SM II: The Standard Model

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1. Building the Standard Model
   - Choice of the gauge group and fermion representations
   - Choice of the pattern of symmetry breaking
   - Charged and Neutral Current Interactions
   - Fermion masses and mixings
   - Gauge and Higgs Bosons Interactions
   - Fixing the SM parameters and Radiative Corrections

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2. Testing the Standard Model
   - The Z line-shape and the global fit
   - Vector and axial Z-couplings and universality
   - LEP2 and the non-Abelian couplings
   - (*) Fermion masses and mixings
   - The Higgs boson: Perturbativity, Triviality and Stability
   - m_H from Radiative Corrections
   - The Higgs Boson Discovery

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2. Testing the Standard Model

Choice of the gauge group

Consider the one generation IVB model plus QED

$$\mathcal{L} = \frac{g}{2\sqrt{2}} \left(J^\mu W_\mu^{(+)} + \mathrm{h.c.} \right) + e J_\mu^{\mathrm{em}} A^\mu$$

$$J_\mu = 2\,(\bar{\nu}_L \gamma_\mu e_L + \bar{u}_L \gamma_\mu d_L)\,, \quad J_\mu^{\mathrm{em}} = -\bar{e}\gamma_\mu e + \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$$

Define the week and electric charges as

$$T_+ = \frac{1}{2}\int d^3x J_0(x)\,,\ T_- = T_+^\dagger,\ Q = \int d^3x J_0^{\mathrm{em}}(x)$$

One can show that $[T_+, T_-] = 2T_3\,, \quad [T_3, T_\pm] = \pm T_\pm$ with

$$T_3 = \frac{1}{2}\int d^3x \left(\nu_L^\dagger \nu_L - e_L^\dagger e_L + u_L^\dagger u_L - d_L^\dagger d_L \right)$$

$SU(2)$ **algebra** in terms of ladder operators!

however T_3 cannot be the charge Q. Define $Y = 2(Q - T_3)$ with

$$J_\mu^Y \equiv -(\bar{\nu}_L\gamma_\mu\nu_L + \bar{e}_L\gamma_\mu e_L) + \frac{1}{3}(\bar{u}_L\gamma_\mu u_L + \bar{d}_L\gamma_\mu d_L) - 2\bar{e}_R\gamma_\mu e_R + \frac{4}{3}\bar{u}_R\gamma_\mu u_R - \frac{2}{3}\bar{d}_R\gamma_\mu d_R$$

and $Y = \int d^3\vec{x}J_0^Y$, which satisfies

$$[Y, T_\pm] = 0\,, \quad [Y, T_3] = 0$$

The gauge group will then be the direct product of $SU(2)$ and a $U(1)$ group with generator Y

$$W_\mu^1\,,\ W_\mu^2\,,\ W_\mu^3 \leftarrow SU(2)_L \otimes U(1)_Y \rightarrow B_\mu$$

4 gauge bosons: prediction of neutral currents!

Choice of the fermion representations

From the composition of the currents the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ multiplets are

$$
\begin{aligned}
L_L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad & \sim (1, 2, -1), \\
e_R \quad & \sim (1, 1, -2), \quad \nu_R \sim (1, 1, 0)? \\
Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad & \sim (3, 2, \tfrac{1}{3}), \\
d_R \quad & \sim (3, 1, -\tfrac{2}{3}), \quad u_R \sim (3, 1, \frac{4}{3})
\end{aligned}
$$

- **Anomalies cancel within a family if $N_C = 3$** $\Rightarrow$ The SM should contain **complete families**.
- **Anomally cancellation fix uniquely all hypercharges!**

The Lagrangian before SSB

Each family contains 5 multiplets (6 if ν_R exist)

$$\mathcal{L}_F = \sum_i i\,\bar{\psi}_i \gamma^\mu D_\mu \psi_i\,, \qquad \psi_i = (Q_L, d_R, u_R, L_L, e_R)$$

$$D_\mu \psi_i \equiv \left(\partial_\mu - ig\vec{T}\vec{W}_\mu - ig'\frac{Y_i}{2}B_\mu\right)\psi_i$$

with $\vec{T} = \vec{\tau}/2$ acting on doublets and $\vec{T} = 0$ acting on singlets
A_μ and Z_μ linear combinations of W^3_μ and B_μ

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$\vec{W}_{\mu\nu} \equiv \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g\,\vec{W}_\mu \times \vec{W}_\nu\,, \quad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

contains non-Abelian $W_3\, W^+ W^-$ **couplings** with the coefficient needed to solve $e^+e^- \to W^+\, W^-$ problems in the IVB.
At this point, W, Z and all fermions are still massless
Need for **SSB.** Physical spectrum **better seen after SSB.**

Spontaneous symmetry breaking

Massive $W^{\pm}, Z$ require SSB with the following patern

$$SU(2)_L \otimes U(1)_Y \xrightarrow{\text{SSB}} U(1)_Q$$

three Goldstone. **Charged fields** should not get VEV
Fermion masses are a doublet with $Y = -1$
$T_3(\overline{e_L} e_R) = 1/2$ while $Y(\overline{e_L} e_R) = -1 \Rightarrow \Phi$ doublet $Y = 1$

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad Y(\Phi) = 1$$

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger \, D^\mu \Phi - V(\Phi), \qquad V(\Phi) = \mu^2 \, \Phi^\dagger \Phi + \lambda \, (\Phi^\dagger \Phi)^2$$

For $\mu^2 < 0$ there is SSB. To preserve the **charge**, $Q\langle \Phi \rangle = 0$

$$\langle \Phi \rangle \equiv \langle 0 | \, \Phi \, | 0 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Exponential parametrization to find the physical spectrum

$$\Phi = \frac{(v+H)}{\sqrt{2}} \exp\left(i \frac{\vec{\tau}}{2} \vec{\theta}/v \right) \left(\begin{array}{c} 0 \\ 1 \end{array} \right)$$

Choice $\vec{\theta} = 0$, (unitary gauge), no unphysical fields

$$\begin{aligned} \mathcal{L}_\Phi &= \left| \left(\partial_\mu - ig\frac{\vec{\tau}}{2}\vec{W}_\mu - i\frac{g'}{2}B_\mu \right) \frac{(v+H)}{\sqrt{2}} \left(\begin{array}{c} 0 \\ 1 \end{array} \right) \right|^2 \\ &\quad - \mu^2 \frac{(v+H)^2}{2} - \lambda \frac{(v+H)^4}{4} \end{aligned}$$

Expanding, one finds the mass terms

$$\begin{aligned} \mathcal{L}_M &= -\lambda v^2 H^2 + \frac{v^2}{8} g^2 \left(W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu} \right) \\ &+ \frac{v^2}{8} \left(g^2 W^3_\mu W^{3\mu} - 2gg' W^3_\mu B^\mu + g'^2 B_\mu B^\mu \right) \end{aligned}$$

the W_μ^1 and W_μ^2 can be combined in fields of definite charge

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i\, W^2\right)$$

the W_μ^3 and B_μ mass terms can be rewritten in matrix form as

$$\frac{v^2}{8}\left(W_\mu^3, B_\mu\right)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

diagonalization leads to two eigenstates:
one massless, which we will identify with the **photon, A_μ**
one massive, which will be identified with the **Z-gauge boson**

$$\begin{aligned} Z_\mu &= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu\,, \qquad \tan\theta_W = g'/g \end{aligned}$$

Mass terms are written as

$$\mathcal{L}_M = -\frac{1}{2} m_H^2 H^2 + m_W^2 W_\mu^+ W^{-\,\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$m_H^2 = 2\lambda v^2 \,, \quad m_W^2 = \frac{v^2}{4} g^2 \,, \quad m_Z^2 = \frac{v^2}{4}(g^2 + g'^2) = \frac{v^2}{4}\frac{g^2}{\cos^2\theta_W}$$

Precise (tree-level) relation between masses and $\cos\theta_W$

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

Custodial symmetry of the Higgs potential for **doublets**
This symmetry does not exist is for other multiplets of scalars
Also broken by B_μ-interactions and Yukawa couplings
(will appear in radiative corrections)

Charged and Neutral Current Interactions

Rewriten the interacctions in terms of W_μ and Z_μ one finds (for one family)

$$\mathcal{L}_{\mathrm{CC}} = \frac{g}{\sqrt{2}} \left\{ W_\mu^+ \left[\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L \right] + \mathbf{h.c.} \right\}$$

$$\mathcal{L}_{\mathrm{NC}} = \mathcal{L}_{\mathrm{QED}} + \mathcal{L}_{\mathrm{NC}}^Z \,, \qquad e = g \sin\theta_W = g' \cos\theta_W$$

while

$$\mathcal{L}_{\mathrm{NC}}^Z = \frac{e}{2\sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu (g_{Vf} - g_{Af}\gamma_5) f \,,$$

where $a_f = T_3^f$ and $v_f = T_3^f \left(1 - 4|Q_f| \sin^2\theta_W\right)$

Fermion masses and mixings

For N_g families of fermions (and no ν_R's) we can write

$$\mathcal{L}_Y = -\bar{L}_L Y_e \Phi e_R - \bar{Q}_L Y_d \Phi d_R - \bar{Q}_L Y_u \tilde{\Phi} u_R + \text{h.c.}$$

where, L_L, Q_L, e_R, d_R, u_R are all vectors in family space and $\tilde{\Phi} = i\tau_2 \Phi^*$ and Y_e, Y_d, Y_u, are $N_g \times N_g$ **matrices**

After SSB ($M_e = Y_e v/\sqrt{2}\,, \quad M_d = Y_d v/\sqrt{2}\,, \quad M_u = Y_u v/\sqrt{2}$)

$$\mathcal{L}_Y = -(1 + \frac{H}{v})\left(\bar{e}_L M_e e_R + \bar{d}_L M_d d_R + \bar{u}_L M_u u_R + \text{h.c.}\right)$$

Diagonalization

$$M_e = V_L D_e V_e^\dagger\,, \qquad L_L \to V_L L_L\,, \quad e_R \to V_e e_R$$

All V's in the leptonic sector can be removed (Not true if ν_R)

Conservation of family lepton numbers

Similarly for M_u

$$M_u = V_Q D_u V_u^\dagger\,, \qquad Q_L \to V_Q Q_L\,, \quad u_R \to V_u u_R$$

But

$$M_d = V D_d V_d^\dagger\,, \qquad d_L \to V d_L\,, \quad d_R \to V_d d_R$$

- All V's removed in the NC quark sector
  No flavor changing neutral currents (GIM mechanism)
- V only in d_L not in u_L; does not leave CC invariant

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \left\{ W_\mu^+ \, [\bar{u}_L \gamma^\mu V d_L + \bar{\nu}_L \gamma^\mu e_L] + \textbf{h.c.} \right\}$$

For N_g generations, V general $N_g \times N_g$ unitary matrix
($N_g(N_g-1)/2$ moduli and $N_g(N_g+1)/2$ phases)
Rest of Lagrangian invariant under

$$u_{aL,R} \to e^{i\alpha_a} u_{aL,R}\,, \quad d_{aL,R} \to e^{i\beta_a} d_{aL,R}$$

$2N_g - 1$ phases removed (baryon number is conserved)

Physical Parameters

$N_g(N_g-1)/2$ angles and $(N_g-1)(N_g-2)/2$ phases

For $N_g=2$: **1 angle** and **0 phases** $\Rightarrow$ **no CP violation**

For $N_g=3$: **3 angles** and **1 phase**: **CP Violation** $N_g \geq 3$

V CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

usually parametrized as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Gauge boson self-interactions

Substitution of $\vec{W}_{\mu\nu}, B_{\mu\nu}$ in the kinetic terms, and rewriting in terms of W_μ, Z_μ, A_μ one obtains a series of trilinear and quartic selfinteractions arising from the non-Abelian part of $\vec{W}_{\mu\nu}$:

- All couplings are dimensionless (trilinear contain one derivative, and quartic do not contain derivatives)
- They contain always (at least) **two** W and **one or two neutral (Z or γ)**

Higgs couplings: Obtained by the following substitution

$$m \to m\left(1+\frac{H}{v}\right)\ , m \neq m_H\ ; \quad m_H^2 \to m_H^2\left(1+\frac{H}{v}+\frac{H^2}{4v^2}\right)$$

- **Produced in** association with **heavy particles**
- **Decay** into the **heaviest** accessible **particles**

Fixing the SM parameters and radiative corrections

Gauge and scalar sector **only 4 for parameters:** g,g',μ^2 and λ.
Best known observables α, G_F, m_Z (and m_H), rest derived

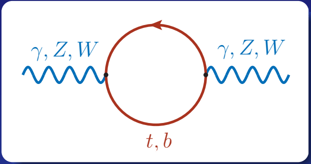
$$\sin^2\theta_W = \frac{1}{2}\left\{1-\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}G_F m_Z^2}}\right\} = 0.21215$$

$$m_W^2 = m_Z^2\cos^2\theta_W = (80.94\,\mathrm{GeV})^2$$

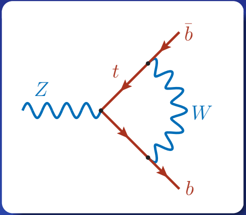
Precision sometimes better than 1%. Rad. Corr. compulsory

- Photonic corrections (ISR important because IR and collinear effects)
- QCD corrections: Very important. Often $N_C \Rightarrow N_C\left\{1+\frac{\alpha_s(s)}{\pi}+\cdots\right\} \approx 3.115$
- Pure electro-weak corrections (Oblique, Vertex, Boxes)

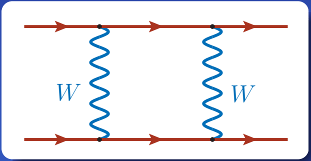
Oblique (gauge boson selfenergies): Dominated by the **running of α** and some large **top-quark** and **Higgs boson** mass contributions



Vertex corrections: usually **small** at large energies except for loops containing the **top quark.** Important in **flavour changing processes**



Boxes: usually **small** at large energies but very important in some low energy and in **flavour changing processes**



Relations modified by radiative corrections. In $\overline{\mathrm{MS}}$ **scheme**

$$s_Z^2 = \frac{g'^2(m_Z)}{g'^2(m_Z)+g^2(m_Z)} = \frac{1}{2}\left\{1-\sqrt{1-\frac{4\pi\hat{\alpha}(m_Z)}{\sqrt{2}G_F m_Z^2\hat{\rho}}}\right\} = 0.231$$

$$m_W^2 = \hat{\rho}c_Z^2 m_Z^2 = (80.34\,\mathrm{GeV})^2, \quad \hat{\rho}\approx 1.009\,, \quad \hat{\alpha}(m_Z) = 1/127.918$$

to be compared with the global fit
$s_Z^2 = 0.23120 \pm 0.00015$ and $m_W = 80.385 \pm 0.015\,\mathrm{GeV}$
Sometimes one can reach precisions **better than 1%** by using **tree level in terms of s_Z, $\hat{\alpha}(m_Z)$ and m_Z**
The dominant **top-quark/Higgs** mass corrections are **only in $\hat{\rho}$**
(Information on m_t and m_H from **precission measurements**!)

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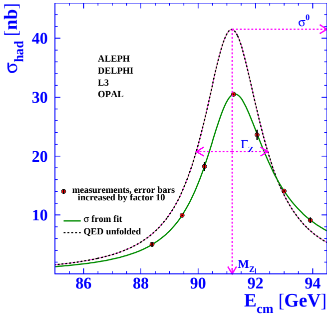
The line-shape of the Z

Close to the Z **peak** the cross section for $e^+e^- \to f\bar{f}$ is completely dominated by the resonance,

$$\sigma^0(e^+e^- \to f\bar{f}) \approx \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2}\frac{s}{(s-m_Z^2)^2+s^2\,\Gamma_Z^2/m_Z^2}$$

LEP gives:



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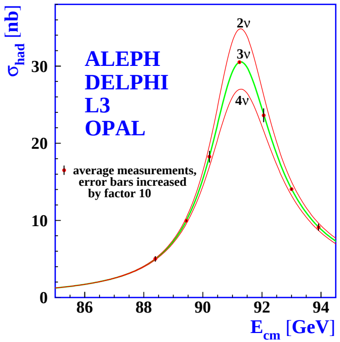


Invisible Z width,
in the SM $\Gamma_{inv} = N_\nu \Gamma_\nu$
$(\Gamma_\nu/\Gamma_\ell)_{SM} = 1.9912(8)$
while mearument
$\Gamma_{inv}/\Gamma_\ell = 5.941 \pm 0.016$
comparing

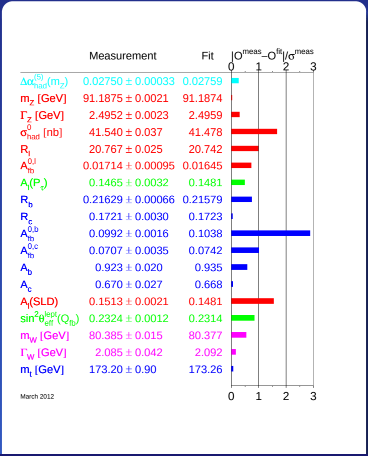
$$N_\nu = 2.984 \pm 0.009$$

# of active light neutrinos

The Global Fit

Expressed in terms of G_F, $\hat{\alpha}(m_Z)$, m_Z, m_t, m_H, $\alpha_s(m_Z)$

$$\chi^2(\text{parameters}) = \sum_i \left(\frac{\mathcal{O}^i_{\text{th}}(\text{parameters}) - \mathcal{O}^i_{\text{exp}}}{\Delta \mathcal{O}^i} \right)^2$$



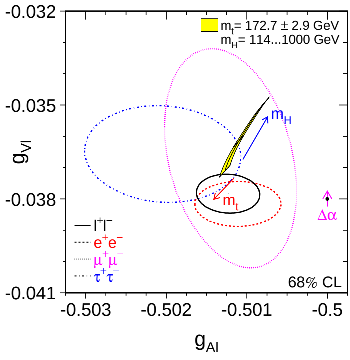
Parameters determined by **minimizing** χ^2

	Global fit
m_Z	$91.1874 \pm 0.0021\,\text{GeV}$
m_H	$99^{+28}_{-23}\,\text{GeV}$
m_t	$173.3 \pm 1\,\text{GeV}$
$\alpha_s(m_Z)$	0.1196 ± 0.0017
$1/\hat{\alpha}(m_Z)$	127.944 ± 0.014

$$\text{Pull}_i = \left(\mathcal{O}^i_{\text{th}}(\text{fit}) - \mathcal{O}^i_{\text{exp}}\right)/\Delta\mathcal{O}^i$$

The couplings of fermions and universality

The **partial Z widths** in the different lepton flavors together with the **asymmetries** allows for a determination of all **lepton neutral-current couplings**, The values of $g_{V\ell}$ and $g_{A\ell}$ can be obtained for $\ell = e, \mu, \tau$ and quarks c, b.



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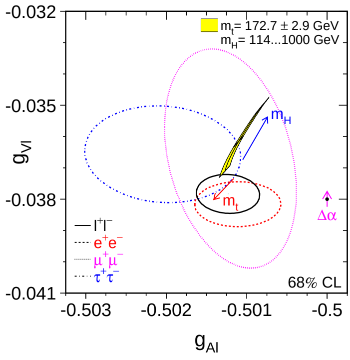

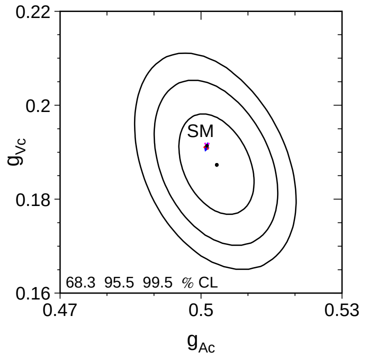

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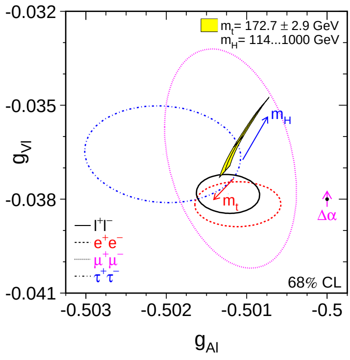
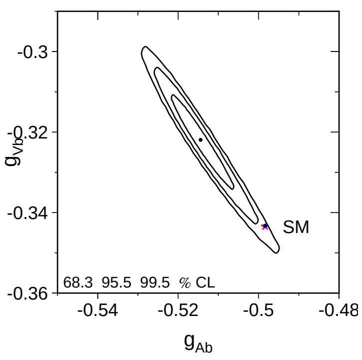
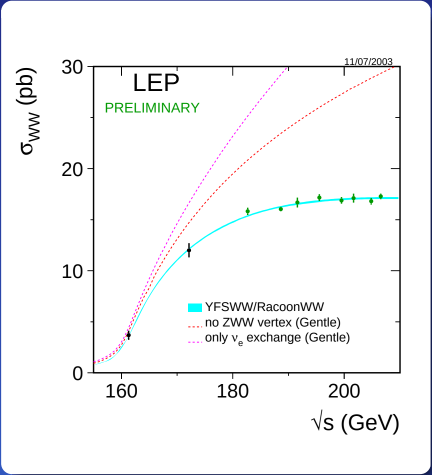




LEP2 and the non-Abelian couplings

The **unitarity problems of the IVB** and the need for non-Abelian couplings were one of the main points that triggered the development of the SM. **Tested at LEP2**



Quark masses

Quarks are not free: different definitions

***u*,*d*,*s* quark masses** determined indirectly from **Lattice QCD** and χPT. The *s* quark mass also determined from its effects in hadronic tau decays. Presented in terms of $\bar{m}(\mu = 2\,\mathrm{GeV})$.
$m_u \approx 2.15 \pm 0.15\,\mathrm{MeV}, \quad m_d \approx 4.7 \pm 0.2\,\mathrm{MeV}, \quad m_s \approx 94 \pm 3\,\mathrm{MeV}$

***c*,*b*, quark masses** determined from **heavy quark bound-states** using **Lattice QCD**, **HQET** and **NRQCD**, we give $\bar{m}(\bar{m})$
$m_c \approx 1.27 \pm 0.03\,\mathrm{GeV}, \quad m_b \approx 4.18 \pm 0.03\,\mathrm{GeV}$

***t* quark mass** determined from **direct production** at Fermilab and from radiative corrections in electroweak observables m_{pole}:
$m_t \approx 174 \pm 1\ \mathrm{GeV}\ , \quad (\mathrm{pole})\ , \quad (\bar{m}_t(\bar{m}_t) \approx 164 \pm 1\ \mathrm{GeV})$

Quark mixings

Mixings obtained from **semileptonic decays** of hadrons $H \to H' l \bar{\nu}_l$ (associated with $d_j \to u_i l^- \bar{\nu}_l$) together with data from **hadronic decays of the W** and from **top decays**

$$|V_{ij}| = \begin{bmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351\,^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412\,^{+0.0011}_{-0.0005} \\ 0.00867\,^{+0.00029}_{-0.00031} & 0.0404\,^{+0.0011}_{-0.0005} & 0.999146\,^{+0.000021}_{-0.000046} \end{bmatrix}$$

Hierarchical pattern $\Rightarrow$ Wolfenstein parametrization

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = |V_{us}| \approx 0.22\,, \qquad A \approx 0.8\,, \qquad \sqrt{\rho^2 + \eta^2} \approx 0.4$$

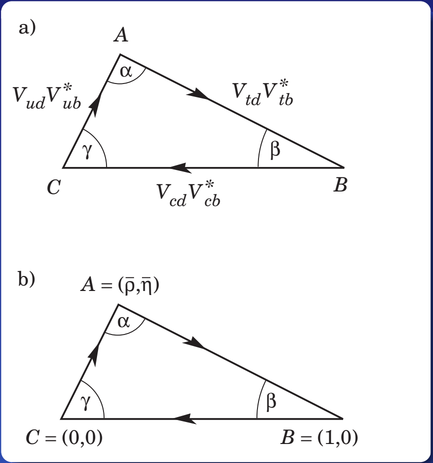
Useful to estimate the size of amplitudes and CP violation

CP Violation

To disentangle CP violation and determine δ, the **only source of CP-violation** in the SM, it is important to use the unitarity of the CKM matrix $\sum_{k=u,c,t} V_{ki} V_{kj}^* = \delta_{ij}$ with $i, j = d, s, b$. For instance, for $i = d$ and $j = b$, we have

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

This is the **Unitarity triangle**
An area $\neq 0$ means CP violation

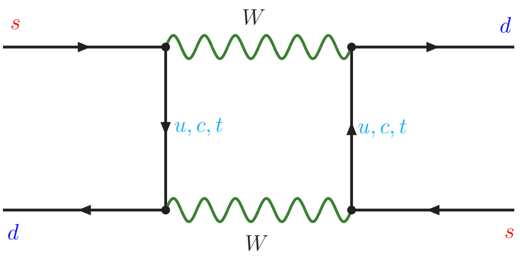


$$\bar\rho + i\bar\eta \equiv -V_{ud} V_{ub}^* / V_{cd} V_{cb}^*$$

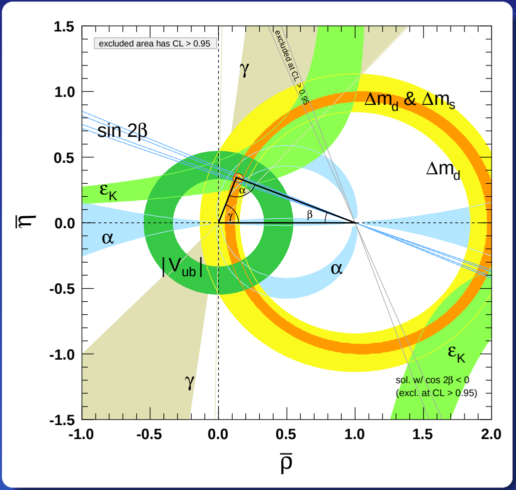
To constrain the sides and angles from the triangle one uses:

- Flavour changing processes which do not violate CP (basically the value of $|V_{ub}|$ and data on B-$\overline{B}$ mixing)
- Data on CP violating processes (ϵ_K and asymmetries in B_d decays which provide $\sin 2\beta$)

Generated by Box diagrams



The unitarity triangle



$$\lambda = 0.22535 \pm 0.00065$$

$$A = 0.811\,^{+0.022}_{-0.012}$$

$$\bar{\rho} = 0.131\,^{+0.026}_{-0.013}$$

$$\bar{\eta} = 0.345\,^{+0.013}_{-0.014}$$

$$\alpha+\beta+\gamma = (178\,^{+11}_{-12})^\circ$$

Charged lepton masses

Charged Lepton masses are all **well known**

$$
\begin{aligned}
m_e &= 0.51099892 \pm 0.00000004\,\mathrm{MeV}\,, \\
m_\mu &= 105.658369 \pm 0.000009\,\mathrm{MeV}\,, \\
m_\tau &= 1777.0 \pm 0.3\,\mathrm{GeV}
\end{aligned}
$$

In the SM we studied there are no **righthanded neutrinos** and there is just **one Higgs doublet**. Then, we can choose M_e diagonal. As a consequence the theory is diagonal in lepton flavour (no CKM in the lepton sector): Individual **lepton numbers are conserved.**

$$
\mu \not\to e\gamma\,, \quad \tau \not\to \mu\gamma\,, \quad \mu \not\to ee\bar{e}, \quad \tau \not\to e\bar{e}\mu
$$

Its non-observation suggested that neutrinos are massless, however...

Intrinsic properties of neutrinos

Before oscillation experiments

- Three types of neutrinos ν_e,ν_μ,ν_τ
- Lepton numbers L_e, L_μ, L_τ conserved separately
  - ν_e produces e's and no μ's
  - No $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to e\gamma$, $\mu \to 3e$
- Total lepton number $L = L_e + L_\mu + L_\tau$ conserved (no $0\nu\beta\beta$)
- ν masses much smaller than charged lepton masses

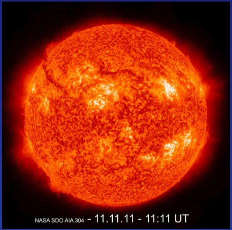
$$m_{\nu_e} < 2\,\mathrm{eV}\,, \quad m_{\nu_\mu} < 170\,\mathrm{KeV}\,, \quad m_{\nu_\tau} < 18\,\mathrm{MeV} \quad \sum_a m_a \lesssim 14\,\mathrm{eV}$$

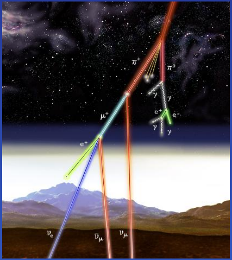
- ν's helicity $-1/2$ and $\bar{\nu}$'s helicity $+1/2$
- Magnetic moments very small: $\mu_\nu < 10^{-10}\mu_B\,, \quad \mu_\nu < 10^{-12}\mu_B$)

After oscillation experiments

- Neutrinos must be massive ($m_\nu \sim 1\,\mathrm{eV}$)
- They mix (with large mixings)
- LFV processes must exist (still not observed)

Solar and atmospheric neutrino problems





The Solar neutrino problem

- The Sun produces ν_e's, whose flux can be calculated using solar models
- The flux of ν_e measured in the earth in all experiments reduced by a factor 0.3–0.5
- Explained by oscillations $\nu_e \to \nu_{\mu,\tau}$

The atmospheric neutrino problem

- π's produced in the atmosphere should give a flux of ν_μ's twice that of ν_e's
- The observed flux of ν_μ's is largely reduced
- Explained in terms of oscillations $\nu_\mu \to \nu_\tau$

Masses of neutrinos in the SM

Simpler solution: add ν_R like in the quark sector

$$\mathcal{L}_{YL} = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R + \mathrm{h.c.}$$

But

- Why m_ν are so small?
- Why omit terms of the form $\overline{\nu_R^c}\nu_R$ in the Lagrangian?

Solution to the two questions: they are not omitted!

$$\mathcal{L}_{YL} \to \mathcal{L}_{YL} = -\bar{L}_L Y_e \Phi e_R - \bar{L}_L Y_\nu \tilde{\Phi} \nu_R - \frac{1}{2}\overline{\nu_R^c} M \nu_R + \mathrm{h.c.}$$

$$\mathcal{L}_{\nu M} = -\frac{1}{2}\left(\bar{\nu}_L, \overline{\nu_R^c}\right) \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \mathrm{h.c.}$$

if $M \gg M_D$ ("see-saw" mechanism):

- 3 **Heavy** Majorana neutrinos $\sim \nu_R$ with masses $\sim M$
- 3 **Light** Majorana neutrinos $\sim \nu_L$ with masses $\sim M_D^2/M$

Dirac and Majorana neutrinos

Dirac: if $M = 0$, $(M_\nu = M_D)$

$$\mathcal{L}_{\rm Dirac} = i\overline{\nu_L}\partial\!\!\!/\nu_L + \overline{\nu_R}\partial\!\!\!/\nu_R - (\overline{\nu_R}M_\nu\nu_L + \text{h.c.})$$

- 4 degrees of freedom
- Conserve total lepton number (NO $0\nu\beta\beta$ decay)
- Less natural (why m_ν are so small)

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$$\mathcal{L}_{\text{Dirac}} = i\overline{\nu_L}\partial\!\!\!/\nu_L + \overline{\nu_R}\partial\!\!\!/\nu_R - (\overline{\nu_R}M_\nu\nu_L + \text{h.c.})$$

- 4 degrees of freedom
- Conserve total lepton number (NO $0\nu\beta\beta$ decay)
- Less natural (why m_ν are so small)

Majorana: if $M \gg M_D$, ($M_\nu = -M_D M^{-1} M_D^T$)

$$\mathcal{L}_{\text{Majorana}} = i\overline{\nu_L}\partial\!\!\!/\nu_L - \frac{1}{2}\left(\overline{\nu_L^c}M_\nu\nu_L + \text{h.c.}\right)$$

- 2 degrees of freedom
- Do not conserve total lepton number ($0\nu\beta\beta$ decay)
- More natural and more CP violating phases

Neutrinos at low energies: Dirac

$$\mathcal{L}_{\rm Dirac} = i\overline{\nu_L}\partial\!\!\!/\nu_L + \overline{\nu_R}\partial\!\!\!/\nu_R - (\overline{\nu_R}M_\nu\nu_L + \text{h.c.}) + \\ -\frac{G_F}{\sqrt{2}}J^\mu J_\mu^\dagger - \frac{G_F}{\sqrt{2}}J_Z^\mu J_{Z\mu} + \mathcal{L}_{\rm MM} + \mathcal{L}_{\rm NSI} + \cdots$$

$$J^\mu = 2\bar{\nu}_L\gamma^\mu e_L + \cdots \,, \qquad J_Z^\mu = \bar{\nu}_L\gamma^\mu\nu_L + \cdots$$

diagonalization

$$\nu_{\alpha L} = V_{\alpha i}\nu_{iL}\,, \quad \nu_{\alpha R} = U_{\alpha i}\nu_{iR}\,, \quad U^\dagger M_\nu V = M_{\rm diag}\,, \quad \nu_i = \nu_{iL} + \nu_{iR}$$

$$J^\mu = 2\bar{\nu}\gamma^\mu V^\dagger P_L e + \cdots \,, \qquad J_Z^\mu = \bar{\nu}_L\gamma^\mu\nu_L + \cdots$$

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}e^{i\delta} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Neutrinos at low energies: Majorana

$$\mathcal{L}_{\rm Dirac} = i\overline{\nu_L}\partial\!\!\!/\nu_L - \frac{1}{2}\left(\overline{\nu_L^c}M_\nu\nu_L + \text{h.c.}\right) +$$

$$-\frac{G_F}{\sqrt{2}}J^\mu J^\dagger_\mu - \frac{G_F}{\sqrt{2}}J^\mu_Z J_{Z\mu} + \mathcal{L}_{\rm MM} + \mathcal{L}_{\rm NSI} + \mathcal{L}_{0\nu\beta\beta} + \cdots$$

$$J^\mu = 2\bar{\nu}_L\gamma^\mu e_L + \cdots \ , \qquad J^\mu_Z = \bar{\nu}_L\gamma^\mu\nu_L + \cdots$$

diagonalization

$$\nu_{\alpha L} = V_{\alpha i}\nu_{iL} \ , \quad V^T M_\nu V = M_{\rm diag} \ , \quad \nu_i = \nu_{iL} + \nu_{iL}^c$$

$$J^\mu = 2\bar{\nu}V^\dagger P_L e_L + \cdots \ , \qquad J^\mu_Z = -\frac{1}{2}\bar{\nu}\gamma^\mu\gamma_5\nu + \cdots$$

$$V_{\text{Majorana}} = V_{\rm Dirac}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Neutrino oscillations in vacuum

If ν's are massive, mass eigenstates are no flavour eigenstates ($W^+ \to \ell_\alpha^+ \nu_\alpha$, $\alpha = e, \mu, \tau$)

$$|\nu_\alpha\rangle = \sum_i V_{\alpha i}^* |\nu_i\rangle$$

where V parametrized as the CKM matrix.
After traveling some distance, L, time evolution gives ($p \gg m_i$)

$$|\nu_\alpha(L)\rangle = \sum_i V_{\alpha i}^* e^{-i m_i^2 L/2E} |\nu_i\rangle \ , \quad P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(L)\rangle|^2$$

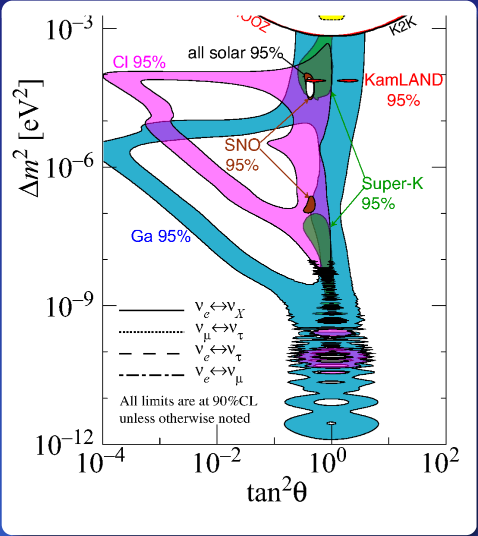
for only 2 flavours

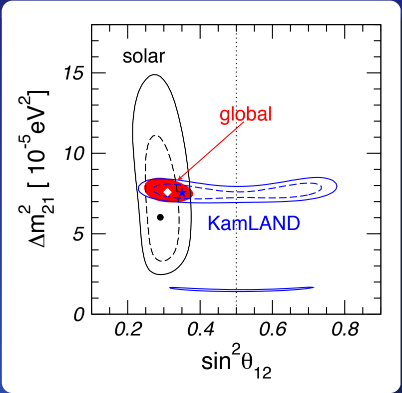
$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \, \sin^2 \left(1.27 \frac{(\Delta m^2/\mathrm{eV})(L/\mathrm{km})}{(E/\mathrm{GeV})} \right)$$

Can be large even if $\Delta m^2/E^2$ is small (magic of oscillations!)
Enhanced in the presence of matter (MSW), as in the Sun

Global results for solar Δm^2

Solar data + reactor neutrinos (KamLAND)



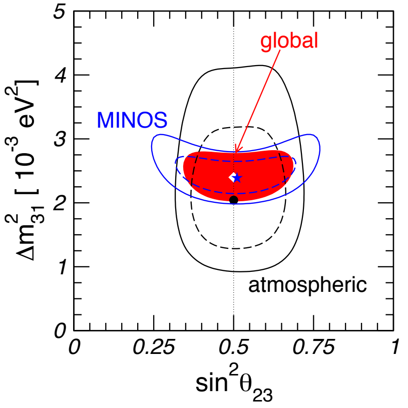


LMA MSW solution

Oscillations $\nu_e \to \nu_{\mu,\tau}$

Global results for Atmospheric Δm^2

Atmospheric + accelerator neutrinos (MINOS, K2K, $\cdots$)



Two solutions:

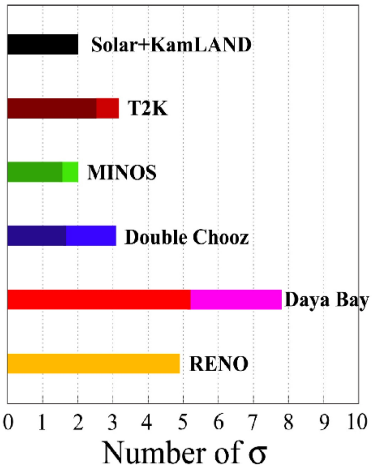
$\Delta m^2_{31} > 0$
Normal hierarchy (NH)

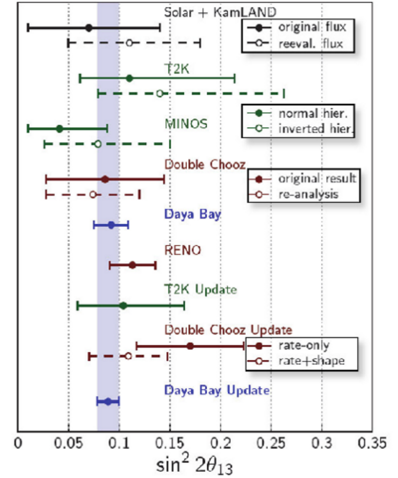
$\Delta m^2_{31} < 0$
Inverted hierarchy (IH)

Oscillation channel

Oscillations $\nu_\mu \to \nu_\tau$

Results on θ_{13}

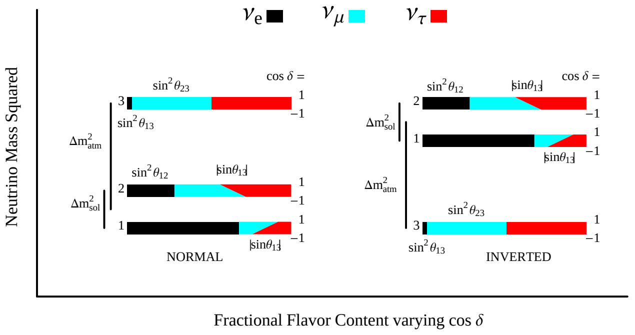




Exclusion of non-zero θ_{13}

By S. Jetter

The two mass orderings



$$\Delta m^2_{21} = 7.5 \times 10^{-5}\,\mathrm{eV}^2 \quad (2.4\%$$

$$\Delta m^2_{31} = \begin{cases} 2.45 \times 10^{-3}\,\mathrm{eV}^2 \\ -2.43 \times 10^{-3}\,\mathrm{eV}^2 \end{cases} \quad (2.8\%)$$

$$\sin^2\theta_{12} = 0.3\ (4\%)$$

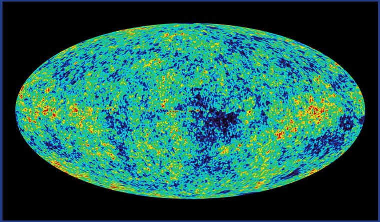
$$\sin^2\theta_{23} = 0.42\ (11\%)$$

$$\sin^2\theta_{13} = 0.023\ (10\%)$$

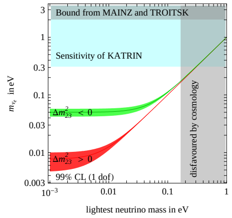
δ still not well determinded from the fits

Absolute mass scale

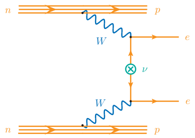
Cosmo: $\sum_i m_{\nu_i} < 0.2\text{–}2\ \mathrm{eV}$



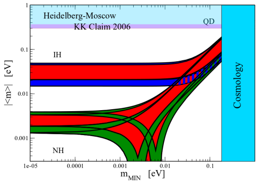
β decay: $(m^2_{\nu_e} = \sum_i |V_{ei}|^2 m^2_{\nu_i})$



Neutrinoless double β decay ($m_{\beta\beta} \lesssim 0.14\text{–}0.38$ eV, future ~ 0.02 eV)



$$\langle m_\nu \rangle = \left| \sum V_{ei}^2 m_i \right|$$



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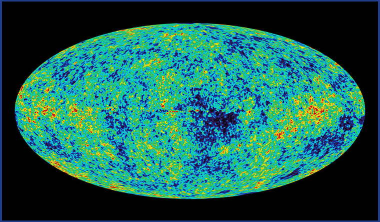

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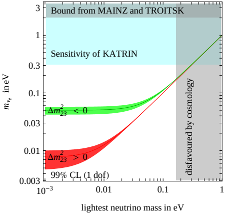

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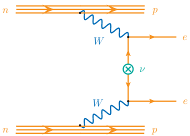

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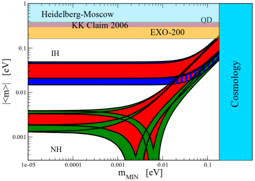

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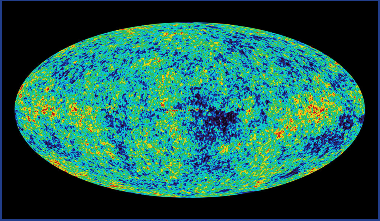

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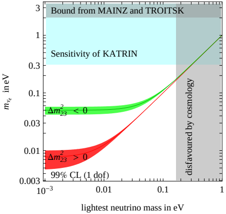

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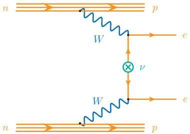

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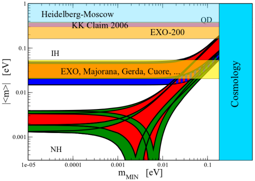

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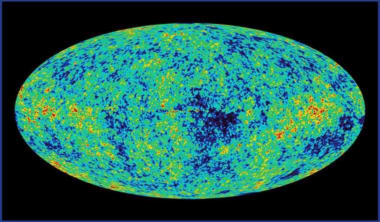

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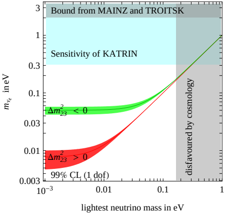

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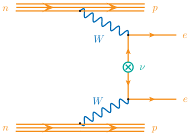
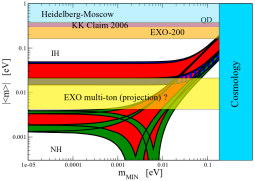




$$\langle m_\nu \rangle = \left|\sum V_{ei}^2 m_i\right|$$

Summary of parameters

$\Delta m^2_{31} \sim \pm 2.4 \times 10^{-3}\,\mathrm{eV}^2$	$\theta_{23} \sim 45^\circ$	Atmos,K2K,MINOS
$\Delta m^2_{21} \sim 7.6 \times 10^{-5}\,\mathrm{eV}^2$	$\theta_{12} \sim 35^\circ$	Solar, KamLAND
	$\theta_{13} \sim 9^\circ$	T2K,MINOS,Double Chooz Daya Bay,RENO
N_ν (active and light)	3	LEP
$m_{\beta\beta} = \sum_i \lvert V^2_{ei} m_{\nu_i} \rvert$	$\lesssim 0.4\,\mathrm{eV}$	HM,IGEX,EXO,...
$m_{\nu_e} = \sum_i \lvert V_{ei} \rvert^2 m^2_{\nu_i}$	$< 2.2\,\mathrm{eV}$	Mainz,Troitsk
$\sum_i m_{\nu_i}$	$\lesssim 1\,\mathrm{eV}$	Cosmology
$\mathrm{sign}(\Delta m^2_{31})$	?	Noνa,NF,BB,SB,...
CP, δ	?	Noνa,NF,BB,SB,...
Dirac or Majorana? (α,β)	?	HM?,$0\nu\beta\beta$
N_s (light sterile)	1, 2 ?	LSND,MiniBooNE,Cosmology
μ_ν/μ_B	$< 10^{-10}, 10^{-12}$	σ_ν, red giants
NSI	$\varepsilon \lesssim 0.01$–$10$	Sun,Atm,LSND,NF,...
LFV $(\mu \to e\gamma, \cdots)$	$< 5.7 \times 10^{-13}$	MEG,COMET/Mu2e,...

Widths of the Higgs into gauge bosons grow like m_H^3

$$\Gamma(H \to W^+W^-) = \frac{G_F m_H^3}{8\pi\sqrt{2}}\,, \quad \Gamma(H \to ZZ) = \frac{G_F m_H^3}{16\pi\sqrt{2}}$$

Requiring $\Gamma_{\rm tot}(H) \leq m_H$ (**perturbativity**) gives

$$m_H \leq 1.6\,\mathrm{TeV}$$

The λ coupling in the scalar potential grows with energy

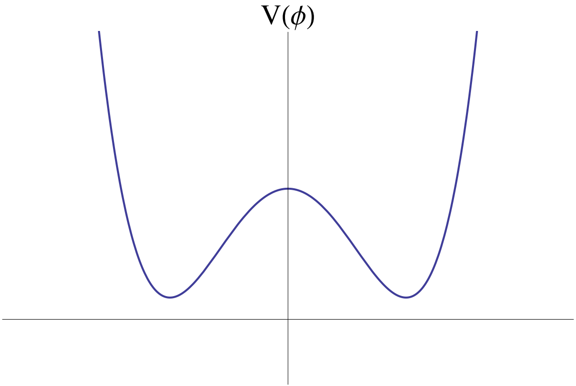
$$\frac{d\lambda}{d\ln q^2} = \frac{3}{4\pi^2}\left(\lambda^2 + \lambda y_t^2 - y_t^4 + \cdots\right)$$

If large,λ diverges at some scale Λ. Taking $\lambda(\Lambda) = \infty$ (the theory only makes sense up to $q^2 \sim \Lambda^2$) one finds

$$\lambda(q^2) = \frac{4\pi^2}{3\log(\Lambda^2/q^2)} \qquad m_H^2 \leq \frac{8\pi^2}{3\sqrt{2}G_F\log(m_H^2/v^2)} \approx (850\,\mathrm{GeV})^2$$

Because the top quark dependence if m_H is very light λ can be driven to negative values making the potential unstable

Stability of the potential



Stability of the Higgs Potential

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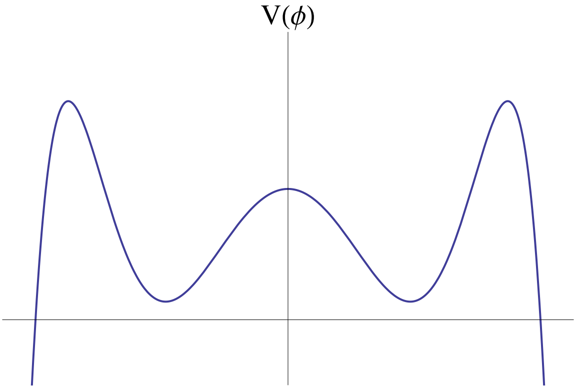
Stability of the potential



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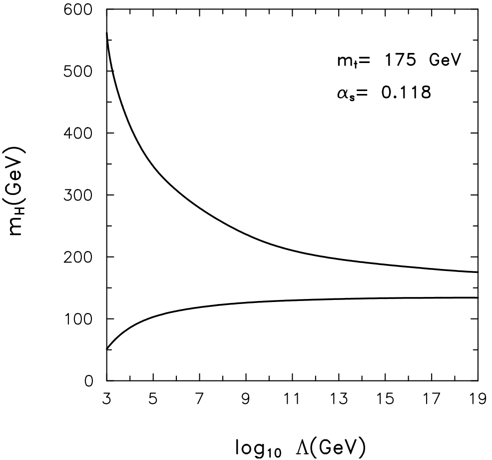
Stability/Triviality bounds



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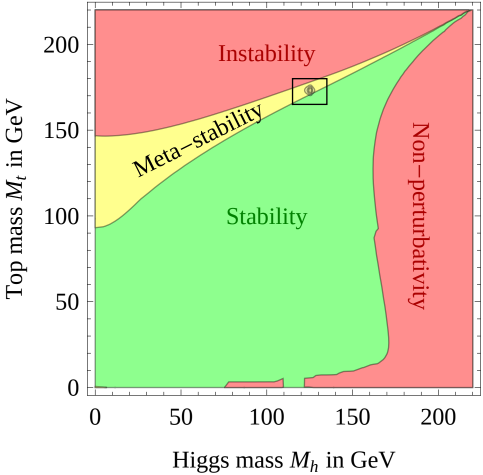
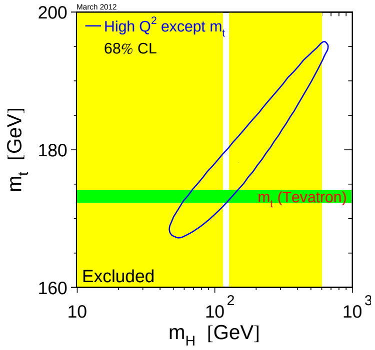
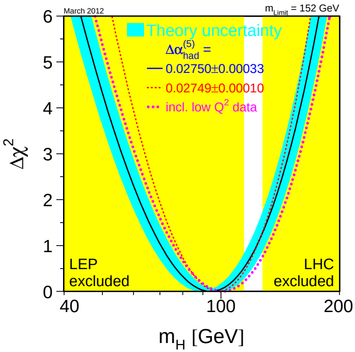


m_H from Radiative Corrections



m_H from Radiative Corrections



From precision data ONLY

$$68\,\mathrm{GeV} < M_H < 155\,\mathrm{GeV} \quad 90\%\,\mathrm{CL}$$

Production of the Higgs Boson

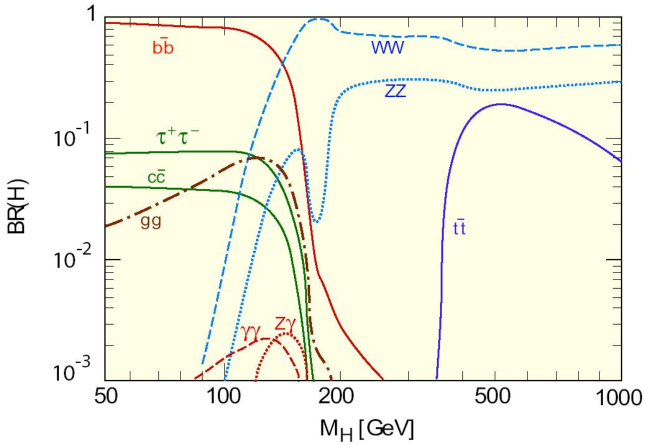
e^+–e^- colliders

- **Bjorken:** $e^+e^- \to Z \to Z\,H$ (Dominant at LEP2)
- WW **fusion:** $e^+e^- \to \nu\bar{\nu}WW \to \nu\bar{\nu}H$
- ZZ **fusion:** $e^+e^- \to e^+e^-(ZZ) \to e^+e^-H$

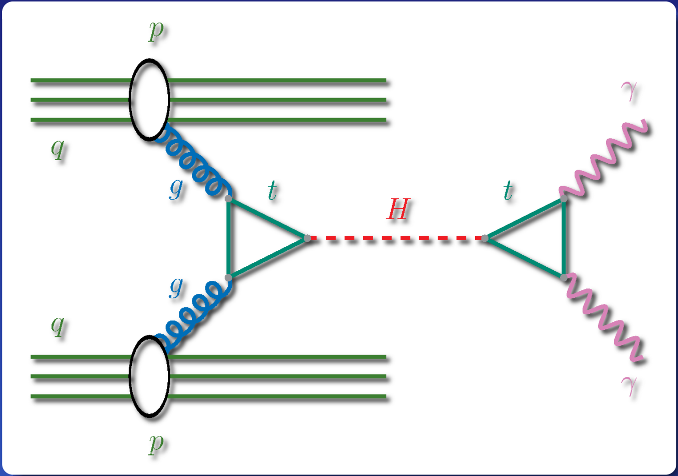
Hadron colliders [proton–(anti)proton collisions]

- **Gluon fusion:** $p\,p \to g\,g \to H$ (Dominant at the LHC)
- VV **fusion:** $pp \to VV \to H$
- **Association with** V**:** $pp \to qq' \to VH$ (Dominant at Tevatron)

Higgs Boson Decays



Higgs Boson Discovery



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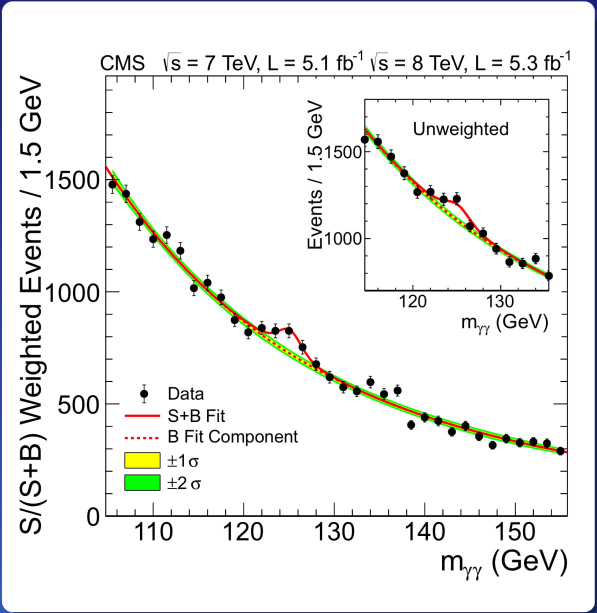

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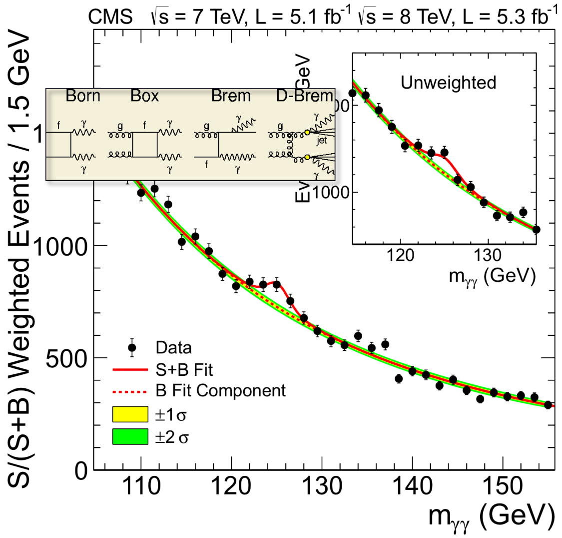

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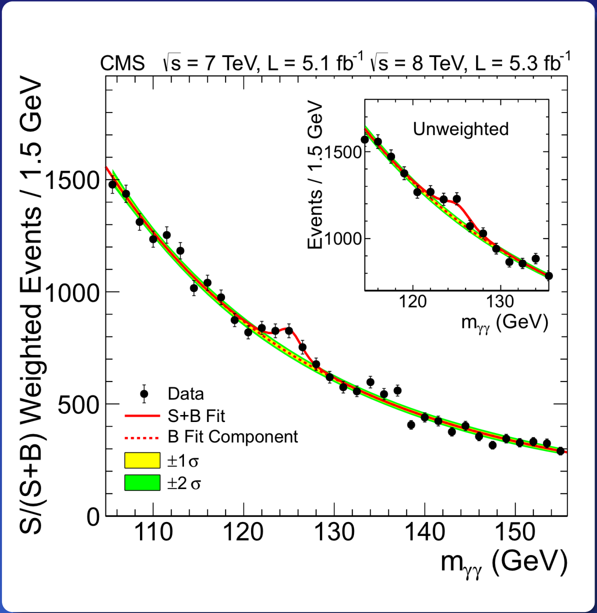
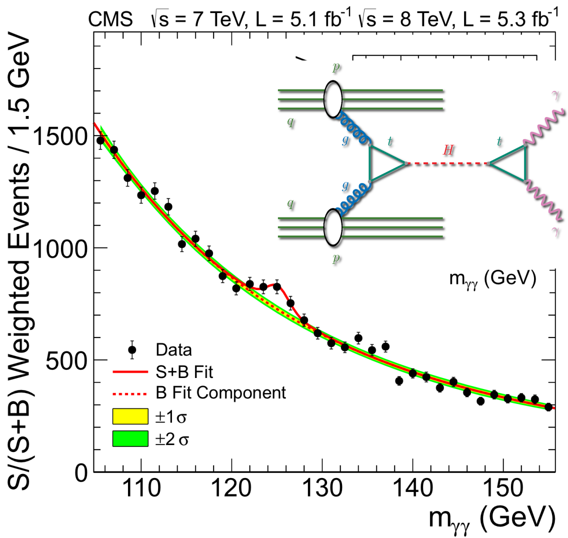
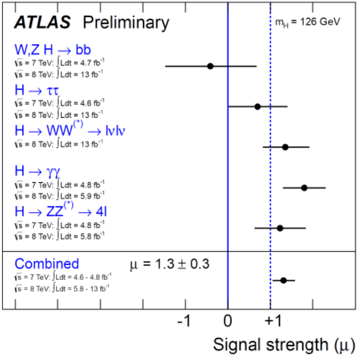
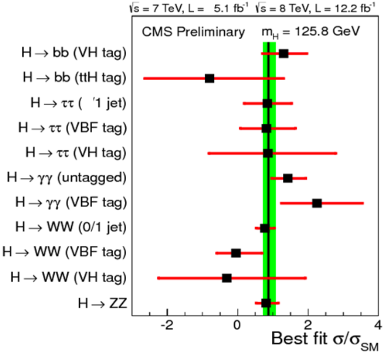



CMS $\sqrt{s}$ = 7 TeV, L = 5.1 fb^{-1} $\sqrt{s}$ = 8 TeV, L = 5.3 fb^{-1}
S/(S+B) Weighted Events / 1.5 GeV
0
500
1000
1500
110
120
130
140
150
$m_{\gamma\gamma}$ (GeV)
Data
S+B Fit
B Fit Component
$\pm 1\sigma$
$\pm 2\sigma$
p
q
g
t
H
t
γ
g
q
p
γ
$m_{\gamma\gamma}$ (GeV)

Best–fit Higgs mass m_H :
126.0 ± 0.4 (stat) ± 0.4 (syst) GeV



- M=125.8 ±0.4 (stat) ± 0.4 (syst) GeV



- σ/σ_{SM}=0.88±0.21

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- Is there a SM of **neutrino masses**?

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