Light states in multi-Higgs potentials with multiple vacua

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One slide take-away summary

[If quartic couplings obey perturbativity constraints]

- nHDMs with CP invariant potential and SCPV:
  there are *at least* 2 neutral and 1 charged scalars, in addition to "the Higgs", which are light, i.e. with masses not much larger than the EW scale $v_{\text{EW}} \sim 246$ GeV.
  Miró, MN & Queiroz ☞ PRD111 (2025)
  Camacho, Miró, MN, Tobarra & Queiroz ☞ arXiv:2605.15270
- General 2HDM:
  if the potential has *two* spontaneous symmetry breaking minima, then *all* the scalars (1 charged and 3 neutral, including "the Higgs") are light.
  Camacho, Miró, MN & Tobarra ☞ arXiv:2606.02710

Motivation – Generic

- *Theoretical* arguments to constrain masses of (new) particles:
  vacuum stability, triviality, perturbativity, . . .
  [short of *experimental* evidence]
  Weinberg ☞ PRL36 (1976)
  Politzer & Wolfram ☞ PLB82 (1979)
  Cabibbo, Maiani, Parisi & Petronzio ☞ NPB158 (1979)
  Dashen & Neuberger ☞ PRL50 (1983)
  Callaway ☞ NPB233 (1984)
- Perturbative unitarity, high energy scattering of bosons
  Lee, Quigg & Thacker ☞ PRL38 (1977), ☞ PRD16 (1977)
  also Dicus & Mathur ☞ PRD7 (1973)
  Langacker & Weldon ☞ PRL52 (1984)
  Weldon ☞ PLB146 (1984)
- 2HDMs
  Ginzburg & Ivanov ☞ PRD72 (2005)
  + many more

Motivation – Specific

- Real 2HDM (i.e. CP invariant potential*) with SCPV:
masses of all the scalars *bounded*

MN, Botella & Branco ☞ EPJC79 (2019)
MN ☞ PRD102 (2020)

- The point: stationarity conditions allow to trade *all* 3 *quadratic* couplings in the potential for *quartics* × vevs^2,
which are bounded.

- Common situation when symmetry requirements allow few quadratic couplings.

- When there is an overabundance of quadratic couplings with respect to stationarity conditions, is a decoupling regime available? (Except for “the Higgs”)

*Invariant under $\Phi \mapsto \Phi^*$.

Outline

Real 2HDM with SCPV – Prelude

Scalar potential for 2 $SU(2)_L$ doublets with $\frac{1}{2}$ hypercharge

$$
\begin{aligned}
\mathcal{V}(\Phi_1,\Phi_2) =& \mathcal{V}_2(\Phi_1,\Phi_2) + \mathcal{V}_4(\Phi_1,\Phi_2),\\
\mathcal{V}_2(\Phi_1,\Phi_2) =& \mu_1^2\,\Phi_1^\dagger\Phi_1 + \mu_2^2\,\Phi_2^\dagger\Phi_2 + \mu_{12}^2\,\left[\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right]\\
\mathcal{V}_4(\Phi_1,\Phi_2) =& \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\
&+ \lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right]\\
&+ (\lambda_6\,\Phi_1^\dagger\Phi_1 + \lambda_7\,\Phi_2^\dagger\Phi_2)\left[\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right]
\end{aligned}
$$

with all quadratic μ_1^2, μ_2^2, μ_{12}^2 and quartic parameters $\lambda_{1,\ldots,7}$ *real*

$\Rightarrow$ invariant under $CP: \Phi_a \mapsto \Phi_a^*$.

Real 2HDM with SCPV – Prelude

Standard procedure

- Expand

$$\Phi_a = \frac{e^{i\theta_a}}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\mathrm{C}_a^+ \\ v_a + \mathrm{R}_a + i\mathrm{I}_a \end{pmatrix}, \quad \langle\Phi_a\rangle = \frac{e^{i\theta_a} v_a}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Compute $\mathrm{V}(v_a, \theta_a) = \mathcal{V}(\langle\Phi_a\rangle)$ and stationarity conditions.
- Compute mass terms

$$-\mathscr{L}_{\text{Mass}} = \vec{\mathbf{C}}^\dagger\, M_\pm^2\, \vec{\mathbf{C}} + \frac{1}{2}\vec{\mathbf{N}}^T\, M_0^2\, \vec{\mathbf{N}}, \qquad -\mathscr{L}_{\text{Mass}} \subset \mathcal{V},$$

where

$$\vec{\mathbf{C}}^\dagger = (\mathrm{C}_1^+, \mathrm{C}_2^+), \qquad \vec{\mathbf{N}}^T = (\mathrm{R}_1, \mathrm{R}_2, \mathrm{I}_1, \mathrm{I}_2)$$

Real 2HDM with SCPV – Prelude

Mass matrices

$$(M_\pm^2)_{a,b} = \left[\frac{\partial^2\mathcal{V}}{\partial \mathrm{C}_a^+ \partial \mathrm{C}_b^-}\right],$$

$$(M_0^2)_{a,b} = \left[\frac{\partial^2\mathcal{V}}{\partial \mathrm{R}_a \partial \mathrm{R}_b}\right], \quad (M_0^2)_{n+a,n+b} = \left[\frac{\partial^2\mathcal{V}}{\partial \mathrm{I}_a \partial \mathrm{I}_b}\right],$$

$$(M_0^2)_{a,n+b} = (M_0^2)_{n+b,a} = \left[\frac{\partial^2\mathcal{V}}{\partial \mathrm{R}_a \partial \mathrm{I}_b}\right]$$

$[f]$: f evaluated at vanishing fields $\mathrm{C}_a^\pm, \mathrm{R}_a, \mathrm{I}_a \to 0$

Real 2HDM with SCPV – Prelude

Stationarity conditions, $\mathrm{V}(v_1, v_2, \theta) = \mathcal{V}(\langle\Phi_1\rangle, \langle\Phi_2\rangle)$,

$$\partial_{v_1}\mathrm{V} = \mu_1^2 v_1 + \mu_{12}^2 v_2 \cos\theta + \lambda_1 v_1^3 + \frac{1}{2}(\lambda_3 + \lambda_4) v_1 v_2^2$$
$$+ \lambda_5 v_1 v_2^2 \cos 2\theta + \frac{3}{2}\lambda_6 v_1^2 v_2 \cos\theta + \frac{1}{2}\lambda_7 v_2^3 \cos\theta = 0$$

$$\partial_{v_2}\mathrm{V} = \mu_2^2 v_2 + \mu_{12}^2 v_1 \cos\theta + \lambda_2 v_2^3 + \frac{1}{2}(\lambda_3 + \lambda_4) v_1^2 v_2$$
$$+ \lambda_5 v_1^2 v_2 \cos 2\theta + \frac{1}{2}\lambda_6 v_1^3 \cos\theta + \frac{3}{2}\lambda_7 v_1 v_2^2 \cos\theta = 0$$

$$\partial_\theta \mathrm{V} = -\mu_{12}^2 v_1 v_2 \sin\theta - \lambda_5 v_1^2 v_2^2 \sin 2\theta - \frac{1}{2}(\lambda_6 v_1^2 + \lambda_7 v_2^2) v_1 v_2 \sin\theta = 0$$

N.B. $\theta = \theta_2 - \theta_1$ is the only physical, CP violating, vacuum phase

Real 2HDM with SCPV – Prelude

- Spontaneous CP Violation, from $\partial_\theta \mathrm{V} = 0$

$$\cos\theta = -\frac{1}{4\lambda_5 v_1 v_2}\left[2\mu_{12}^2 + \lambda_6 v_1^2 + \lambda_7 v_2^2\right] \neq \pm 1$$

- Express quadratic parameters as quartics × 2 powers of v_1, v_2: μ_{12}^2 with $\partial_\theta \mathrm{V} = 0$, then μ_1^2 with $\partial_{v_1} \mathrm{V} = 0$, and μ_2^2 with $\partial_{v_2} \mathrm{V} = 0$.
- Quartic parameters bounded by perturbativity considerations
  $\Rightarrow$ bounds on quadratic parameters $\sim \mathcal{O}(1) v_{\mathrm{EW}}^2$.

$\Rightarrow$ *all* mass^2 matrix elements bounded $< \mathcal{O}(1) v_{\mathrm{EW}}^2$,
that is, all physical scalars are *light*.

- The crux: we have 3 quadratic parameters and 3 stationarity conditions.

Now, from 2HDM to nHDM.

Real nHDM with SCPV – Setup

Real nHDM scalar potential

$$\begin{aligned}\mathcal{V}(\Phi_a) = &\sum_{a=1}^{n} \mu_a^2 \Phi_a^\dagger \Phi_a + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \mu_{ab}^2 \mathcal{H}_{ab} + \sum_{a=1}^{n} \lambda_a (\Phi_a^\dagger \Phi_a)^2 \\ &+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \lambda_{a,b} (\Phi_a^\dagger \Phi_a)(\Phi_b^\dagger \Phi_b) + \sum_{a=1}^{n}\sum_{b=1}^{n-1}\sum_{c=b+1}^{n} \lambda_{a,bc} (\Phi_a^\dagger \Phi_a)\mathcal{H}_{bc} \\ &+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd} \mathcal{H}_{ab}\mathcal{H}_{cd} \\ &+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd}^{\mathcal{A}} \mathcal{A}_{ab}\mathcal{A}_{cd}\end{aligned}$$

$$\mathcal{H}_{ab} = \frac{1}{2}\left(\Phi_a^\dagger \Phi_b + \Phi_b^\dagger \Phi_a\right) \qquad \mathcal{A}_{ab} = \frac{1}{2}\left(\Phi_a^\dagger \Phi_b - \Phi_b^\dagger \Phi_a\right)$$

With μ_a^2, μ_{ab}^2, λ_a, $\lambda_{a,b}$, $\lambda_{a,bc}$, $\lambda_{ab,cd}$, $\lambda_{ab,cd}^{\mathcal{A}}$ real, CP invariant.

Real nHDM with SCPV – Setup

- Quadratic couplings, $n\ \mu_a^2 + n(n-1)/2\ \mu_{ab}^2$: $n(n+1)/2$.
- Stationarity conditions: $2n-1$.
- Omitting Goldstones, $n-1$ charged and $2n-1$ neutral scalars.

n	2	3	4	5	6	7
$n(n+1)/2$	3	6	10	15	21	28
$2n-1$	3	5	7	9	11	13
$(n-1)(n-2)/2$	0	1	3	6	10	15

- Number of quadratic couplings in excess of the number of stationarity conditions.
- Can they make all (new) scalar masses $\gg v_{\text{EW}}$?

Real nHDM with SCPV – Setup

Standard procedure (again)

- Expand fields around vacuum.
- Compute $\mathrm{V}(v_a, \theta_a) = \mathcal{V}(\langle\Phi_a\rangle)$ and stationarity conditions.
- Mass terms

$$-\mathscr{L}_{\text{Mass}} = \vec{\mathbf{C}}^{\dagger}\, M_{\pm}^2\, \vec{\mathbf{C}} + \frac{1}{2}\vec{\mathbf{N}}^T\, M_0^2\, \vec{\mathbf{N}}, \qquad -\mathscr{L}_{\text{Mass}} \subset \mathcal{V}\,,$$

where now

$$\vec{\mathbf{C}}^{\dagger} = (\mathrm{C}_1^+, \ldots, \mathrm{C}_n^+), \qquad \vec{\mathbf{N}}^T = (\mathrm{R}_1, \ldots, \mathrm{R}_n, \mathrm{I}_1, \ldots, \mathrm{I}_n)$$

  - $M_{\pm}^2$ hermitian $n \times n$ matrix,
  - M_0^2 real symmetric $2n \times 2n$ matrix.

Real nHDM with SCPV – Setup

Stationarity conditions (focus on quadratics)

$$\partial_{\theta_1} V = -\frac{1}{2}\sum_{b=2}^{n} \mu^2_{1b} s_{1b} v_1 v_b + \text{Quartics}$$

$$\partial_{\theta_j} V = \frac{1}{2}\sum_{a=1}^{j-1} \mu^2_{aj} s_{aj} v_a v_j - \frac{1}{2}\sum_{b=j+1}^{n} \mu^2_{jb} s_{jb} v_j v_b + \text{Quartics}$$

$$\partial_{\theta_n} V = \frac{1}{2}\sum_{a=1}^{n-1} \mu^2_{an} s_{an} v_a v_n + \text{Quartics}$$

$$\text{N.B.} \ \sum_{j=1}^{n} \partial_{\theta_j} V = 0$$

where $c_{ab} = \cos(\theta_a - \theta_b)$, $s_{ab} = \sin(\theta_a - \theta_b)$.
We can, for example, trade all μ^2_{1j} for other quadratics and quartics.

Real nHDM with SCPV – Setup

Stationarity conditions (focus on quadratics)

$$\partial_{v_1}\mathrm{V} = \mu_1^2 v_1 + \frac{1}{2}\sum_{b=2}^{n}\mu_{1b}^2 c_{1b} v_b + \text{Quartics}$$

$$\partial_{v_j}\mathrm{V} = \mu_j^2 v_j + \frac{1}{2}\sum_{a=1}^{j-1}\mu_{aj}^2 c_{aj} v_a + \frac{1}{2}\sum_{b=j+1}^{n}\mu_{jb}^2 c_{jb} v_b + \text{Quartics}$$

$$\partial_{v_n}\mathrm{V} = \mu_n^2 v_n + \frac{1}{2}\sum_{a=1}^{n-1}\mu_{an}^2 c_{an} v_a + \text{Quartics}$$

Trade all μ_j^2 for other quadratics and quartics.
$\Rightarrow$ All n μ_a^2's and all $n-1$ μ_{1j}^2 quadratics removed, we are left with $(n-1)(n-2)/2$ quadratics μ_{ab}^2, $a \geq 2$, $b > a$.
Next: illustrative numerical exercises.

Real nHDM with SCPV – Numerical exercise

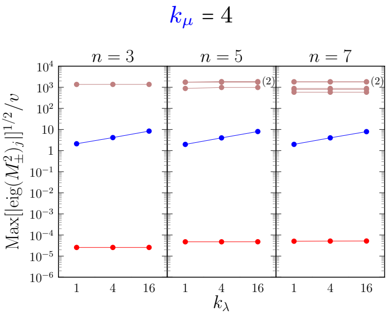
Numerical exercises

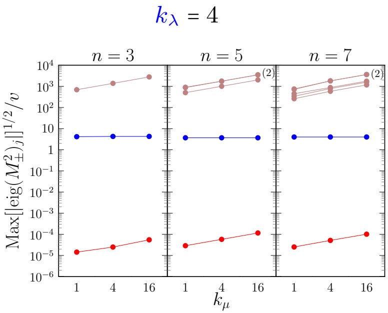
- Random $\mu^2_{ab} \in [-1;+1] \times k_\mu \times 10^{10}$ GeV^2 $(a \geq 2,\ b > a)$.
- Random $\lambda_a, \lambda_{a,b}, \lambda_{a,bc}, \lambda_{ab,cd}, \lambda^{\mathcal{A}}_{ab,cd} \in [-1;+1] \times k_\lambda$.
- Random v_a with $v_1^2 + \ldots + v_n^2 \equiv v^2 = v^2_{\text{EW}} \simeq 246^2$ GeV^2.
- Random $\theta_a \in [-\pi;+\pi]$.
- Discard cases in which the stationarity conditions yield quadratics outside $[-1;+1] \times k_\mu \times 10^{10}$ GeV^2.
- Compute the resulting "mass^2 matrices".
- Order eigenvalues according to their absolute values.
- No requirement on positivity of the eigenvalues (local minimum).
- No requirement on boundedness from below of the potential*.
- Repeat and keep the largest value of each |eigenvalue|.
- Results in the following plots.

*No need to sound the alarm because of the absence of these two requirements

Real nHDM with SCPV – Numerical exercise

Charged mass matrix

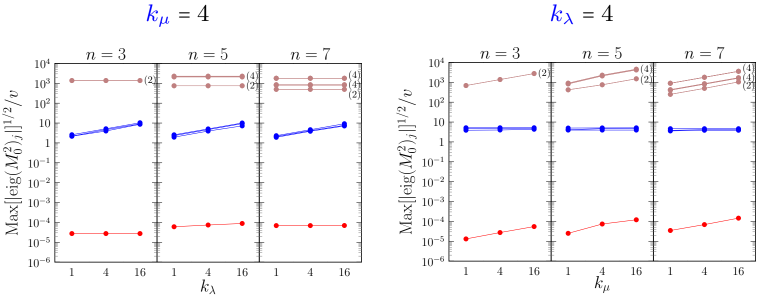




- Numerical zero: would-be Goldstone.
- One light state, sensitive to k_λ, insensitive to k_μ.
- $n-2$ heavy states, insensitive to k_λ, sensitive to k_μ.

Real nHDM with SCPV – Numerical exercise

Neutral mass matrix



- Numerical zero: would-be Goldstone.
- Three light states, sensitive to k_λ, insensitive to k_μ.
- $2n - 4$ heavy states, insensitive to k_λ, sensitive to k_μ.

Real nHDM with SCPV – Numerical exercise

Recap

- As expected, massless would-be Goldstones.
- As expected, heavy states, insensitive to k_λ, sensitive to k_μ.
- Unexpected, light $\mathcal{O}(v_{\text{EW}})$ states,

sensitive to k_λ, insensitive to k_μ.

How can they ignore $\mu^2_{ab} \gg v^2_{\text{EW}}$?

Real nHDM with SCPV – Analysis

Short of analytic black sorcery[☆],

how do we make sense of these results?

- Consider the limit where all the quartic couplings are negligible:

$$\mathcal{V}(\Phi_a) \to \mathcal{V}_2(\Phi_a) = \sum_{a=1}^{n} \mu_a^2 \Phi_a^\dagger \Phi_a + \sum_{a=1}^{n-1} \sum_{b=a+1}^{n} \mu_{ab}^2 \mathcal{H}_{ab}$$

- The key: focus on *null eigenvectors* of the mass matrices?

(since they ignore $\mu_{ab}^2 \gg v^2$)

- Then, treat quartic couplings as a perturbation.

☆Obtain the eigenvalues and eigenvectors of the mass matrices for generic n

Real nHDM with SCPV – Analysis

Stationarity conditions with no quartics

$$\partial_{\theta_1} \mathrm{V}_2 = -\frac{1}{2}\sum_{b=2}^{n} \mu_{1b}^2 s_{1b} v_1 v_b$$

$$\partial_{\theta_j} \mathrm{V}_2 = \frac{1}{2}\sum_{a=1}^{j-1} \mu_{aj}^2 s_{aj} v_a v_j - \frac{1}{2}\sum_{b=j+1}^{n} \mu_{jb}^2 s_{jb} v_j v_b$$

$$\partial_{\theta_n} \mathrm{V}_2 = \frac{1}{2}\sum_{a=1}^{n-1} \mu_{an}^2 s_{an} v_a v_n \qquad \text{N.B.} \ \sum_{j=1}^{n} \partial_{\theta_j} \mathrm{V} = 0$$

$$\partial_{v_1} \mathrm{V}_2 = \mu_1^2 v_1 + \frac{1}{2}\sum_{b=2}^{n} \mu_{1b}^2 c_{1b} v_b$$

$$\partial_{v_j} \mathrm{V}_2 = \mu_j^2 v_j + \frac{1}{2}\sum_{a=1}^{j-1} \mu_{aj}^2 c_{aj} v_a + \frac{1}{2}\sum_{b=j+1}^{n} \mu_{jb}^2 c_{jb} v_b$$

$$\partial_{v_n} \mathrm{V}_2 = \mu_n^2 v_n + \frac{1}{2}\sum_{a=1}^{n-1} \mu_{an}^2 c_{an} v_a$$

Real nHDM with SCPV – Analysis

Mass terms

$$V_2(\Phi_a)\big|_{\text{dim}=2} = \sum_{a=1}^{n} \mu_a^2 \left(\mathrm{C}_a^-\mathrm{C}_a^+ + \frac{1}{2}\left[\mathrm{R}_a^2 + \mathrm{I}_a^2\right]\right) + \frac{1}{2}\sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \mu_{ab}^2 \begin{pmatrix} c_{ab}[\mathrm{C}_a^-\mathrm{C}_b^+ + \mathrm{C}_b^-\mathrm{C}_a^+] - i s_{ab}[\mathrm{C}_a^-\mathrm{C}_b^+ - \mathrm{C}_b^-\mathrm{C}_a^+] \\ c_{ab}[\mathrm{R}_a\mathrm{R}_b + \mathrm{I}_a\mathrm{I}_b] + s_{ab}[\mathrm{R}_a\mathrm{I}_b - \mathrm{R}_b\mathrm{I}_a] \end{pmatrix}$$

$$V_2(\Phi_a)\big|_{\text{dim}=2} = \left(\mathrm{C}_1^- \ldots \mathrm{C}_n^-\right) M_\pm^2 \begin{pmatrix} \mathrm{C}_1^+ \\ \vdots \\ \mathrm{C}_n^+ \end{pmatrix} + \frac{1}{2}\left(\mathrm{R}_1 \ldots \mathrm{R}_n\ \mathrm{I}_1 \ldots \mathrm{I}_n\right) M_0^2 \begin{pmatrix} \mathrm{R}_1 \\ \vdots \\ \mathrm{R}_n \\ \mathrm{I}_1 \\ \vdots \\ \mathrm{I}_n \end{pmatrix}$$

Real nHDM with SCPV – Analysis

Mass matrices, $\theta_{ab} \equiv \theta_a - \theta_b$

$$M_{\pm}^2 = \begin{pmatrix} \mu_1^2 & \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 & \mu_2^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{2n}}\mu_{2n}^2 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \mu_{n-1}^2 & \frac{1}{2}e^{i\theta_{n-1n}}\mu_{n-1n}^2 \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 & \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 & \cdots & \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 & \mu_n^2 \end{pmatrix}$$

$$M_0^2 = \begin{pmatrix} \mathrm{Re}\left(M_{\pm}^2\right) & \mathrm{Im}\left(M_{\pm}^2\right) \\ -\mathrm{Im}\left(M_{\pm}^2\right) & \mathrm{Re}\left(M_{\pm}^2\right) \end{pmatrix}, \qquad \begin{cases} \mathrm{Re}\left(M_{\pm}^2\right)^T = \mathrm{Re}\left(M_{\pm}^2\right) \\ \mathrm{Im}\left(M_{\pm}^2\right)^T = -\mathrm{Im}\left(M_{\pm}^2\right) \end{cases}$$

Null eigenvector $\vec{u} \in \mathbb{C}^n$ of $M_{\pm}^2$

$$M_{\pm}^2 \vec{u} = \vec{0}_n$$

Real nHDM with SCPV – Analysis

One can read $M_{\pm}^2\,\vec{u} = \vec{0}_n$ as

$$\left(\mathrm{Re}\left(M_{\pm}^2\right) + i\mathrm{Im}\left(M_{\pm}^2\right)\right)\left(\mathrm{Re}\left(\vec{u}\right) + i\mathrm{Im}\left(\vec{u}\right)\right) = \mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right) - \mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right) + i\left(\mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right) + \mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right)\right) = \vec{0}_n$$

that is

$$\mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right) - \mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right) = \vec{0}_n$$
$$\mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right) + \mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right) = \vec{0}_n$$

which means

$$\begin{pmatrix} \mathrm{Re}\left(M_{\pm}^2\right) & \mathrm{Im}\left(M_{\pm}^2\right) \\ -\mathrm{Im}\left(M_{\pm}^2\right) & \mathrm{Re}\left(M_{\pm}^2\right) \end{pmatrix}\begin{pmatrix} \mathrm{Re}\left(\vec{u}\right) \\ -\mathrm{Im}\left(\vec{u}\right) \end{pmatrix} = \begin{pmatrix} \vec{0}_n \\ \vec{0}_n \end{pmatrix}$$
$$\begin{pmatrix} \mathrm{Re}\left(M_{\pm}^2\right) & \mathrm{Im}\left(M_{\pm}^2\right) \\ -\mathrm{Im}\left(M_{\pm}^2\right) & \mathrm{Re}\left(M_{\pm}^2\right) \end{pmatrix}\begin{pmatrix} \mathrm{Im}\left(\vec{u}\right) \\ \mathrm{Re}\left(\vec{u}\right) \end{pmatrix} = \begin{pmatrix} \vec{0}_n \\ \vec{0}_n \end{pmatrix}$$

Real nHDM with SCPV – Analysis

- If there is a null eigenvector $\vec{u} \in \mathbb{C}^n$ of $M^2_{\pm}$
  $\Rightarrow$ two null eigenvectors $\begin{pmatrix} \mathrm{Re}(\vec{u}) \\ -\mathrm{Im}(\vec{u}) \end{pmatrix}, \begin{pmatrix} \mathrm{Im}(\vec{u}) \\ \mathrm{Re}(\vec{u}) \end{pmatrix} \in \mathbb{R}^{2n}$ of M^2_0
- The first null eigenvector $\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ of $M^2_{\pm}$,
  corresponds to the charged Goldstone since

$$\begin{pmatrix} \mu_1^2 & \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 & \mu_2^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{2n}}\mu_{2n}^2 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \mu_{n-1}^2 & \frac{1}{2}e^{i\theta_{n-1n}}\mu_{n-1n}^2 \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 & \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 & \cdots & \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 & \mu_n^2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n-1} \\ v_n \end{pmatrix} =$$

$$\begin{pmatrix} \mu_1^2 v_1 + \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 v_2 + \ldots + \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 v_n \\ \vdots \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 v_1 + \ldots + \frac{1}{2}e^{-i\theta_{12}}\mu_{1j-1}^2 v_{j-1} + \mu_j^2 v_j + \frac{1}{2}e^{i\theta_{jj+1}}\mu_{jj+1}^2 v_{j+1} + \ldots + \frac{1}{2}e^{i\theta_{jn}}\mu_{2n}^2 v_n \\ \vdots \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 v_1 + \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 v_2 + \ldots + \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 v_{n-1} + \mu_n^2 v_n \end{pmatrix}$$

Real nHDM with SCPV – Analysis

That is

$$M_{\pm}^2 \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} \partial_{v_1} \mathrm{V}_2 - \frac{i}{v_1}\partial_{\theta_1} \mathrm{V}_2 \\ \vdots \\ \partial_{v_j} \mathrm{V}_2 - \frac{i}{v_j}\partial_{\theta_j} \mathrm{V}_2 \\ \vdots \\ \partial_{v_n} \mathrm{V}_2 - \frac{i}{v_n}\partial_{\theta_n} \mathrm{V}_2 \end{pmatrix} = \vec{0}_n$$

- In the neutral sector it gives the neutral Goldstone and "the Higgs"

$$\vec{r}_G^{\,T} = (\vec{0}_n, v_1, \ldots, v_n), \qquad \vec{r}_h^{\,T} = (v_1, \ldots, v_n, \vec{0}_n)$$

Real nHDM with SCPV – Analysis

- There is *another* null eigenvector

$$\vec{c}_0^{\,T} = (e^{i2\theta_1} v_1, \ldots, e^{i2\theta_j} v_j, \ldots, e^{i2\theta_n} v_n)$$

- Explicitly

$$\begin{pmatrix} \mu_1^2 & \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 & \mu_2^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{2n}}\mu_{2n}^2 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \mu_{n-1}^2 & \frac{1}{2}e^{i\theta_{n-1n}}\mu_{n-1n}^2 \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 & \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 & \cdots & \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 & \mu_n^2 \end{pmatrix} \begin{pmatrix} e^{i2\theta_1} v_1 \\ e^{i2\theta_2} v_2 \\ \vdots \\ e^{i2\theta_{n-1}} v_{n-1} \\ e^{i2\theta_n} v_n \end{pmatrix}$$

$$M_\pm^2\, \vec{c}_0 = \begin{pmatrix} e^{i2\theta_1}(\partial_{v_1}\mathrm{V}_2 + \frac{i}{v_1}\partial_{\theta_1}\mathrm{V}_2) \\ \vdots \\ e^{i2\theta_j}(\partial_{v_j}\mathrm{V}_2 + \frac{i}{v_j}\partial_{\theta_j}\mathrm{V}_2) \\ \vdots \\ e^{i2\theta_n}(\partial_{v_n}\mathrm{V}_2 + \frac{i}{v_n}\partial_{\theta_n}\mathrm{V}_2) \end{pmatrix} = \vec{0}_n \qquad \checkmark$$

Real nHDM with SCPV – Analysis

Null eigenvectors of mass matrices with no quartics in $\mathcal{V}$

- Charged

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}, \qquad \begin{pmatrix} e^{i2\theta_1} v_1 \\ \vdots \\ e^{i2\theta_n} v_n \end{pmatrix}$$

- Neutral

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 \\ \vdots \\ 0 \\ v_1 \\ \vdots \\ v_n \end{pmatrix}, \qquad \begin{pmatrix} v_1 \cos 2\theta_1 \\ \vdots \\ v_n \cos 2\theta_n \\ -v_1 \sin 2\theta_1 \\ \vdots \\ -v_n \sin 2\theta_n \end{pmatrix}, \qquad \begin{pmatrix} v_1 \sin 2\theta_1 \\ \vdots \\ v_n \sin 2\theta_n \\ v_1 \cos 2\theta_1 \\ \vdots \\ v_n \cos 2\theta_n \end{pmatrix}$$

- Not orthogonal! But no problem, one can always orthonormalize

Real nHDM with SCPV – Analysis

Why is there a second null eigenvector? (With no quartics)

- Important observation: the CP conjugate vevs $\langle\Phi_a^*\rangle = \langle\Phi_a\rangle^*$ give a *different* candidate vacuum!
- With no quartics, *both* vacua give null eigenvectors of $M_\pm^2$ and M_0^2 computed around the chosen vacuum.

"Quartics restored"

- The first null eigenvector gives the charged and neutral would-be Goldstones + "the Higgs".
- The second null eigenvector gives 1 charged and 2 neutral states with masses^2 ~ $\mathcal{O}((\lambda' s)\, v^2)$.
- This is in full agreement with the numerical results!
- Key ingredient: the existence of a second minimum of the potential.

General 2HDM – Setup

Scalar potential

$$\begin{aligned}\mathcal{V}(\Phi_1,\Phi_2) =& \mathcal{V}_2(\Phi_1,\Phi_2)+\mathcal{V}_4(\Phi_1,\Phi_2),\\ \mathcal{V}_2(\Phi_1,\Phi_2) =& \mu_1^2\,\Phi_1^\dagger\Phi_1+\mu_2^2\,\Phi_2^\dagger\Phi_2+\mu_{12}^2\,\Phi_1^\dagger\Phi_2+\mu_{12}^{2\,*}\,\Phi_2^\dagger\Phi_1\\ \mathcal{V}_4(\Phi_1,\Phi_2) =& \lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ &+\lambda_5(\Phi_1^\dagger\Phi_2)^2+\lambda_5^*(\Phi_2^\dagger\Phi_1)^2\\ &+(\lambda_6\,\Phi_1^\dagger\Phi_2+\lambda_6^*\,\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_1)+(\lambda_7\,\Phi_1^\dagger\Phi_2+\lambda_7^*\,\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\end{aligned}$$

with real μ_1^2, μ_2^2, λ_1, λ_2, λ_3, λ_4, and complex μ_{12}^2, λ_5, λ_6, λ_7.
Compute $\mathrm{V}(v_1,v_2,\theta)=\mathcal{V}(\langle\Phi_1\rangle,\langle\Phi_2\rangle)$.
More (real) quadratic parameters than stationarity conditions.

General 2HDM – Setup

Stationarity conditions (one last time. . .),

$$\partial_{v_1} \mathrm{V} = \mu_1^2 v_1 + \mathrm{Re}\left(\mu_{12}^2 e^{i\theta}\right) v_2 + \lambda_1 v_1^3 + \frac{1}{2}(\lambda_3 + \lambda_4) v_1 v_2^2$$
$$+ \mathrm{Re}\left(\lambda_5 e^{i2\theta}\right) v_1 v_2^2 + \frac{3}{2}\mathrm{Re}\left(\lambda_6 e^{i\theta}\right) v_1^2 v_2 + \frac{1}{2}\mathrm{Re}\left(\lambda_7 e^{i\theta}\right) v_2^3 = 0$$

$$\partial_{v_2} \mathrm{V} = \mu_2^2 v_2 + \mathrm{Re}\left(\mu_{12}^2 e^{i\theta}\right) v_1 + \lambda_2 v_2^3 + \frac{1}{2}(\lambda_3 + \lambda_4) v_1^2 v_2$$
$$+ \mathrm{Re}\left(\lambda_5 e^{i2\theta}\right) v_1^2 v_2 + \frac{1}{2}\mathrm{Re}\left(\lambda_6 e^{i\theta}\right) v_1^3 + \frac{3}{2}\mathrm{Re}\left(\lambda_7 e^{i\theta}\right) v_1 v_2^2 = 0$$

$$\partial_{\theta} \mathrm{V} = -\,\mathrm{Im}\left(\mu_{12}^2 e^{i\theta}\right) v_1 v_2 - \mathrm{Im}\left(\lambda_5 e^{i2\theta}\right) v_1^2 v_2^2 - \mathrm{Im}\left((\lambda_6 v_1^2 + \lambda_7 v_2^2) e^{i\theta}\right) \frac{v_1 v_2}{2} = 0$$

General 2HDM – Setup

- Physical scalars, charged

$$(C_1^-, C_2^-)^T = U\,(G^-, H^-)^T \qquad U^\dagger M_\pm^2 U = \mathrm{Diag}(0, M_{H^\pm}^2)$$

with $U \in U(2)$ (rotation into the "Higgs basis")

$$M_{H^\pm}^2 = \mu_1^2 + \mu_2^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 + \frac{1}{2}\lambda_3 v^2 + \mathrm{Re}\left((\lambda_6 + \lambda_7)e^{i\theta}\right) v_1 v_2$$

$$= \frac{v^2}{2 v_1 v_2 \sin\theta} \mathrm{Im}\left(2\mu_{12}^2 + 2(\lambda_5 - \lambda_4)e^{i\theta} v_1 v_2 + (\lambda_6 v_1^2 + \lambda_7 v_2^2)e^{i\theta}\right)$$

General 2HDM – Setup

- Physical scalars, neutral

$$(\mathrm{R}_1, \mathrm{R}_2, \mathrm{I}_1, \mathrm{I}_2)^T = O\,(G^0, h, \mathrm{H}_1^0, \mathrm{H}_2^0)^T$$

$$O^T\, M_0^2\, O = \mathrm{Diag}(0, m_h^2, M_{\mathrm{H}_1^0}^2, M_{\mathrm{H}_2^0}^2)$$

with real $O \in O(4)$

- We choose $m_h^2 \leq M_{\mathrm{H}_1^0}^2 \leq M_{\mathrm{H}_2^0}^2$, with h "the Higgs".
- Sum of masses, $\mathrm{Tr}\left\{M_0^2\right\}$,

$$m_h^2 + M_{\mathrm{H}_1^0}^2 + M_{\mathrm{H}_2^0}^2 =$$

$$2(\mu_1^2 + \mu_2^2) + 4(\lambda_1 v_1^2 + \lambda_2 v_2^2) + (\lambda_3 + \lambda_4) v^2 + 4\mathrm{Re}\left((\lambda_6 + \lambda_7) e^{i\theta}\right) v_1 v_2$$

- If we can have $\mu_1^2 + \mu_2^2 \gg v_{\mathrm{EW}}^2$,

  there is a decoupling regime for the new scalars.

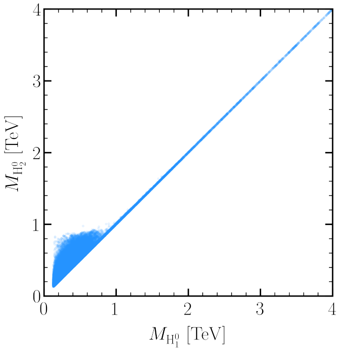
General 2HDM – Numerical exercise

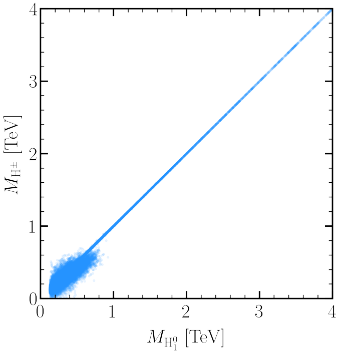
Simple numerical exercise

- Generate random values of λ's, ensure boundedness from below.
- Generate random values of μ^2's.
- Minimize repeatedly $\text{V}(v_1, v_2, \theta)$ with random starting points. Remove repeated minima.
- Reject if there is no spontaneous symmetry breaking.
- We are left with potentials with either one or two minima (as vacuum, we take the deepest).
  In agreement with Ivanov ☞ PRD75 (2007), ☞ PRD77 (2008)
- Parameter redefinitions to set
  (i) $v^2 = v_1^2 + v_2^2 = v_{\text{EW}}^2$ and
  (ii) mass of the lightest neutral state = m_h.
- Apply perturbative unitarity constraints on the quartic parameters.
- Plot masses for both cases, one or two minima, separately.

General 2HDM – Numerical exercise

Masses of the new scalars, potentials with one minimum

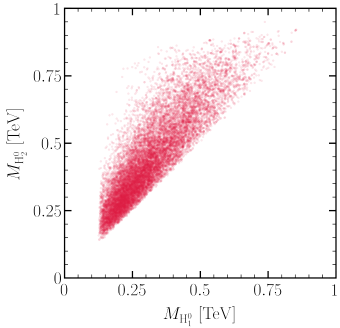


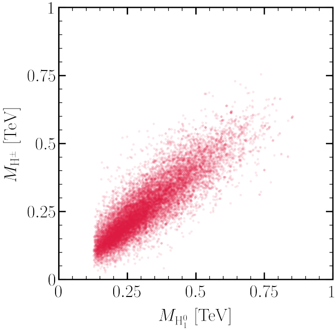


- Decoupling regime feasible,
  where all new scalars have (degenerate) masses > 1 TeV.

General 2HDM – Numerical exercise

Masses of the new scalars, potentials with two minima





- No decoupling regime, all new scalars with masses < 1 TeV!

General 2HDM – Analysis

Making sense of the results for two minima

- There are two minima with vevs $\{v_1, v_2 e^{i\theta}\}$ and $\{v_1', v_2' e^{i\theta'}\}$.
- Stationarity conditions fulfilled by $\{v_1, v_2, \theta\}$ and $\{v_1', v_2', \theta'\}$.
- Take for example $\partial_\theta \mathrm{V} = 0$,

$$-\operatorname{Im}\left(\mu_{12}^2 e^{i\theta}\right) v_1 v_2 - \operatorname{Im}\left(\lambda_5 e^{i2\theta}\right) v_1^2 v_2^2 - \operatorname{Im}\left((\lambda_6 v_1^2 + \lambda_7 v_2^2) e^{i\theta}\right) \frac{v_1 v_2}{2} = 0$$

$$-\operatorname{Im}\left(\mu_{12}^2 e^{i\theta'}\right) v_1' v_2' - \operatorname{Im}\left(\lambda_5 e^{i2\theta'}\right) {v_1'}^2 {v_2'}^2 - \operatorname{Im}\left((\lambda_6 {v_1'}^2 + \lambda_7 {v_2'}^2) e^{i\theta'}\right) \frac{v_1' v_2'}{2} = 0$$

Assume $v_1 v_2 \neq 0$, $v_1' v_2' \neq 0$ (OK in a generic $\{\Phi_1, \Phi_2\}$ basis)

General 2HDM – Analysis

Making sense of the results for two minima

- $\mathrm{Im}\left(\mu_{12}^2\right)$ is not free anymore!

$$\mathrm{Re}\left(\mu_{12}^2\right)=\frac{1}{\sin(\theta'-\theta)}\left([E]\cos\theta'-[E']\cos\theta\right)$$

$$\mathrm{Im}\left(\mu_{12}^2\right)=\frac{1}{\sin(\theta'-\theta)}\left([E']\sin\theta-[E]\sin\theta'\right)$$

$$\text{with}\quad [E]=\mathrm{Im}\left(\lambda_5 e^{i2\theta}\right)v_1v_2+\frac{1}{2}\mathrm{Im}\left((\lambda_6 v_1^2+\lambda_7 v_2^2)e^{i\theta}\right)$$

$$[E']=\mathrm{Im}\left(\lambda_5 e^{i2\theta'}\right)v_1'v_2'+\frac{1}{2}\mathrm{Im}\left((\lambda_6 v_1'^2+\lambda_7 v_2'^2)e^{i\theta'}\right)$$

- *All* quadratic parameters are now *bounded* by perturbativity requirements on the λ's, the whole spectrum is bounded!

General 2HDM – Bonus

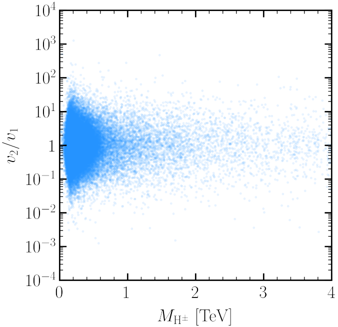
- It is clear that "Masses > 1 TeV"

  $\Rightarrow$ potential with a single minimum.
- Can one say something for "Masses < 1 TeV"?
- Masses are of no help, what about other quantities,

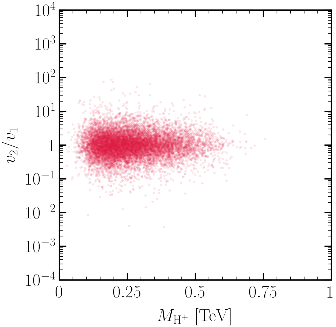
  for example v_2/v_1?

General 2HDM – Bonus

v_2/v_1 vs. $M_{H^\pm}$, generic basis



Potentials with one minimum



Potentials with *two* minima

- Not interesting, but expected: v_2/v_1 has no physical meaning, one can rotate the $\{\Phi_1, \Phi_2\}$ basis and v_2/v_1 changes accordingly.

General 2HDM – Bonus

- Not the end of the story, there is a *particular* basis...
- Introduce $(\Phi_1^\dagger,\Phi_2^\dagger)=(\Phi_1'^\dagger,\Phi_2'^\dagger)\mathcal{U}^\dagger$, $\mathcal{U}\in U(2)$, such that

$$\mathcal{V}_2=\begin{pmatrix}\Phi_1^\dagger & \Phi_2^\dagger\end{pmatrix}\begin{pmatrix}\mu_1^2 & \mu_{12}^2\\ \mu_{12}^{2\,*} & \mu_2^2\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix}=\begin{pmatrix}\Phi_1'^\dagger & \Phi_2'^\dagger\end{pmatrix}\begin{pmatrix}\mu_{\mathrm{d}1}^2 & 0\\ 0 & \mu_{\mathrm{d}2}^2\end{pmatrix}\begin{pmatrix}\Phi_1'\\ \Phi_2'\end{pmatrix}$$

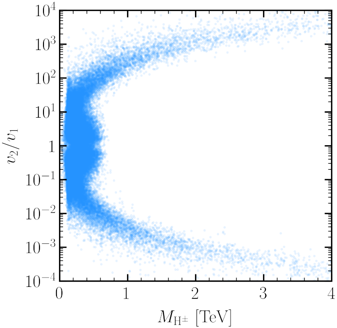
that is, diagonalize the hermitian matrix of quadratic parameters to obtain the real eigenvalues

$$\mu_{\mathrm{d}1,2}^2=\frac{1}{2}\left(\mu_1^2+\mu_2^2\pm\sqrt{(\mu_1^2-\mu_2^2)^2+4|\mu_{12}^2|^2}\right)$$

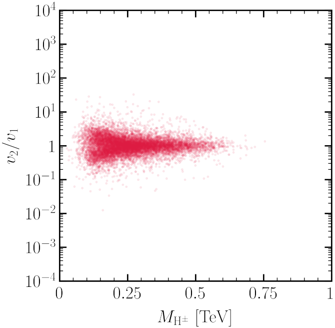
and the rotation $\mathcal{U}$, and now represent v_2'/v_1' vs. $M_{\mathrm{H}^\pm}$.

General 2HDM – Bonus

v_2/v_1 vs. $M_{H^\pm}$, *particular* basis



Potentials with one minimum



Potentials with *two* minima

- Decoupling requires very large or very small values of v_2/v_1.
- For $10^{-2} < \frac{v_2}{v_1} < 10^2$, still no clue,

  but for $\frac{v_2}{v_1} < 10^{-2}$ or $\frac{v_2}{v_1} > 10^2$, the potential has one minimum.

General 2HDM – Bonus conundrum

- In the *particular* basis there are just 2 quadratic parameters, are they fixed through the 3 stationarity conditions and thus the decoupling regime *cannot* arise?

$$\partial_{v_1}\mathrm{V} = \mu^2_{\mathrm{d}1} v_1 + \lambda_1 v_1^3 + \frac{1}{2}(\lambda_3+\lambda_4) v_1 v_2^2 + \mathrm{Re}\left(\lambda_5 e^{i2\theta}\right) v_1 v_2^2 + \frac{3}{2}\mathrm{Re}\left(\lambda_6 e^{i\theta}\right) v_1^2 v_2 + \frac{1}{2}\mathrm{Re}\left(\lambda_7 e^{i\theta}\right) v_2^3 = 0$$

$$\partial_{v_2}\mathrm{V} = \mu^2_{\mathrm{d}2} v_2 + \lambda_2 v_2^3 + \frac{1}{2}(\lambda_3+\lambda_4) v_1^2 v_2 + \mathrm{Re}\left(\lambda_5 e^{i2\theta}\right) v_1^2 v_2 + \frac{1}{2}\mathrm{Re}\left(\lambda_6 e^{i\theta}\right) v_1^3 + \frac{3}{2}\mathrm{Re}\left(\lambda_7 e^{i\theta}\right) v_1 v_2^2 = 0$$

$$\partial_{\theta}\mathrm{V} = -\,\mathrm{Im}\left(\lambda_5 e^{i2\theta}\right) v_1^2 v_2^2 - \mathrm{Im}\left((\lambda_6 v_1^2 + \lambda_7 v_2^2) e^{i\theta}\right) \frac{v_1 v_2}{2} = 0$$

General 2HDM – Bonus conundrum

- $\mu^2_{\text{d}1}$ and $\mu^2_{\text{d}2}$ absent from $\partial_\theta \text{V} = 0$,
  but still 2 equations for 2 parameters.
- Notice that $\mu^2_{\text{d}1}$ and $\mu^2_{\text{d}2}$ appear multiplied by vevs:
  one can have either $\mu^2_{d1} \gg v^2_{\text{EW}}$ or $\mu^2_{d2} \gg v^2_{\text{EW}}$ (not both),
  provided $v_1 \ll v_{\text{EW}}$ or $v_2 \ll v_{\text{EW}}$ (not both),
  exactly as the plot shows!

Conclusions

- In both the real nHDM with SCPV and the general 2HDM
  - There is an "overabundance" of free quadratic couplings in $\mathcal{V}$, stationarity conditions cannot remove them all.
  - One could have expected that besides "the Higgs" (+ Goldstones), all scalars could have large masses.

That is not the case

- In the real nHDM (at least)
  one charged and two new neutral scalars have $\mathcal{O}(v_{\text{EW}})$ masses
  - Analysis in the absence of quartic couplings, with *two* null eigenvectors corresponding to would-be Goldstones of the two CP conjugate minima.
  - Reintroducing quartic couplings, zero masses lifted (except for the would-be Goldstones).

Conclusions

- In the general 2HDM
  the charged and the two new neutral
  have $\mathcal{O}(v_{\text{EW}})$ masses if the potential has two minima.
  - Analysis directly in terms of stationarity conditions
    involving both minima.
- General 2HDM Bonus
  - Besides the decoupling regime, particular basis with diagonal quadratic parameters can help telling apart one vs. two minima through v_2/v_1.
  - Potential conundrum: 2 quadratic parameters in diagonal basis vs. 2 stationarity conditions, and yet decoupling possible $\Rightarrow$ hierarchy of vevs.

ευχαριστώ !

Thank you!

Comments, questions  ?

Backup

Backup – Real nHDM with SCPV – Setup

Real nHDM scalar potential, $\mathrm{V}(v_a, \theta_a) = V(\langle\Phi_a\rangle)$

$$
\begin{aligned}
4\mathrm{V}(v_a,\theta_a) = {} & 2\sum_{a=1}^{n}\mu_a^2 v_a^2 + 2\sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\mu_{ab}^2 c_{ab} v_a v_b + \sum_{a=1}^{n}\lambda_a v_a^4 \\
& + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\lambda_{a,b} v_a^2 v_b^2 + \sum_{a=1}^{n}\sum_{b=1}^{n-1}\sum_{c=b+1}^{n}\lambda_{a,bc} c_{bc} v_a^2 v_b v_c \\
& + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd} c_{ab} c_{cd} v_a v_b v_c v_d \\
& + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd}^{\mathcal{A}} s_{ab} s_{cd} v_a v_b v_c v_d
\end{aligned}
$$

Backup – General 2HDM – Numerical exercise

Parameter redefinitions to set good vev and Higgs mass.

- After minimizing a given potential V with the minimum at $\langle\Phi_1\rangle = \frac{v_{1,0}}{\sqrt{2}}\binom{0}{1}$, $\langle\Phi_2\rangle = \frac{v_{2,0}e^{i\theta_0}}{\sqrt{2}}\binom{0}{1}$, rescale

$$\mu_a^2, \mu_{ab}^2 \mapsto \frac{v_{\text{EW}}^2}{v_0^2}\,\mu_a^2, \mu_{ab}^2, \qquad v_0^2 \equiv v_{1,0}^2 + v_{2,0}^2$$

- Compute the mass matrices. With m_0^2 the smallest non-vanishing eigenvalue of M_0^2, rescale

$$\mu_a^2, \mu_{ab}^2, \lambda_x \mapsto \frac{m_h^2}{m_0^2}\,\mu_a^2, \mu_{ab}^2, \lambda_x$$