IDPASC School of Flavour
2-6 May 2013

$K \to \pi\pi$ **within factorization**

The amplitudes for the decays of the kaons into two pions are given, in the isospin limit, by:

$$\begin{aligned}\mathcal{A}\left(K^0 \to \pi^+\pi^-\right) &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}}\, A_2\, e^{i\chi_2}, \\ \mathcal{A}\left(K^0 \to \pi^0\pi^0\right) &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}, \\ \mathcal{A}\left(K^+ \to \pi^+\pi^0\right) &= \frac{3}{2}\, A_2\, e^{i\chi_2},\end{aligned}$$

in terms of the isospin amplitudes A_0 and A_2 and the corresponding phase-shifts χ_0, χ_2:

- By evaluating the leading Feynman diagram in the Standard Model, given by Figure 1, obtain the result:

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(\overline{s}\gamma_\mu(1-\gamma_5)u\right)\left(\overline{u}\gamma^\mu(1-\gamma_5)d\right),$$

  in the limit of $M_W^2 \gg q^2$.

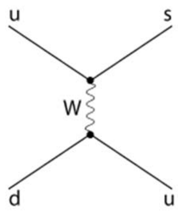

Figure 1:

- This result can be hadronized by assuming factorization of the SU(3) currents. Show that one can write:

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(V_\mu^{4-i5} - A_\mu^{4-i5}\right)\left(V^{\mu(1+i2)} - A^{\mu(1+i2)}\right),$$

  where $V_\mu^i = \overline{q}\gamma_\mu \frac{\lambda^i}{2} q$ and $A_\mu^i = \overline{q}\gamma_\mu\gamma_5 \frac{\lambda^i}{2} q$.

- Obtain the isospin amplitudes A_0 and A_2 using the $\mathcal{O}(p^2)$ chiral Lagrangian:

$$\mathcal{L} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle.$$

  The notation is the one in the slides of the Kaon Physics talk. To hadronize the currents you can use that $V_\mu^i = \frac{\delta\mathcal{L}}{\delta v_i^\mu}$ and, analogously, for the axial-vector current.

- Check that $A_0/A_2 = \sqrt{2}$.