The Standard Model of Electroweak Interactions

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Outline

- SM I: Weak Interactions and Gauge Teories
- SM II: Construction of the SM
- SM III: Testing the SM

References

Links

- PDG `http://pdg.lbl.gov/2012/reviews/contents_sports.html`
- LEP EWWG `http://lepewwg.web.cern.ch/LEPEWWG/Welcome.html`
- TQC Course `http://eeemaster.uv.es/course/view.php?id=2`

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- Ten Lectures on the ElectroWeak Interactions,
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1. One-slide Introduction to QFT

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2. Introduction to weak interactions
   - μ and β decays
   - The $V - A$ model
   - The Intermediate Vector Boson Hypothesis
   - Ingredients for a theory of Weak Interactions

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   - QED as a gauge theory
   - Non-Abelian Gauge Invariance
   - Chiral Fermions and Quantization

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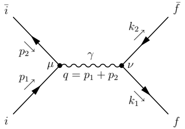
4. Spontaneous Symmetry Breaking
   - SSB of discrete symmetries
   - Goldstone Theorem
   - The Higgs Mechanism

One-Slide Introduction to QFT

Feynman Diagrams



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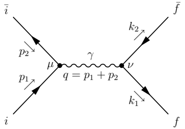

The Lagrangian

$$\mathcal{L}_{\rm QED} = \underbrace{\bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Free EOM}} + \underbrace{eA_\mu\bar{\psi}\gamma^\mu\psi}_{\text{Interaction}}$$

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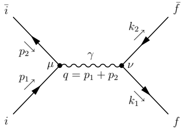

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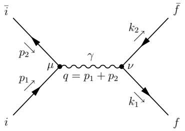

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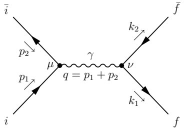

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More in the bibliography and in the QFT Course

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4 Spontaneous Symmetry Breaking

The Standard Model

Binding of nuclei and radioactivity require two additional short-range forces:

- **Strong Interactions**: Keep nucleus bound.
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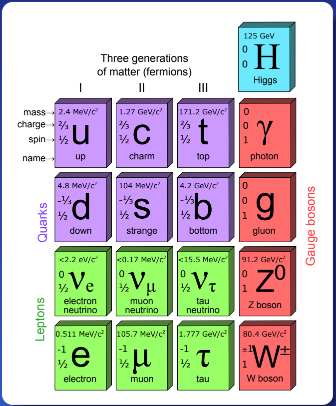

3 families of matter (chiral fermions)

$$\text{Quarks}: \quad Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; d_R, \; u_R$$

$$\text{Leptons}: \quad L_L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \; e_R, \; \nu_R?$$

3 types of gauge bosons (spin 1)

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

- Strong (8 masless gluons, g)
- Electromag. (1 massless photon γ)
- Weak (3 massive Z, W^+, W^-)

1 Higgs boson (spin 0) needed for SSB

$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ decay

$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ could be described the most general four-fermion interaction
(Scalar,Pseudo-scalar,Vector, Axial-Vector, Tensor)
Experimentally the amplitude only involves left-handed fermions, with an effective interaction of the $V - A$ type:

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e\right] \left[\bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\mu\right]$$

G_F **(Fermi coupling constant)** fixed by the μ decay width.
One obtains

$$G_F = (1.16639 \pm 0.00002) \times 10^{-5}\,{\rm GeV}^{-2} \approx \frac{1}{(293\,{\rm GeV})^2}$$

Beta decay

- Weak transitions $n \to p e^- \bar{\nu}_e$ and $p \to n e^+ \nu_e$ (in nuclei) can be described by the effective interaction

$$\mathcal{L}_{\rm eff} = -\frac{G}{\sqrt{2}} \left[\bar{p}\gamma^\alpha(1 - g_A\gamma_5)n\right] \left[\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e\right]$$

where $G \approx 0.975\, G_F$, $g_A = 1.2573 \pm 0.0028$

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Beta decay

- Weak transitions $n \to pe^- \bar{\nu}_e$ and $p \to ne^+ \nu_e$ (in nuclei) can be described by the effective interaction

$$\mathcal{L}_{\rm eff} = -\frac{G}{\sqrt{2}} [\bar{p}\gamma^\alpha(1 - g_A\gamma_5)n] \, [\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e]$$

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Universal interaction at the quark-lepton level:

$$\mathcal{L}_{\rm eff} = -\frac{G}{\sqrt{2}} [\bar{u}\gamma^\alpha(1 - \gamma_5)d] \, [\bar{e}\gamma_\alpha(1 - \gamma_5)\nu_e]$$

g_A understood as a **QCD** correction.

$\Delta S = 1$ transitions and ν flavors

$\Delta S = 1$ decays [$K \to (\pi) l^- \bar{\nu}_l$, $\Lambda \to p e^- \bar{\nu}_e$, ...] show:

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Neutrino flavors

$\bar{\nu}_\mu$ can produce μ^+ but never e^+

$$\bar{\nu}_\mu X \to \mu^+ X' \,, \qquad\qquad \bar{\nu}_\mu X \not\to e^+ X'$$

$\bar{\nu}_e$ produces e^+ but never $\mu^+ \Longrightarrow$ the neutrino partners of the electron and the muon are two different particles: $\nu_e \neq \nu_\mu$.

The $V-A$ model

All previous facts can be described by:

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} J^\mu J^\dagger_\mu$$

with

$$\begin{aligned} J^\mu &= = \bar{u}\gamma^\mu(1-\gamma_5)\left[\cos\theta_C d + \sin\theta_C s\right] \\ &+ \bar{\nu}_e\gamma^\mu(1-\gamma_5)e + \bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu \end{aligned}$$

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The different strength of $\Delta S = 0$ and $\Delta S = 1$ processes parametrized by θ_C, $\sin\theta_C \equiv G^{\Delta S=1}/G_F \approx 0.22$.
Correctly **describes the weak decays** $\pi^+ \to \pi^0 e^+ \nu_e$, $\pi^- \to l^- \bar{\nu}_l$: strong helicity suppression in $\pi^- \to l^- \bar{\nu}_l$.

Problems of the V-A model

- **Unitarity**: G_F is a dimensionful quantity ($[G_F] = M^{-2}$) : **cross-sections increase with energy**:

$$\sigma(\nu_\mu e^- \to \mu^- \nu_e) \approx G_F^2 s/\pi \,.$$

At large values of s, tree-level unitarity is violated. The unitarity bound, $\sigma < 2\pi/s$, only satisfied if

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- **Renormalizability**: Higher-order transitions such as $\nu_\mu e^- \to \mu^- \bar{\nu}_e \to \nu_\mu e^-$ are divergent with **divergences** that cannot be absorbed in the parameters of the model.
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The $V - A$ model can only be a **low-energy effective description** of some more fundamental theory.

The Intermediate Vector Boson (IVB) Hypothesis

QED has a **dimensionless coupling** $\Longrightarrow$ renormalizable
Can one do the same for the Weak Interactions?

$$\mathcal{L}_{\rm QED} = eJ^{\mu}_{\rm QED}A_{\mu} \Longrightarrow \mathcal{L}_{\rm IVB} = \frac{g}{2\sqrt{2}}\left(J^{\mu}W^{\dagger}_{\mu} + \mathrm{h.c.}\right)$$

$V - A$ interaction **generated by W exchange**

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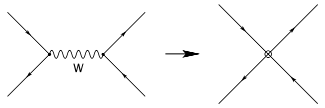

$$\frac{-g_{\mu\nu} + q_{\mu} q_{\nu} / m_W^2}{q^2 - m_W^2} \quad \xrightarrow{q^2 \ll m_W^2} \quad \frac{g_{\mu\nu}}{m_W^2}$$

$$\frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}} \,, \quad g < 1 \Rightarrow m_W < 123\,\text{GeV}$$

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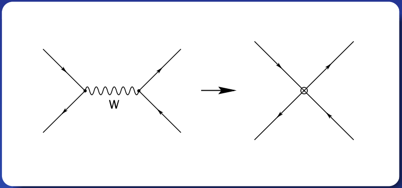

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$\nu \ell^- \to \nu \ell^-$ has much better behaviour at high-energies

$$G_F \to G_F \frac{m_W^2}{m_W^2 - q^2}$$

Problems of the IVB

Problems reappear in processes with external W bosons:

$$\sigma(\nu_e\bar{\nu}_e, e^- e^+ \to W^+ W^-) \overset{s\to\infty}{\propto} G_F^2 s$$

from $q_\mu q_\nu / m_W^2$ piece in the sum over polarizations of W
Implies that the theory is **not renormalizable**
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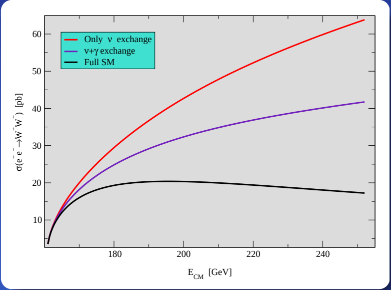
Solution: additional diagrams and additional particles



Cancellation can be realized with **a neutral** intermediate **boson Z**

Important implications **(neutral-currents)**

$\nu_\mu e^- \to \nu_\mu e^-$ and $\nu_\mu p \to \nu_\mu p$.

Confirmed in 1973!

Ingredients for a theory of Weak Interactions

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- Electroweak **unification**: $g_W/2\sqrt{2} \sim g_Z/2\sqrt{2} \sim e$, i.e. $g^2/4\pi \sim 8\alpha$. Implies

$$m_W \sim \left(\frac{\sqrt{2}g^2}{8G_F}\right)^{1/2} \sim \left(\frac{4\pi\alpha\sqrt{2}}{G_F}\right)^{1/2} \sim 100\,\mathrm{GeV}$$

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- Should allow for CP **violation**

Ingredients for a theory of Weak Interactions

- **photon** γ and three massive spin–1 bosons $W^\pm, Z$
- Electroweak **unification**: $g_W/2\sqrt{2} \sim g_Z/2\sqrt{2} \sim e$, i.e. $g^2/4\pi \sim 8\alpha$. Implies

$$m_W \sim \left(\frac{\sqrt{2}g^2}{8G_F}\right)^{1/2} \sim \left(\frac{4\pi\alpha\sqrt{2}}{G_F}\right)^{1/2} \sim 100\,\mathrm{GeV}$$

- **Universality** of couplings
- The $W^\pm$ field couples only to **left-handed** particles
- The Z **boson** has only **flavor-diagonal couplings**
- **Lepton-number** is conserved to good accuracy
- Should allow for CP **violation**
- **Renormalizability**

Quantum field theories can have global invariances.
For instance the free Dirac Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

is invariant under a **global phase transformation** ($\alpha \equiv$const.)

$$\psi \rightarrow \psi' = e^{i\alpha Q}\psi$$

Noether theorem $\Rightarrow$ charge is conserved.

QED as a gauge theory

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Noether theorem $\Rightarrow$ charge is conserved.
Global invariances, however, require that the field is transformed exactly in the same way in the whole universe.
More reasonable to think that **fundamental symmetries should be local**, with parameters depending on the position.
That is the **gauge principle**.

However, the free Dirac Lagrangian is not invariant under the

local gauge transformation

$$\psi \to \psi' = e^{i\alpha(x)Q}\psi$$

since

$$\mathcal{L}_\psi \to \mathcal{L}'_\psi = \bar{\psi}\,(i\,\gamma^\mu\,(\partial_\mu + i\,Q\partial_\mu\alpha) - m)\,\psi$$

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To preserve the local **gauge invariance** one must introduce the **gauge field A_μ** through the **minimal coupling**

$$\partial_\mu\psi \Rightarrow D_\mu\psi \equiv \left(\partial_\mu - ieQA_\mu\right)\psi$$

and require that A_μ transforms like

$$A_\mu \longrightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$$

then $D_\mu\psi$ transforms nicely

$$D_\mu\psi \longrightarrow (D_\mu\psi)' \equiv e^{i\alpha(x)Q}D_\mu\psi$$

The coupling between ψ (*e.g.* electrons) and the gauge field A_μ (photon) arises naturally when we **promote** the **global phase invariance** of free Dirac Lagrangian to a **local gauge symmetry**.

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Gauge Kinetic Term

To complete the theory we must add a kinetic term also for the gauge field:
Quadratic in the field and **gauge invariant**. Only possibility

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

$F_{\mu\nu}$ is the gauge invariant electromagnetic strength tensor

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As QED these theories will be **renormalizable** and will be **universal** (particles with same quantum numbers couple with the same strength).

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The free Dirac Lagrangian is invariant under a global
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We will require that the Lagrangian is invariant under

$$\psi \to \psi' = U(x)\, \psi$$

with

$$U \equiv \exp\left[i\, T^a \alpha^a(x)\right]$$

T^a are the generators of the group in the representation furnished by ψ and satisfy

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As in the Abelian case, we must introduce **one gauge field for each generator**, and define the

Covariant derivative

$$D_\mu \equiv \partial_\mu - igT^a A^a_\mu\,, \quad D_\mu\psi \longrightarrow (D_\mu\psi)' = UD_\mu\psi$$

Gauge invariance will be preserved as long as

$$T^a A^a_\mu \longrightarrow T^a A'^{\,a} = U \left(T^a A^a_\mu + \frac{i}{g}\partial_\mu \right) U^{-1}$$

or, in infinitesimal form, *i.e.* for $U \approx 1 + i\, T^a \alpha^a(x)$,

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The Field Strength tensor

Using the covariant derivative we can generalize the field strength tensor for a non-Abelian Lie group,

$$-igT^a F^a_{\mu\nu} \equiv [D_\mu, D_\nu]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, C_{abc}\, A^b_\mu A^c_\nu$$

which transforms (infinitessimally) as

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- At difference with the Abelian case, pure **non-Abelian gauge theory is NOT A FREE THEORY** and contains triple and quartic self-interactions

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- Note however that parity or other symmetries (charge) could force the fields to be combined into Dirac fields
- Note that ordinary **Dirac mass terms** require the existence of the **two chiralities** $\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$

Some comments on quantization

Non-Abelian gauge theories are **renormalizable**

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No problem for global symmetries (this explains $\pi^0 \to \gamma\gamma$)

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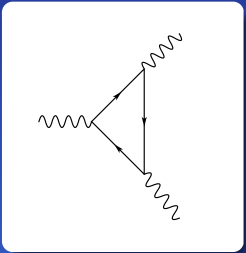

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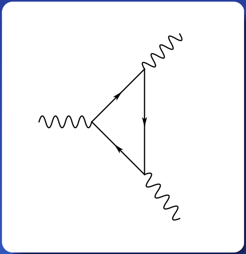

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This should cancel

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$T = 0\,^{\circ}\mathrm{C}$, ice cristals only have hexagonal symmetry
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Isotropy **is broken spontaneously**

This can happen in **quantum mechanical systems** with **infinite degrees of freedom** (quantum field theory)

Exercise: SSB of discrete symmetries

Let us take a self-interacting real field with Lagrangian,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

with ***potential***

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\,, \qquad \lambda > 0$$

Invariant under the transformation $\phi \to -\phi$
Ground state (ϕ_0) obtained by minimizing the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\left[(\partial_0\phi)^2 + (\nabla\phi)^2\right] + V(\phi)$$

The **minimum** is found for $\phi_0 =$ *constant* satisfying

$$\phi_0(\mu^2 + \lambda\phi_0^2) = 0$$

For $\mu^2 > 0$, just one invariant minimum at $\phi_0 = 0$

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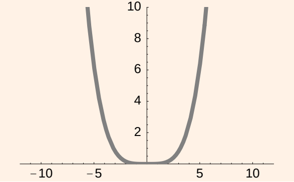

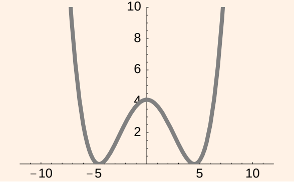

and the symmetry is *spontaneously broken (SB):*
the Lagrangian $\mathcal{L}$ is invariant but the vacuum is *not*

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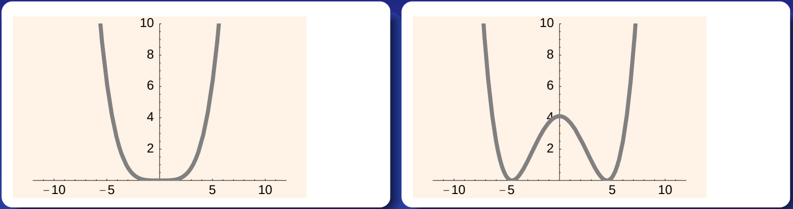

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the Lagrangian $\mathcal{L}$ is invariant but the vacuum is *not*
Perturbations defined about the true ground-state: $\phi' \equiv \phi - v$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi'\partial^\mu\phi' - \frac{1}{2}\left(\sqrt{-2\mu^2}\right)^2\phi'^{\,2} - \lambda\, v\, \phi'^{\,3} - \frac{1}{4}\lambda\phi'^{\,4}$$

ϕ' with positive mass, $m_{\phi'} = \sqrt{-2\mu^2}$, but **symmetry broken**
It is **Hidden** (reduced number of parameters!)

SSB of a continuous global symmetry

Consider a complex self-interacting scalar field,

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

with a potential,

$$V(\phi) = \mu^2(\phi^\dagger \phi) + \lambda(\phi^\dagger \phi)^2$$

It is **invariant under the *global phase* transformation**

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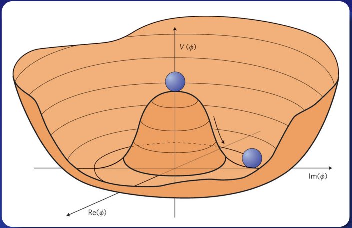
For $\mu^2 < 0$ the minimum is at $v = |\phi_0| = \sqrt{-\mu^2/2\lambda}$ and it is not unique. There is a **continuum of degenerate states.**



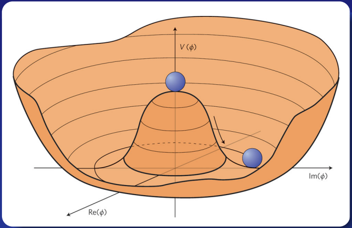
Choose one of the minima (**linear** parametrization)

$$\phi = \frac{(v + \phi_1' + i\phi_2')}{\sqrt{2}}$$

The Lagrangian is then

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1'\partial^\mu\phi_1' - \frac{1}{2}(-2\mu^2)\phi_1'^{\,2} + \frac{1}{2}\partial_\mu\phi_2'\partial^\mu\phi_2' + \cdots$$

Describes a massive scalar field ϕ_1', $m^2_{\phi'} = -2\mu^2 > 0$, and a massless scalar boson, ϕ_2', the **Goldstone boson**



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Alternative **non-linear** parametrization

$$\phi = \frac{(v + \rho(x))}{\sqrt{2}} e^{i\theta(x)/v}\,, \qquad \theta(x) \to \theta(x) + \alpha v$$

Only derivatives of $\theta(x)$ in $\mathcal{L} \Longrightarrow \theta(x)$ has no mass term

Goldstone Theorem

Exact continuous global symmetry broken spontaneously:
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Consider Φ with N_G components,

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the conserved currents are

$$j_\mu^a = \left(\partial_\mu \Phi^T\right) iT^a \Phi \,, \qquad \partial^\mu j_\mu^a = 0$$

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if $\langle\Phi\rangle = \text{constant} \neq 0$, expand around $\Phi' = \Phi - \langle\Phi\rangle$

Conservation of the currents implies

$$\partial^2 \left(\Phi^T T^a \langle\Phi\rangle\right) + \text{interaction terms} = 0$$

therefore the fields

$$\theta^a \equiv \Phi^T T^a \langle\Phi\rangle\,, \qquad \text{such that } T^a\langle\Phi\rangle \neq 0$$

satisfy the **massless equation of motion** $\Rightarrow$ **are massless**

What if the symmetry is a **local gauge symmetry**?

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Consider again the charged self-interacting scalar Lagrangian with the potential $V(\phi)$, and let us require a invariance under the *local* phase transformation,

$$\phi \to \exp\left[i\,\alpha(x)Q\right]\phi$$

In order to make the Lagrangian invariant, we introduce a ***gauge boson*** A_μ and the *covariant derivative* D_μ

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - ieQA_\mu$$

then the Lagrangian is

$$\mathcal{L} = (D_\mu\phi)^\dagger\, D^\mu\phi - V(\phi^\dagger\phi)$$

SSB occurs for $\mu^2 < 0$, with the vacuum $\langle|\phi|\rangle$ given as before. This time we will chose the **exponential parametrization** of the scalar field

$$\phi \equiv \frac{(v + \rho(x))}{\sqrt{2}} e^{i\theta(x)/v}$$

But now there is an important difference, since the symmetry is local we have that

$$\begin{array}{rcl} \theta(x) & \to & \theta'(x) = \theta(x) + Q\alpha(x)/v \\ A_\mu(x) & \to & A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x) \end{array}$$

leaves the Lagrangian invariant.

Without lose of generality, we can **choose the gauge** in such a way that $\theta(x) = 0$, removing it completely from the theory. In this gauge the Lagrangian is just

$$\mathcal{L} = \frac{1}{2}\left|\partial_\mu \rho - ieQA_\mu(v+\rho)\right|^2 - V(\frac{1}{2}(v+\rho)) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

When expanding this we immediately see that **the gauge boson has obtained a mass**

$$m_A^2 = e^2 Q^2 v^2$$

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The total **number of degrees of freedom unchanged**

Initial $\mathcal{L}$		Final $\mathcal{L}$	
ϕ charged scalar :	2	ρ neutral scalar :	1
A_μ massless vector :	2	A_μ massive vector :	3
	4		4

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- The $N_G - N_{G'}$ Goldstone bosons will be eaten by $N_G - N_{G'}$ **gauge bosons** that will become **massive**
- The G' subgroup will **remain unbroken** with $N_{G'}$ **massless gauge bosons**

Initial $\mathcal{L}$	Final $\mathcal{L}$
ϕ charged scalar : 2	ρ neutral scalar : 1
A_μ massless vector : 2	A_μ massive vector : 3
4	4

The Goldstone boson has been **eaten** by the **gauge boson** to give him the longitudinal degree of freedom

Generalized to **non-Abelian groups**: $G \to G'$

- The $N_G - N_{G'}$ Goldstone bosons will be eaten by $N_G - N_{G'}$ **gauge bosons** that will become **massive**
- The G' subgroup will **remain unbroken** with $N_{G'}$ **massless gauge bosons**

We will see this in action when we discuss the SSB of the SM.