Omega_h \rangle$$

$$\Pi^{\mu\nu}_{ij,A}(q) \equiv i \int d^4x \, e^{iqx} \, \langle \Omega_h | \, T A^\mu_{ij}(x) A^\nu_{ij}(0)^\dagger \, | \Omega_h \rangle$$

$$V^\mu_{ij} = \overline{\psi}_j \gamma^\mu \psi_i$$

$$A^\mu_{ij} = \overline{\psi}_j \gamma^\mu \gamma_5 \psi_i$$

i,j = flavour indices

J

$$\Pi^{\mu\nu}_{ij,V/A}(q) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) \Pi^{(1)}_{ij,V/A}(q^2) + q^\mu q^\nu \, \Pi^{(0)}_{ij,V/A}(q^2)$$

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left(\Pi^{(J)}_{ud,V}(s)+\Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2\left(\Pi^{(J)}_{us,V}(s)+\Pi^{(J)}_{us,A}(s)\right)$$

spectral functions

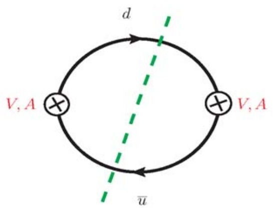
$$\mathrm{Im}\Pi^{(J)}{}_{ud(s),U} = \frac{1}{2\pi}\, u_J$$

$$U = V, A \longrightarrow u_J = v_J, \mathrm{a}_J$$

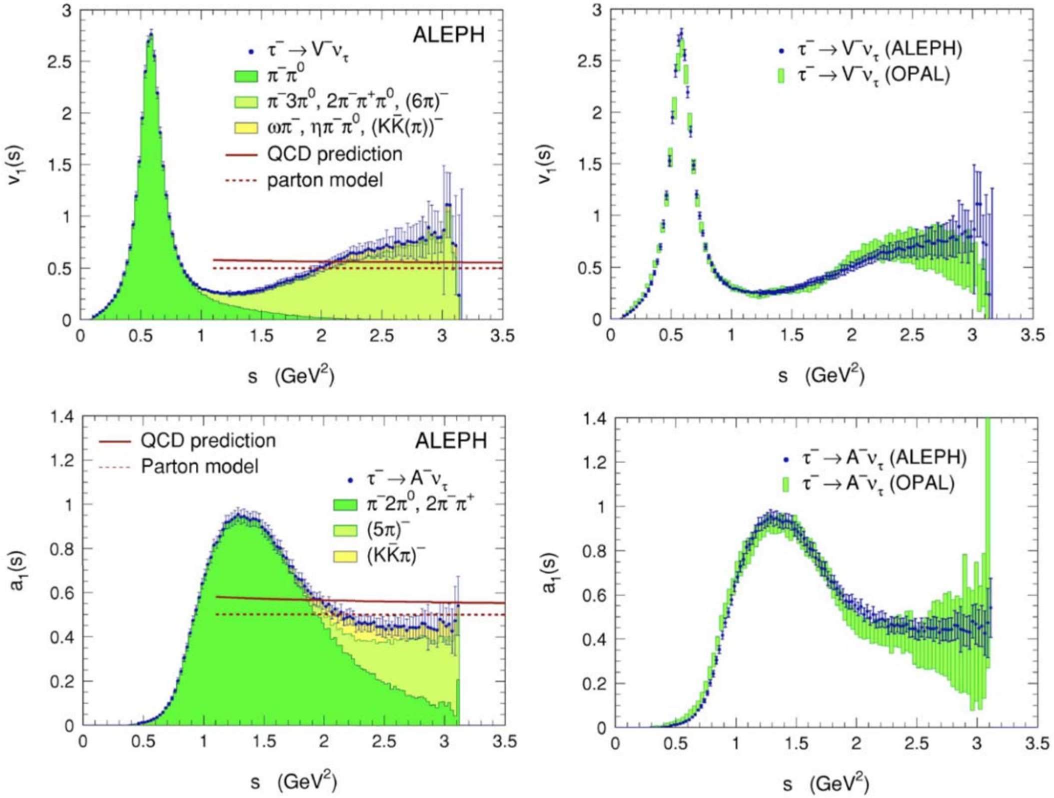
$$R_{\tau,V} = 1.783\,(11)_{exp}(2)_{V/A}$$

$$R_{\tau,A} = 1.695\,(11)_{exp}(2)_{V/A}$$

$$R_{\tau,S} \underset{[8]}{=} 0.1615\,(40)$$

$\mathrm{Im}\Pi^{(1)}_{V,A} =$  $= \dfrac{N_C}{12\pi}$

$$v_1 = \mathrm{a}_1 = \frac{N_C}{6} \qquad v_0 \simeq 0 \qquad \mathrm{a}_0 \propto \delta\left(s - M_\pi^2\right)$$



[7]

Working on the theoretical prediction of R_τ .... to get $\alpha_S(M_\tau)$,$|V_{us}|$ .....

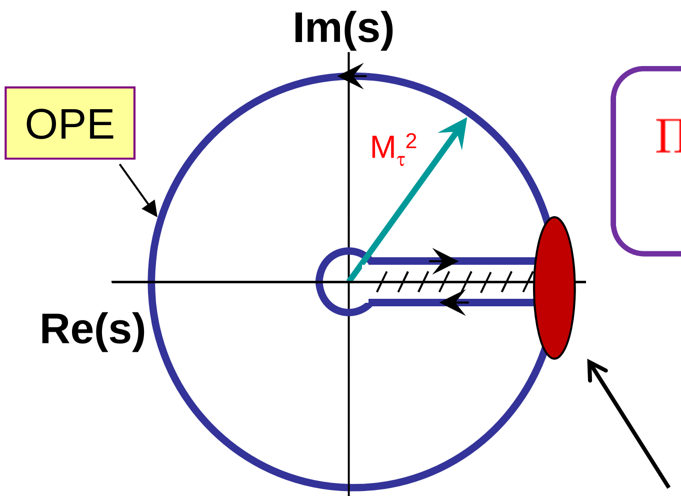
$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

Cauchy's Theorem

$$\frac{1}{\pi}\int_0^{s_0} ds\, f(s)\, \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, f(s)\, \Pi(s)$$

- $\Pi(s)$ analytic everywhere except on the positive real axis
- $f(s)$ analytic

$$R_\tau \overset{[9]}{=} 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\Pi^{(0+1)}(s) - \frac{2s}{M_\tau^2}\Pi^{(0)}(s)\right]$$

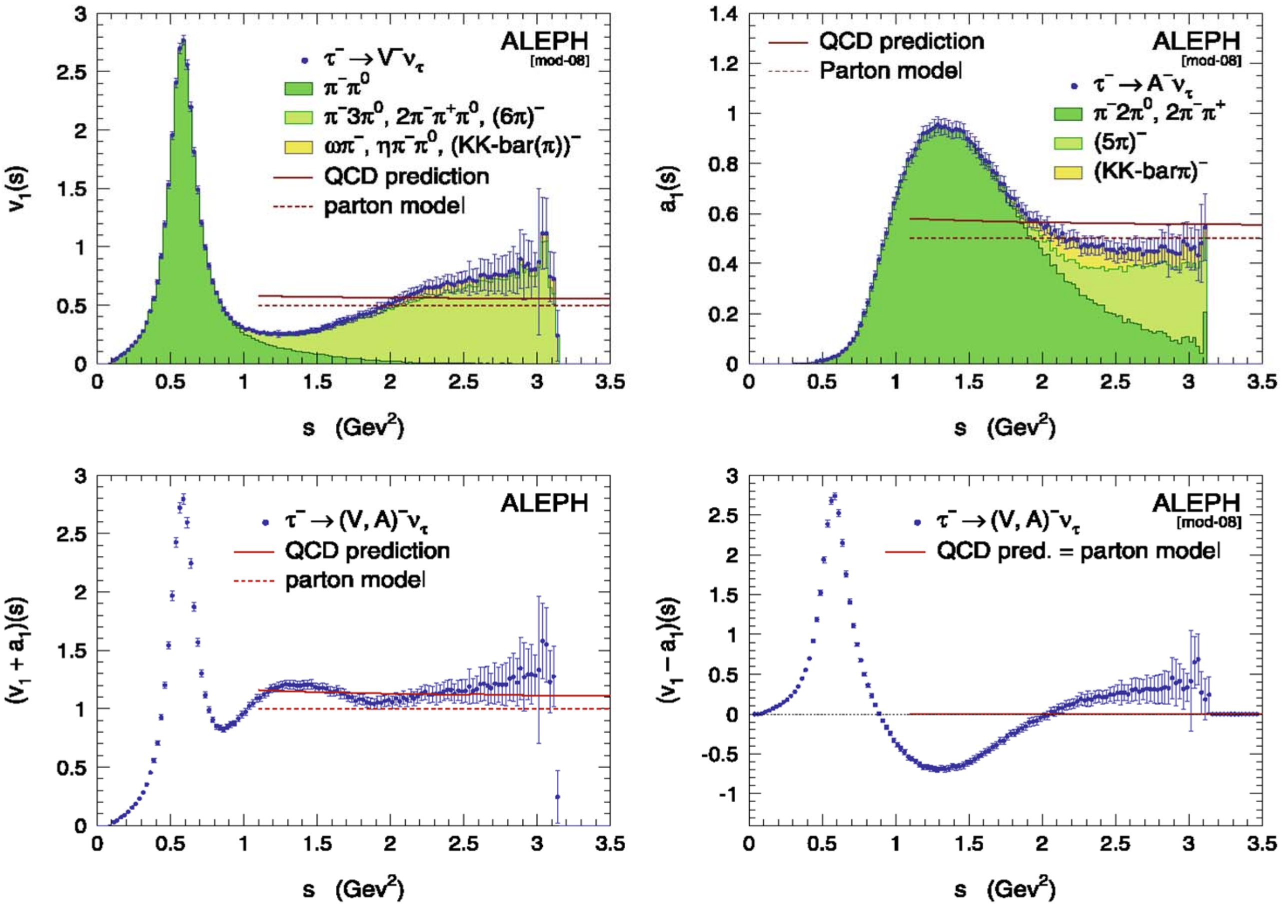


Operator Product Expansion (OPE)

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim \mathcal{O}=D} C_D^{(J)}(s,\mu)\, \langle \mathcal{O}_D(\mu)\rangle$$

$D=0$ $\rightarrow$ perturbative (expansion in $\alpha_S(\mu)$)

$D>0$ $\rightarrow$ non-perturbative (expansion in condensates)



$$(V - A)\Big|_{\chi} \quad \propto \quad \text{non-perturbative} \qquad (m_u = m_d = m_s = 0)$$

$$(V + A) \quad \propto \quad \left(\text{perturbative} + \frac{1}{M_\tau^6}\,\text{non-perturbative}\right)$$

$$\alpha_S(M_\tau)$$

$$R_{\tau,V+A} = N_C\, |V_{ud}|^2\, S_{\rm EW}\, \{1 + \delta_P + \delta_{NP}\}$$

$$S_{\rm EW} = 1.0201\,(3)$$

[3,10]

$$S_{\rm EW} \simeq 1 + \frac{3\,\alpha}{4\,\pi} \ln\left(\frac{M_Z^2}{M_\tau^2}\right)\left[\frac{4}{3} - \frac{\alpha_S}{3\,\pi}\right]$$

Perturbative contribution

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim \mathcal{O}=D} C_D^{(J)}(s,\mu)\, \langle \mathcal{O}_D(\mu) \rangle$$

$m_q = 0$ [8,11,12]

$$\delta_P = \sum_{n=1} K_n\, A^{(n)}(\alpha_S)$$

$$A^{(n)}(\alpha_S) \equiv \frac{1}{2\pi i} \oint_{|s|=M_\tau^2} \frac{ds}{s} \left(\frac{\alpha_S(-s)}{\pi}\right)^n \left(1 - 2\frac{s}{M_\tau^2} + 2\frac{s^3}{M_\tau^6} - \frac{s^4}{M_\tau^8}\right)$$

K_n known up to $\mathcal{O}\left({\alpha_S}^4\right) \longrightarrow N_F = 3$

$$\begin{cases} K_0 = K_1 = 1 \\ K_2 = 1.63982 \\ K_3^{\overline{\rm MS}} = 6.37101 \\ K_4^{\overline{\rm MS}} = 49.07570 \end{cases}$$

$$a_\tau \equiv \alpha_S(M_\tau)/\pi$$

Fixed-order perturbation theory (FOPT)

Expansion of $A^{(n)}(\alpha_S)$ in powers of $\alpha_S(M_\tau^2)$

$$\delta_P = a_\tau + 5.2\, a_\tau^2 + 26.4\, a_\tau^3 + 127.1\, a_\tau^4$$

Contour-improved perturbation theory (CIPT)

Using the exact solution for $\alpha_S(s)$ given by the RG β-function equation

$$\delta_P = 1.4\, a_\tau + 2.5\, a_\tau^2 + 9.7\, a_\tau^3 + 64.3\, a_\tau^4$$

Non-perturbative contributions

$$\delta_{NP} = \frac{-1}{2\pi i}\oint_{|s|=M_\tau^2}\frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)\sum_{D\geq 2}\frac{1}{(-s)^{D/2}}C_D(s,\mu)\,\langle\mathcal{O}_D(\mu)\rangle$$

$$\delta_{NP}\Big|_{C_D=\text{constant}} \simeq \frac{1}{M_\tau^2}C_2\langle\mathcal{O}_2\rangle - \frac{3}{M_\tau^6}C_6\,\langle\mathcal{O}_6\rangle - \frac{2}{M_\tau^8}C_8\,\langle\mathcal{O}_8\rangle + \ldots$$

[9, 13]

$$C_2\,\langle\mathcal{O}_2\rangle \;\propto\; \left[1+\frac{16}{3}\frac{\alpha_S(M_\tau)}{\pi}\right]\left(m_u^2(M_\tau)+m_d^2(M_\tau)\right)$$

$$C_4\,\langle\mathcal{O}_4\rangle \;\propto\; \left(\frac{\alpha_S(M_\tau)}{\pi}\right)^2 \langle(\alpha_S/\pi)G_{\mu\nu}G^{\mu\nu}\rangle\,,$$

$$\langle m_u\overline{\psi}_u\psi_u + m_d\overline{\psi}_d\psi_d\rangle\,,\ldots.$$

$$C_6\,\langle\mathcal{O}_6\rangle \;\propto\; \frac{\alpha_S(M_\tau^2)}{\pi}\,\langle\overline{\psi}_u\Gamma\psi_d\overline{\psi}_d\Gamma\psi_u\rangle\,,\ldots.$$

$$C_8\,\langle\mathcal{O}_8\rangle \;\propto\; \langle(\alpha_S/\pi)G_{\mu\nu}G^{\mu\nu}\rangle^2\,,\ldots.$$

$$\langle\overline{\psi}_i\psi_i\rangle(2\,\mathrm{GeV}) =$$

$-(283(2)\,\mathrm{MeV})^3$ [Lattice] [14]

$-(267(16)\,\mathrm{MeV})^3$ [Pheno] [15]

$$\langle\frac{\alpha_S}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle \simeq$$

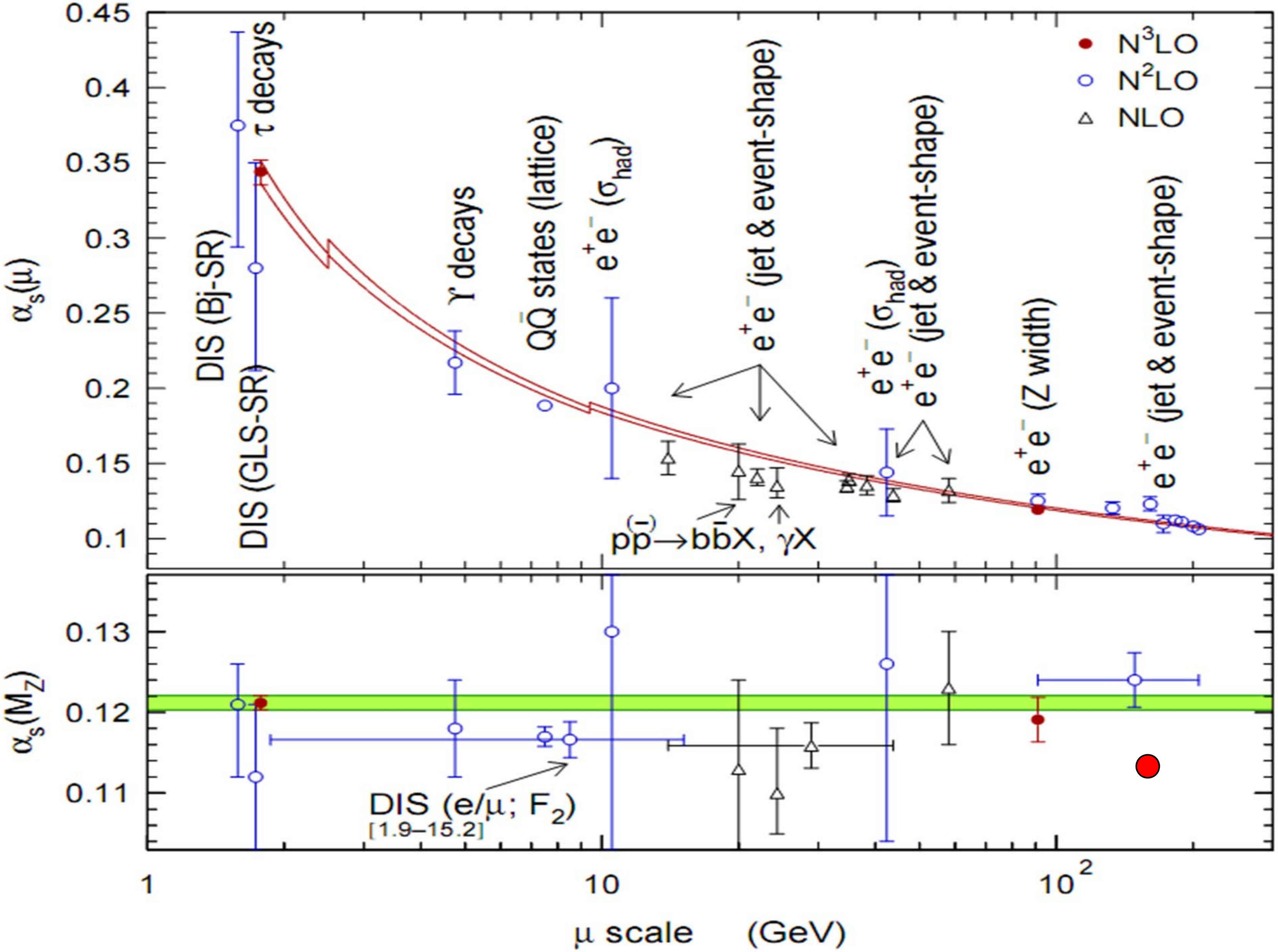
$0.012\,\mathrm{GeV}^4$ [Sum Rules] [13]

$\alpha_S(M_\tau)$ Analyses

Reference	Method	δ_P	δ_{NP}	$\alpha_S(M_\tau)$	$\alpha_S(M_Z)$
Baikov et al. [12]	CIPT, FOPT	0.1998 (43)	-	0.332 (16)	0.1202 (19)
Davier et al. [8]	CIPT	0.2066 (70)	-0.0059 (14)	0.344 (09)	0.1212 (11)
Beneke-Jamin [16]	BSR + FOPT	0.2042 (50)	-0.007 (03)	0.316 (06)	0.1180 (08)
Maltman-Yavin [17]	PWM + CIPT	-	+0.012 (18)	0.321 (13)	0.1187 (16)
Menke [18]	CIPT, FOPT	0.2042 (50)	-	0.342 (11)	0.1213 (12)
Narison [19]	CIPT, FOPT	-	-	0.324 (08)	0.1192 (10)
Caprini-Fischer [20]	BSR + CIPT	0.2037 (54)	-	0.322 (16)	-
Abbas et al. [21]	IFOPT	0.2037 (54)	-	0.338 (10)	-
Cvetic et al. [22]	β_{exp} + CIPT	0.2040 (40)	-	0.341 (08)	0.1211 (10)
Boito et al. [23]	CIPT, DV FOPT, DV	- -	-0.002 (12) -0.004 (12)	0.347 (25) 0.325 (18)	0.1216 (27) 0.1191 (22)
Pich [24]	CIPT, FOPT	0.1995 (33)	-0.0059 (14)	0.329 (13)	0.1198 (15)

CIPT : Contour-improved perturbation theory
FOPT : Fixed-order perturbation theory
BSR : Borel summation of renormalon series
IFOPT: Improved FOPT

β_{exp} : Expansion in derivatives of α_s
PWM : Pinched-weight moments
DV : Duality violation



N³LO
N²LO
NLO
τ decays
DIS (Bj-SR)
DIS (GLS-SR)
ϒ decays
Q̄Q states (lattice)
e⁺e⁻ (σ_had)
e⁺e⁻ (jet & event-shape)
e⁺e⁻ (σ_had)
e⁺e⁻ (jet & event-shape)
e⁺e⁻ (Z width)
e⁺e⁻ (jet & event-shape)
pp̄ → bb̄X, γX
DIS (e/μ; F₂)
[1.9–15.2]
α_s(μ)
α_s(M_Z)
μ scale (GeV)

m_S and $|V_{us}|$ from inclusive tau data decays

$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} ds \left(1-\frac{s}{M_\tau^2}\right)^k \left(\frac{s}{M_\tau^2}\right)^l \frac{dR_\tau}{ds} = R_{\tau,V+A}^{kl} + R_{\tau,S}^{kl}$$

moments

(notice that $R_\tau^{00} = R_\tau$)

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = N_C S_{\rm EW} \sum_{D\geq 2} \left(\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)}\right)$$

The most relevant contributions come from D=2,4 : $\delta^{(2)} \propto \frac{m^2}{M_\tau^2}, \quad \delta^{(4)} \propto \frac{m\langle \overline{q}q\rangle}{M_\tau^4}$

$$\delta R_\tau^{kl}\big|_{\rm theo} = f\left(|V_{ud}|, |V_{us}|, m_s\right)$$

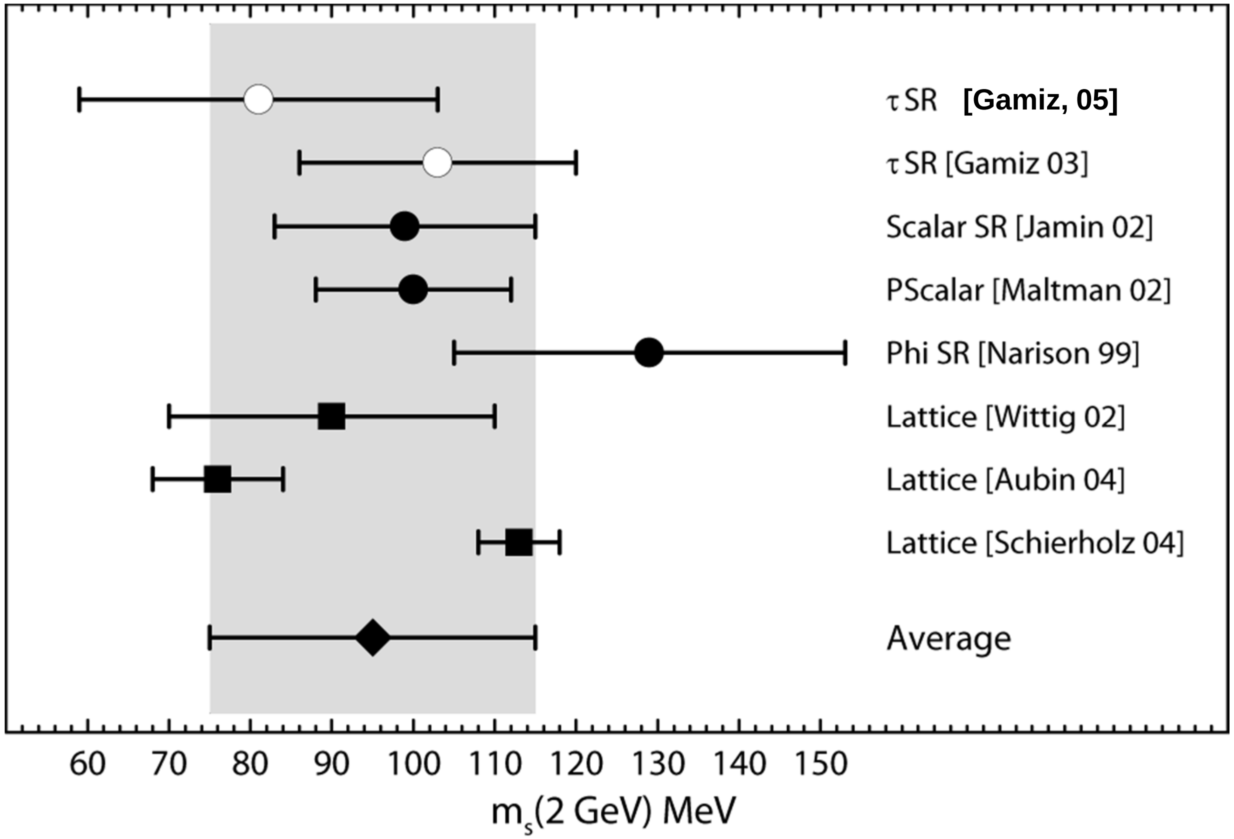
$$\begin{aligned} \delta R_\tau^{00}\big|_{\rm theo} &= 0.240(32) \\ R_{\tau,V+A}^{00} &= 3.4671(84) \\ R_{\tau,S}^{00} &= 0.162(28) \\ |V_{ud}| &= 0.97425(22) \end{aligned} \Bigg\} \; |V_{us}| = 0.2173(20)_{exp}(10)_{th}$$

[25, 26]

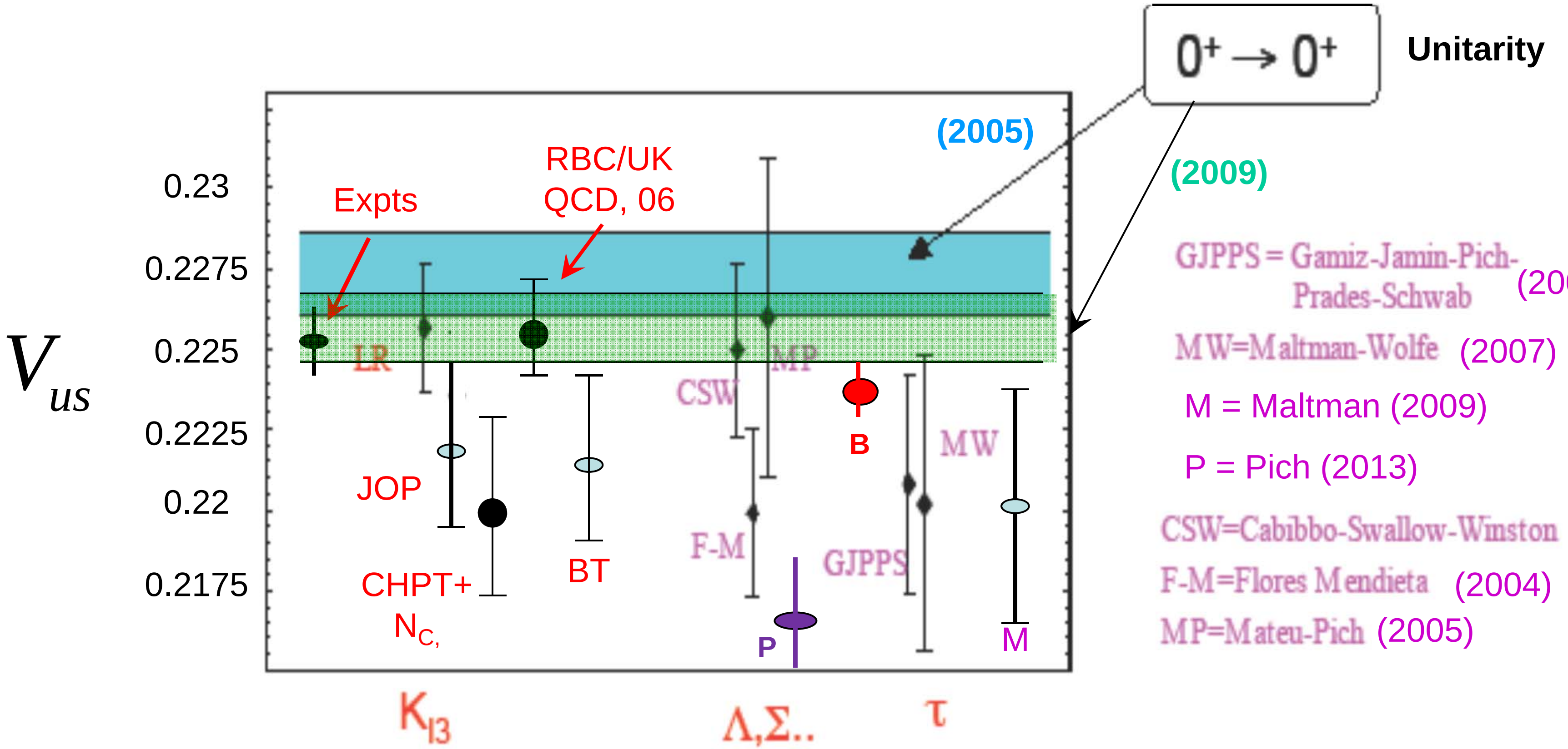
Joint fit

$$\begin{aligned} m_s(2\,\rm GeV) &\simeq 76\,\rm MeV \\ |V_{us}| &\simeq 0.2196 \end{aligned}$$

$$m_s(2\,\mathrm{GeV})|_{\mathrm{average}} = (95 \pm 20)\ \mathrm{MeV}$$



V_{us}



GJPPS = Gamiz-Jamin-Pich-Prades-Schwab (2005)

MW=Maltman-Wolfe (2007)

M = Maltman (2009)

P = Pich (2013)

CSW=Cabibbo-Swallow-Winston (2004)

F-M=Flores Mendieta (2004)

MP=Mateu-Pich (2005)

LR = Leutwyler-Roos (1984)

JOP = Jamin-Oller-Pich (2004)

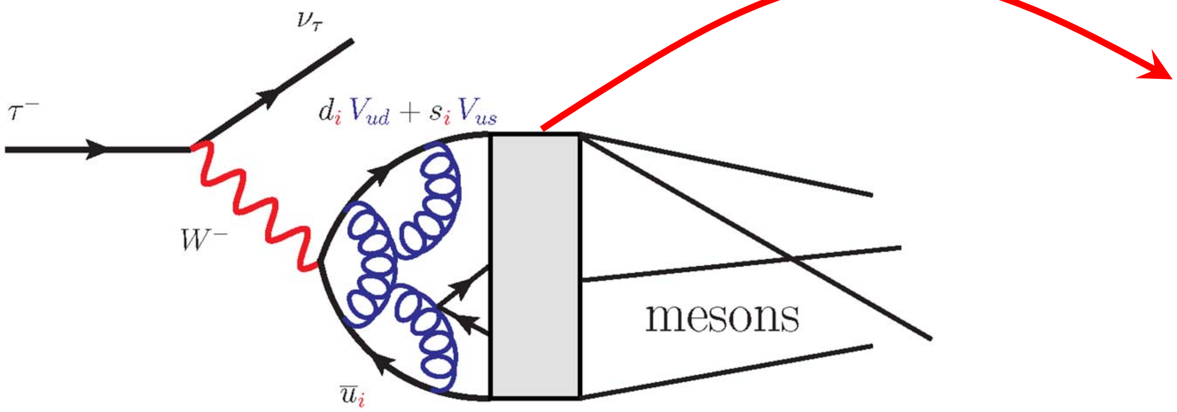
BT = Bijnens-Talavera (2003)

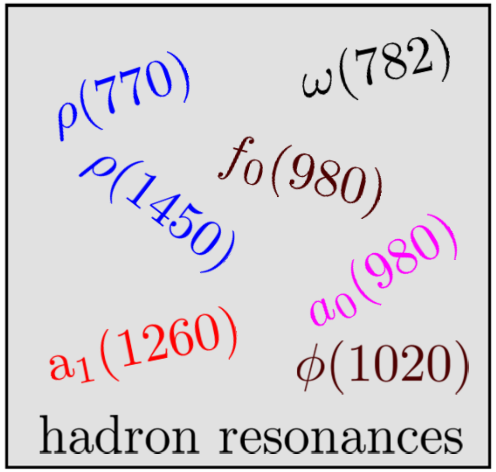
CHPT+N_C = Cirigliano et al (2006)

Expts = FLAVIAnet WG (2010)

B = A. Bazavov et al. (2012)

2.2 Exclusive hadron decays





$$\mathcal{M}(\tau \to \nu_\tau \mathrm{H}) = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}} \, \overline{u}_{\nu_\tau} \gamma^\mu (1-\gamma_5) u_\tau \, \langle H | \, (V_\mu - A_\mu) \, e^{i \, \mathrm{L_{QCD}}} | \Omega_{\mathrm{H}} \rangle$$

$$\langle H | \, (V_\mu - A_\mu) \, e^{i \, \mathrm{L_{QCD}}} | \Omega_{\mathrm{H}} \rangle = \sum_i (\text{Lorentz structure})^i{}_\mu \, F_i(Q^2, s, \ldots)$$

form factors

$$d\Gamma(\tau \to \nu_\tau H) = \frac{G_F^2}{4 M_\tau} |V_{\mathrm{CKM}}|^2 \, L_{\mu\nu} \, H^{\mu\nu} \, d\mathrm{PS}$$

$$\begin{cases} L_{\mu\nu} H^{\mu\nu} = \sum_X L_X W_X \\ W_X \equiv \textbf{structure functions} \end{cases}$$

Examples

$\mathrm{H} \;=\; PP$ $\qquad\qquad P = \pi, K, \eta, \eta'$

$$\langle P_1 P_2 | V_\mu \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \;=\; F_V(q^2) \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (p_1 - p_2)^\nu + F_S(q^2)\, q_\mu$$

$$q = p_1 + p_2 \qquad\qquad \partial^\mu V_\mu \;\propto\; (m_i - m_j)\, \overline{q}_i \, q_j$$

$$\langle \pi^- \pi^0 | V_\mu \, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle \;=\; F_V(q^2)\, (p_- - p_0)_\mu$$

Vector form factor

$\mathrm{H} \;=\; PPP$

$$\langle P_1^- P_2^- P_3^+ | \, (V_\mu - A_\mu)\; e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle =$$

$$\left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) \left[F_1(Q^2, s, t)\, (p_1 - p_3)^\nu + F_2(Q^2, s, t)\, (p_2 - p_3)^\nu \right]$$

$$+ F_3(Q^2, s, t)\, Q_\mu \;+\; i F_4(Q^2, s, t)\, \varepsilon_{\mu\alpha\beta\gamma}\, p_3^\alpha p_2^\beta p_1^\gamma$$

$$Q = p_1 + p_2 + p_3$$
$$s = (p_2 + p_3)^2$$
$$t = (p_1 + p_3)^2$$

F_3: $\pi\pi\pi, KK\pi, m_\pi = 0$

F_4: $\pi\pi\pi,\, SU(2)_I$

$$\tau \to \pi\pi\pi\nu_\tau$$

$$F_2(Q^2, s, t) = F_1(Q^2, t, s)$$

Bose symmetry, Axial-Vector only

$$\tau \to KK\pi\nu_\tau$$

Vector and Axial-Vector

Phenomenological Lagrangians : Tree Level

$$\mathcal{L}_\chi^2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle$$

$$\mathcal{L}_R = \sum_i \lambda_i \, \mathcal{O}_R^i(R,\phi)$$

$\mathcal{L}_R^K$ (kinetic)

Resonance Chiral Theory

RχT

[27,28]

Chiral Perturbation Theory

Resonance Fields

Large $- N_C$

$$u_\mu = i\left[u^\dagger\,(\partial_\mu - i\,r_\mu)\,u - u\,(\partial_\mu - i\,\ell_\mu)\,u^\dagger\right]$$

$$\chi_+ = u^\dagger \chi u^\dagger + u\chi^\dagger u, \qquad \chi = 2B_0\,(s + i\,p)$$

$$u = \exp\left(\frac{i}{F\sqrt{2}}\,\Pi(\Phi)\right)$$

$F =$ decay constant of the pion

$$B_0^2\,F = -\langle \overline{u} u \rangle$$

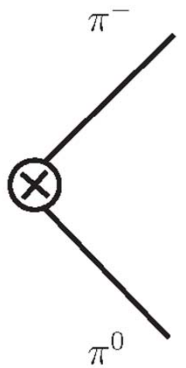
$$\mathcal{L}_R = \frac{F_V}{2\sqrt{2}}\,\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + \ldots$$

$$f_{\mu\nu}^{\pm} = uF_{\mu\nu}^L u^\dagger \pm u^\dagger F_{\mu\nu}^R u, \qquad F_{\mu\nu}^L = \partial_\mu \ell_\nu - \partial_\nu \ell_\mu - i\,[\ell_\mu, \ell_\nu]$$

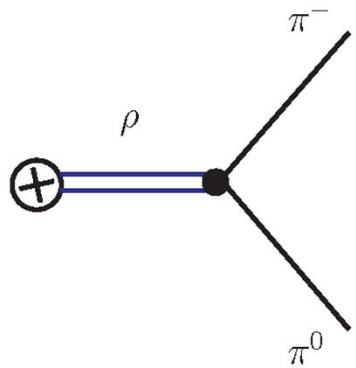
$\tau^- \to \pi^- \pi^0 \nu_\tau$ [29,30,31]

$$\langle \pi^- \pi^0 | V_\mu \, e^{i L_{QCD}}) | \Omega_h \rangle \; = \; \sqrt{2}\, F_V(q^2)\; (p_- - p_0)_\mu$$

$F_V(q^2) \; =$



$+$



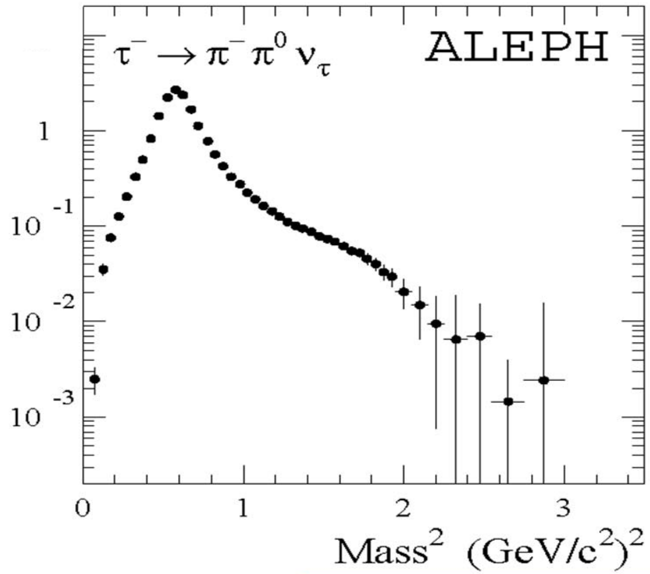
$$= \; 1 \; + \; \frac{F_V\, G_V}{F^2} \, \frac{q^2}{M_V^2 - q^2}$$

1– Short-distance constraints

$$F_V(q^2) \underset{q^2 \to \infty}{\longrightarrow} \frac{1}{q^2} \qquad \longrightarrow \qquad F_V\, G_V \, = \, F^2$$

2– Off-shell widths of resonances

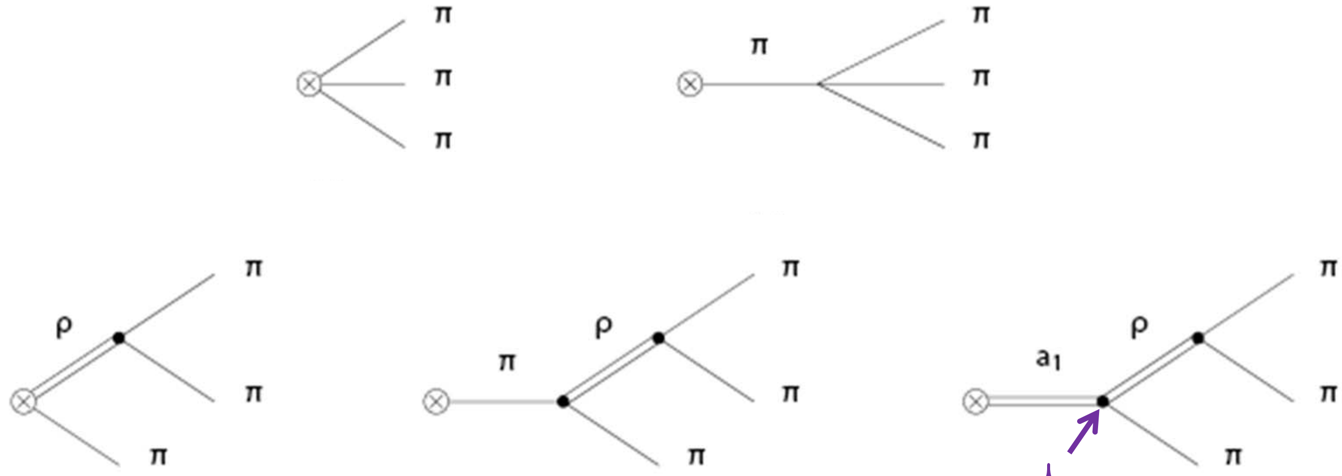
$$M_V^2 \; \longrightarrow \; M_V^2 - i M_V \Gamma_V(q^2)$$



$$\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$$

[32]

$$F_1(Q^2, s, t) \;=$$



$\lambda_0,\ \lambda',\ \lambda''$

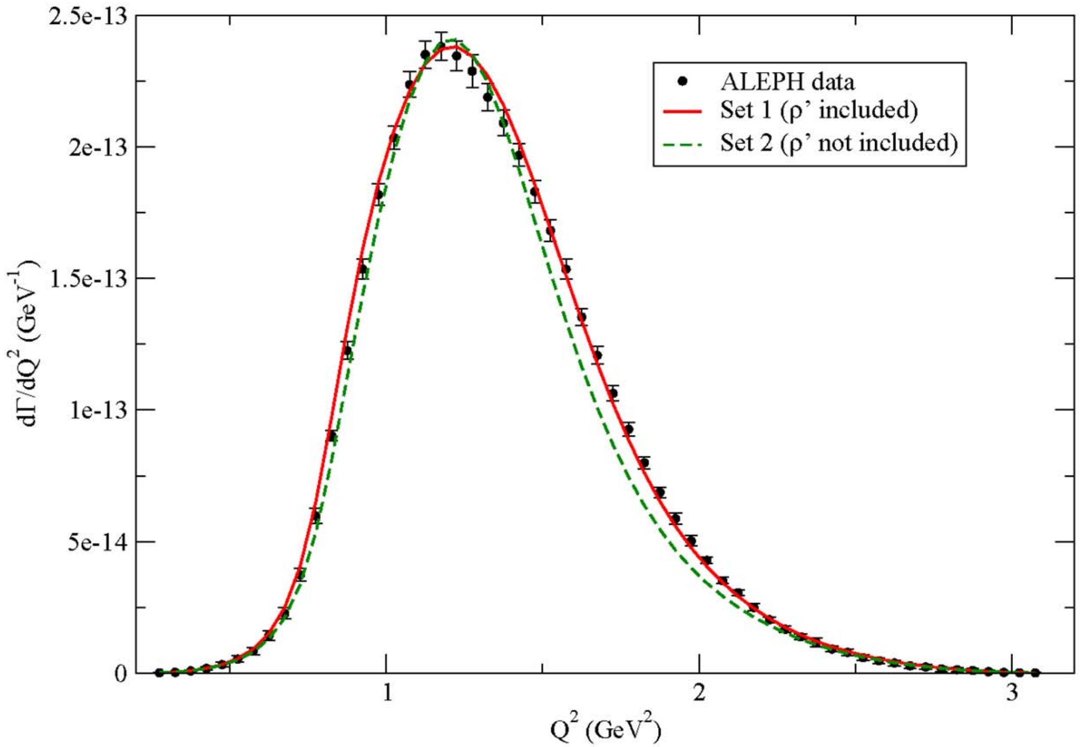
$$H(Q^2,x) = -\lambda_0 \frac{M_\pi^2}{Q^2} + \lambda' \frac{x}{Q^2} + \lambda''$$

$$F_1(Q^2,s,t) = -\frac{2\sqrt{2}}{3F} + \frac{\sqrt{2}F_V G_V}{3F^3}\left[\frac{3s}{s-M_V^2} - \left(\frac{2G_V}{F_V}-1\right)\left(\frac{2Q^2-2s-u}{s-M_V^2} + \frac{u-s}{t-M_V^2}\right)\right]$$
$$+\frac{4F_A G_V}{3F^3}\frac{Q^2}{Q^2-M_A^2}\left[-(\lambda'+\lambda'')\frac{3s}{s-M_V^2} + H(Q^2,s)\frac{2Q^2+s-u}{s-M_V^2} + H(Q^2,t)\frac{u-s}{t-M_V^2}\right]$$

Short-distance constraints

$$\mathrm{Im}\,\Pi_A(q^2) \underset{q^2\to\infty}{\longrightarrow} 0$$

$$\begin{cases} \lambda' = \dfrac{M_A}{2\sqrt{2}M_V} \\ \lambda'' = \dfrac{M_A^2 - 2M_V^2}{2\sqrt{2}M_V M_A} \\ \lambda_0 = (\lambda'+\lambda'')/4 \end{cases}$$



Set 1

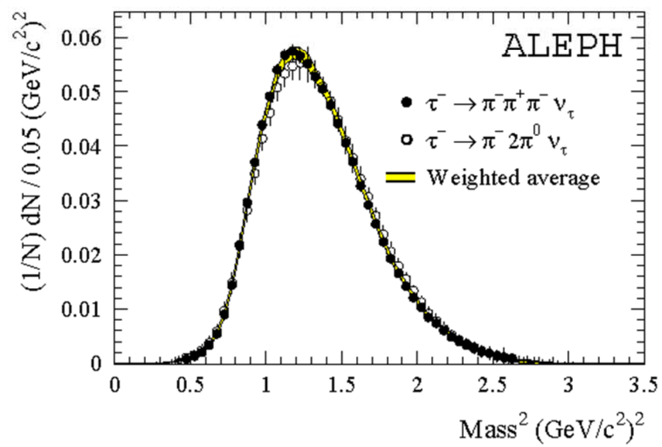
$$F_V = 0.180\,\text{GeV}, \qquad F_A = 0.149\,\text{GeV}$$
$$M_V = 0.775\,\text{GeV}, \qquad M_A = 1.120\,\text{GeV}$$

Set 2

$$F_V = 0.206\,\text{GeV}, \qquad F_A = 0.145\,\text{GeV}$$
$$M_V = 0.775\,\text{GeV}, \qquad M_A = 1.115\,\text{GeV}$$

$$\Gamma(\tau \to \pi\pi\pi\nu_\tau)\Big|_{\text{theo}} = 2.09 \times 10^{-13}\,\text{GeV}$$

$$\Gamma(\tau \to \pi\pi\pi\nu_\tau)\Big|_{\text{exp}} = 2.11(02) \times 10^{-13}\,\text{GeV}$$



1. Inclusive decays: full hadron spectra. Precision physics.

$$\tau^- \rightarrow \nu_\tau \, (\bar{u}d, \bar{u}s)$$

→ Study of Standard Model parameters : $\alpha_S(M_\tau)$, $|V_{us}|$, m_S

2. Exclusive decays: specific hadron spectrum. Approximate physics

$$\tau^- \rightarrow \nu_\tau \, (PP, PPP, ...)$$

P = pseudoscalar meson

→ Study of form factors, resonance parameters (M_R, Γ_R), hadronization of QCD currents.

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