which, from eq. (91), has the divergence

$$\partial_\mu D^\mu(x) = m_A\, \bar{q}^A_\alpha(x)\, q^A_\alpha(x) \tag{I.95}$$

and is thus conserved in the limit of massless quarks. This is the Belinfante dilatation current. Its canonical counterpart is the Noether current of another symmetry of the Q.C.D. Lagrangian in the limit of massless quarks: dilatation or scale invariance. As we will see later on, in the quantum theory eqs. (91) and (95) will have a further contribution, called the trace anomaly, much the same as what happened in eq. (47) with the axial current.

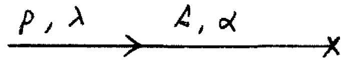
There is still one term allowed by gauge invariance and renormalizability which can be added to the Q.C.D. Lagrangian

$$\mathcal{L}_\theta(x) \equiv -\frac{\theta g^2}{64\pi^2}\, \epsilon^{\mu\nu\rho\sigma}\, F^a_{\mu\nu}(x)\, F^a_{\rho\sigma}(x) \tag{I.96}$$

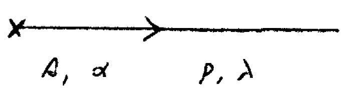
As we saw in (49) this term is a total divergence and leaves therefore the equations of motion unchanged. It is irrelevant for perturbation theory. Its contribution to the Q.C.D. action is proportional to the winding number and it is thus responsible for instanton effects. It influences the structure of the Q.C.D. vacuum through tunneling. We will omit it, because we will work almost exclusively within perturbation theory. Notice that it violates the P and CP symmetries, but these effects can be made very small by taking θ very small. Parity and charge conjugation are otherwise symmetries of the Q.C.D. Lagrangian, as corresponds to a theory of the strong interactions.

Let us finally give the Feynman rules for Q.C.D.

Quark

Incoming 

$u^A_\alpha(\vec{p}, \lambda)$

Outgoing 

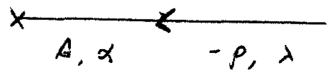
$\bar{u}^A_\alpha(\vec{p}, \lambda)$

Antiquark

Incoming

$$\bar{v}^{A}_{\alpha}(\vec{p},\lambda)$$

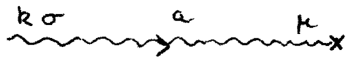
Outgoing



$$v^{A}_{\alpha}(\vec{p},\lambda)$$

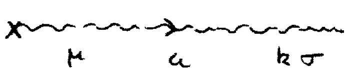
Gluon

Incoming



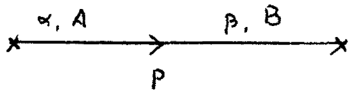
$$\epsilon^{a}_{\mu}(\vec{k},\sigma)$$

Outgoing



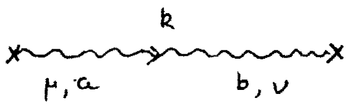
$$\epsilon^{a*}_{\mu}(\vec{k},\sigma)$$

Quark propagator



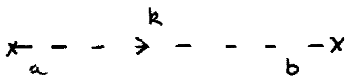
$$+\frac{i}{(2\pi)^4}\,\frac{1}{\not{p}-m_A+i\eta}\,\delta_{\alpha\beta}\,\delta_{AB}$$

Gluon propagator



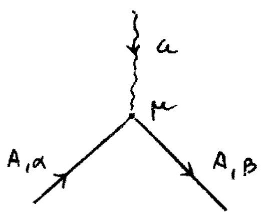
$$-\frac{i}{(2\pi)^4}\left[g_{\mu\nu}-(1-a)\frac{k_\mu k_\nu}{k^2+i\eta}\right]\frac{\delta_{ab}}{k^2+i\eta}$$

Ghost propagator



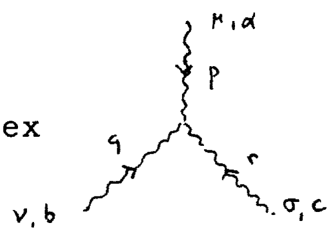
$$-\frac{i}{(2\pi)^4}\,\frac{\delta_{ab}}{k^2+i\eta}$$

Fermionic vertex



$$+\,g\left(\frac{\lambda_a}{2}\right)_{\beta\alpha}\gamma^{\mu}$$

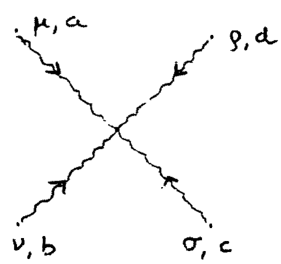
Triple gluon vertex



$$-ig\,f_{abc}\,[\,g_{\mu\nu}\,(p-q)_\sigma$$

$$+\,g_{\nu\sigma}\,(q-r)_\mu+g_{\sigma\mu}\,(r-p)_\nu\,]$$

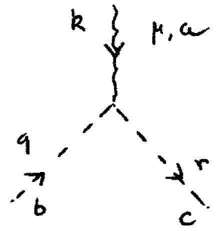
Quartic gluon vertex



$$-g^2\,[\,f_{abe}\,f_{cde}\,(g_{\mu\sigma}\,g_{\nu\rho}-g_{\mu\rho}\,g_{\nu\sigma})$$

$$+\,f_{ace}\,f_{bde}\,(g_{\mu\nu}\,g_{\sigma\rho}-g_{\mu\rho}\,g_{\nu\sigma})$$

$$+\,f_{ade}\,f_{cbe}\,(g_{\mu\sigma}\,g_{\nu\rho}-g_{\mu\nu}\,g_{\sigma\rho})\,]$$

Ghost vertex 

$-ig f_{abc} r^{\mu}$

The ghost propagator can be taken with opposite sign, as long as one also changes the sign of the ghost vertex. Physical Green's functions are unchanged, because ghosts only appear in loops.

Furthermore we must take into account the following rules

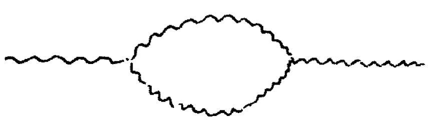
i) For each vertex write a factor $(2\pi)^4 \delta\left(\sum_{in} p - \sum_{out} p\right)$

ii) Multiply the obtained expression by $(2\pi)^{-4} i^{n+1}$, where n is the number of vertices.

iii) Integrate over all internal momenta and extract a δ - function expressing the total energy-momentum conservati

iv) Multiply by (-1) for each internal loop of quarks or ghosts.

v) Take into account the correct statistical factors



1/2!

1/3!

vi) The arrows along quark and ghost lines should point all into the same directions.

vii) One should put a relative minus sign in contributions coming from diagrams which only differ by the exchange of identical external fermions.

This leads to the T matrix elements, where the S and T matrices are related by

$$S = 1 - i\,(2\pi)^4\,\delta\Big(\sum_{in} p_i - \sum_{out} p_f\Big)\,T \qquad \text{(I.97)}$$

The choices of the gauge parameter $a = 0$ and $a = 1$ are called Landa and Feynman gauge, respectively.