ArXiv: 1911.06822 [hep-ph] BSM Journal Club, 15/01/20, Luiz Vale Silva (IFIC, UV - CSIC)

Standard-model prediction of ϵ_K with manifest CKM unitarity

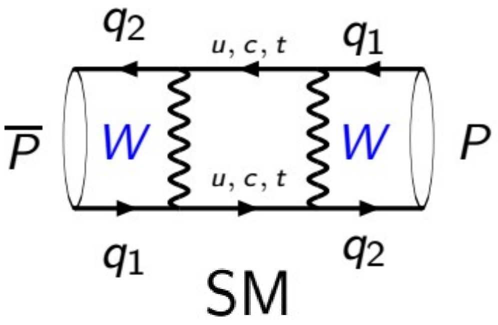
Joachim Brod,[1,*] Martin Gorbahn,[2,†] and Emmanuel Stamou[3,‡]

[1] *Department of Physics, University of Cincinnati, Cincinnati, OH 45221, USA*
[2] *Department of Mathematical Sciences, University of Liverpool, Liverpool, L69 7ZL, UK*
[3] *Institut de Théorie des Phénomenes Physiques, EPFL, Lausanne, Switzerland*

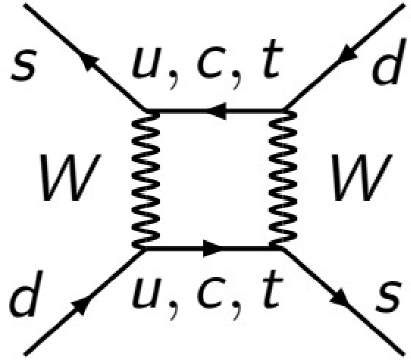
(Dated: November 19, 2019)

The parameter ϵ_K describes CP violation in the neutral kaon system and is one of the most sensitive probes of new physics. The large uncertainties related to the charm-quark contribution to ϵ_K have so far prevented a reliable standard-model prediction. We show that CKM unitarity enforces a unique form of the $|\Delta S=2|$ weak effective Lagrangian in which the short-distance theory uncertainty of the imaginary part is dramatically reduced. The uncertainty related to the charm-quark contribution is now at the percent level. We present the updated standard-model prediction $\epsilon_K = 2.16(6)(8)(15) \times 10^{-3}$, where the errors in brackets correspond to QCD short-distance and long-distance, and parametric uncertainties, respectively.

- ϵK: discovery of CP Violation
- ϵK: CP Violation in the mixing of kaons
- BSM: new contributions to ϵK



ϵK w/o perturbative QCD corrections



$$= i \frac{G_F^2 M_W^2}{4\pi^2} \sum_{i,j=c,t} \underbrace{V^L_{is}\, V^{L*}_{id}\, V^L_{js}\, V^{L*}_{jd}}_{\text{unitarity: no } u \text{ index}} \; \underbrace{S(x_i, x_j)}_{\text{loop function}} \; Q_V, \quad x_i = \frac{m_i^2}{M_W^2}$$

$x_u \sim 0$

$x_c \sim 3 \cdot 10^{-4}$

$x_t \sim 4$

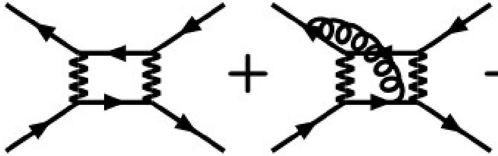
Weak Left-Handed Currents: $Q_V = \bar{s}\gamma^\mu P_L d \cdot \bar{s}\gamma_\mu P_L d$

i, j	t, t	c, t	c, c
$S(x_i, x_j)$	$f_{tt}(x_t)$	$x_c[-\log x_c + f_{ct}(x_t)]$	x_c

- CKM unitarity: Vus.(Vud)* = -Vts.(Vtd)* -Vcs.(Vcd)*
- S: Inami-Lim functions
- GIM mechanism: S \propto xt, xc (xu=0)

Taking into account QCD corrections

BSM JC, L. Vale Silva



$$+ \ldots \overset{\text{OPE}}{=} \sum_i \underbrace{C_i(\mu)}_{\textit{short distances}} \times \underbrace{Q_i(\mu)}_{\textit{long distances}}$$

$C_i(\mu_{\text{high}})$ at high energies,
$\mu_{\text{high}} = \mathcal{O}(M_{W',W}, M_H, m_t)$
perturbative

$\langle P | Q_i | \bar{P} \rangle \equiv \langle Q_i(\mu_{\text{low}}) \rangle$ at low energies
μ_{low} a few GeV
non-perturbative, Lattice QCD

$$\frac{d}{d \log \mu} \begin{pmatrix} C_1 \\ C_2 \\ \vdots \end{pmatrix} \overset{\text{RGE}}{=} \gamma^{\text{T}} \cdot \begin{pmatrix} C_1 \\ C_2 \\ \vdots \end{pmatrix}$$

γ: anom. dim. matrix of Q_i

$$\frac{d}{d \log \mu} \alpha_s \overset{\text{LO}}{=} -\beta_0 \frac{\alpha_s^2}{2\pi}$$

$$C(\mu_{\text{high}})$$

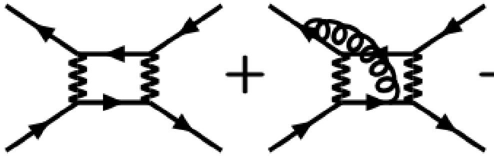
$\downarrow$ running

$$C(\mu_{\text{low}}) \overset{\text{LO}}{=} \underbrace{\left(\frac{\alpha_s(\mu_{\text{low}})}{\alpha_s(\mu_{\text{high}})} \right)^{\frac{\gamma}{2\beta_0}}}_{\left[\sum_{n=0}^{\infty} \left(\beta_0 \frac{\alpha_s(\mu_{\text{low}})}{2\pi} \log \left(\frac{\mu_{\text{low}}}{\mu_{\text{high}}} \right) \right)^n \right]^{\frac{\gamma}{2\beta_0}}} C(\mu_{\text{high}})$$

large $\alpha_s \cdot \log(\mu_{\text{low}}/\mu_{\text{high}})$ factors: Short-Distance QCD effects

Impact of the SD QCD corrections in the SM

The quest for SM precision has required to calculate Short-Distance QCD corrections up to high orders



$$+ \ldots = i\frac{G_F^2 M_W^2}{4\pi^2} \sum_{i,j=c,t} V^L_{iq_1} V^{L*}_{iq_2} V^L_{jq_1} V^{L*}_{jq_2} \times S(x_i, x_j)\, \bar{\eta}_{ij}\, Q_V$$

$\bar{\eta}_{ij}$ Short-Distance, perturbative QCD corrections

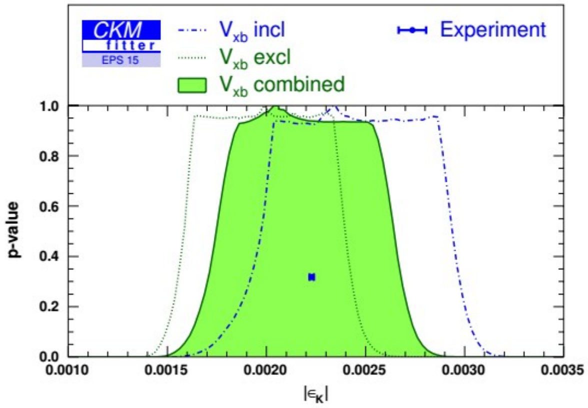
$\bar{\eta}_{ij} = 1 + \mathcal{O}(\alpha_s^m \cdot \log^n x_c) + \ldots$: mandatory for phenomenology

SM	t,t	c,t	c,c
$\eta^K_{\rm LO}$ [Gilman, Wise '83]	0.59	0.37	0.74
$\eta^K_{\rm NLO}$ [Buras etal '90, Herrlich, Nierste '94 '96]	0.577(7)	$0.47^{+0.03}_{-0.04}$	$1.3^{+0.3}_{-0.2}$
$\eta^K_{\rm NNLO}$ [Brod, Gorbahn '10 '12]	-	0.50 ± 0.05	1.9 ± 0.8 ☹
$\eta^B_{\rm NLO}$ [Buchalla etal '96]	0.551(2)		

Prediction of ϵK

$$\epsilon_K = \frac{G_F^2 F_K^2 M_K M_W^2}{12\sqrt{2}\pi^2 \Delta M_{K_{\rm exp}}} \kappa_\varepsilon e^{i\phi_\varepsilon} \widehat{\mathcal{B}}_K \sum_{i,j=c,t} \mathrm{Im}\{\lambda_i^{sd}\lambda_j^{sd}\}\eta_{ij} S\left(\frac{\overline{m}_i^2}{M_W^2}, \frac{\overline{m}_j^2}{M_W^2}\right)$$

- QCD: perturbative & non-perturbative (e.g., BK) effects
- |ϵK| \propto 1*ηtt + 0.7*ηct + 0.08*ηcc (roughly)



Error budget “c-t” (old evaluation!):

$$\begin{aligned} |\varepsilon_K|_{\rm SM} = \; & 2.09 \cdot 10^{-3} (1 \pm 4\ \%|_A \pm 2\ \%|_\lambda \pm 1\ \%|_{\bar\rho} \pm 3\ \%|_{\bar\eta} \\ & \pm 2\ \%|_{\widehat{\mathcal{B}}_K} \pm 7\ \%|_{\eta_{cc}} \pm 4\ \%|_{\eta_{ct}} \pm 1\ \%|_{\eta_{tt}} \\ & \pm 2\ \%|_{\overline{m}_c} \pm 0.5\ \%|_{\overline{m}_t} \pm 1\ \%|_{F_K} \pm 3\ \%|_{\kappa_\varepsilon}) \end{aligned}$$

CKM unitarity: go from “c-t”, to “u-t”

1.) λc = -λt -λu

😡 $\mathscr{C}^{uu}_{S2} \equiv \mathcal{C}_1, \; \mathscr{C}^{tt}_{S2} \equiv \mathcal{C}_2, \text{ and } \mathscr{C}^{ut}_{S2} \equiv \mathcal{C}_3.$

$$\mathcal{L}^{\Delta S=2}_{f=3} = -\frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_c^2 C^{cc}_{S2}(\mu) + \lambda_t^2 C^{tt}_{S2}(\mu) + \lambda_c\lambda_t C^{ct}_{S2}(\mu)\right] Q_{S2} + \text{h.c.} + \ldots, \quad (5)$$

$$\mathcal{L}^{\Delta S=2}_{f=3} = -\frac{G_F^2 M_W^2}{4\pi^2}\left[\lambda_u^2 \mathscr{C}^{uu}_{S2}(\mu) + \lambda_t^2 \mathscr{C}^{tt}_{S2}(\mu) + \lambda_u\lambda_t \mathscr{C}^{ut}_{S2}(\mu)\right] Q_{S2} + \text{h.c.} + \ldots, \quad (4)$$

$$C^{cc}_{S2} \equiv \mathcal{C}_1, \; C^{ct}_{S2} \equiv 2\mathcal{C}_1 + \mathcal{C}_3, \text{ and } 2C^{tt}_{S2} \equiv 2\mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3.$$

2.) It turns out that ηuu, or C1, has a bad convergence (unknown reason)

3.) Same phase convention for |ΔS|=1,2 transitions: “uu” does not contr. to ϵK

- The calculation of η̃tt is essentially the same as for ηtt, i.e., η̃tt ~ ηtt
- The calculation of ηut is analogous to the one of ηct
- |ϵK| \propto 1*ηtt + 0.6*ηut (roughly)

"c-t" vs. "u-t" parameterizations

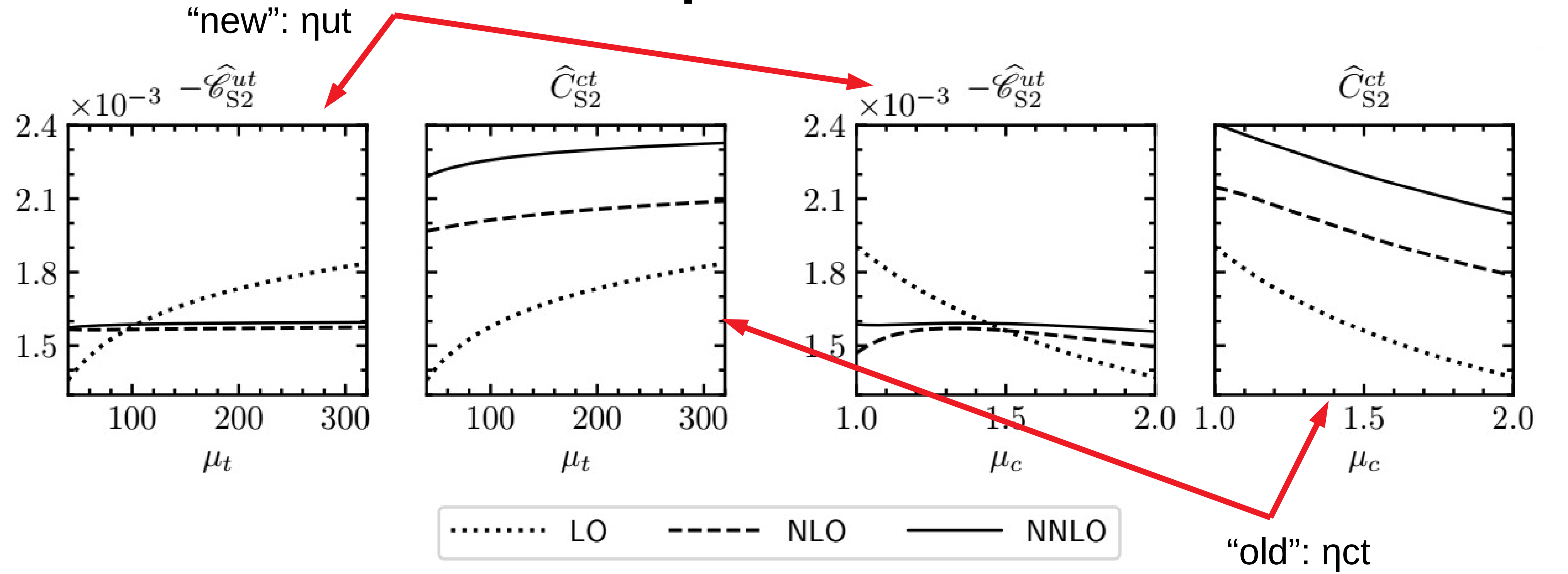


FIG. 1: Comparison of Wilson coefficients in u-t (1^{st} and 3^{rd} plot) and c-t unitarity (2^{nd} and 4^{th} plot). Shown is the residual renormalization-scale dependence of the RI Wilson coefficients as a proxy for their theory uncertainty. In the two plots on the left the five-flavour threshold, μ_t, is varied, while in the two on the right the three-flavour threshold, μ_c, is varied (see text for further details).

Error budget “u-t”

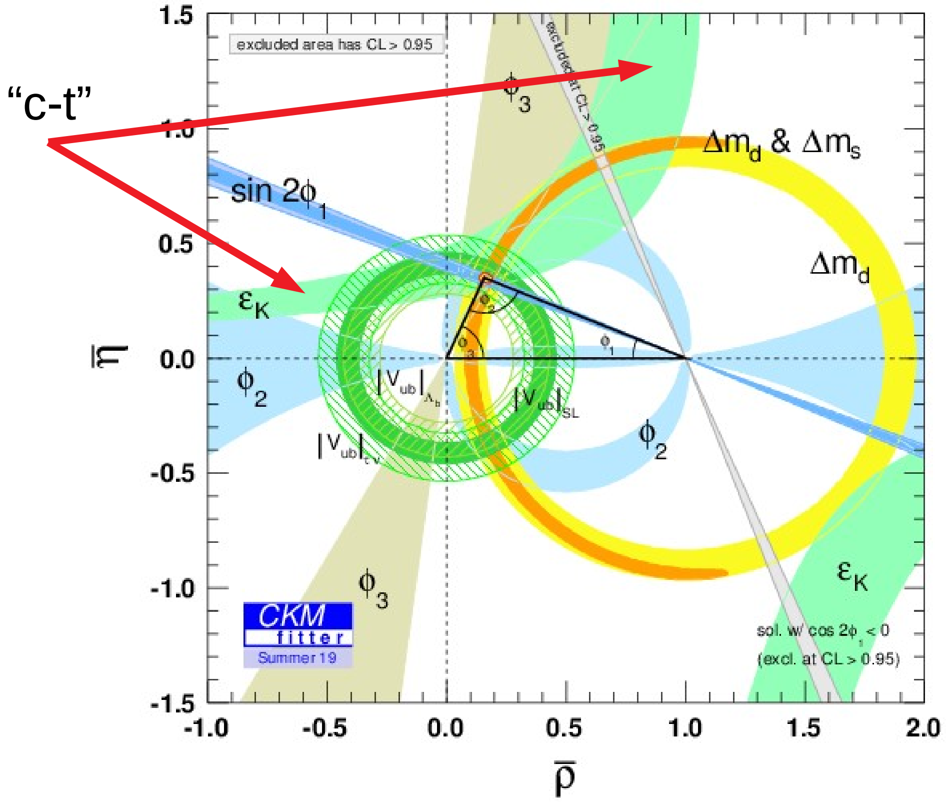
$$\eta_{tt}^{\rm NLL} = 0.55(1 \pm 4.2\%_{\rm scales} \pm 0.1\%_{\alpha_s})\,, \qquad (16)$$
$$\eta_{ut}^{\rm NNLL} = 0.402(1 \pm 1.3\%_{\rm scales} \pm 0.2\%_{\alpha_s} \pm 0.2\%_{m_c})\,.$$

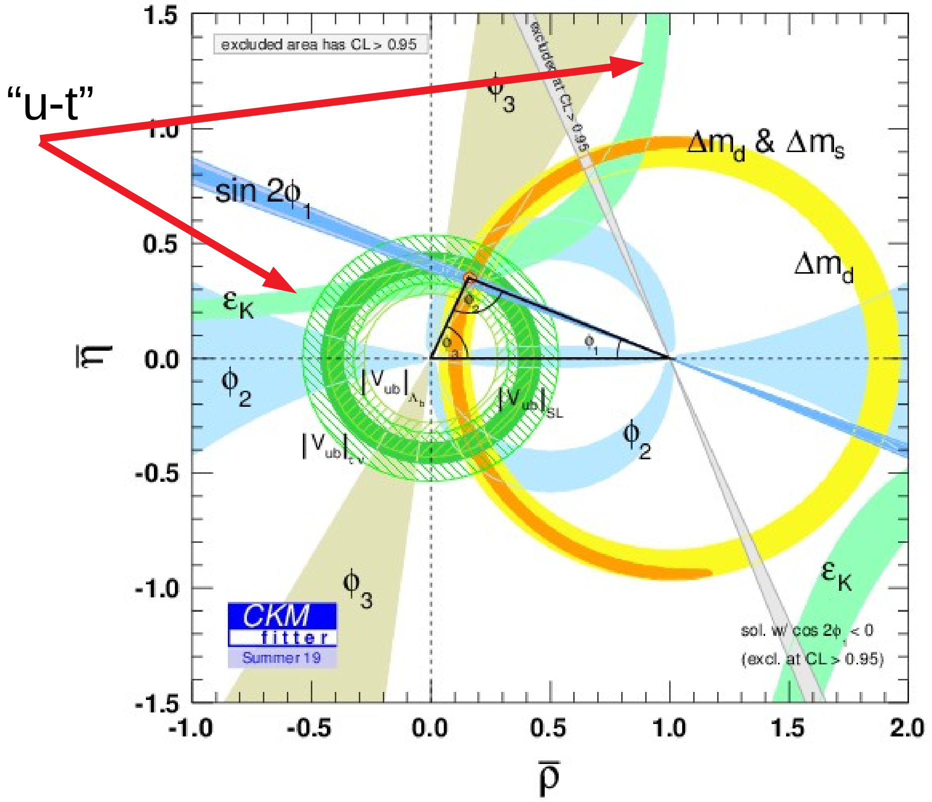
$$\epsilon_K\ :\quad {\rm CV}\times\Big(1 \pm 7.1\%|_{\rm param} \pm 3.0\%|_{\eta_{tt}} \pm 0.4\%|_{\eta_{ut}} \pm 1.2\%|_{\hat{B}_K} \pm 2.4\%|_{\xi_s} \pm 2.1\%|_{\kappa_\epsilon}\Big)$$

ArXiv: 1911.06822 [hep-ph]

Room for improvement:

- param: e.g., CKM matrix elements
- Likely (?), Brod & Gorbahn et al. will calculate ηtt to the NNLO
- BK: bag parameter
- $\xi_s = (F_{B_s}\sqrt{\widehat{B}_s})/(F_{B_d}\sqrt{\widehat{B}_d}) = 1.206(17)$



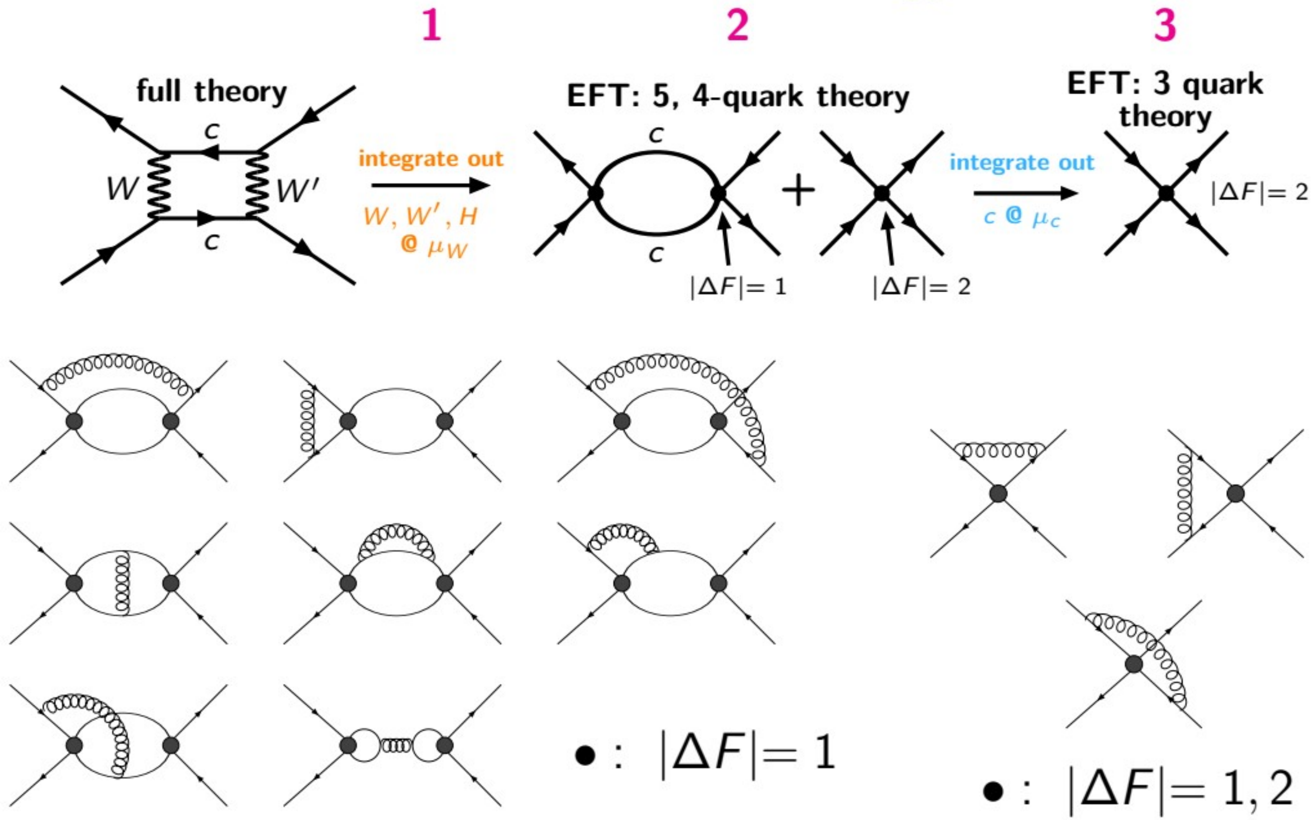


- Experimental measurement: 2.228*(1 +/- 0.5%)/10^3
- ϵK with new param. still in very good agreement with global fit

More details @
arXiv: 1911.06822 [hep-ph]

Spare

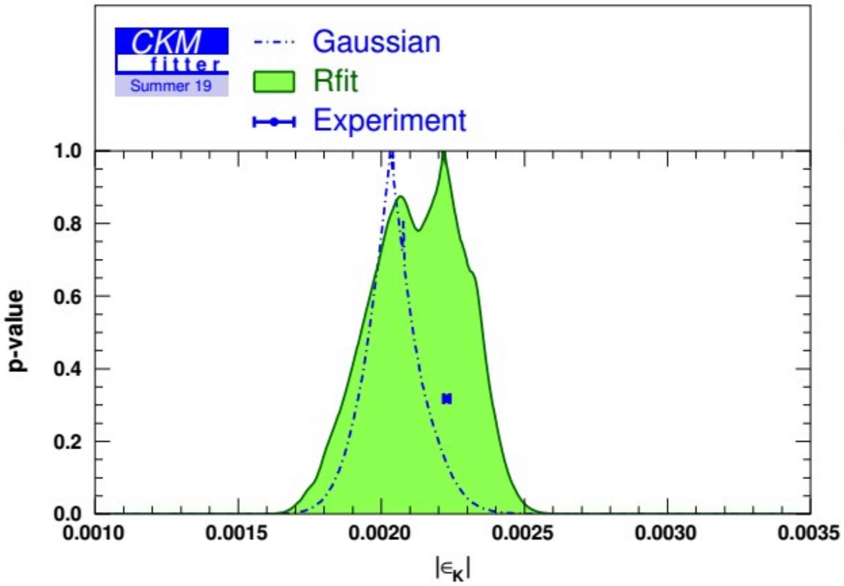
Overview of the calculation for $\bar{\eta}_{cc}^{LR}$ @ NLO



Spare

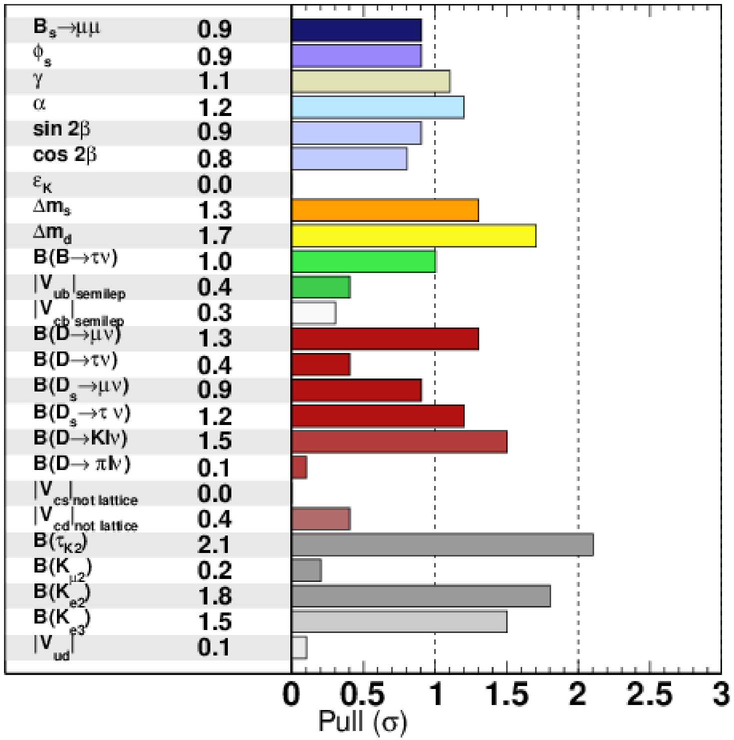
From time to time, question about compatibility of ϵ_K with the rest of the fit, related to the fact that ϵ_K has a strong dependence on

- B_K : role of theoretical uncertainties
- $|V_{cb}|$: inclusive, exclusive or average



- Rfit versus Gaussian treatment of theoretical uncertainties
- agreement of prediction with experiment in both cases

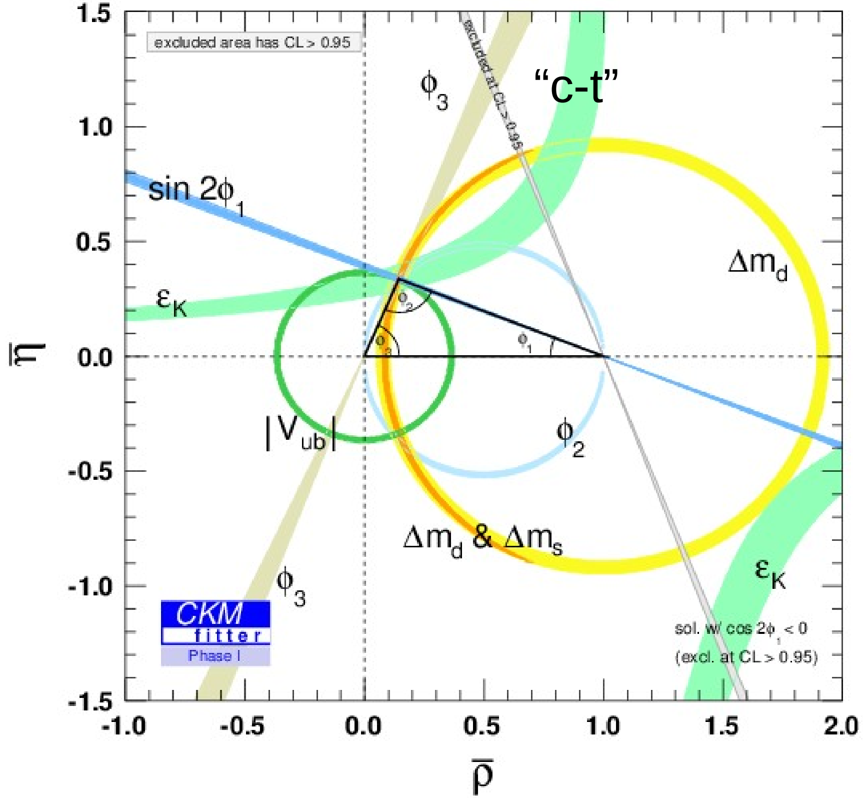
Spare



B_s→μμ 0.9
φ_s 0.9
γ 1.1
α 1.2
sin 2β 0.9
cos 2β 0.8
ε_K 0.0
Δm_s 1.3
Δm_d 1.7
B(B→τν) 1.0
|V_ub|_semilep 0.4
|V_cb|_semilep 0.3
B(D→μν) 1.3
B(D→τν) 0.4
B(D_s→μν) 0.9
B(D_s→τ ν) 1.2
B(D→Klν) 1.5
B(D→ πlν) 0.1
|V_cs|_not lattice 0.0
|V_cd|_not lattice 0.4
B(τ_K2) 2.1
B(K_μ2) 0.2
B(K_e2) 1.8
B(K_e3) 1.5
|V_ud| 0.1
0
0.5
1
1.5
2
2.5
3
Pull (σ)

Future bounds (LHCb, Belle II, ...)

LHCb at 23 fb-1, CMS/ATLAS at 300 fb-1, and Belle II at 50 ab-1.



LHCb at 300 fb-1, CMS/ATLAS at 3000 fb-1, and Belle II at 50 ab-1.