Theory of Flavour physics and CP violation

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IDPASC School on Flavour Physics, May 2013

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Rare particle decay delivers blow to supersymmetry

By Lucie Bradley
Cosmos Online

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SYDNEY: The popular physics theory of supersymmetry has been called into question by new results from CERN.

Physicists working at CERN's Large Hadron Collider (LHC) near Geneva, Switzerland, have announced the discovery of an extremely rare type of particle decay.

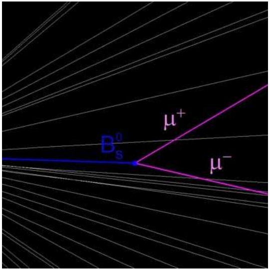
While discoveries are usually accompanied by excitement there is also a tinge of uncertainty surrounding this latest finding from CERN. It has dealt a hefty blow to the popular physics theory of supersymmetry.

The results were presented at the Hadron Collider Physics Symposium in Kyoto, Japan, and will also be submitted to the journal *Physical Review Papers*.

A three in one billion chance

Scientists have been searching for this type of particle decay for the last decade and so the results from CERN have "generated a lot of excitement now that it has been found," according to physicist Mark Kruse, from Duke University, North Carolina, USA. "And it hasn't ruled out supersymmetry – just some of the more favoured variants of it."

The traditional theory of subatomic physics is known as the Standard Model, but it is unable to explain everything observed in the world around us, including gravity and dark matter. Supplementary theories exist to help explain these inconsistencies. Of these theories, supersymmetry, which proposes that 'superparticles' exist – massive versions of those particles that are already known – is arguably the most popular.



A typical decay of the Bs (B sub s) meson into two muons. The two muons traversed the whole LHCb detector, which originated from the B0s decay point 14 mm from the proton-proton collision. Credit: LHCb

COSMOS Magazine

Contents

Basics

Flavour physics

studies transitions between fermions of different generations.

flavour = fermion species

$$\begin{pmatrix} u_L, u_L, u_L \\ d_L, d_L, d_L \end{pmatrix} \quad \begin{pmatrix} c_L, c_L, c_L \\ s_L, s_L, s_L \end{pmatrix} \quad \begin{pmatrix} t_L, t_L, t_L \\ b_L, b_L, b_L \end{pmatrix}$$

$$\begin{matrix} u_R, u_R, u_R \\ d_R, d_R, d_R \end{matrix} \quad \begin{matrix} c_R, c_R, c_R \\ s_R, s_R, s_R \end{matrix} \quad \begin{matrix} t_R, t_R, t_R \\ b_R, b_R, b_R \end{matrix}$$

$$\begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix}$$

$$e_R \quad \mu_R \quad \tau_R$$

Flavour quantum numbers:

quantum number	d	u	s	c	b	t	e,ν_e	μ,ν_μ	τ,ν_τ
D	-1	0	0	0	0	0	0	0	0
U	0	1	0	0	0	0	0	0	0
strangeness S	0	0	-1	0	0	0	0	0	0
charm C	0	0	0	1	0	0	0	0	0
beauty B	0	0	0	0	-1	0	0	0	0
T	0	0	0	0	0	1	0	0	0
electron number L_e	0	0	0	0	0	0	1	0	0
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lepton number $L = L_e + L_\mu + L_\tau$

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antifermions have opposite quantum numbers

Flavour quantum numbers are respected by the strong interaction, so we can use them to categorise hadrons. E.g. a B^+ meson has $B = U = 1$, shorthand notation:

$$B^+ \sim \overline{b}u$$

For a $B_d \equiv B^0$ (with $B = -D = 1$) we write

$$B_d \sim \overline{b}d$$

Some flavoured mesons

charged:

$$K^+ \sim \bar{s}u, \quad D^+ \sim c\bar{d}, \quad D_s^+ \sim c\bar{s}, \quad B^+ \sim \bar{b}u, \quad B_c^+ \sim \bar{b}c,$$
$$K^- \sim s\bar{u}, \quad D^- \sim \bar{c}d, \quad D_s^- \sim \bar{c}s, \quad B^- \sim b\bar{u}, \quad B_c^- \sim b\bar{c},$$

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In flavour physics only the ground-state hadrons which decay weakly rather than strongly are interesting.
Weakly decaying baryons are less interesting, because they are produced in smaller rates and are theoretically harder to cope with.

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The neutral K, D, B_d and B_s mesons mix with their antiparticles, $\overline{K}$, $\overline{D}$, $\overline{B}_d$ and $\overline{B}_s$ thanks to the weak interaction (quantum-mechanical two-state systems).

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$\Rightarrow$ gold mine for fundamental parameters

Strong isospin: Instead of U and D use (I, I_3):

Fundamental doublets ($I = \frac{1}{2}$): $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

For $m_u = m_d$ the QCD lagrangian is invariant under SU(2) rotations of $\begin{pmatrix} u \\ d \end{pmatrix}$ and $\begin{pmatrix} \overline{d} \\ -\overline{u} \end{pmatrix}$.

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Owing to $m_d - m_u \ll \Lambda_{\rm had} \sim 500\,\text{MeV}$, strong isospin holds to $\sim 2\%$ accuracy. E.g. $M_{B_d} - M_{B^+} = (0.37 \pm 0.24)\,\text{MeV}$.

Isospin triplet:

$$\pi^+ \sim u\overline{d}, \qquad \pi^0 \sim \frac{u\overline{u} - d\overline{d}}{\sqrt{2}}, \qquad \pi^- \sim d\overline{u}.$$

Compare with spin triplet

$$\uparrow\uparrow, \qquad \frac{\uparrow\uparrow + \downarrow\downarrow}{\sqrt{2}}, \qquad \downarrow\downarrow$$

Flavour–$SU(3)$:

Since $m_s - m_{u,d} < \Lambda_{\rm had}$ we can try to enlarge isospin–SU(2) to SU(3)$_{\rm F}$ with fundamental triplet $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$

U-spin subgroup: SU(2) rotations of $\begin{pmatrix} d \\ s \end{pmatrix}$

Pedestrian's use of U-spin:
(i) Draw all diagrams contributing to some process.
(ii) Replace $s \leftrightarrow d$ to connect the hadronic interaction in different processes.

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Accuracy of $\mathrm{SU(3)_F}$: 30% per $s \leftrightarrow d$ exchange.

Electroweak interaction

Gauge group:

$SU(2) \times U(1)_Y$

doublets: $Q_L^j = \begin{pmatrix} u_L^j \\ d_L^j \end{pmatrix}$ und $L^j = \begin{pmatrix} \nu_L^j \\ \ell_L^j \end{pmatrix}$

$j = 1, 2, 3$ labels the generation.

Examples: $Q_L^3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $L^1 = \begin{pmatrix} \nu^{eL} \\ e_L \end{pmatrix}$

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Important: Only left-handed fields couple to the W boson.

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- three gauge interactions
- Yukawa interaction of Higgs with quarks and leptons
- Higgs self-interaction

Yukawa interaction

Higgs doublet $H = \begin{pmatrix} G^+ \\ v + \frac{h^0 + iG^0}{\sqrt{2}} \end{pmatrix}$ with $v = 174\,\mathrm{GeV}$.

Charge-conjugate doublet: $\widetilde{H} = \begin{pmatrix} v + \frac{h^0 - iG^0}{\sqrt{2}} \\ -G^- \end{pmatrix}$

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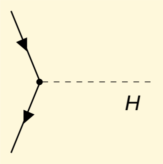

Yukawa lagrangian:

$$-L_Y = Y^d_{jk}\, \overline{Q}^j_L\, H\, d^k_R \; + \; Y^u_{jk}\, \overline{Q}^j_L\, \widetilde{H}\, u^k_R \; + \; Y^l_{jk}\, \overline{L}^j_L\, H\, e^k_R \; + \; \text{h.c.}$$

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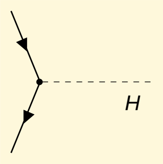

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Here neutrinos are (still) massless.
The Yukawa matrices Y^f are arbitrary complex 3×3 matrices.
The mass matrices $M^f = Y^f v$ are not diagonal!

$\Rightarrow$ $u^j_{L,R}, d^j_{L,R}$ do not describe physical quarks!
We must find a basis in which Y^f is diagonal!

Any matrix can be diagonalised by a bi-unitary transformation. Start with

$$\widehat{Y}^u = S_Q^\dagger Y^u S_u \quad \text{with } \widehat{Y}^u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad \text{and } y_{u,c,t} \geq 0$$

This can be achieved via

$$Q_L^j = S_{jk}^Q Q_L^{k\prime}, \qquad u_R^j = S_{jk}^u u_R^{k\prime}$$

with unitary 3×3 matrices S^Q, S^u.
This transformation leaves L_{gauge} invariant (“flavour-blindness of the gauge interactions”)!

Next diagonalise Y^d:

$$\widehat{Y}^d = V^\dagger S_Q^\dagger Y^d S_d \quad \text{with } \widehat{Y}^d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad \text{and } y_{d,s,b} \geq 0$$

with unitary 3×3 matrices V, S^d.
Via $d_R^j = S_{jk}^d d_R^{k\prime}$ we leave L_{gauge} unchanged, while

$$-L_Y^{\text{quark}} = \overline{Q}_L V \widehat{Y}^d H\, d_R \; + \; \overline{Q}_L \widehat{Y}^u \widetilde{H}\, u_R \; + \; \text{h.c.}$$

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To diagonalise $M^d = V \widehat{Y}^d v$ transform

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The transformation $d_L^j = V_{jk} d_L^{k\prime}$ changes the W-boson couplings in L_{gauge}:

$$L_W = \frac{g_2}{\sqrt{2}} \left[\overline{u}_L V \gamma^\mu \, d_L \, W_\mu^+ \; + \; \overline{d}_L V^\dagger \gamma^\mu \, u_L \, W_\mu^- \right]$$

The Z-boson couplings stay flavour-diagonal because of $V^\dagger V = 1$.

V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

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$\Rightarrow$ Add a ν_R to the SM to mimick the quark sector or add a Majorana mass term $Y^M \dfrac{\overline{L}HH^T L^c}{M}$.

The lepton mixing matrix is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

Discrete symmetries

Parity transformation P: $\vec{x} \to -\vec{x}$

Charge conjugation C: Exchange particles and antiparticles, e.g. $e^- \leftrightarrow e^+$

Time reversal T: $t \to -t$

C and P

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1964: CP is not a symmetry of the microscopic laws of nature!

$\Rightarrow$ Also the T symmetry must be violated, there is a microscopic arrow of time!

K and M

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Example: A π^0 meson has $P = -1$ and $C = +1$. A π^+ has $P = -1$, but is no eigenstate of C, because $C|\pi^+\rangle = |\pi^-\rangle$.

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$$\left[H_{\text{strong}}, \mathrm{P}\right] = \left[H_{\text{strong}}, \mathrm{C}\right] = \left[H_{\text{strong}}, \mathrm{T}\right] = 0$$

$\Rightarrow$ We can assign C and P quantum numbers, which can be $+1$ or -1, to hadrons.

Example: A π^0 meson has $P = -1$ and $C = +1$. A π^+ has $P = -1$, but is no eigenstate of C, because $C|\pi^+\rangle = |\pi^-\rangle$.

Also QED respects C,P, and T.

Parity violation

1956: $\theta - \tau$ *puzzle*:

A seemingly degenerate pair (θ, τ) of two mesons with P$= +1$ and P$= -1$, weakly decaying as

$$
\begin{array}{lllll}
\text{“}\theta\text{”} & \to & \pi\pi & \qquad & P = +1 \\
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$$K^+ = \text{“}\theta\text{”} = \text{“}\tau\text{”}.$$

Maximal P violation

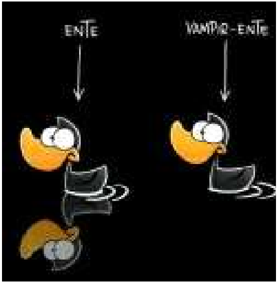
In the SM only left-handed fields feel the charged weak interaction, no couplings of the W-boson to u_R^j, d_R^j, and e_R^j.

Early monograph on parity violation:

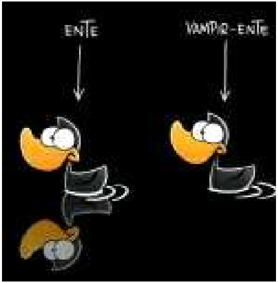
Early monograph on parity violation:

Lewis Carroll:
Alice through the looking glass

Maximal parity violation



Maximal parity violation



Charge conjugation C maps left-handed (particle) fields on right-handed (antiparticle) fields and vice versa:

$$\psi_L \xleftrightarrow{C} \psi_L^C, \qquad \text{where} \quad \psi_L^C \equiv (\psi^C)_R \text{ is right-handed.}$$

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But: Nothing prevents CP and T from being good symmetries...

...except experiment!

CP violation

Neutral K mesons:

K_{long} and K_{short} (linear combinations of K and $\overline{K}$).

Dominant decay channels:

$$K_{\text{long}} \to \pi\pi\pi \qquad \text{CP} = -1$$

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1964: Christenson, Cronin, Fitch and Turlay observe

$$K_{\text{long}} \to \pi\pi$$

and therefore discover CP violation.

$$\epsilon_K \equiv \frac{\langle(\pi\pi)_{I=0}|H_{weak}|K_{\text{long}}\rangle}{\langle(\pi\pi)_{I=0}|H_{weak}|K_{\text{short}}\rangle} = (2.229 \pm 0.010) \cdot 10^{-3} e^{i0.97\pi/4}.$$


M. C. E S C H E R.

CP violation in the SM

Example: W coupling to b and u:

$$L_W = \frac{g_2}{\sqrt{2}} \left[V_{ub} \overline{u}_L \gamma^\mu \, b_L \, W^+_\mu \; + \; V^*_{ub} \overline{b}_L \gamma^\mu \, u_L \, W^-_\mu \right]$$

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CP transformation
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Hence
$$L_W \xrightarrow{CP} \frac{g_2}{\sqrt{2}} \left[V_{ub} \overline{b}_L \gamma^\mu u_L W_\mu^- + V_{ub}^* \overline{u}_L \gamma^\mu b_L W_\mu^+ \right]$$

Is CP violated?

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Is CP violated? Not yet...
Rephasing $b_L \to e^{i\phi} b_L$, $u_L \to e^{i\phi'} u_L$ amounts to

$$L_W \xrightarrow{CP+\text{reph.}} \frac{g_2}{\sqrt{2}} \left[V_{ub} e^{i(\phi'-\phi)} \overline{b}_L \gamma^\mu u_L W^-_\mu \; + \; V^*_{ub} e^{i(\phi-\phi')} \overline{u}_L \gamma^\mu b_L W^+_\mu \right],$$

so that we can achieve $V_{ub} e^{i(\phi'-\phi)} = V^*_{ub}$.

Alternatively we could have used the rephasing to render V_{ub} real from the beginning.

Observation by Kobayashi and Maskawa:
A unitary $n \times n$ matrix has $\frac{n(n+1)}{2}$ phases. In an n-generation SM one can eliminate $2n-1$ phases from V by rephasing the quark fields. The remaining $\frac{(n-1)(n-2)}{2}$ phases are physical, CP-violating parameters of the theory!

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n	$\frac{(n-1)(n-2)}{2}$
1	0
2	0
3	1
4	3

Kobayashi-Maskawa phase δ_{KM}

CKM metrology

The Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

involves 4 parameters: 3 angles and the KM phase δ_{KM}.
Best way to parametrise V: Wolfenstein expansion

Expand the CKM matrix V in $V_{us} \simeq \lambda = 0.2246$:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(1+\frac{\lambda^2}{2}\right)(\overline{\rho}-i\overline{\eta}) \\ -\lambda - iA^2\lambda^5\overline{\eta} & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 - iA\lambda^4\overline{\eta} & 1 \end{pmatrix}$$

with the Wolfenstein parameters λ, A, $\overline{\rho}$, $\overline{\eta}$

CP violation $\Leftrightarrow \overline{\eta} \neq 0$

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with the Wolfenstein parameters λ, A, $\overline{\rho}$, $\overline{\eta}$

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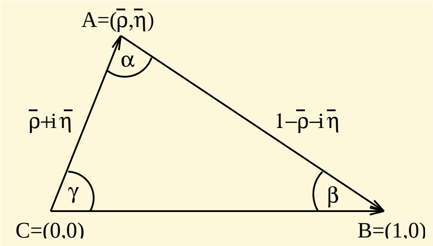
Unitarity triangle:

Exact definition:

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = \left|\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right| e^{i\gamma}$$



In the SM the flavour violation only occurs in the couplings of $W^{\pm}_{\mu}$ and $G^{\pm}$ to fermions.

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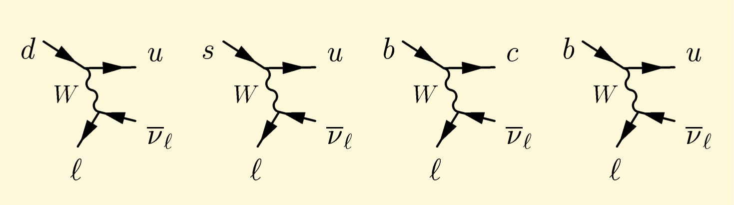
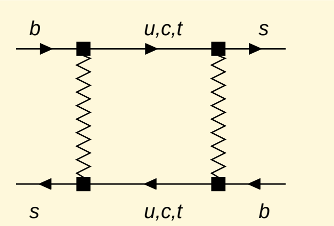
Semileptonic decays:



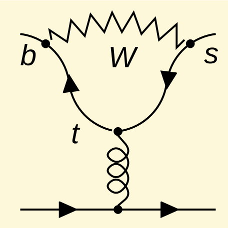
determining $|V_{ud}|$ $|V_{us}|$ $|V_{cb}|$ $|V_{ub}|$.

Flavour-changing neutral current (FCNC) processes

Examples:



$B_s - \overline{B}_s$ mixing



penguin diagram

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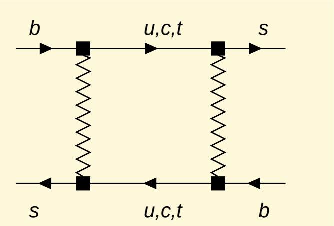

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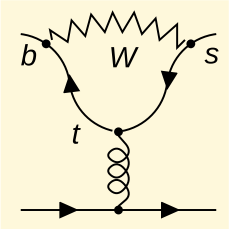

penguin diagram

FCNC processes are the only possibility to gain information on V_{td} and V_{ts}. However: FCNC processes are highly sensitive to physics beyond the SM.

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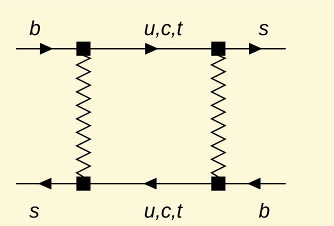
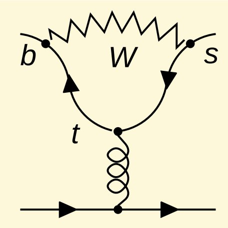


$\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing



penguin diagram

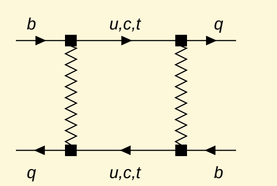
FCNC processes are the only possibility to gain information on V_{td} and V_{ts}. However: FCNC processes are highly sensitive to physics beyond the SM.

In principle can determine all parameters λ, A, $\overline{\rho}$, $\overline{\eta}$ from tree-level processes.

$\Rightarrow$ View FCNC processes as new physics analysers rather than ways to measure V_{td} and V_{ts}.

$\mathrm{B}-\overline{\mathrm{B}}$ mixing basics

Consider $\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing with $q = d$ or $q = s$:
A meson identified ("tagged") as a B_q at time $t = 0$ is described by $|B_q(t)\rangle$.



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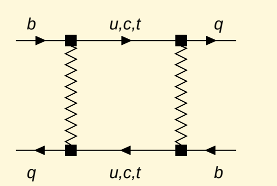

For $t > 0$:

$$|B_q(t)\rangle = \langle B_q|B_q(t)\rangle|B_q\rangle + \langle \overline{B}_q|B_q(t)\rangle|\overline{B}_q\rangle + \ldots,$$

with "..." denoting the states into which $B_q(t)$ can decay.

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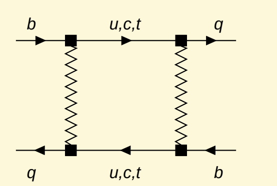

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Analogously: $|\overline{B}_q(t)\rangle$ is the ket of a meson tagged as a $\overline{B}_q$ at time $t = 0$.

Schrödinger equation:

$$i\frac{d}{dt}\begin{pmatrix}\langle B_q|B_q(t)\rangle\\ \langle \overline{B}_q|B_q(t)\rangle\end{pmatrix} = \left(M^q - i\,\frac{\Gamma^q}{2}\right)\begin{pmatrix}\langle B_q|B_q(t)\rangle\\ \langle \overline{B}_q|B_q(t)\rangle\end{pmatrix}$$

with the 2×2 mass and decay matrices $M^q = M^{q\dagger}$ and $\Gamma^q = \Gamma^{q\dagger}$.

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3 physical quantities in $\mathrm{B_q}-\overline{\mathrm{B}}_{\mathrm{q}}$ mixing:

$$\left|M_{12}^q\right|, \quad \left|\Gamma_{12}^q\right|, \quad \phi_q \equiv \arg\left(-\frac{M_{12}^q}{\Gamma_{12}^q}\right)$$

Diagonalise $M^q - i\,\frac{\Gamma^q}{2}$ to find the two mass eigenstates:

$$\begin{aligned} \text{Lighter eigenstate:} \quad |B_L\rangle &= p|B_q\rangle + q|\overline{B}_q\rangle. \\ \text{Heavier eigenstate:} \quad |B_H\rangle &= p|B_q\rangle - q|\overline{B}_q\rangle \end{aligned}$$

with masses $M^q_{L,H}$ and widths $\Gamma^q_{L,H}$.
Further $|p|^2 + |q|^2 = 1$.

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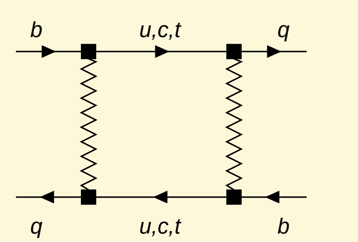
Relation of Δm_q and $\Delta\Gamma_q$ to $|M^q_{12}|$, $|\Gamma^q_{12}|$ and ϕ_q:

$$\begin{aligned} \Delta m_q &= M_H - M_L \simeq 2|M^q_{12}|, \\ \Delta\Gamma_q &= \Gamma_L - \Gamma_H \simeq 2|\Gamma^q_{12}|\cos\phi_q \end{aligned}$$

M_{12}^{q} stems from the dispersive (real) part of the box diagram, internal t.
Γ_{12}^{q} stems from the absorpive (imaginary) part of the box diagram, internal c, u.



Solve the Schrödinger equation to find the desired $\mathrm{B_q} - \overline{\mathrm{B}}_\mathrm{q}$ oscillations:

$$
\begin{aligned}
|\langle B_q|B_q(t)\rangle|^2 = |\langle \bar{B}_q|\bar{B}_q(t)\rangle|^2 &= \frac{e^{-\Gamma_q t}}{2}\left[\cosh\frac{\Delta\Gamma_q\, t}{2} + \cos\left(\Delta m_q\, t\right)\right] \\
|\langle \bar{B}_q|B_q(t)\rangle|^2 \simeq |\langle B_q|\bar{B}_q(t)\rangle|^2 &\simeq \frac{e^{-\Gamma_q t}}{2}\left[\cosh\frac{\Delta\Gamma_q\, t}{2} - \cos\left(\Delta m_q\, t\right)\right]
\end{aligned}
$$

with $\Gamma_q \equiv \dfrac{\Gamma_L^q + \Gamma_H^q}{2}$

Time-dependent decay rate:

$$\Gamma(B_q(t) \to f) = \frac{1}{N_B} \frac{d\,N(B_q(t) \to f)}{d\,t},$$

where $d\,N(B_q(t) \to f)$ is the number of $B_q(t) \to f$ decays within the time interval $[t, t + d\,t]$.
N_B is the number of B_q's present at time $t = 0$.

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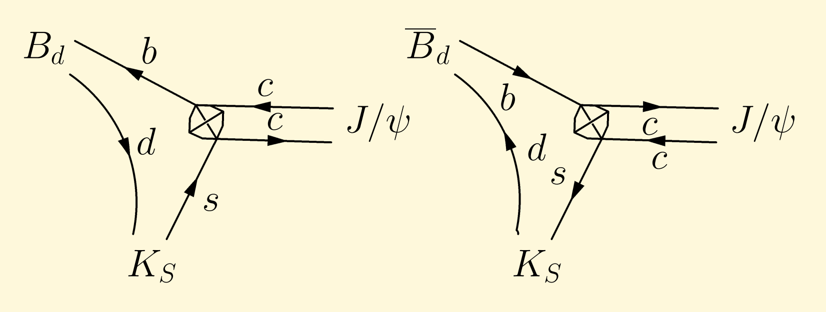
N_B is the number of B_q's present at time $t = 0$.

With $|\bar{f}\rangle \equiv CP|f\rangle$ define the time-dependent CP asymmetry:

$$a_f(t) = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to \bar{f})}$$

Example 1:

$B_d \to J/\psi K_S$ $\Rightarrow$ $|\bar{f}\rangle = -|f\rangle$ (CP-odd eigenstate)



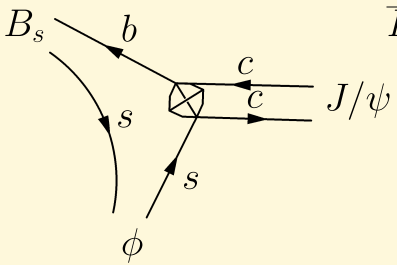
$$a_{J/\psi K_S}(t) \simeq -\sin(2\beta)\sin(\Delta m_d t),$$

where

$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

Example 2:

$B_s \to (J/\psi\phi)_{L=0}$ $\Rightarrow$ $|\bar{f}\rangle = |f\rangle$ (CP-even eigenstate)



$$a_{(J/\psi\phi)_{L=0}}(t) = -\frac{\sin(2\beta_s)\sin(\Delta m_s t)}{\cosh(\Delta\Gamma_s t/2) - \cos(2\beta_s)\sinh(\Delta\Gamma_s t/2)},$$

where $$\beta_s = \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right] \simeq \lambda^2\overline{\eta}$$

The Wolfenstein parameters λ and A are well determined from the semileptonic decays $K \to \pi \ell^+ \nu_\ell$ and $B \to X_c \ell^+ \nu_\ell$, $\ell = e, \mu$.

Metrology of the unitarity triangle:
The apex $(\bar{\rho},\bar{\eta})$ is currently constrained from the following experimental input:

- $|V_{ub}| \propto \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ measured in $B \to \pi \ell \nu_\ell$, $B \to X_u \ell \nu_\ell$ and $B^+ \to \tau^+ \nu_\tau$.

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- $\Delta m_d \propto \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$
- $\Delta m_d / \Delta m_s \propto \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$

Metrology of the unitarity triangle:
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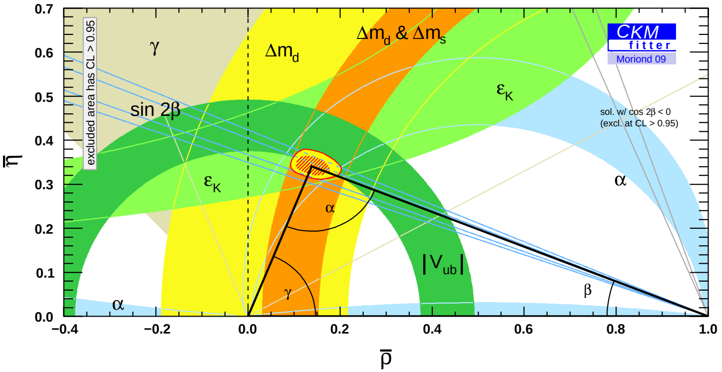
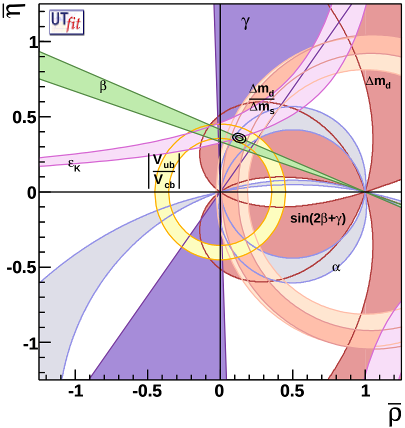


Statistical method: Rfit, a Frequentist approach.

Global fit in the SM from UTfit:



Statistical method: Bayesian.

Flavour experiments

B,D,τ: BaBar, BELLE (upgrade: BELLE-II)
CDF, DØ
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…plus many neutrino experiments

Future: Project X at Fermilab for rare K and μ decays.

New physics

In the LHC era CKM metrology is less important and constraints on physics beyond the SM is the main focus of flavour physics.

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Spectacular: In FCNC transitions of charged leptons the GIM suppression factor is even m_ν^2/M_W^2!

$\Rightarrow$ The SM predictions for charged-lepton FCNCs are essentially zero!

The suppression of FCNC processes in the Standard Model is not a consequence of the $SU(3) \times SU(2)_L \times U(1)_Y$ symmetry. It results from the particle content of the Standard Model and the accidental smallness of most Yukawa couplings. It is absent in generic extensions of the Standard Model.

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Examples:

extra Higgses	$\Rightarrow$	Higgs-mediated FCNC's at tree-level, helicity suppression possibly absent,
squarks/gluinos	$\Rightarrow$	FCNC quark-squark-gluino coupling, no CKM/GIM suppression,
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$\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ and $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing and rare decays like $B_{s,d} \to \mu^+\mu^-$ and $K \to \pi\nu\overline{\nu}$ are sensitive to scales above $\Lambda \sim 100\,\mathrm{TeV}$.

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Flavour physics will explore their couplings to quarks.

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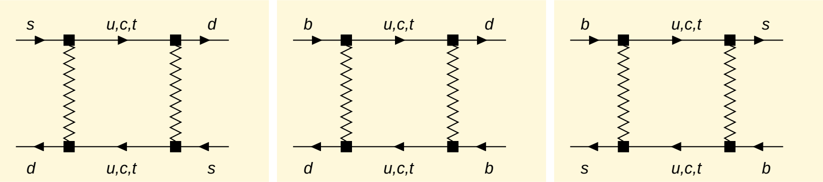
If ATLAS and CMS find no further new particles:

Flavour physics probes scales of new physics exceeding 100 TeV.

New-physics analysers:

- Global fit to UT: overconstrain $(\bar{\rho}, \bar{\eta})$,
  probes FCNC processes $\mathrm{K}-\overline{\mathrm{K}}$, $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ and $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing.



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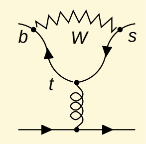
- Penguin decays: $B \to X_s\gamma$, $B \to X_s\ell^+\ell^-$, $B \to K\pi$, $B_d \to \phi K_{\mathrm{short}}$, $B_s \to \mu^+\mu^-$, $K \to \pi\nu\bar\nu$.



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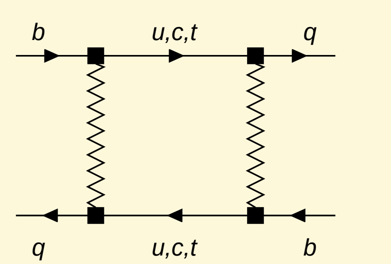
- CKM-suppressed or helicity-suppressed tree-level decays: $B^+ \to \tau^+\nu$, $B \to \pi\ell\nu$, $B \to D\tau\nu$, probe charged Higgses and right-handed W-couplings.

$\mathrm{B}-\overline{\mathrm{B}}$ mixing and new physics

$\mathrm{B_q}-\overline{\mathrm{B}}_\mathrm{q}$ mixing with $q=d$ or $q=s$:

New physics can barely affect Γ_{12}^q, which stems from tree-level decays.

M_{12}^q is very sensitive to virtual effects of new heavy particles.



Generic new physics

The phase $\phi_s = \arg(-M_{12}^s/\Gamma_{12}^s)$ is negligibly small in the Standard Model:

$$\phi_s^{\mathrm{SM}} = 0.2^\circ.$$

Define the complex parameter Δ_s through

$$M_{12}^s \equiv M_{12}^{\mathrm{SM},s} \cdot \Delta_s\,, \qquad \Delta_s \equiv |\Delta_s| e^{i\phi_s^\Delta}.$$

In the Standard Model $\Delta_s = 1$. Use $\phi_s = \phi_s^{\mathrm{SM}} + \phi_s^\Delta \simeq \phi_s^\Delta$.

Confront the LHCb-CDF average

$$\Delta m_s \quad = \quad (17.719 \pm 0.043)\ \mathrm{ps}^{-1}$$

with the SM prediction:

$$\Delta m_s = \left(18.8 \pm 0.6_{V_{cb}} \pm 0.3_{m_t} \pm 0.1_{\alpha_s} \right) \mathrm{ps}^{-1} \, \frac{f_{B_s}^2 \, B_{B_s}}{(220\,\mathrm{MeV})^2}$$

Largest source of uncertainty: $f_{B_s}^2 \, B_{B_s}$ from lattice QCD.

Here f_{B_s} is the B_s decay constant and $f_{B_s}^2 \, B_{B_s}$ parametrises a hadronic matrix element calculated with lattice QCD.

With

$$f_{B_s} = (229 \pm 2 \pm 6)\,\text{MeV}, \qquad B_{B_s} = 0.85 \pm 0.02 \pm 0.02$$

find $\Delta m_s^{\rm SM} = (17.3 \pm 1.5)\,\text{ps}^{-1}$ entailing

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Too good to be true: prediction is based on many calculation of f_{B_s} and the prejudice $B_{B_s} = 0.85 \pm 0.02 \pm 0.02$.

Flavour-specific decay: $B_s \to f$ is allowed, while
$\bar{B}_s \to f$ is forbidden

CP asymmetry in flavour-specific decays (semileptonic CP asymmetry):

$$a_{\mathrm{fs}}^s = \frac{\Gamma(\bar{B}_s(t) \to f) - \Gamma(B_s(t) \to \bar{f})}{\Gamma(\bar{B}_s(t) \to f) + \Gamma(B_s(t) \to \bar{f})}$$

with e.g. $f = X\ell^+\nu_\ell$ and $\bar{f} = \bar{X}\ell^-\bar{\nu}_\ell$. Untagged rate:

$$a_{\mathrm{fs,unt}}^s \equiv \frac{\int_0^\infty dt \left[\Gamma(\overset{(-)}{B_s} \to \mu^+ X) - \Gamma(\overset{(-)}{B_s} \to \mu^- X)\right]}{\int_0^\infty dt \left[\Gamma(\overset{(-)}{B_s} \to \mu^+ X) + \Gamma(\overset{(-)}{B_s} \to \mu^- X)\right]} \simeq \frac{a_{\mathrm{fs}}^s}{2}$$

Relation to M_{12}^s:

$$a_{fs}^s = \frac{|\Gamma_{12}^s|}{|M_{12}^s|} \sin\phi_s = \frac{|\Gamma_{12}^s|}{|M_{12}^{\mathrm{SM},s}|} \cdot \frac{\sin\phi_s}{|\Delta_s|} = (4.4 \pm 1.2) \cdot 10^{-3} \cdot \frac{\sin\phi_s}{|\Delta_s|}$$

A. Lenz, UN, 2006,2011,2012

Dilepton events:

Compare the number N_{++} of decays $(B_s(t), \bar{B}_s(t)) \to (f, f)$ with the number N_{--} of decays to $(\bar{f}, \bar{f})$.

Then $a_{\mathrm{fs}}^s = \dfrac{N_{++} - N_{--}}{N_{++} + N_{--}}$.

At the Tevatron all b-flavoured hadrons are produced. Still only those events contribute to $(N_{++} - N_{--})/(N_{++} + N_{--})$, in which one of the b hadronises as a B_d or B_s and undergoes mixing.

New physics

M_{12}^s is highly sensitive to new physics, unlike the tree-level decay $b \to c\bar{c}s$ responsible for $B_s \to J/\psi\phi$ and Γ_{12}^s.

It is plausible to consider a generic scenario, in which the M_{12} elements in $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$, $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$, and $\mathrm{K}-\overline{\mathrm{K}}$ mixing are affected by new-physics, while all other quantities entering the global fit to the UT are as in the Standard-Model.

Recall: In the Standard Model

$$\phi_s = 0.22^\circ \pm 0.06^\circ \quad \text{and} \quad \phi_d = -4.3^\circ \pm 1.4^\circ.$$

A new-physics contribution to $\arg M_{12}^q$ may enhance

$$|a_{\mathrm{fs}}^q| \propto \sin \phi_q$$

to a level observable at current experiments.

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But: Precise data on CP violation in $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$ preclude large NP contributions to $\arg\phi_d$ and $\arg\phi_s$.

New physics

Trouble maker:

$$\begin{aligned} A_{\mathrm{SL}} &= (0.532 \pm 0.039) a_{\mathrm{fs}}^{d} + (0.468 \pm 0.039) a_{\mathrm{fs}}^{s} \\ &= (-7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3} \qquad \text{DØ 2011} \end{aligned}$$

This is 3.9σ away from $a_{\mathrm{fs}}^{\mathrm{SM}} = (-0.24 \pm 0.03) \cdot 10^{-3}$.

A. Lenz, UN 2006,2011

Global analysis of $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing with A. Lenz and the CKMfitter Group (J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker, S. Monteil, V. Niess) arXiv:1008.1593, 1203.0238

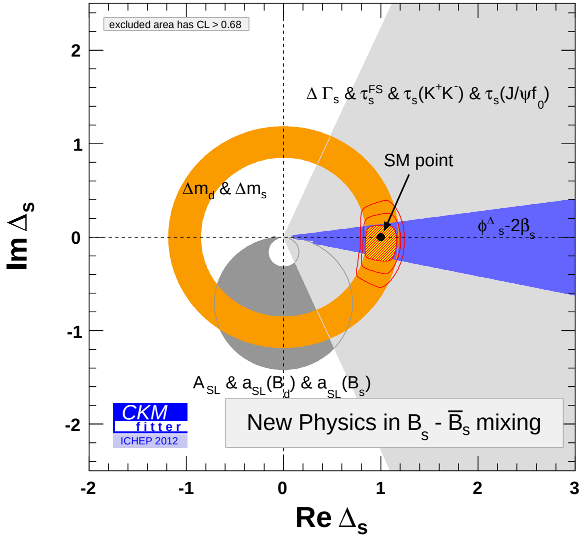
Rfit method: No statistical meaning is assigned to systematic errors and theoretical uncertainties.

We have performed a simultaneous fit to the Wolfenstein parameters and to the new physics parameters Δ_s and Δ_d,

$$\Delta_q \equiv \frac{M_{12}^q}{M_{12}^{q,\mathrm{SM}}}, \qquad \Delta_q \equiv |\Delta_q| e^{i\phi_q^\Delta},$$

and further permitted NP in $\mathrm{K}-\overline{\mathrm{K}}$ mixing as well.

CKMfitter September 2012 update of 1203.0238:



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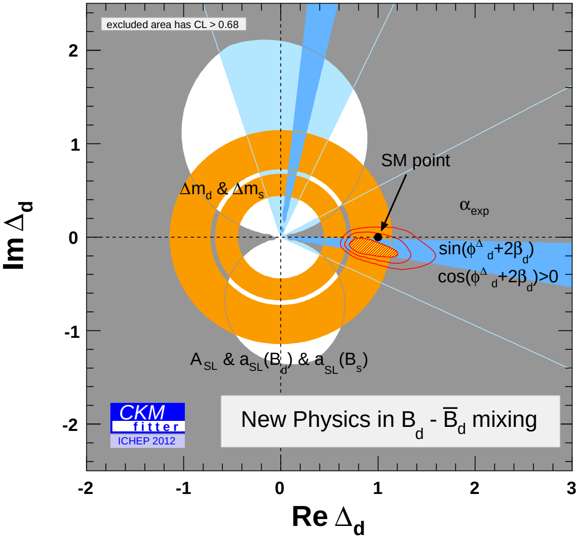

A_{SL} and WA for $B(B \to \tau\nu)$ prefer small $\phi_d^\Delta < 0$.

Pull value for A_{SL}: 3.3σ

$\Rightarrow$ Scenario with NP in M_{12}^q only cannot accomodate the DØ measurement of A_{SL}.

The Standard Model point $\Delta_s = \Delta_d = 1$ is disfavoured by 1σ, down from the 2010 value of 3.6σ.

Supersymmetry

The MSSM has many new sources of flavour violation, all in the supersymmetry-breaking sector.

No problem to get big effects in $B_s - \overline{B}_s$ mixing, but rather to suppress the big effects elsewhere.

Squark mass matrix

Diagonalise the Yukawa matrices Y^u_{jk} and Y^d_{jk}

$\Rightarrow$ quark mass matrices are diagonal, super-CKM basis

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E.g. Down-squark mass matrix:

$$M^2_{\tilde d} = \begin{pmatrix}
\left(M^{\tilde d}_{1L}\right)^2 & \Delta^{\tilde d\,LL}_{12} & \Delta^{\tilde d\,LL}_{13} & \Delta^{\tilde d\,LR}_{11} & \Delta^{\tilde d\,LR}_{12} & \Delta^{\tilde d\,LR}_{13} \\
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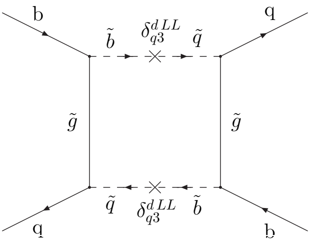
Not diagonal! $\Rightarrow$ new FCNC transitions.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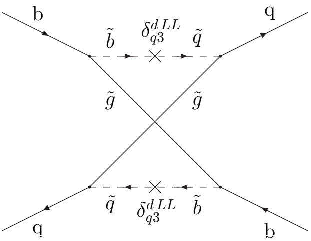
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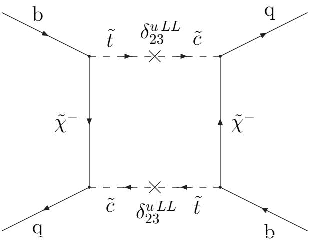
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q
b

Model-independent analyses constrain

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using data on FCNC (and also charged-current) processes.

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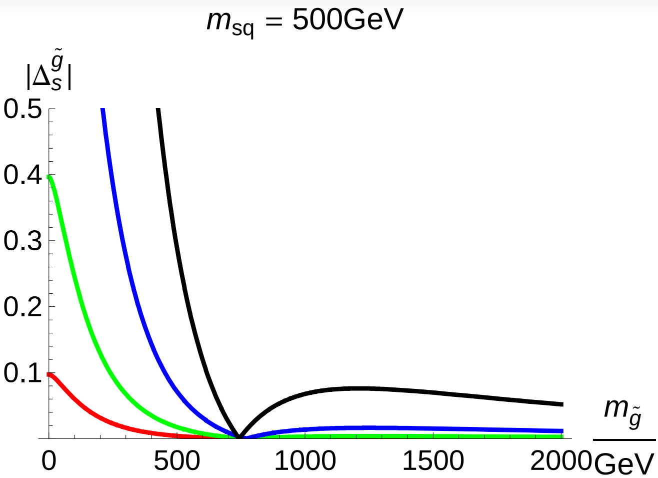
- To derive meaningful bounds on $\delta_{ij}^{q\,LR}$ chirally enhanced higher-order contributions must be taken into account.

A. Crivellin, UN, 2009



The gluino contribution vanishes for $M_{\tilde{g}} \approx 1.5 M_{\tilde{q}}$, independently of the size of $\Delta_{23}^{d\,LL}$ (curves correspond to 4 different values).

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Could they instead be helpful to understand the SM flavour puzzle?

We observe: Flavour violation is small in the quark sector, because the Yukawa matrices possess an approximate $SU(2) \times SU(2) \times SU(3)$ flavour symmetry. In the exact symmetry limit only the top quark has mass and $V = 1$.

What causes the small deviations leading to $V \neq 1$?

Flavour violation from trilinear terms

Origin of the SUSY flavour problem: Misalignment of squark mass matrices with Yukawa matrices.
Unorthodox solution: Set Y^u_{ij} and Y^d_{ij} to zero, except for $(i,j)=(3,3)$.

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Radiative flavour violation: S. Weinberg 1972

flavour from soft SUSY terms:

W. Buchmüller, D. Wyler	1983,
T. Banks	1988,
F. Borzumati, G.R. Farrar, N. Polonsky, S.D. Thomas	1998, 1999
J. Ferrandis, N. Haba	2004

Today: Strong constraints from FCNCs probed at B factories.

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But: Radiative flavour violation in the MSSM is still viable, albeit only with A^d_{ij} and A^u_{ij} entering

$$M^{\tilde{d}\,LR}_{ij} \;=\; A^d_{ij} v_d + \delta_{i3}\delta_{j3} y_b \mu v_u , \qquad M^{\tilde{u}\,LR}_{ij} \;=\; A^u_{ij} v_u + \delta_{i3}\delta_{j3} y_t \mu v_d .$$

Andreas Crivellin, UN, PRD 79 (2009) 035018

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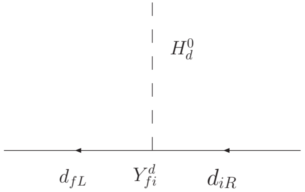
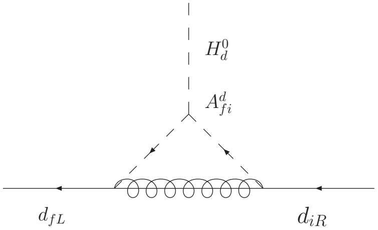




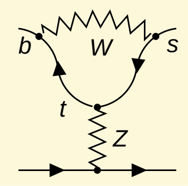
Electric dipole moments

Darkest corner of the MSSM: The phases of A^q_{ii} and μ generate too large EDMs. If light quark masses are generated radiatively through soft SUSY-breaking terms, this “supersymmetric CP problem” is substantially alleviated:

- The phases of A^q_{ii} and m_q are aligned, i.e. zero.
- The phase of μ (essentially) does not enter the EDMs at the one-loop level, because the Yukawa couplings of the first two generations are zero.

Borzumati, Farrar, Polonsky, Thomas 1998,1999

$B_{d,s} \to \mu^+\mu^-$



LHCb 2013:

$$B(B_s \to \mu^+\mu^-) = \left(3.2^{+1.5}_{-1.2}\right) \cdot 10^{-9}$$

$$B(B_d \to \mu^+\mu^-) < 9.4 \cdot 10^{-10} \quad @95\%\ \mathrm{CL}$$

Theory:

$$B\left(B_s \to \mu^+\mu^-\right) = (3.52 \pm 0.08) \cdot 10^{-9} \quad \times$$

$$\frac{\tau_{B_s}}{1.519\,\mathrm{ps}} \left[\frac{|V_{ts}|}{0.040}\right]^2 \left[\frac{f_{B_s}}{230\ \mathrm{MeV}}\right]^2$$

Lattice QCD results of ETMC, HPQCD and FNAL/MILC (1107.1441, 1112.3051, 1202.4914). Personal combination:

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$B_s \to f$ decays involve two exponentials:

$$\Gamma(\overset{(-)}{B_s} \to f, t) = A_f e^{-\Gamma_L t} + B_f e^{-\Gamma_H t},$$

since $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing leads to a sizable decay-width difference $\Gamma_L - \Gamma_H = \Delta\Gamma_s = (0.078 \pm 0.022)\,\text{ps}^{-1}$.

$\Rightarrow$ correct $B(B_s \to \mu^+\mu^-)$ for this De Bruyn et al, 1204.1737

Supersymmetry

COSMOS Magazine 14 Nov 2012:

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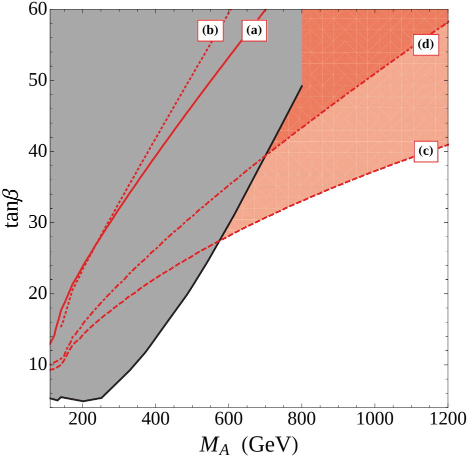
M_A: mass of the pseudoscalar Higgs boson A^0
$\tan\beta$: ratio of the two Higgs-vevs of the MSSM:

$$B(B_s \to \mu^+\mu^-) \propto \frac{\tan^6\beta}{M_A^4}$$

$\Rightarrow$ $B_s \to \mu^+\mu^-$ places lower bounds on M_A for large values of $\tan\beta$, similarly to searches for $A^0 \to \tau^+\tau^-$ at ATLAS and CMS.

MSSM



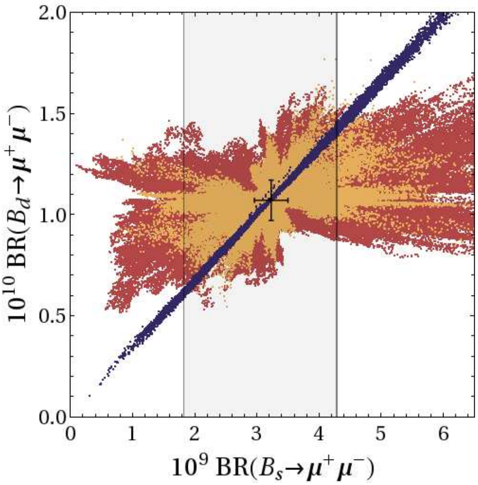
$M_3 = 3M_2 = 6M_1 = 1.5$ TeV
$m_{\tilde{f}} = 2$ TeV
$A_b = A_t = A_\tau$,
so that
$m_h = 125$ GeV.

a) $\mu = 1$ TeV, $A_t > 0$,
b) $\mu = 4$ TeV, $A_t > 0$,
c) $\mu = -1.5$ TeV, $A_t > 0$,
d) $\mu = 1$ TeV, $A_t < 0$,

Excluded areas:
Gray: $A^0, H^0 \to \tau^+\tau^-$
Red: $B_s \to \mu^+\mu^-$

Altmannshofer et al., 1211.1976

Partial Compositeness



Models with non-elementary Higgs und additional non-elementary fermions
$\Rightarrow$ FCNC-Z-couplings

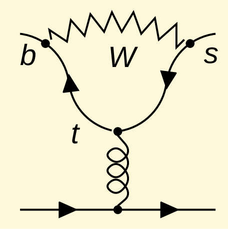
Red, brown, blue:
Three models; the blue model has a $U(2)^3$-flavour symmetry.

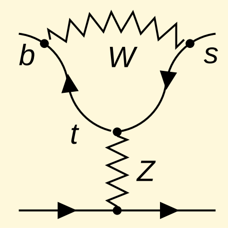
Straub, 1302.4651

$B_s \to \phi\pi^0, \phi\rho^0$

QCD penguins do not contribute to $B_s \to \phi\pi^0$ and $B_s \to \phi\rho^0$:
Strong isospin: $I(B_s) = I(\phi) = 0$
and $I(\pi^0) = I(\rho^0) = 1$
$b \to s$ QCD penguin diagrams are $\Delta I = 0$ transitions.
Tree diagrams are suppressed by $R_u \lambda^2$.

$B_s \to \phi\pi^0$ and $B_s \to \phi\rho^0$ therefore probe Z penguins.





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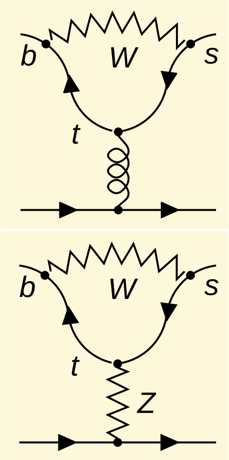

New physics can enhance the branching fractions by a factor of 5 over the SM values:

$$B(B_s \to \phi\pi^0) = \left(1.6^{+1.1}_{-0.3}\right) \cdot 10^{-7},$$
$$B(B_s \to \phi\rho^0) = \left(4.4^{+2.7}_{-0.7}\right) \cdot 10^{-7}$$

Hofer et al., 1011.6319, 1212.4785

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- New physics of order 20% in the $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ and $\mathrm{B_d}-\overline{\mathrm{B}}_\mathrm{d}$ mixing amplitudes is still allowed.
- Supersymmetry with non-minimal flavour violation gains attractivity as it comes with (multi-)TeV squark masses. The deviation of V from the unit matrix may come from supersymmetric loops (Radiative Flavour Violation).

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- Suggestion for LHCb and Belle-II: Study $B_s \to \phi\pi^0, \phi\rho^0$ to look for isospin-violating new physics (electroweak penguins).

Penguins: Wake-up call for new physics?