Kaon Physics

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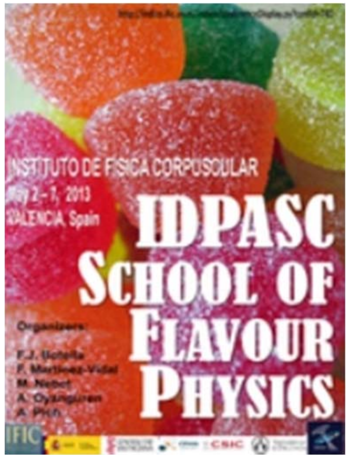
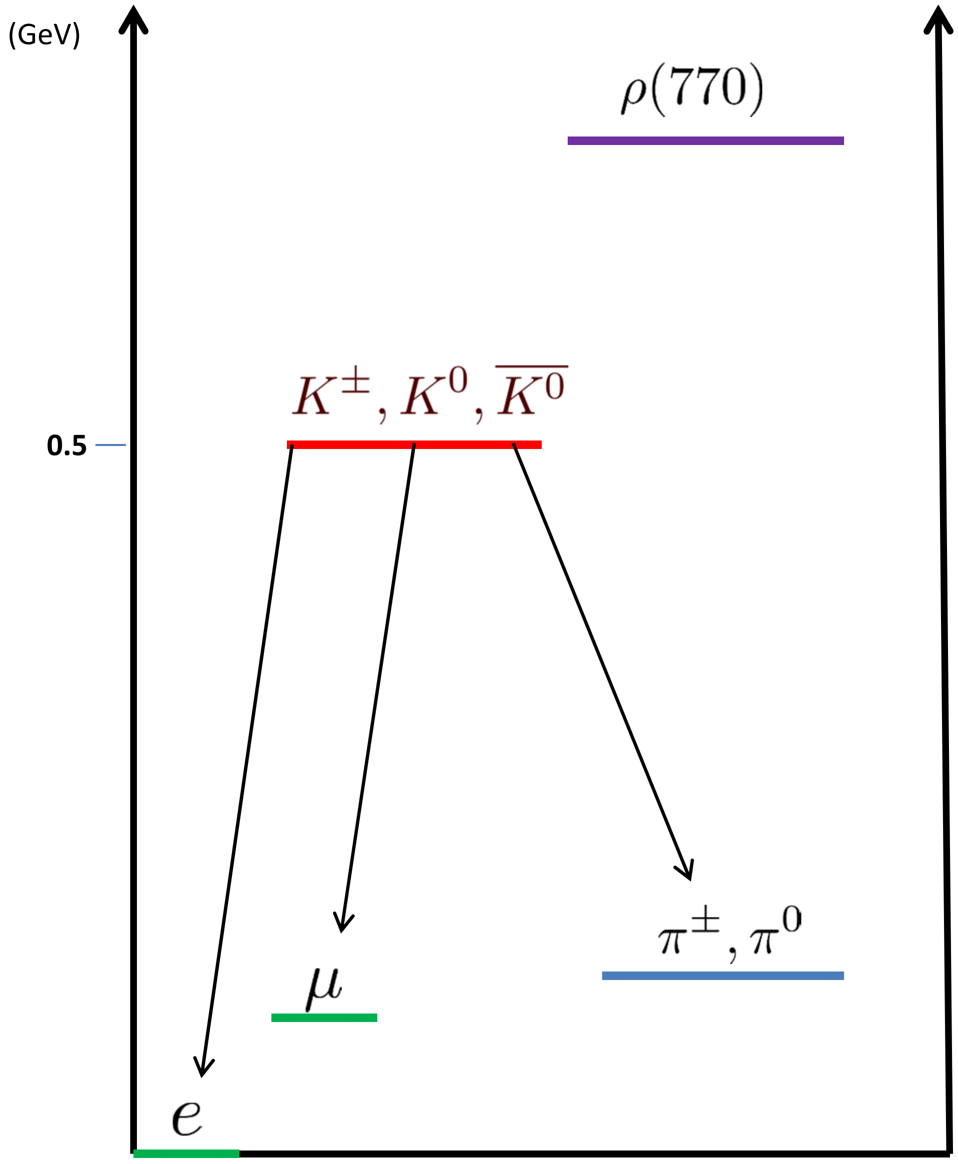
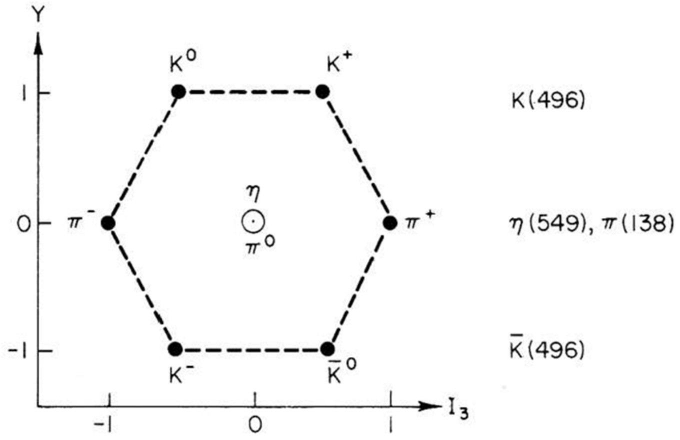




$$M_{K^{\pm}} = 493.677(16)\,\mathrm{MeV}$$
$$M_{K^0} = 497.614(24)\,\mathrm{MeV}$$



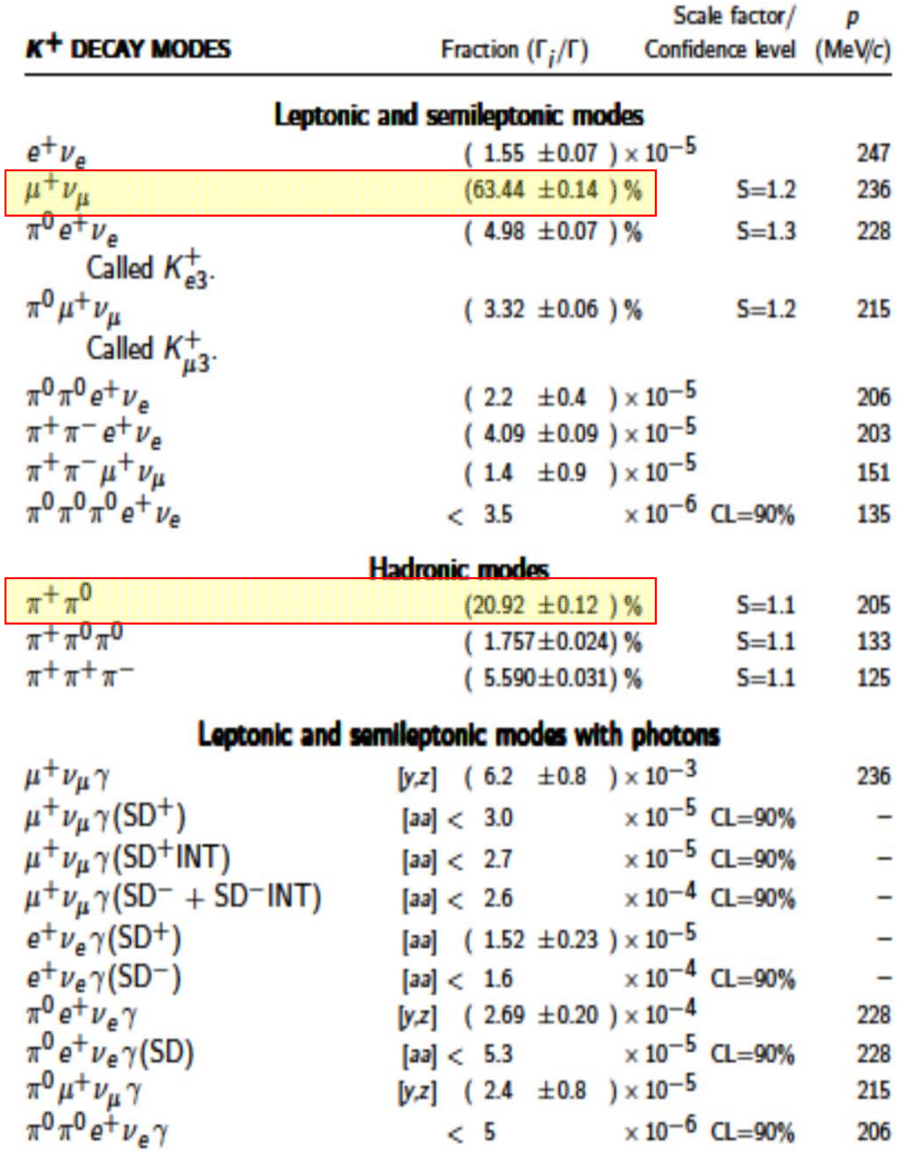
$$\tau_{K^{\pm}} = 1.2380(21) \times 10^{-8}\,\mathrm{s}$$
$$c\tau_{K^{\pm}} = 371.2\,\mathrm{cm}$$

$$\tau_{K_S} = 0.89564(33) \times 10^{-10}\,\mathrm{s}$$
$$c\tau_{K_S} = 2.7\,\mathrm{cm}$$

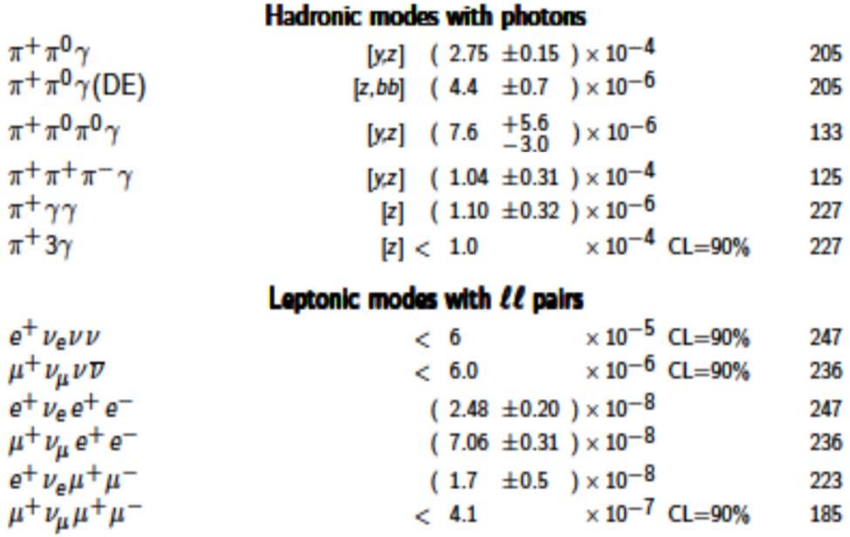
$$\tau_{K_L} = 5.116(21) \times 10^{-8}\,\mathrm{s}$$
$$c\tau_{K_L} = 1534\,\mathrm{cm}$$

K$^+$

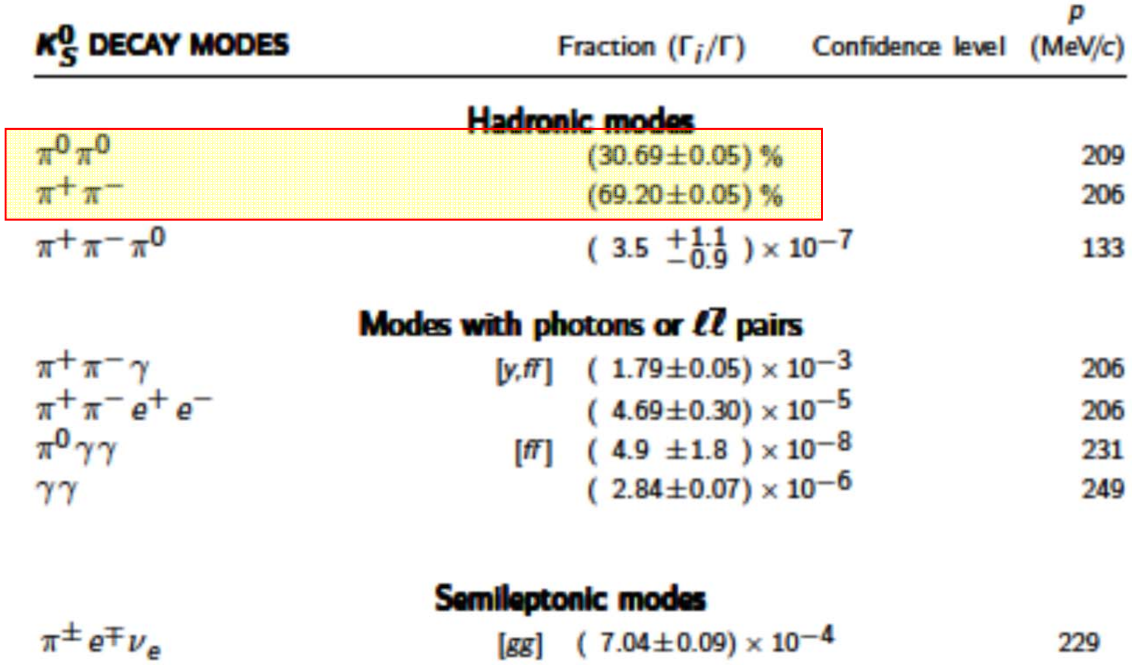
$$K^+ \to \mu^+ \nu_\mu$$
$$K^+ \to \pi^+ \pi^0$$



K^+ DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Leptonic and semileptonic modes				
$e^+\nu_e$		$(1.55 \pm 0.07) \times 10^{-5}$		247
$\mu^+\nu_\mu$		(63.44 ± 0.14) %	S=1.2	236
$\pi^0 e^+\nu_e$		(4.98 ± 0.07) %	S=1.3	228
Called K^+_{e3}.				
$\pi^0\mu^+\nu_\mu$		(3.32 ± 0.06) %	S=1.2	215
Called $K^+_{\mu 3}$.				
$\pi^0\pi^0 e^+\nu_e$		$(2.2 \pm 0.4) \times 10^{-5}$		206
$\pi^+\pi^- e^+\nu_e$		$(4.09 \pm 0.09) \times 10^{-5}$		203
$\pi^+\pi^-\mu^+\nu_\mu$		$(1.4 \pm 0.9) \times 10^{-5}$		151
$\pi^0\pi^0\pi^0 e^+\nu_e$		$< 3.5 \times 10^{-6}$	CL=90%	135
Hadronic modes				
$\pi^+\pi^0$		(20.92 ± 0.12) %	S=1.1	205
$\pi^+\pi^0\pi^0$		(1.757 ± 0.024) %	S=1.1	133
$\pi^+\pi^+\pi^-$		(5.590 ± 0.031) %	S=1.1	125
Leptonic and semileptonic modes with photons				
$\mu^+\nu_\mu\gamma$	[y,z]	$(6.2 \pm 0.8) \times 10^{-3}$		236
$\mu^+\nu_\mu\gamma(\mathrm{SD}^+)$	[aa]	$< 3.0 \times 10^{-5}$	CL=90%	–
$\mu^+\nu_\mu\gamma(\mathrm{SD}^+\mathrm{INT})$	[aa]	$< 2.7 \times 10^{-5}$	CL=90%	–
$\mu^+\nu_\mu\gamma(\mathrm{SD}^- + \mathrm{SD}^-\mathrm{INT})$	[aa]	$< 2.6 \times 10^{-4}$	CL=90%	–
$e^+\nu_e\gamma(\mathrm{SD}^+)$	[aa]	$(1.52 \pm 0.23) \times 10^{-5}$		–
$e^+\nu_e\gamma(\mathrm{SD}^-)$	[aa]	$< 1.6 \times 10^{-4}$	CL=90%	–
$\pi^0 e^+\nu_e\gamma$	[y,z]	$(2.69 \pm 0.20) \times 10^{-4}$		228
$\pi^0 e^+\nu_e\gamma(\mathrm{SD})$	[aa]	$< 5.3 \times 10^{-5}$	CL=90%	228
$\pi^0\mu^+\nu_\mu\gamma$	[y,z]	$(2.4 \pm 0.8) \times 10^{-5}$		215
$\pi^0\pi^0 e^+\nu_e\gamma$		$< 5 \times 10^{-6}$	CL=90%	206



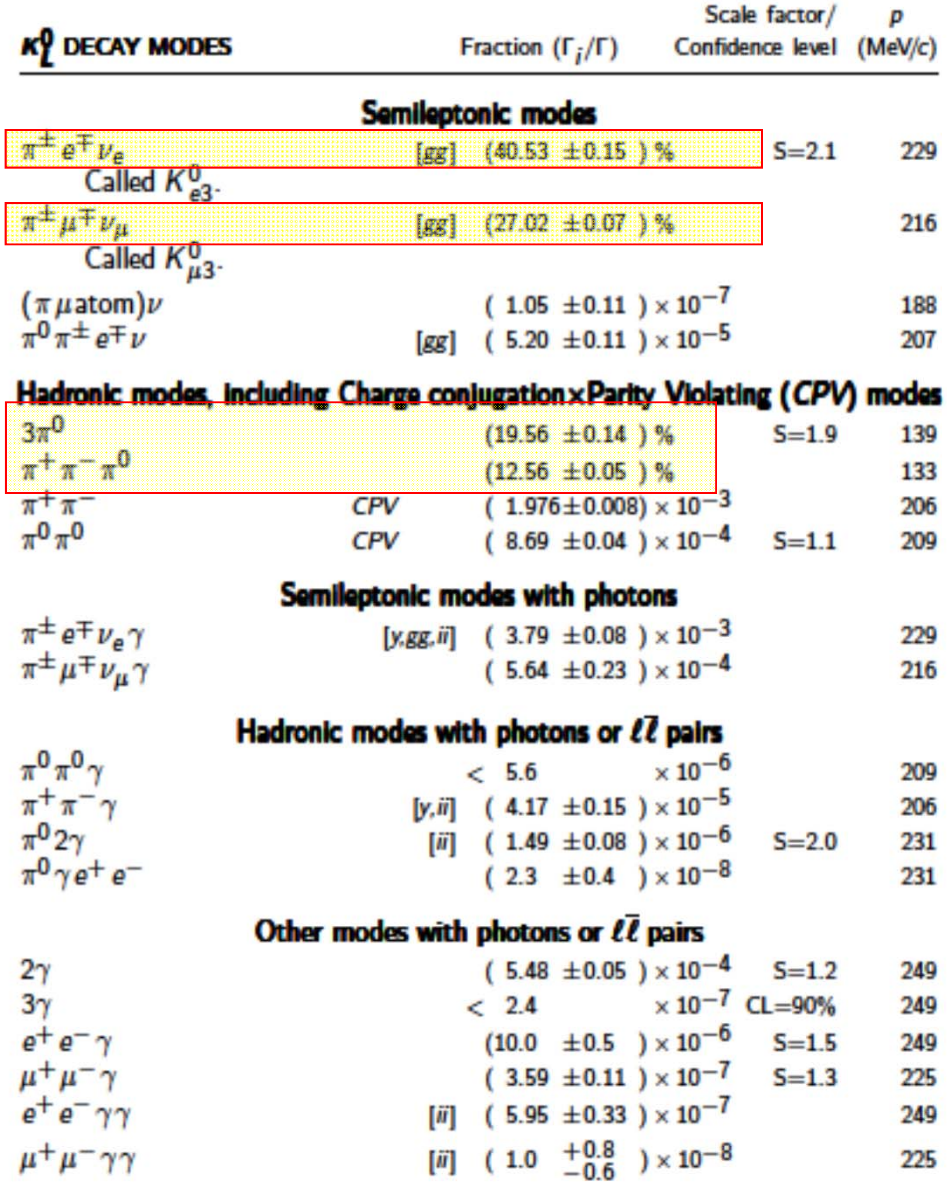
Hadronic modes with photons				
$\pi^+\pi^0\gamma$	[y,z]	$(2.75 \pm 0.15) \times 10^{-4}$		205
$\pi^+\pi^0\gamma(\mathrm{DE})$	[z,bb]	$(4.4 \pm 0.7) \times 10^{-6}$		205
$\pi^+\pi^0\pi^0\gamma$	[y,z]	$(7.6 ^{+5.6}_{-3.0}) \times 10^{-6}$		133
$\pi^+\pi^+\pi^-\gamma$	[y,z]	$(1.04 \pm 0.31) \times 10^{-4}$		125
$\pi^+\gamma\gamma$	[z]	$(1.10 \pm 0.32) \times 10^{-6}$		227
$\pi^+ 3\gamma$	[z]	$< 1.0 \times 10^{-4}$	CL=90%	227
Leptonic modes with $\ell\ell$ pairs				
$e^+\nu_e\nu\nu$		$< 6 \times 10^{-5}$	CL=90%	247
$\mu^+\nu_\mu\nu\overline{\nu}$		$< 6.0 \times 10^{-6}$	CL=90%	236
$e^+\nu_e e^+ e^-$		$(2.48 \pm 0.20) \times 10^{-8}$		247
$\mu^+\nu_\mu e^+ e^-$		$(7.06 \pm 0.31) \times 10^{-8}$		236
$e^+\nu_e\mu^+\mu^-$		$(1.7 \pm 0.5) \times 10^{-8}$		223
$\mu^+\nu_\mu\mu^+\mu^-$		$< 4.1 \times 10^{-7}$	CL=90%	185



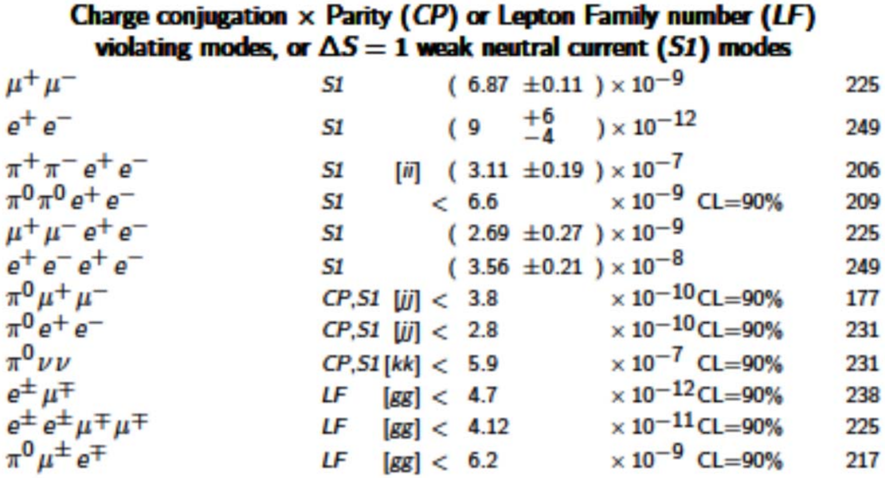
K^0_S DECAY MODES		Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
Hadronic modes				
$\pi^0\pi^0$		(30.69 ± 0.05) %		209
$\pi^+\pi^-$		(69.20 ± 0.05) %		206
$\pi^+\pi^-\pi^0$		$(3.5\ ^{+1.1}_{-0.9})\times10^{-7}$		133
Modes with photons or $\ell\bar{\ell}$ pairs				
$\pi^+\pi^-\gamma$	[y,ff]	$(1.79\pm0.05)\times10^{-3}$		206
$\pi^+\pi^-e^+e^-$		$(4.69\pm0.30)\times10^{-5}$		206
$\pi^0\gamma\gamma$	[ff]	$(4.9\ \pm1.8\ )\times10^{-8}$		231
$\gamma\gamma$		$(2.84\pm0.07)\times10^{-6}$		249
Semileptonic modes				
$\pi^\pm e^\mp\nu_e$	[gg]	$(7.04\pm0.09)\times10^{-4}$		229

$$K_S \to \pi\pi$$

K_S



K^0_L DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Semileptonic modes				
$\pi^{\pm} e^{\mp} \nu_e$	[gg]	(40.53 ± 0.15) %	S=2.1	229
Called K^0_{e3}.				
$\pi^{\pm} \mu^{\mp} \nu_\mu$	[gg]	(27.02 ± 0.07) %		216
Called $K^0_{\mu 3}$.				
$(\pi \mu \text{atom}) \nu$		$(1.05 \pm 0.11) \times 10^{-7}$		188
$\pi^0 \pi^{\pm} e^{\mp} \nu$	[gg]	$(5.20 \pm 0.11) \times 10^{-5}$		207
Hadronic modes, including Charge conjugation×Parity Violating (*CPV*) modes				
$3\pi^0$		(19.56 ± 0.14) %	S=1.9	139
$\pi^+ \pi^- \pi^0$		(12.56 ± 0.05) %		133
$\pi^+ \pi^-$	*CPV*	$(1.976 \pm 0.008) \times 10^{-3}$		206
$\pi^0 \pi^0$	*CPV*	$(8.69 \pm 0.04) \times 10^{-4}$	S=1.1	209
Semileptonic modes with photons				
$\pi^{\pm} e^{\mp} \nu_e \gamma$	[y,gg,ii]	$(3.79 \pm 0.08) \times 10^{-3}$		229
$\pi^{\pm} \mu^{\mp} \nu_\mu \gamma$		$(5.64 \pm 0.23) \times 10^{-4}$		216
Hadronic modes with photons or $\ell\bar{\ell}$ pairs				
$\pi^0 \pi^0 \gamma$		$< 5.6 \times 10^{-6}$		209
$\pi^+ \pi^- \gamma$	[y,ii]	$(4.17 \pm 0.15) \times 10^{-5}$		206
$\pi^0 2\gamma$	[ii]	$(1.49 \pm 0.08) \times 10^{-6}$	S=2.0	231
$\pi^0 \gamma e^+ e^-$		$(2.3 \pm 0.4) \times 10^{-8}$		231
Other modes with photons or $\ell\bar{\ell}$ pairs				
2γ		$(5.48 \pm 0.05) \times 10^{-4}$	S=1.2	249
3γ		$< 2.4 \times 10^{-7}$	CL=90%	249
$e^+ e^- \gamma$		$(10.0 \pm 0.5) \times 10^{-6}$	S=1.5	249
$\mu^+ \mu^- \gamma$		$(3.59 \pm 0.11) \times 10^{-7}$	S=1.3	225
$e^+ e^- \gamma\gamma$	[ii]	$(5.95 \pm 0.33) \times 10^{-7}$		249
$\mu^+ \mu^- \gamma\gamma$	[ii]	$(1.0 \; {}^{+0.8}_{-0.6}) \times 10^{-8}$		225



Charge conjugation × Parity (*CP*) or Lepton Family number (*LF*) violating modes, or $\Delta S = 1$ weak neutral current (*S1*) modes

$\mu^+ \mu^-$	*S1*		$(6.87 \pm 0.11) \times 10^{-9}$		225
$e^+ e^-$	*S1*		$(9 \; {}^{+6}_{-4}) \times 10^{-12}$		249
$\pi^+ \pi^- e^+ e^-$	*S1*	[ii]	$(3.11 \pm 0.19) \times 10^{-7}$		206
$\pi^0 \pi^0 e^+ e^-$	*S1*		$< 6.6 \times 10^{-9}$	CL=90%	209
$\mu^+ \mu^- e^+ e^-$	*S1*		$(2.69 \pm 0.27) \times 10^{-9}$		225
$e^+ e^- e^+ e^-$	*S1*		$(3.56 \pm 0.21) \times 10^{-8}$		249
$\pi^0 \mu^+ \mu^-$	*CP,S1*	[jj]	$< 3.8 \times 10^{-10}$	CL=90%	177
$\pi^0 e^+ e^-$	*CP,S1*	[jj]	$< 2.8 \times 10^{-10}$	CL=90%	231
$\pi^0 \nu \nu$	*CP,S1*	[kk]	$< 5.9 \times 10^{-7}$	CL=90%	231
$e^{\pm} \mu^{\mp}$	*LF*	[gg]	$< 4.7 \times 10^{-12}$	CL=90%	238
$e^{\pm} e^{\pm} \mu^{\mp} \mu^{\mp}$	*LF*	[gg]	$< 4.12 \times 10^{-11}$	CL=90%	225
$\pi^0 \mu^{\pm} e^{\mp}$	*LF*	[gg]	$< 6.2 \times 10^{-9}$	CL=90%	217

$$K_L \to \pi \ell \nu_\ell$$

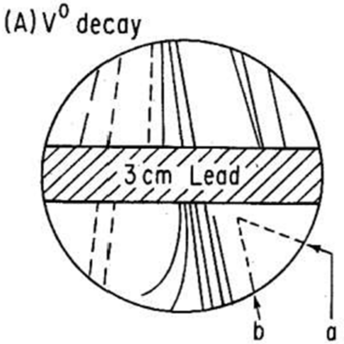
$$K_L \to \pi\pi\pi$$

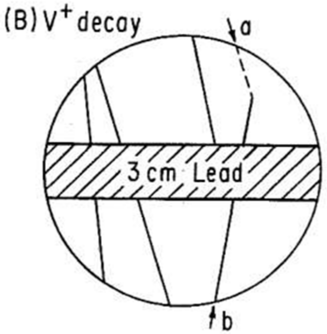
$\boxed{K_L}$

Discovery of kaon meson (strangeness)

Rochester, Butler (1947) [2]

- Cosmic ray particles which were just like pions except for their long lifetime.
- Always produced in pairs
- Mass ~ 0.5 GeV

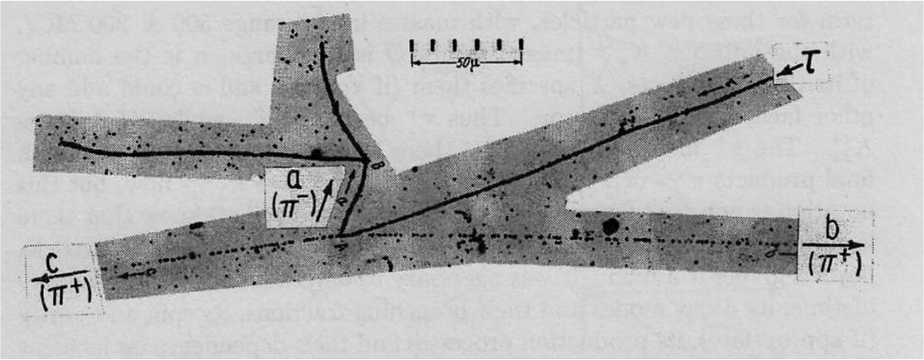




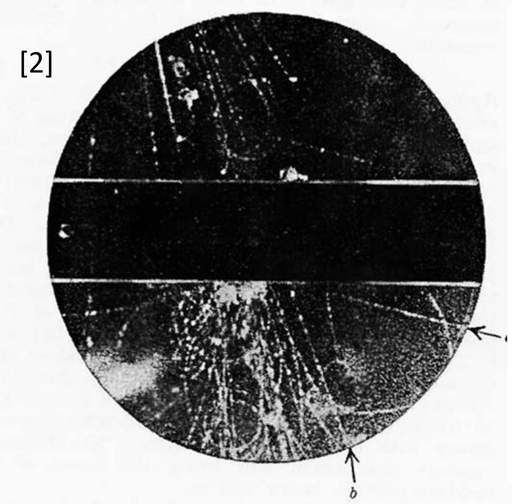
$K_S \to \pi^+\pi^-$

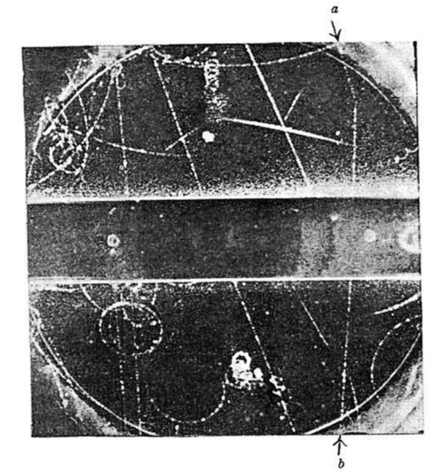
$K^+ \to \mu^+\nu_\mu$

$K^+ \to \pi^+\pi^+\pi^-$ [3]



[2]





$$\tau^\pm \to \pi^\pm\pi^+\pi^- \qquad \tau'^\pm \to \pi^\pm\pi^0\pi^0$$

$$\theta^\pm \to \pi^\pm\pi^0 \qquad \theta^0 \to \pi^+\pi^-$$

$$V^0 \to \pi^+\pi^- \qquad V^+ \to \nu^+\nu_\mu$$

$$M_\tau \simeq M_{\tau'} \simeq M_{\theta^+} \simeq M_{\theta^0} \simeq M_{V^0} \simeq M_{V^+}$$

$$M \simeq 0.5\,\mathrm{GeV}$$

The τ - θ puzzle

$$\left.\begin{array}{l}\tau^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-} \\ \theta^{\pm} \to \pi^{\pm}\pi^{0}\end{array}\right\} \quad \begin{array}{c} M_\tau \simeq M_\theta \\ \Gamma_\tau \simeq \Gamma_\theta \end{array} \quad \Longrightarrow \quad \tau = \theta \;\; ?$$

Parity

$$\theta^{\pm} \to \pi^{\pm}\pi^{0}$$

$$\left.\begin{array}{l}\eta_P(\theta) = \eta_P(\pi^{\pm}\pi^{0}) = (-1)^2(-1)^J = (-1)^J \\ \theta^0 \to \pi^0\pi^0 \;\; \longrightarrow \;\; \text{even spin}\end{array}\right\} \quad \eta_P(\theta) = +1$$

$$\tau^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$$

$$\eta_P(\tau) = \eta_P(\pi^{\pm}\pi^{+}\pi^{-}) = (-1)^3(-1)^{L+\ell} = (-1)^{\ell+1}$$

L ≡ relative orbital angular momentum of the two identical pions. L even (Bose symmetry)

ℓ ≡ orbital angular momentum of the third (”odd”) relative to the center of mass of identical pions

J^P	L	ℓ	2π
0^-	0	0	no
1^+	0	1	no
1^-	2	2	yes
2^+	2	1	yes
2^-	0	2	no
2^-	2	0	no
3^+	0	3	no
3^+	2	1	no
3^-	2	2	yes

$L = 0,\ \ell = 0$ → $\eta_P(\tau) = -1$

$\eta_P(\theta) = +1$ $\eta_P(\tau) = -1$

$$\begin{aligned} \eta_P(\theta) &= \eta_P(\pi^\pm \pi^0) \\ \eta_P(\tau) &= \eta_P(\pi^\pm \pi^+ \pi^-) \end{aligned}$$

??

→ Violation of Parity

Lee & Yang (1956) [4]

No convincing evidence of Parity Symmetry in $\beta -$ decays

Wu et al (1957) [5]

Experimental evidence of Parity Violation

The remainder of the lecture [1]

1. Survey on kaon decays
2. Nonleptonic decays: $\Delta I = 1/2\,\mathrm{rule}$
3. CP-violation
4. Rare decays: $K \to \pi \nu \overline{\nu}$, $K \to \pi \ell^+ \ell^-$

1. Survey on kaon decays

Non- Rare versus Rare Decays

BR $\gtrsim$ 10^{-5}

Decay	BR
$K^+ \to \pi^+ \nu_\mu$	0.6355 (11)
$K^+ \to \pi^0 \mu^+ \nu_\mu$	0.03353 (34)
$K_L \to \pi^\pm e^\mp \nu_e$	0.4055 (12)
$K^+ \to \pi^+ \pi^0$	0.2066 (8)
$K^+ \to \pi^+ \pi^+ \pi^-$	0.0559 (4)
$K_S \to \pi^0 \pi^0$	0.3069 (5)
$K_S \to \pi^+ \pi^-$	0.6920 (5)
$K_L \to \pi^0 \pi^0 \pi^0$	0.1952 (12)
$K_L \to \pi^+ \pi^- \pi^0$	0.1254 (5)
$K^+ \to \pi^+ \pi^0 \gamma$	2.75 (15) x 10^{-4}
$K_L \to \gamma\gamma$	5.47 (4) x 10^{-4}
$K_L \to \pi^+ \pi^- \gamma$	4.15 (15) x 10^{-5}
$K_S \to \pi^+ \pi^- \gamma$	1.79 (5) x 10^{-3}

Semileptonic Decays

Non-leptonic decays

Radiative decays

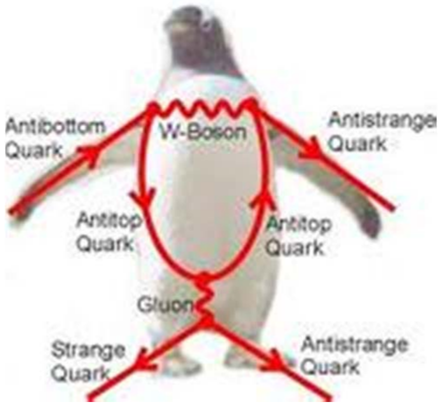
Non- Rare versus Rare Decays

BR $\lesssim 10^{-5}$

Decay	BR x 10^5
$K^+ \to \pi^+\gamma\gamma$	0.110 (32)
$K^+ \to \pi^+e^+e^-\gamma$	1.19 (13) x 10^{-3}
$K^+ \to \pi^+e^+e^-$	0.0300 (9)
$K_S \to \gamma\gamma$	0.263 (17)
$K_S \to \pi^0\mu^+\mu^-$	2.9 (1.5) x 10^{-4}
$K_L \to \pi^0\gamma\gamma$	0.1273 (34)
$K_L \to \mu^+\mu^-\gamma$	0.0359 (11)
$K_L \to e^+e^-$	9 $(^{+6}_{-4})$ x 10^{-7}
$K_L \to \pi^+\pi^-e^+e^-$	0.0311 (19)
$K_L \to \mu^+\mu^-e^+e^-$	2.69 (27) x 10^{-4}
$K_L \to \pi^0\mu^+\mu^-$	$<$ 1.8 x 10^{-5} (90% C.L.)
$K^+ \to \pi^+\nu\overline{\nu}$	1.7 (1.1) x 10^{-5}
$K_L \to \pi^0\nu\overline{\nu}$	$<$ 6.7 x 10^{-3} (90% C.L.)

Non- Rare versus Rare Decays

ΔS = 1 weak neutral current modes (FCNC)



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Non- Rare versus Rare Decays

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$K_L \to \pi^0\nu\overline{\nu}$	< 6.7 x 10^{-3} (90% C.L.)

Tiniest branching ratio ever measured …. as today (BNL E871)

Non- Rare versus Rare Decays

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$K^+ \to \pi^+\pi^+\pi^-$	0.0559 (4)
$K_S \to \pi^0\pi^0$	0.3069 (5)
$K_S \to \pi^+\pi^-$	0.6920 (5)
$K_L \to \pi^0\pi^0\pi^0$	0.1952 (12)
$K_L \to \pi^+\pi^-\pi^0$	0.1254 (5)
$K^+ \to \pi^+\pi^0\gamma$	2.75 (15) x 10^{-4}
$K_L \to \gamma\gamma$	5.47 (4) x 10^{-4}
$K_L \to \pi^+\pi^-\gamma$	4.15 (15) x 10^{-5}
$K_S \to \pi^+\pi^-\gamma$	1.79 (5) x 10^{-3}

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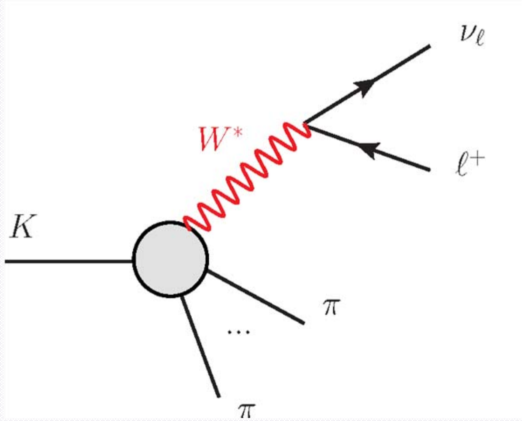
$K^+ \to \pi^+\nu\bar{\nu}$	1.7 (1.1) x 10^{-5}
$K_L \to \pi^0\nu\bar{\nu}$	< 6.7 x 10^{-3} (90% C.L.)

A look on Kaon Decays

Type



Processes

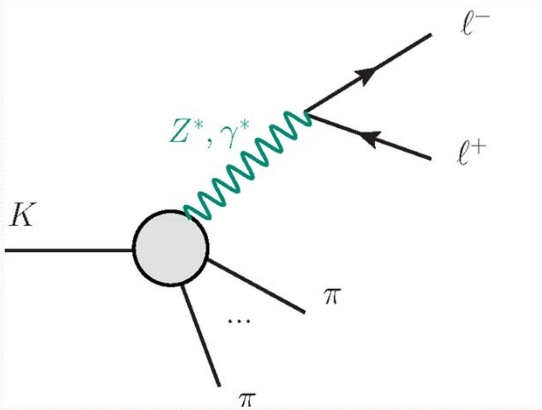
$$K^+ \to \ell^+ \nu_\ell\,(\gamma), \quad K \to \pi \ell^+ \nu_\ell\,(\gamma)$$

$$K \to \pi\pi \ell^+ \nu_\ell\,(\gamma), \quad K \to \pi\pi\pi e^+ \nu_e\,(\gamma)$$

Main Features

Low-energy dominated. Hadronization of electrically charged currents: χPT

Type



Processes

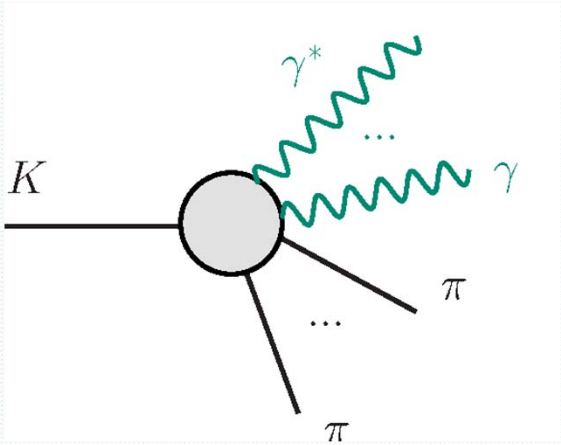
$$K \to \ell^+ \ell^-, \quad K \to \pi \ell^+ \ell^-$$

$$K \to \pi\pi \ell^+ \ell^-$$

Main Features

$\gamma^* >> Z^*$ → Low-energy dominated, FCNC

Type



Processes

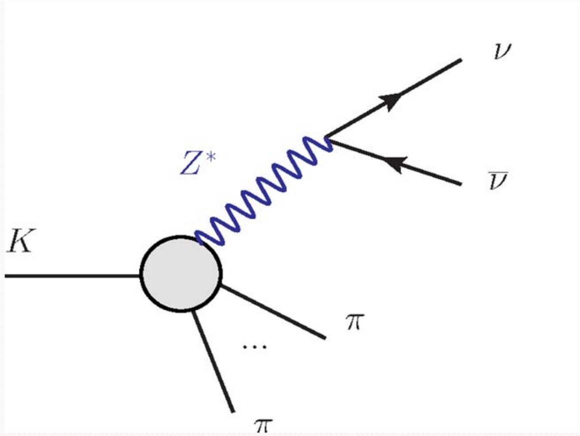
$$K \to \gamma\gamma^{(*)}, \quad K \to \pi\gamma\gamma^{(*)}$$

$$K \to \pi\pi\gamma^{(*)} \quad K \to \ell^+\ell^-\ell^+\ell^-$$

Main Features

Low-energy dominated, FCNC

Type



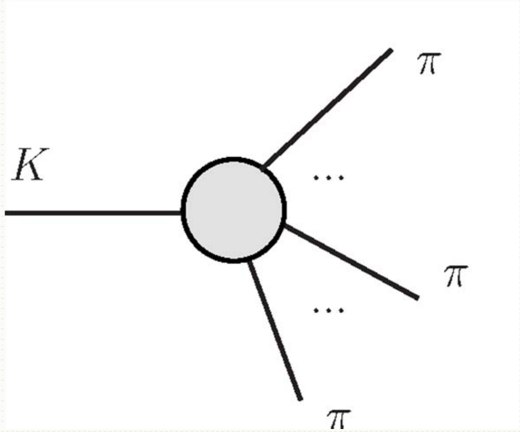
Processes

$$K \to \pi\nu\overline{\nu}, \quad K_L \to \gamma\nu\overline{\nu}$$

$$K \to \pi\pi\nu\overline{\nu}$$

Main Features

High-energy dominated, FCNC

Type	Processes
	$K \to \pi\pi, \quad K \to \pi\pi\pi$
	Main Features
	Low-energy dominated, Δ I = 1/2, ε'

State of the art

Low-energy dominated processes

Chiral Perturbation Theory framework

Up to $\mathcal{O}(p^4)$ √

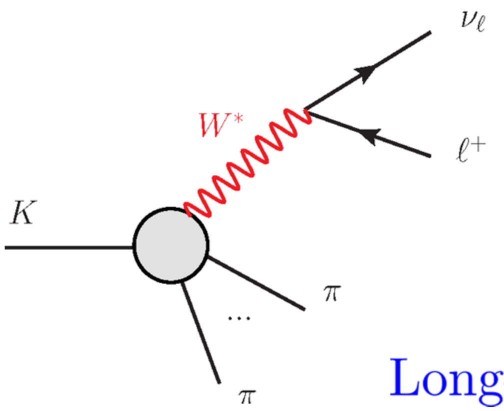
Dominating $\mathcal{O}(p^6)$ contributions √

High-energy dominated processes

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{13} C_i(\mu)\, Q_i(\mu)$$

Short-distance kaon dynamics

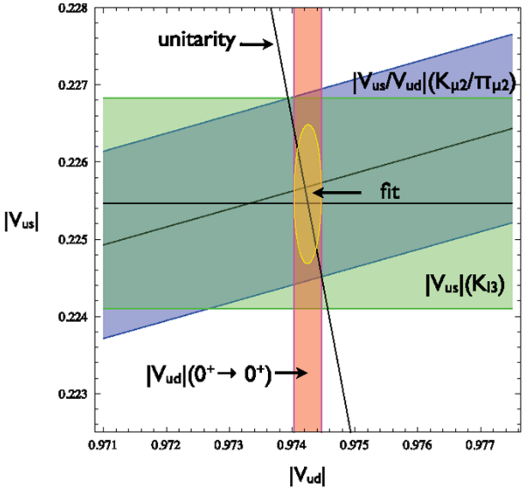
Semileptonic effective lagrangian



$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}}\, S_{\rm EW}{}^{1/2}\, \left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right]\left[\bar{u}_i\gamma^\mu(1-\gamma_5)V_{ij}d_j\right] + \text{h.c.}$$

$$S_{\rm EW} = 1 + \frac{2\alpha}{\pi}\left(1-\frac{\alpha_S}{4\pi}\right)\ln\frac{M_Z}{M_\rho} + \mathcal{O}\left(\frac{\alpha\alpha_S}{\pi^2}\right) = 1.0223(5)$$

Long-distance dominated ⟶ Chiral Perturbation Theory



$$K_{\ell 2(\gamma)} \equiv K \to \ell\,\nu_\ell(\gamma)$$

$$K_{\ell 3(\gamma)} \equiv K \to \pi\,\ell\,\nu_\ell(\gamma)$$

$$K_{\ell 4(\gamma)} \equiv K \to \pi\pi\ell\,\nu_\ell(\gamma)$$

$$K_{e5(\gamma)} \equiv K \to \pi\pi\pi\, e\,\nu_e(\gamma)$$

$$|V_{us}|(K_{\ell 3}) = 0.2255\,(5)_{\rm exp}\,(12)_{\rm th}$$

$a_0,\ a_2$ scattering lengths

$\pi\pi \to \pi\pi$

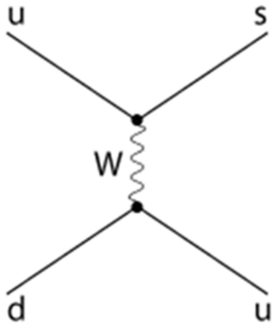
IFIC - Instituto de Física Corpuscular

$|\Delta S| = 1$ Effective Lagrangian

Very different mass scales : $M_\pi < M_K \ll M_W$ → Large logs

Operator Product Expansion and Renormalization Group → $M_W \to \mu < m_c$

$$\boxed{\mathcal{L}_{\text{eff}}\,(|\Delta S| = 1) = -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^* \sum_i^{13} C_i(\mu)\; Q_i(\mu)}$$



$$Q_2 = \underbrace{[\overline{s}\gamma^\mu(1-\gamma_5)u]}_{\mathbf{8}}\;\underbrace{[\overline{u}\gamma_\mu(1-\gamma_5)d]}_{\mathbf{8}}$$

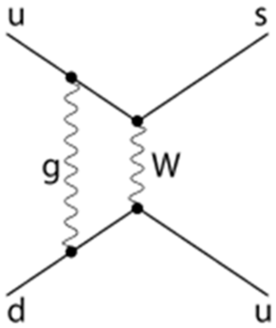
$SU(3)$ QCD currents

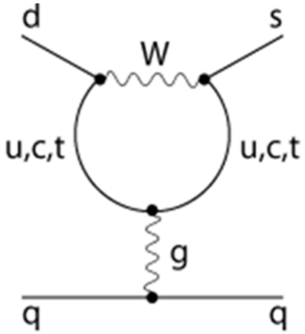
$$SU(3)\;:\quad (\mathbf{8}\otimes\mathbf{8})_{\text{symm}} = \cancel{\mathbf{1}} \oplus \mathbf{8} \oplus \mathbf{27}$$

$$SU(2)_I\;:\quad 1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2}$$

$$\left.\right\} \quad \mathbf{8}\left(\Delta I = \frac{1}{2}\right) \qquad \mathbf{27}\left(\Delta I = \frac{1}{2}, \frac{3}{2}\right)$$

QCD corrections





$$Q_1 = \left[\overline{s}^{\alpha}\gamma^{\mu}(1-\gamma_5)u^{\beta}\right]\left[\overline{u}^{\beta}\gamma_{\mu}(1-\gamma_5)d^{\alpha}\right]$$

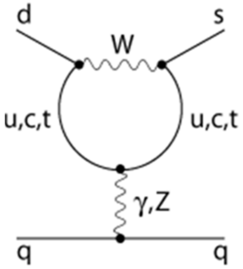
$$Q_3 = \left[\overline{s}\gamma^{\mu}(1-\gamma_5)d\right]\sum_{q=u,d,s}\left[\overline{q}\gamma_{\mu}(1-\gamma_5)q\right]$$

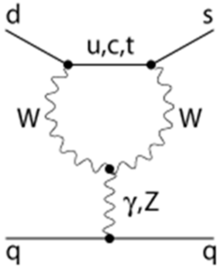
$$Q_4 = \left[\overline{s}^{\alpha}\gamma^{\mu}(1-\gamma_5)d^{\beta}\right]\sum_{q=u,d,s}\left[\overline{q}^{\beta}\gamma_{\mu}(1-\gamma_5)q^{\alpha}\right]$$

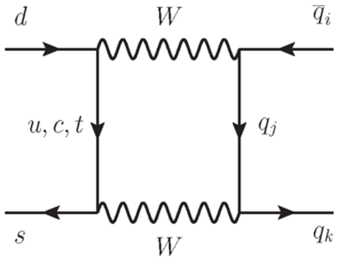
$$Q_5 = \left[\overline{s}\gamma^{\mu}(1-\gamma_5)d\right]\sum_{q=u,d,s}\left[\overline{q}\gamma_{\mu}(1+\gamma_5)q\right]$$

$$Q_6 = \left[\overline{s}^{\alpha}\gamma^{\mu}(1-\gamma_5)d^{\beta}\right]\sum_{q=u,d,s}\left[\overline{q}^{\beta}\gamma_{\mu}(1+\gamma_5)q^{\alpha}\right]$$

Electromagnetic and Z-penguins, W-boxes







$$Q_7 = \frac{3}{2}\left[\overline{s}\gamma^{\mu}(1-\gamma_5)d\right]\sum_{q=u,d,s} e_q\left[\overline{q}\gamma_{\mu}(1+\gamma_5)q\right]$$

$$Q_8 = \frac{3}{2}\left[\overline{s}^{\alpha}\gamma^{\mu}(1-\gamma_5)d^{\beta}\right]\sum_{q=u,d,s} e_q\left[\overline{q}^{\beta}\gamma_{\mu}(1+\gamma_5)q^{\alpha}\right]$$

$$Q_9 = \frac{3}{2}\left[\overline{s}\gamma^{\mu}(1-\gamma_5)d\right]\sum_{q=u,d,s} e_q\left[\overline{q}\gamma_{\mu}(1-\gamma_5)q\right]$$

$$Q_{10} = \frac{3}{2}\left[\overline{s}^{\alpha}\gamma^{\mu}(1-\gamma_5)d^{\beta}\right]\sum_{q=u,d,s} e_q\left[\overline{q}^{\beta}\gamma_{\mu}(1-\gamma_5)q^{\alpha}\right]$$

$e_q \equiv$ charge of the q quark in units of $e = \sqrt{4\pi\alpha}$

$E > \mu$ **Wilson coefficients :** $C_i(\mu)$

$$C_i(\mu) = C_i\left(M_Z, M_W, m_t, m_b, m_c, \mu\right)$$

$$C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu) \qquad \tau = -V_{td}V_{ts}^* / \left(V_{ud}V_{us}^*\right) \qquad \cancel{CP} \propto y_i(\mu)$$

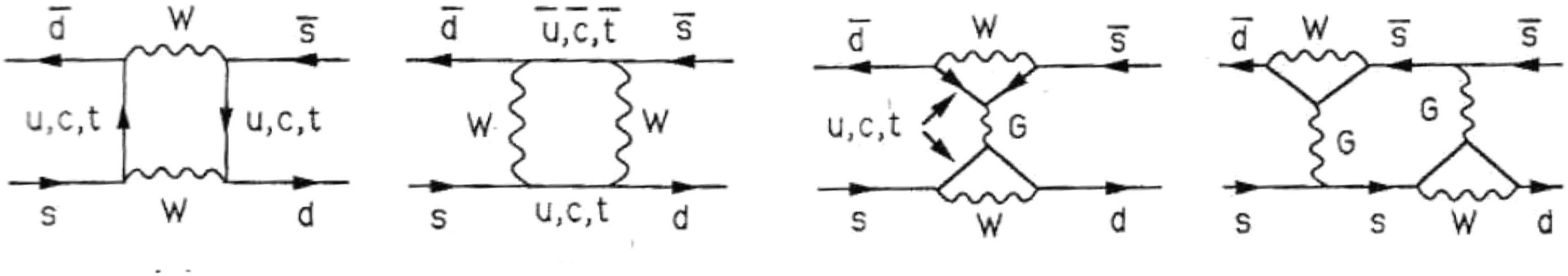
Known at the NLO : $\mathcal{O}\left(\alpha_S^n t^n\right),\ \mathcal{O}\left(\alpha_S^{n+1} t^n\right)$ $\qquad t = \ln(M_1/M_2),\quad M_1, M_2 \geq \mu$

$E < \mu$ **Matrix elements :** $\langle\, h \,|\, \mathcal{O}_i(\mu) \,|\, K \,\rangle$

Methods : Lattice Gauge Theory, QCD sum rules, functional bosonization, dynamical models, $1/N_C$ expansion, ...

Operators	$SU(3)_L \otimes SU(3)_R$
$Q_2 - Q_1, Q_3, Q_4, Q_5, Q_6$	$(8_L, 1_R)\,, \quad \Delta I = 1/2$
$2Q_2 + 3Q_1 - Q_3$	$(27_L, 1_R)\,, \quad \Delta I = 1/2, 3/2$
$Q_7,\ Q_8$	$(8_L, 1_R)\,, (8_L, 8_R)$
$Q_9,\ Q_{10}$	$(8_L, 1_R)\,, (27_L, 1_R)$

Mixing $K^0 - \overline{K^0}$: $\Delta S = 2$ Effective Lagrangian



$$\mathcal{L}_{\text{eff}}\,(|\Delta S| = 2) = -\frac{G_F^2\, M_W^2}{(4\pi)^2}\, C_{\Delta S=2}(\mu)\, \left[\bar{s}\gamma^\mu (1-\gamma_5) d\right] \left[\bar{s}\gamma_\mu (1-\gamma_5) d\right]$$

$$C_{\Delta S=2}(\mu) \;\simeq\; \lambda_c^2\, G_1\,(x_c) + \lambda_t^2\, G_2\,(x_t) + 2\,\lambda_c\lambda_t\, G_3\,(x_c, x_t)$$

$$\begin{aligned} x_i &= m_i^2/M_W^2 \\ \lambda_i &= V_{id}^* V_{is} \end{aligned}$$

$$\langle\, \overline{K^0}\, | Q_{\Delta S=2}\, |\, K^0\, \rangle \;=\; \frac{16}{3}\, F_K^2\, M_K^2\, B_K(\mu)$$

$$\hat{B}_K \;=\; \alpha_S(\mu^2)^{-2/9}\, B_K(\mu^2)$$

	Vacuum Saturation	$N_C \to \infty$	Lattice	Lattice
$\hat{B}_K$	1	$3/4$	0.724(30)	0.749(27)

[6]

Long-distance kaon dynamics

Chiral Perturbation Theory (weak)

- Chiral symmetry of massless QCD (spontaneously broken)

$$SU(3)_L \otimes SU(3)_R \to SU(3)_{L+R}$$

- Perturbative expansion : $p^2/\Lambda_\chi^2, \qquad \Lambda_\chi \sim 4\pi F,\; M_\rho \longrightarrow 1\,\mathrm{GeV}$

- $\mathcal{O}(p^2)$ Lagrangian :

$$\begin{aligned}\mathcal{L}^{(2)} \;&=\; G_8\, F^4\, \langle\, \lambda\, (D^\mu U)^\dagger D_\mu U\,\rangle && (8_L, 1_R)\\ &\quad + G_{27}\, F^4 \left([L_\mu]_{23}\, [L^\mu]_{11} \;+\; \frac{2}{3}\; [L_\mu]_{21}\, [L^\mu]_{13} \right) && (27_L, 1_R)\end{aligned}$$

$$U = \exp\left(i \frac{\sqrt{2}}{F}\, \Phi \right), \qquad D_\mu U = \partial_\mu U - i\, r_\mu\, U + i\, U\, \ell_\mu$$

$$L_\mu = i\, U^\dagger D_\mu U, \qquad \lambda = (\lambda_6 - i\, \lambda_7)/2 \qquad [\overline{s} \to \overline{d}]$$

G_8, G_{27} determined from phenomenology $\left[G_{8,27} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \, g_{8,27} \right]$

Chiral order	Isospin	g_8	g_{27}
LO	IC	4.96	0.285
LO	IV	4.99	0.253
NLO	IC	3.62 (28)	0.286 (28)
NLO	IV	3.61 (28)	0.297 (28)

IC = Isospin conserving, IV = Isospin violating

$$\left. \frac{g_8}{g_{27}} \right|_{\mathrm{NLO}} \simeq 13 \quad \left\{ \begin{array}{l} \text{Octet enhancement} \\ \Delta I = \frac{1}{2} \ \text{rule} \end{array} \right.$$

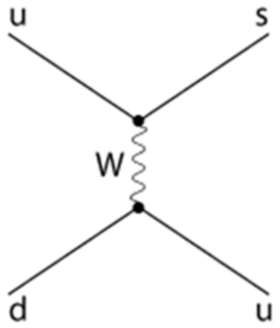
$$\boxed{\Delta S = 2}$$

$$\mathcal{L}^{(2)}_{\Delta S=2} = \frac{G_F^2 M_W^2}{(4\pi)^2} \, g_{\Delta S=2} \, F^4 \, \langle \lambda U^\dagger D^\mu U \rangle \langle \lambda U^\dagger D_\mu U \rangle \qquad (27_L, 1_R)$$

2. Nonleptonic kaon decays : $\Delta I = 1/2\,\mathrm{rule}$

$\boxed{K \to \pi\pi}$ (isospin limit)

$$\begin{aligned}
\mathcal{A}(K^0 \to \pi^+\pi^-) &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2} \\
\mathcal{A}(K^0 \to \pi^0\pi^0) &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} \\
\mathcal{A}(K^+ \to \pi^+\pi^0) &= \frac{3}{2} A_2\, e^{i\chi_2}
\end{aligned}$$



$SU(3)$ QCD currents

$$\mathcal{A} = \frac{g^2}{8} V_{ud} V_{us}^* \, (\bar{s}\gamma_\mu(1-\gamma_5)u) \frac{-g^{\mu\nu}}{q^2 - M_W^2} (\bar{u}\gamma_\nu(1-\gamma_5)d)$$

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \, (\bar{s}\gamma_\mu(1-\gamma_5)u)\, (\bar{u}\gamma^\mu(1-\gamma_5)d) \quad \longleftarrow \quad Q_2$$

factorization

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(V_\mu^{4-i5} - A_\mu^{4-i5}\right) \left(V^{\mu\,1+i2} - A^{\mu\,1+i2}\right)$$

$$\begin{aligned}
V_\mu^i &= \bar{q}\,\gamma_\mu \frac{\lambda_i}{2}\, q \\
A_\mu^i &= \bar{q}\,\gamma_\mu\gamma_5 \frac{\lambda_i}{2}\, q \\
q &= (u, d, s)^T
\end{aligned}$$

χPT

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle$$

$$u_\mu = i \left[u^\dagger \left(\partial_\mu - i\, (v+a)_\mu \right) u - u \left(\partial_\mu - i\, (v-a)_\mu \right) u^\dagger \right]$$

$$\begin{aligned} v_\mu &= v_\mu^i \frac{\lambda^i}{2} \\ a_\mu &= a_\mu^i \frac{\lambda^i}{2} \end{aligned}$$

$$\begin{aligned} V^{\mu\, i} &= \frac{\delta \mathcal{L}^{(2)}}{\delta v_\mu^i} = \frac{F^2}{4} \langle \lambda^i \left(u\, u_\mu\, u^\dagger - u^\dagger u_\mu u \right) \rangle \\ A^{\mu\, i} &= \frac{\delta \mathcal{L}^{(2)}}{\delta a_\mu^i} = \frac{F^2}{4} \langle \lambda^i \left(u\, u_\mu\, u^\dagger + u^\dagger u_\mu u \right) \rangle \end{aligned}$$

$$\left.\begin{aligned} A_0 &= \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \frac{2\sqrt{2}}{3} F \left(M_K^2 - M_\pi^2 \right) \\ A_2 &= \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \frac{2}{3} F \left(M_K^2 - M_\pi^2 \right) \end{aligned}\right\} \quad \boxed{\frac{A_0}{A_2} = \sqrt{2}}$$

$$\left.\frac{A_0}{A_2}\right|_{\text{pheno}} = 21.63 \pm 0.04$$

	10^8 A_0	10^8 A_2
Theory	**3.54**	**2.50**
Phenomenology	**27.04 (1)**	**1.210 (2)**

$$\boxed{\Delta I = 1/2 \begin{cases} \text{rule} \\ \text{problem} \end{cases}}$$

3. CP Violation

Kaon CP-violating observables : ε, ε'

$$CP\,|\,K^0\,\rangle \;=\; +|\,\overline{K^0}\,\rangle$$

$$\left.\begin{aligned} |\,K_1\,\rangle &= \frac{1}{\sqrt{2}}\Big(|\,K^0\,\rangle + |\,\overline{K^0}\,\rangle\Big) \\ |\,K_2\,\rangle &= \frac{1}{\sqrt{2}}\Big(|\,K^0\,\rangle - |\,\overline{K^0}\,\rangle\Big) \end{aligned}\right\}\qquad \begin{aligned} CP\,|\,K_1\,\rangle &= +|\,K_1\,\rangle \longrightarrow \pi\pi \\ CP\,|\,K_2\,\rangle &= -|\,K_2\,\rangle \longrightarrow \pi\pi\pi \end{aligned}$$

$$|\,K_S\,\rangle = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}\,(|\,K_1\,\rangle + \tilde{\epsilon}\,|\,K_2\,\rangle) \longrightarrow \pi\pi$$

$$|\,K_L\,\rangle = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}\,(|\,K_2\,\rangle + \tilde{\epsilon}\,|\,K_2\,\rangle) \longrightarrow \pi\pi\pi$$

Br	$\pi^0\pi^0$	$\pi^+\pi^-$	$\pi^+\pi^-\pi^0$	$\pi^0\pi^0\pi^0$
K_S	30.69(5) %	69.20(5) %	$3.5(1.1)\times 10^{-7}$	$< 1.2\times 10^{-7}$
K_L	$8.64(6)\times 10^{-4}$	$1.967(10)\times 10^{-3}$	12.54(5) %	19.52(12) %

$$\boxed{\eta_{+-} \equiv \frac{\mathcal{A}\left(K_L \to \pi^+\pi^-\right)}{\mathcal{A}\left(K_S \to \pi^+\pi^-\right)} = \varepsilon + \varepsilon'}$$

$$\boxed{\eta_{00} \equiv \frac{\mathcal{A}\left(K_L \to \pi^0\pi^0\right)}{\mathcal{A}\left(K_S \to \pi^0\pi^0\right)} = \varepsilon - 2\,\varepsilon'}$$

$$\varepsilon \simeq \tilde{\epsilon} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} = \tilde{\epsilon}_{\mathrm{WY}}$$

Indirect CP violation

$$\varepsilon' \simeq \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_0)}\, \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0} \left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]$$

Direct CP violation

Indirect CP violation

$$\varepsilon = f\left(\hat{B}_K, V_{\mathrm{CKM}}, m_c, m_t, \ldots\right) \xrightarrow{\text{Theory}} \boxed{|\varepsilon| = 1.90(26) \times 10^{-3}}$$

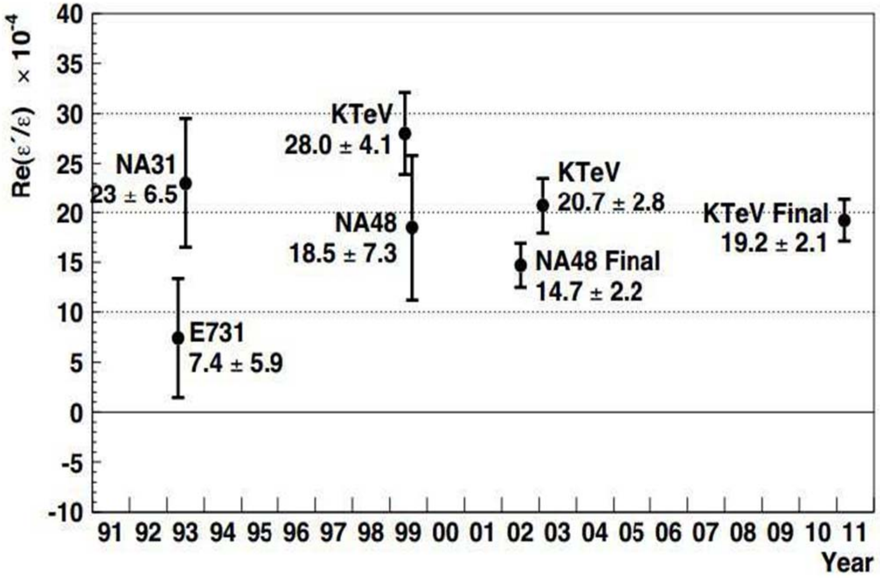
$$\boxed{|\varepsilon|_{\mathrm{exp}} = 2.228(11) \times 10^{-3} \qquad \arg\left(\varepsilon\right)_{\mathrm{exp}} = 44(7)^\circ}$$

$$\frac{\Gamma\left(K_L \to \pi^-\ell^+\nu_\ell\right) - \Gamma\left(K_L \to \pi^+\ell^-\overline{\nu}_\ell\right)}{\Gamma\left(K_L \to \pi^-\ell^+\nu_\ell\right) + \Gamma\left(K_L \to \pi^+\ell^-\overline{\nu}_\ell\right)} = \frac{2\,\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}$$ √

Direct CP violation

$$\varepsilon' \simeq \frac{i}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0} \left[\frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2} - \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \right]$$

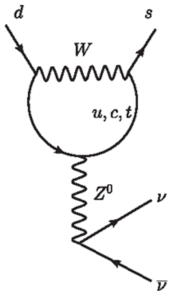
$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) \quad \begin{cases} \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{exp}} = 16.8\,(1.4) \times 10^{-4} \\ \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{theo\ SM}} = 19\,(11) \times 10^{-4} \end{cases}$$

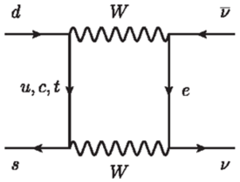


$$\begin{aligned} \arg(\varepsilon') &= \chi_2 - \chi_0 + \frac{\pi}{2} \\ &= 42.5\,(9)^\circ \end{aligned}$$

4. Rare decays: $K \to \pi\nu\overline{\nu}$

$K \to \pi \nu \overline{\nu}$





$$A(s \to d\nu\overline{\nu}) \sim \frac{m_t^2}{M_W^2}\lambda_t + \frac{m_c^2}{M_W^2}\ln\frac{M_W}{m_c}\lambda_c + \frac{\Lambda_{QCD}^2}{M_W^2}\lambda_u, \qquad \lambda_q = V_{qd}V_{qs}^*$$

~ 68%　　~ 29%　　~ 3%

“hard” GIM

$\lambda_t + \lambda_c + \lambda_u = 0$

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \, C_{13}(\mu) \; [\overline{s}\gamma^\mu(1-\gamma_5)d] \, [\overline{\nu}\gamma_\mu(1-\gamma_5)\nu]$$

Our knowledge on $C_{13}(\mu)$

- NLO QCD effects (top)
- Two-loop electroweak corrections (top)
- NNLO QCD effects (charm)
- NLO electroweak corrections (charm)

Our knowledge on $Q_{13}(\mu)$

- Matrix element can be related with $K_{\ell 3}$ form factors
- Long-distance corrections

$$\boxed{\mathrm{BR}\left(K_L \to \pi^0 \nu\overline{\nu}\right) = \kappa_L \left(\frac{\mathrm{Im}\,\lambda_t}{\lambda^5}\, X\right)^2 (1-\delta_\epsilon)}$$

$\kappa_L = (2.231 \pm 0.013) \times 10^{-10} (\lambda/0.225)^8$	Hadronic matrix element $K_{\ell 3}$
$X = 1.469 \pm 0.017$	Top-quark contribution
$\delta_\epsilon = \sqrt{2}\|\varepsilon\| \left[1 + P_c/(A^2\lambda) - \rho\right]/\eta$	$K^0 - \overline{K}^0$ contribution
$P_c = 0.38 \pm 0.04$	Dimension-6 charm contribution

$$\boxed{\mathrm{BR}(K_L \to \pi^0 \nu\overline{\nu})\big|_{\mathrm{theo}} = (2.4 \pm 0.4) \times 10^{-11}}$$

$$\boxed{\mathrm{BR}(K_L \to \pi^0 \nu\overline{\nu}) < 2.6 \times 10^{-8} \qquad (90\%\ \mathrm{C.L.})}$$

$$\boxed{\mathrm{BR}\left(K^+ \to \pi^+ \nu\bar{\nu}\right) = \kappa_+ \, (1 + \Delta_{EM}) \left[\left(\frac{\mathrm{Im}\, \lambda_t}{\lambda^5} \, X \right)^2 + \left(\frac{\mathrm{Re}\, \lambda_t}{\lambda^5} \, X + \frac{\mathrm{Re}\, \lambda_c}{\lambda} \, (P_c + \delta P_{c,u}) \right)^2 \right]}$$

$\kappa_+ = (5.173 \pm 0.025) \times 10^{-11} (\lambda/0.225)^8$	Hadronic matrix element $K_{\ell 3}$
$X = 1.469 \pm 0.017$	Top-quark contribution
$\Delta_{EM} = -0.003$	EM correction ($E_\gamma^{\mathrm{cms}} < 20\,\mathrm{MeV}$)
$P_c = 0.38 \pm 0.04$	Dimension-6 charm contribution
$\delta P_{c,u} = 0.04 \pm 0.02$	Long-distance + dimension-8 charm

$$\boxed{\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})\big|_{\mathrm{theo}} = (0.78 \pm 0.08) \times 10^{-10}}$$

$$\boxed{\mathrm{BR}(K^+ \to \pi^+ \nu\bar{\nu})\big|_{\mathrm{exp}} = \left(1.73^{+1.15}_{-1.05}\right) \times 10^{-10}}$$

4. Rare decays: $K_L \to \pi^0 \ell^+ \ell^-$

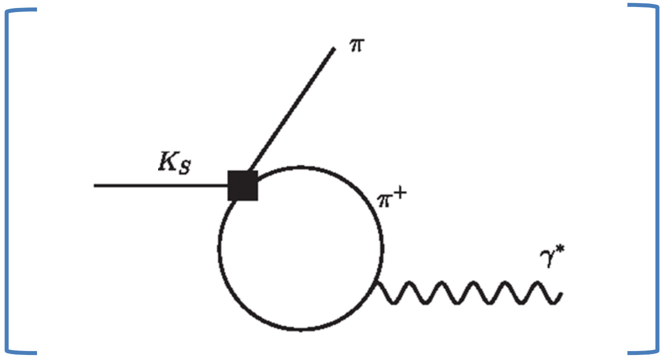
$\boxed{K_L \to \pi^0 \ell^+ \ell^-}$

1. Direct CP-violating transition

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left[C_{7V}(\mu)\, [\bar{s}\gamma^\mu (1-\gamma_5) d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu \ell] \right. \\ \left. + C_{7A}(\mu)\, [\bar{s}\gamma^\mu (1-\gamma_5) d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu \gamma_5 \ell] \right]$$

2. Indirect CP-violating transition due to $K^0 - \overline{K}^0$ oscillation

$$A_2 \left(K_L \to \pi^0 \ell^+ \ell^- \right) = \varepsilon \times$$

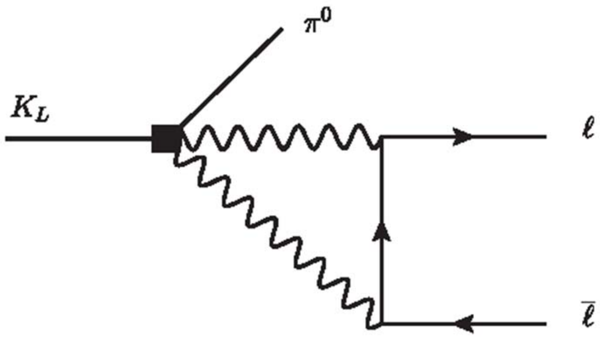


$$\mathrm{BR}\left(K_L \to \pi^0 e^+ e^-\right)\Big|_{\mathrm{CPV}} = 10^{-12} \times \left[15.7\,|a_S|^2 \pm 6.2\,|a_S|\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 2.4\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2\right]$$

$$\mathrm{BR}\left(K_L \to \pi^0 \mu^+ \mu^-\right)\Big|_{\mathrm{CPV}} = 10^{-12} \times \left[3.7\,|a_S|^2 \pm 1.6\,|a_S|\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right) + 1.0\left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2\right]$$

$$K_S \to \pi^0 \ell^+ \ell^- \quad \longrightarrow \quad a_S \sim 1$$

3. CP-conserving contribution from $K_L \to \pi^0 \gamma\gamma \to \pi^0 \ell^+ \ell^-$

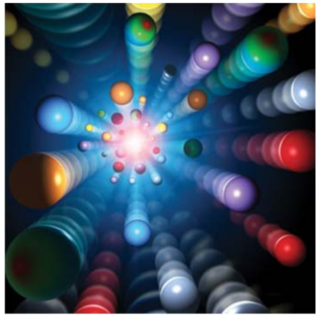


Assuming positive interference between the CP-V contributions (theoretically preferred) …

	$\mathrm{BR}\,(K_L \to \pi^0 e^+ e^-)$	$\mathrm{BR}\,(K_L \to \pi^0 \mu^+ \mu^-)$
CP-V	$(3.1 \pm 0.9) \times 10^{-11}$	$(1.4 \pm 0.5) \times 10^{-11}$
CP-C	~ 0	$(5.2 \pm 1.6) \times 10^{-12}$
KTeV (90% C.L.)	$< \ 2.8 \times 10^{-10}$	$< \ 3.8 \times 10^{-10}$

Epilogue

Present and Future Experiments



Experiment	Kaon Physics Main Goal
NA48 (CERN),KTeV (Fermilab)	$K_{\ell 3},\ K_{\ell 4},\ K \to \pi\pi/\pi\pi\pi,\ K \to \pi\gamma\gamma,$ $K \to \pi\ell^+\ell^-\ ,\ \ \varepsilon'$
NA62 (CERN)	$K^+ \to \pi^+\nu\overline{\nu}\,,\ K^+ \to \pi^+\gamma\gamma$
K^{0}TO (J-PARC)	$K_L \to \pi^0\nu\overline{\nu}$
TREK (J-PARC)	$K^+ \to \pi^0\mu^+\nu_\mu$
KLOE-2 (KLOE) (DAΦNE)	CP issues, radiative decays
KLOD (IHEP, Protvino)	$K_L \to \pi^0\nu\overline{\nu}$
OKA (ISTRA+) (IHEP, Protvino)	Kaon decays (BR ~ $10^{-3} – 10^{-8}$)
Project – X (Fermilab)	$K \to \pi\nu\overline{\nu},\ \ K_L \to \pi^0\ell^+\ell^-$

1. Kaon decays provide an excellent framework to settle SM predictions and, consequently, might foresee hints of BSM effects.

2. Short-distance dominated processes (namely with a "neutrino Dalitz pair") are clean and can be predicted accurately.
They are/will be the goal of present/future flavour facilities.

3. Most of long-distance dominated rare decays can also be predicted within a 30 % in the branching ratios. This is not precision physics but enough for the present and foreseen experimental status.
In general, it will be difficult to increase the accuracy in the theoretical predictions of these processes.

4. Semileptonic (charged current) processes have an excellent status.
Theoretical analyses reach a few percent accuracy in most cases.

5. Non-leptonic kaon decays are still an open issue.

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