IDPASC School of Flavour
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Vector form factor of the pion

The vector form factor of the pion, $F_V(q^2)$ determines the dynamics of the $\tau^- \to \pi^-(p_-)\pi^0(p_0)\nu_\tau$ process. It is defined by:

$$\langle \pi^-\pi^0 | V_\mu^{1-i2} | \Omega_h \rangle \;=\; \sqrt{2}\, F_V(q^2)\,(p_- - p_0)_\mu \,,$$

where $q_\mu = (p_- + p_0)_\mu$ and $V_\mu^i = \overline{q}\gamma_\mu \frac{\lambda^i}{2} q$ is the SU(3) vector current (λ^i are the Gell-Mann matrices and $q = (u,d,s)^T$).

1/ Calculate the leading result for the vector form factor as given by the following phenomenological Lagrangian:

$$\mathcal{L}_1 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle + \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu \rangle .$$

Here $\langle ... \rangle$ indicates a trace in the flavour space and

$$\begin{aligned} u_\mu &= i\left[u^\dagger\left(\partial_\mu - i r_\mu\right)u - u\left(\partial_\mu - i\ell_\mu\right)u^\dagger\right], & u &= \exp\left(i\frac{1}{\sqrt{2}F}\Phi\right), \\ \chi_+ &= 2B_0\left[u^\dagger(s+ip)u^\dagger + u(s-ip)u\right], & f_+^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u, \\ F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu], & F_L^{\mu\nu} &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu,\ell^\nu], \end{aligned}$$

where $r_\mu = (v+a)_\mu$, $\ell_\mu = (v-a)_\mu$, s and p are external currents and $v_\mu = v_\mu^i \lambda^i/2$ and $a_\mu = a_\mu^i\lambda^i/2$ (index i summed over) are the vector and axial-vector external currents, and

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}.$$

Vector meson fields $V_{\mu\nu}$ in $\mathcal{L}_1$ are given by the antisymmetric tensor representation. Its use (wave function and propagator) are given in Appendix A of reference: G. Ecker et al, Nucl. Phys. B321 (1989) 311.

2/ Repeat the calculation above with the phenomenological Lagrangian:

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle + \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + i\frac{g_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu \rangle$$

where now we use the Proca representation for the vector fields, V_μ, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$.

3/ Compare the asymptotic behaviour ($q^2 \to \infty$) in the results for the vector form factor given by $\mathcal{L}_1$ and $\mathcal{L}_2$. Determine the corresponding constraints on the couplings of the phenomenological Lagrangians.

4/ You have found a problem in the above point when using $\mathcal{L}_2$ to evaluate the vector form factor. How the problem can be solved?