# Model-independent upper limits on lepton number violating states from neutrino mass 

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#### Abstract

We propose a model-independent framework to classify and study neutrino mass models and their phenomenology. The idea is to introduce one particle beyond the Standard Model which couples to leptons and carries lepton number together with the lowest-dimensional operator which violates lepton number by two units and contains this particle. The resulting contribution to neutrino masses can be translated to a robust upper bound on the mass of the new particle. We compare it to the stronger but less robust upper bounds from Higgs naturalness and discuss several lower bounds.


## INTRODUCTION

Neutrino oscillation experiments established the need for massive neutrinos and large mixings in the lepton sector. At the same time tritium beta decay experiments, cosmology and experiments searching for neutrinoless double beta decay put strong constraints on the absolute scale of neutrino mass. Despite tremendous progress in neutrino physics in recent years, the origin of neutrino mass remains a mystery. An elegant explanation of small neutrino masses is by linking their smallness to the breaking of lepton number $(\mathcal{L})$, the number of leptons minus antileptons, at a high scale $\Lambda$.

This lead to a plethora of explicit models such as the tree-level seesaw models $1-11$ and models at loop level (see Refs. 12-19 for the first one- and two-loop models and recent reviews). There are also several systematic studies of neutrino mass generation [20-31], in particular studies of Majorana neutrino mass generation in terms of effective operators that break lepton number by two units $(\Delta \mathcal{L}=2) \quad[23-30]$, which provide an efficient way to efficiently study neutrino mass generation, but do not allow to study other phenomenology such as lepton flavor violation or collider constraints.

Here we propose a hybrid approach in order to use the best of both schemes and to allow for a simpler classification. It is based on the following premises: (i) In any model of Majorana neutrino masses there is at least one new particle of mass $M$ which directly couples to leptons and carries lepton number (and in some cases also baryon number $\mathcal{B}$ ). We assume that this is the lightest beyond the Standard Model (SM) particle involved in the generation of neutrino masses. (ii) Following the common lore in quantum field theory that everything not forbidden is mandatory, lepton number is violated by two units ( $\Delta \mathcal{L}=2$ ) via operators which contain the new particle. (iii) Neutrino masses are generated from the $\Delta \mathcal{L}=2$ interactions of the new particle. We assume that this contribution dominates and generates the scale of neutrino mass, $m_{\nu} \gtrsim \sqrt{\Delta m_{\mathrm{atm}}^{2}} \simeq 0.05 \mathrm{eV}$. The latter can be estimated [25] and recast into a conservative upper bound
on $M$ Whereas neutrino mass is based on a $\Delta \mathcal{L}=2$ operator, other low-energy phenomenology is mainly determined by the renormalizable $\Delta \mathcal{L}=0$ interaction.

## UPPER BOUNDS

For Majorana neutrinos the dominant contribution to neutrino masses generally originates from the unique dimension 5 operator $\mathcal{O}_{1} \equiv L H L H$, the so-called Weinberg operator [23], where $L(H)$ is the SM lepton (Higgs) doublet. After electroweak symmetry breaking it leads to $m_{\nu} \simeq c_{1} v^{2} / \Lambda$, with $\langle H\rangle=(0, v)^{T}, v \simeq 174 \mathrm{GeV}$ and $c_{1} / \Lambda$ the Wilson coefficient of $\mathcal{O}_{1}$. The smallness of neutrino mass is generally linked to the hierarchy $v \ll \Lambda$, known as the seesaw mechanism [1-11. For $c_{1} \sim \mathcal{O}(1)$ the scale $\Lambda$ has to be sufficiently small, $\Lambda \lesssim 6 \times 10^{14}$ GeV , so that $m_{\nu} \gtrsim 0.05 \mathrm{eV}$. Some models may feature an additional suppression of the parameter $c$. It may be due an almost conserved $\mathcal{L}$ like in type-II seesaw model $\left(\epsilon=\mu / m_{\Delta}\right)$ [6-10, 13], inverse seesaw scenarios $\left(\epsilon=\mu / m_{R}\right)$ 32, 33, or in the (Generalized) Scotogenic model $\left(\epsilon=\lambda_{5}\right)$ 34 $\sqrt[36]{ }$. In all these cases lepton number is restored in the limit $\epsilon \rightarrow 0$. Similarly, in models where the Weinberg operator is absent but $\mathcal{O}_{1}^{\prime n} \equiv \operatorname{LHLH}\left(H^{\dagger} H\right)^{n}$ is generated, neutrino masses are suppressed by $\left(v^{2} / \Lambda^{2}\right)^{n}$ 37]. Finally neutrinos may be massless at tree level and only be generated at loop level. Hence it is hetter to narameterize neutrino mase hy

$$
\begin{equation*}
m_{\nu} \simeq \frac{c_{\mathrm{R}} v^{2}}{\left(16 \pi^{2}\right)^{\ell} \Lambda}, \quad \text { with } \quad c_{\mathrm{R}} \simeq \prod_{i} g_{i} \times \epsilon \times\left(\frac{v^{2}}{\Lambda^{2}}\right)^{n} \tag{1}
\end{equation*}
$$

where $i$ runs over the couplings $g_{i}$ and $\ell$ is the loop order at which neutrino mass is generated. The couplings $g_{i}$ are subject to perturbativity constraints, which naively demands them to be at most order one. For low-scale

[^0]models rare processes typically constrain the couplings to be even smaller, naively $g_{i} \lesssim \mathcal{O}(0.1)$. The number of couplings increases with the loop order. A conservative estimate yields that there are at least $2 \ell$ couplings in an $\ell$ loop diagram and thus $\Lambda \lesssim 4 \times 10^{10}\left(2 \times 10^{6}\right)$ [200] GeV for neutrino masses generated at one (two) [three] loop order. Neutrino mass generation at higher loop order is thus theoretically disfavored. These simple estimates however do not allow to distinguish further between different models and thus it is desirable to go beyond.

As outlined in the introduction the generation of Majorana neutrino mass requires the introduction of at least one new particle which couples to leptons and the existence of a $\Delta \mathcal{L}=2$ operator. This allows to obtain a conservative upper limit for the mass of the lightest new particle by demanding that the atmospheric neutrino mass scale eV is generated. In the estimate we use third generation SM Yukawa couplings and order one values for the new unknown couplings. In the case of a model with several new particles, our analysis applies to the lightest particle of the model which typically generates the largest onntribution to neutrino mass.

In Tab. [I] ve list all possible particles with lepton number (first column) which couple to leptons at the renormalizable level. We restrict ourselves to masses above the electroweak scale for the consistency of the effective field theory (EFT) approach. The first four particles induce neutrino mass at tree level via the well-known see-saw mechanisms (type-I [1-5], type-II [6-10, 13], type-III [11]) and a new one generated via the mixing $m \bar{L}_{1} L$ of a new vector-like lepton doublet $L_{1}$ with the SM one. Notice that there is no symmetry that allows the new Weinberglike operator $L_{1} H L H$ and forbids the usual one. However, this contribution may dominate for $m / M \lesssim 1$, which induces large mixing with the SM leptons and is therefore constrained by measurements in the charged lepton sector. The remaining particles generate neutrino masses radiatively.

The second column displays the renormalizable coupling of the new particle and defines the lepton number of the new particle. The third column shows the lowestdimensional and simplest $\Delta \mathcal{L}=2$ operator. In case there are multiple operators we choose the one which yields the most conservative constraint (see below). In some sense, our approach is technically equivalent to studying the simplest models for each type of particle and deriving their upper bound. The fourth column (named BL) lists the odd-dimensional [29, 38] $\Delta \mathcal{L}=2$ operator which is generated after integrating out the new particle, as provided in Refs. [24, 25]. The loop order $\ell$ at which neutrino masses are generated is given in the fifth column. The sixth column provides an estimate for neutrino mass by closing off loops of SM particles following Ref. [25]: Each loop contributes $(4 \pi)^{-2}$, chirality-flips are proportional to the SM Yukawa coupling, and $W$-bosons contribute $g^{2} / 2$. The Weinberg operator is induced via matching at
loop-level, with neutrino masses generated in the form of Eq. (1). As we are interested in conservative upper limits, we neglect any additional suppression and set $\epsilon=1$. Generating the neutrino mass scale translates into an upper bound on $\Lambda$ and consequently on $M$, as the EFT requires $M \leq \Lambda$. This bound is conservative and shown in the last column. We note that the upper limits derived are applicable to all models involving a particular particle, as long it is the lightest one, which is phenomenologically the most interesting possibility. In the cases where several $\mathrm{SU}(2)$ contractions in the $\Delta \mathcal{L}=2 \mathrm{SM}$ operators are possible we select the ones that yield the most conservative upper limit.

The upper limits on the mass in Tab. $\mathbb{1}$ are robust, model-independent and conservative within our assumptions, but not necessarily the strongest possible bounds for a particular model, because there may be extra suppressions as discussed above. The bounds span several orders of magnitude, in the range $\left[10^{6}, 10^{15}\right] \mathrm{GeV}$. Limits for dominant couplings to the first two families are obtained by a simple rescaling. Relaxing the perturbativity conditions on the couplings pushes all bounds up. Clearly, the most promising particle to search for is a doubly-charged scalar due to its low upper limit, and also its large electric charge. The Zee-Babu model [15, 16, 39] 44 is the simplest model which contains it.

Higgs naturalness. Generally the new particles also contribute to the Higgs mass $m_{H}$. Thus the requirement of a low fine-tuning of the Higgs mass translates into an upper bound for the mass of the new particle as a function of its couplings. We define the theory at the scale $\Lambda$ and estimate the leading log-enhanced contribution for each case.

Scalar particles with electroweak charges and mass $M$ contribute to the Higgs mass via their Higgs portal coupling $\lambda$ at one-loop order, $\delta m_{H}^{2} \simeq$ $-\lambda N_{w} N_{c} M^{2} \ln \left(M^{2} / \Lambda^{2}\right) /\left(16 \pi^{2}\right)$, where $N_{c}\left[N_{w}\right]$ denotes the dimension of the $\mathrm{SU}(3)$ [ $\mathrm{SU}(2)]$ representation. Even if absent at tree level, $\lambda$ is generated at one-loop order by gauge boson loops, $\delta \lambda \simeq 3\left(Y^{2} g^{4}+\right.$ $\left.C_{2} g^{4}\right) \ln \left(M^{2} / \Lambda^{2}\right) /\left(32 \pi^{2}\right)$, with the $\mathrm{SU}(2)$ Casimir invariant $C_{2}$ and hypercharge $Y$. Thus naturalness poses a limit on the scalar mass

$$
\begin{equation*}
M\left|\ln \frac{M}{\Lambda}\right| \lesssim \frac{16 \pi^{2}\left|\delta m_{H}^{2}\right|_{\max }^{1 / 2}}{\sqrt{6 N_{c}\left(3 D g^{4}+N_{w} Y^{2} g^{\prime 4}\right)}} \tag{2}
\end{equation*}
$$

where $D$ is the $\mathrm{SU}(2)$ Dynkin index and $\left|\delta m_{H}^{2}\right|_{\text {max }}^{1 / 2}$ is the maximum correction to the Higgs mass that is considered natural. In the type-II seesaw model, the trilinear coupling $\mu$ also contributes to the Higgs mass, $\delta m_{H}^{2} \simeq 12 \mu^{2} \ln \left(M^{2} / \Lambda^{2}\right) /\left(16 \pi^{2}\right)$ [45, 46], which translates into an upper bound $\mu|\ln (M / \Lambda)|^{1 / 2} \lesssim \sqrt{2 / 3} \pi\left|\delta m_{H}^{2}\right|_{\max }^{1 / 2}$. A similar bound can be obtained in the Zee model [47.

New fermions with mass $M$ and Yukawa coupling $y$ contribute to the Higgs mass at one-loop order,

| Particle | $\Delta \mathcal{L}=0$ | $\|\Delta \mathcal{L}\|=2$ | BL | $\ell$ | $m_{\nu}$ | Upper bound |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{N} \sim(1,1,0)_{F}^{-1,0}$ | $y \bar{N} H L$ | $M \bar{N} \bar{N}$ | $\mathcal{O}_{1}$ | 0 | $\frac{y^{2} v^{2}}{M}$ | $M \lesssim 10^{15} \mathrm{GeV}$ |
| $\Delta \sim(1,3,1)_{S}^{-2,0}$ | $y L \Delta L$ | $\mu H \Delta^{\dagger} H$ | $\mathcal{O}_{1}$ | 0 | $v^{2}$ | $M \lesssim<10^{15} \mathrm{GeV}$ |
| $\bar{\Sigma}_{0} \sim(1,3,0)_{F}^{-1,0}$ | $y \bar{\Sigma}_{0} L H$ | $M \bar{\Sigma}_{0} \bar{\Sigma}_{0}$ | $\mathcal{O}_{1}$ | 0 | $\frac{y^{2} v^{2}}{M}$ | $M \lesssim 10^{15} \mathrm{GeV}$ |
| $L_{1} \sim(1,2,-1 / 2)_{F}^{1,0}$ | $m \bar{L}_{1} L$ | $\frac{c}{\Lambda} L_{1} H L H$ | $\mathcal{O}_{1}$ | 0 | $\frac{c m}{M} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{15} \mathrm{GeV}$ |
|  | $y H^{\dagger} \bar{e} L_{1}$ | $\frac{c}{\Lambda^{2}} \bar{L}_{1} \bar{u} \bar{d}^{\dagger} L^{\dagger}$ | $\mathcal{O}_{8}^{\dagger}$ | 2 | $\frac{c y y_{u} y_{d} y_{l}}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{7} \mathrm{GeV}$ |
| $h \sim(1,1,1)_{S}^{-2,0}$ | $y$ LLh | $\frac{c}{\Lambda} h^{\dagger} \bar{e} L H$ | $\mathcal{O}_{2}$ | 1 | $\frac{c y y_{l}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{10} \mathrm{GeV}$ |
| $k \sim(1,1,2)_{S}^{-2,0}$ | $y \bar{e}^{\dagger} \bar{e}^{\dagger} k$ | $\frac{c}{\Lambda^{3}} k^{\dagger} L L L^{\dagger} L^{\dagger}$ | $\mathcal{O}_{9}^{\dagger}$ | 2 | $\frac{c y y_{1}^{2}}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{6} \mathrm{GeV}$ |
| $\bar{E} \sim(1,1,1)_{F}^{-1,0}$ | $y \bar{E} L H^{\dagger}$ | $\frac{c}{\Lambda^{4}} L E H Q^{\dagger} \bar{u}^{\dagger} H$ | $\mathcal{O}_{6}$ | 2 | $\frac{c y y u}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{10} \mathrm{GeV}$ |
| $\bar{\Sigma}_{1} \sim(1,3,1)_{F}^{-1,0}$ | $y H^{\dagger} \bar{\Sigma}_{1} L$ | $\frac{c}{\Lambda^{2}} L H H \Sigma_{1} H$ | $\mathcal{O}_{1}^{\prime 1}$ | 2 | $\frac{c y}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{10} \mathrm{GeV}$ |
| $L_{2} \sim(1,2,-3 / 2)_{F}^{1,0}$ | $y H \bar{e} L_{2}$ | $\frac{c}{\Lambda^{2}} \bar{L}_{2} L L L$ | $\mathcal{O}_{2}$ | 1 | $\frac{c y y_{l}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{11} \mathrm{GeV}$ |
| $X_{2} \sim(1,2,3 / 2)_{V}^{-2,0}$ | $y \bar{e}^{\dagger} \bar{\sigma}^{\mu} L X_{2 \mu}$ | $\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} \bar{d} X_{2 \mu}^{\dagger} H$ | $\mathcal{O}_{8}$ | 2 | $\frac{c y y_{u} y_{d} y_{e}}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{7} \mathrm{GeV}$ |
| $\tilde{R}_{2} \sim(3,2,1 / 6)_{S}^{-1,1}$ | $y \bar{d} L \tilde{R}_{2}$ | ${ }_{\Lambda} \tilde{R}_{2}$ QLH | $\mathcal{O}_{3}$ | 1 |  | $M \lesssim<10^{9} \mathrm{GeV}$ |
| $R_{2} \sim(3,2,7 / 6)_{S}^{-1,1}$ | $y \bar{e}^{\dagger} Q^{\dagger} R_{2}$ | $\frac{c}{\Lambda^{3}} R_{2}^{\dagger} L^{\dagger} L L \bar{d}^{\dagger}$ | $\mathcal{O}_{10}^{\dagger}$ | 2 | $\frac{c y y_{d} y_{l}}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{7} \mathrm{GeV}$ |
|  | $y \bar{u} L R_{2}$ | $\frac{c}{\Lambda^{3}} R_{2}^{\dagger} L^{\dagger} L L \bar{d}^{\dagger}$ | $\mathcal{O}_{15}^{\dagger}$ | 3 | $\frac{c y y_{d} y_{u} g^{2}}{2(4 \pi)^{6}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{6} \mathrm{GeV}$ |
| $S_{1} \sim(\overline{3}, 1,1 / 3)_{S}^{-1,-1}$ | $y L$ | ${ }_{\Lambda}^{c} S_{1}^{\dagger} L H \bar{d}$ | $\mathcal{O}_{3_{b}}$ | 1 | $\frac{c y y_{d}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{11} \mathrm{GeV}$ |
|  | $y \bar{u}^{\dagger} \bar{e}^{\dagger} S_{1}$ | $\frac{c}{\Lambda} S_{1}^{\dagger} L H \bar{d}$ | $\mathcal{O}_{8}$ | 2 | $\frac{c y y_{l} y_{u} y_{d}}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{7} \mathrm{GeV}$ |
| $S_{3} \sim(\overline{3}, 3,1 / 3)_{S}^{-1,-1}$ |  | $\frac{c}{\Lambda} \bar{d} L S_{3}^{\dagger} H$ | $\mathcal{O}_{3}$ | 1 | $\frac{c y y_{d}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{11} \mathrm{GeV}$ |
| $\tilde{S}_{1} \sim(\overline{3}, 1,4 / 3)_{S}^{-1,1}$ | $y \bar{e}^{\dagger} \bar{d}^{\dagger} \tilde{S}_{1}$ | $\frac{c}{\Lambda^{3}} \tilde{S}_{1}^{\dagger} L^{\dagger} L^{\dagger} L^{\dagger} Q^{\dagger}$ | $\mathcal{O}_{10}^{\dagger}$ | 2 | $\frac{c y y_{d} y_{l}}{(4 \pi)^{4}} \frac{v^{2}}{\Lambda}$ | $M \lesssim<10^{7} \mathrm{GeV}$ |
| $V_{2} \sim(\overline{3}, 2,5 / 6)_{V}^{-1,1}$ | $y \bar{d}^{\dagger} \bar{\sigma}^{\mu} V_{2 \mu} L$ | $\frac{c}{\Lambda^{5}} Q^{\dagger} \bar{\sigma}^{\mu} L V_{2 \mu}^{\dagger} H \bar{e} L H$ | $\mathcal{O}_{23}$ | 3 | $\begin{aligned} & \frac{c y y_{d} y_{l}}{(4 \pi)_{l}} \frac{v^{2}}{\Lambda} \\ & c y g^{2} \end{aligned}$ | $M \lesssim 10^{4} \mathrm{GeV}$ |
|  |  |  | ${ }_{44_{a, b, d}}$ | 3 |  |  |
| $\tilde{V}_{2} \sim(\overline{3}, 2,-1 / 6)_{V}^{-1,1}$ | $y \bar{u}^{\dagger} \bar{\sigma}^{\mu} \hat{V}_{2 \mu} L$ | $\frac{c}{\Lambda} Q^{\dagger} \bar{\sigma}^{\mu} L H \hat{V}_{2 \mu}^{\dagger}$ | $\mathcal{O}_{4}$ | 1 | $\frac{c y y_{u}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{12} \mathrm{GeV}$ |
| $U_{1} \sim(3,1,2 / 3)_{V}^{-1,1}$ | $y Q^{\dagger} \bar{\sigma}^{\mu} U_{1 \mu} L$ | $\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L H U_{1 \mu}^{\dagger}$ | $\mathcal{O}_{4 a}$ | 1 | $\frac{c y y_{u}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{12} \mathrm{GeV}$ |
|  | $y \bar{d} \sigma^{\mu} U_{1 \mu} \bar{e}^{\dagger}$ | $\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L H U_{1 \mu}^{\dagger}$ | $\mathcal{O}_{8}$ | 2 | $\frac{c y y_{u} y_{d} y_{l}}{(4 \pi)^{4}}, \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{7} \mathrm{GeV}$ |
| $U_{3} \sim(3,3,2 / 3)_{V}^{-1,1}$ | $y Q^{\dagger} \bar{\sigma}^{\mu} U_{3 \mu} L$ | $\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L U_{3 \mu}^{\dagger} H$ | $\mathcal{O}_{4}{ }_{\text {a }}$ | 1 | $\frac{c y y_{u}}{(4 \pi)^{2}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{12} \mathrm{GeV}$ |
| $\tilde{U}_{1} \sim(3,1,5 / 3)_{V}^{-1,1}$ | $y \bar{u} \sigma^{\mu} \bar{e}^{\dagger} \tilde{U}_{1 \mu}$ | $\frac{c}{\Lambda^{5}} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L H \tilde{U}_{1 \mu}^{\dagger} \bar{e} L H$ | $\mathcal{O}_{46}$ | 3 | $\frac{c y g^{2}}{2(4 \pi)^{6}} \frac{v^{2}}{\Lambda}$ | $M \lesssim 10^{7} \mathrm{GeV}$ |

Doubly-charged

TABLE I. Particles with quantum numbers $\left(\mathrm{SU}(3)_{\mathrm{c}}, \mathrm{SU}(2)_{\mathrm{L}}, \mathrm{U}(1)_{\mathrm{Y}}\right)_{N}^{\mathcal{L}, 3 \mathcal{B}}$ that couple to SM leptons at the renormalizable level, where $N=F, S, V$ denotes whether it is a fermion, scalar or vector. Fermions are 2-component Weyl fermions. The corresponding Dirac partner is denoted by a bar on top of the same symbol. The interaction with leptons is shown in the second column. We do not show the $\mathrm{SU}(2)$ contractions. The third column shows the lowest-dimensional $\Delta \mathcal{L}=2$ operator. After integrating out the particle, the operator in the fourth column is generated. The fifth column provides the lowest loop order at which neutrino mass is generated and the sixth column shows an estimate for it following Ref. [25. From perturbativity considerations, $c, y \lesssim \mathcal{O}(1)$, and using couplings to the third family, this translates into an upper bound on $M$ which is shown in the last column. $W$-bosons in the loop lead to a further suppression by $g^{2} / 2 \simeq 0.2$.
$\delta m_{H}^{2} \simeq 4 N_{c} C|y|^{2} M^{2} \ln \left(M^{2} / \Lambda^{2}\right) /\left(16 \pi^{2}\right)$, with $C=2$ for the electroweak triplets $\bar{\Sigma}_{i}$ and $C=1$ for the electroweak doublet and singlet fermions in Tab. I] Particles with electroweak charges also contribute at twoloop order to the Higgs mass, $\delta m_{H}^{2} \simeq 8 \kappa N_{c}\left(3 D g^{4}+\right.$ $\left.N_{w} Y^{2} g^{\prime 4}\right) M^{2} \ln \left(M^{2} / \Lambda^{2}\right) /\left(16 \pi^{2}\right)^{2}$. For Dirac (Majorana) fermions $\kappa=1(1 / 2)$. Thus naturalness demands the
fermion masses to obey

$$
\begin{align*}
& M\left|\ln \frac{M}{\Lambda}\right|^{1 / 2} \lesssim \frac{2 \pi\left|\delta m_{H}^{2}\right|_{\max }^{1 / 2}}{|y| \sqrt{2 N_{c} C}}  \tag{3}\\
& M\left|\ln \frac{M}{\Lambda}\right|^{1 / 2} \lesssim \frac{4 \pi^{2}\left|\delta m_{H}^{2}\right|_{\max }^{1 / 2}}{\sqrt{\kappa N_{c}\left(3 D g^{4}+N_{w} Y^{2} g^{\prime 4}\right)}} . \tag{4}
\end{align*}
$$

Vector bosons. For models with vector bosons Higgs naturalness is very model-dependent, because there are additional contributions depending on how their mass is generated.

In Fig. 1 we show the model-independent upper limits from neutrino mass as blue bars and indicate the upper


FIG. 1. Summary plot of the upper limits. The blue bars illusurate the robust, model-independent and conservative upper limits from neutrino masses. The horizontal black lines indicate the upper limits from Higgs naturalness. Hatching indicates parameter space excluded by non-observation of $\mathcal{B}$ violating nucleon decays.
limits from Higgs naturalness by horizontal black lines. The renormalization scale is set to the maximally-allowed value of $\Lambda$ from neutrino masses. The electroweak twoloop contribution generally dominates if present. For $\bar{N}$ there is only the one-loop contribution. In this case we use the neutrino mass scale to fix the Yukawa coupling $y$. The Higgs naturalness limits for the three seesaw models are consistent with previous results [48/50] taking the different choice of renormalization scale into account.

## LOWER BOUNDS

Charged lepton flavor/universality violation. In the type-I seesaw model charged lepton flavor violation is suppressed due to unitarity. The doubly-charged scalars $\Delta^{++}$and $k$ induce tri-lepton decays at tree level and pose a stringent constraint on the involved Yukawa couplings $y / M \lesssim\left(g / m_{W}\right)\left[B R_{\lim }\left(l \rightarrow l_{1} l_{2} \bar{l}_{3}\right) / B R(l \rightarrow\right.$ $\left.\left.l^{\prime} \nu \bar{\nu}^{\prime}\right)\right]^{1 / 4}$ in terms of the branching ratios $B R$ and the current limit $B R_{\text {lim }}$. First-generation leptoquarks may induce $\mu-e$ conversion at tree level and thus $y / M \lesssim$ $\left(B R_{\text {lim }} \omega_{\text {capt }} /\left(4 C_{N}\right)\right)^{1 / 4}$ where $\omega_{\text {capt }}$ denotes the capture rate and $C_{N} \sim(0.01-0.1) m_{\mu}^{5}$ parameterizes the nuclear physics. The contribution to radiative leptonic muon and tau decays can be estimated by $B R\left(l \rightarrow l^{\prime} \gamma\right) / B R(l \rightarrow$ $\left.l^{\prime} \nu \bar{\nu}^{\prime}\right) \sim 3 \alpha_{e m} y^{4} /\left(16 \pi G_{F}^{2} M^{4}\right)$, but may be further enhanced if the fermion in the loop is heavier than the decaying lepton.

Mixing with SM leptons leads to a non-unitary PMNS matrix [51/56]. For the type-I and type-III seesaw models, it is generally small due to its relation to neutrino masses. However in extended models like the inverse seesaw model or in models with new fermions with
weak charges $\left(\bar{E}, L_{i}\right)$, there may be large deviations (See e.g. [56]).

Lepton number violation. In the early Universe sphaleron processes together with processes mediated by a $\Delta(\mathcal{B}-\mathcal{L})=2$ operator may erase the baryon asymmetry of the Universe [57. This imposes lower bounds on the scale $\Lambda$ due to interactions mediated by either (i) the BL operators (fourth column) or (ii) the $\Delta \mathcal{L}=2$ operator (third column), if the new particle is relativistic and the $\Delta \mathcal{L}=0$ interaction (second column) rate is faster than the Hubble rate. In particular, in order for a $\mathcal{B}-\mathcal{L}$ asymmetry generated at $T_{\mathcal{B}-\mathcal{L}}$ not to be washed-out, for order one couplings the requirement reads $\Lambda \gtrsim\left[M_{p} T_{\mathcal{B}-\mathcal{L}}^{2 d-9} /\left(20 \mathrm{PS}_{n}\right)\right]^{1 /(2 d-8)}$, where $M_{p}$ is the Planck scale, $d$ is the dimension of the operator (third and/or fourth column) and $\mathrm{PS}_{n}$ denotes the $n$-particle phase space factor. For example, for two massless final state particles and $T_{\mathcal{B}-\mathcal{L}}=10^{6}, 10^{10}, 10^{13} \mathrm{GeV}$, this reads $\Lambda \gtrsim 10^{11}, 10^{13}, 10^{14} \mathrm{GeV}$ for the Weinberg operator and roughly $\Lambda \gtrsim 10^{7}, 10^{10}, 10^{13} \mathrm{GeV}$ for other operators of dimension $d \leq 11$. Notice that a lower limit on $M$ can be derived for order one couplings from the upper limit on $\Lambda$ from neutrino masses, if the exact combination of powers of $\Lambda$ and $M$ is known.

In addition, the new particles may generate new sizable contributions to neutrinoless double beta decay and other low-scale probes of the violation of lepton number, e.g. via mixing (type-I and III seesaws) or due to new couplings like in the type-II seesaw.

Baryon number violation. There are stringent limits on $\mathcal{B}$-violating ( $\mathcal{B}-\mathcal{L}$ conserving) dimension- 6 operators with first generation quarks (See e.g. [58-63]) due to nucleon decays [23, 64, 65] such as $p \rightarrow e^{+} \pi^{0}$, $p \rightarrow \bar{\nu} \pi^{+}$and $n \rightarrow \bar{\nu} \pi^{0}$, whose lower limit on the lifetime is $\mathcal{O}\left(10^{33}\right)$ y [66, 67]. Unless $B$-conservation is imposed, the scalar leptoquarks $S_{1}, S_{3}$ and $\tilde{S}_{1}$ have diquark couplings like $S_{1} \bar{d} \bar{u}, S_{1,3} Q^{\dagger} Q^{\dagger}, \tilde{S}_{1} \bar{u} \bar{u}$ and thus induce nucleon decay. The vectors $V_{2}$ and $\tilde{V}_{2}$ also have diquark couplings $\bar{u} \bar{\sigma}^{\mu} V_{2 \mu} Q^{\dagger}$ and $\bar{d} \sigma^{\mu} \tilde{V}_{2 \mu} Q^{\dagger}$, respectively, which mediate nucleon decay together with the other couplings shown in the table. For $\tilde{S}_{1}$, the antisymmetry of the coupling implies that the decay proceeds into three leptons via W-exchange 61, suppressed by $V_{t d} y_{t}$ if coupled to the top quark. The lower limits on the mass are $M \gtrsim 10^{16}\left(10^{11}\right) \mathrm{GeV}$ for $\mathcal{O}(\underset{\tilde{S}}{1})$ couplings for one lepton (three leptons in the case of $\tilde{S}_{1}$ ) in the final state, which are in tension with the neutrino mass bounds.

There are also diquark couplings for $R_{2}, \tilde{R}_{2}$, generated by the $\mathcal{B}+\mathcal{L}$ conserving dimension- 5 operators, $\tilde{R}_{2} Q H^{\dagger} Q / \Lambda^{\prime}$ and $H^{\dagger} R_{2} \bar{d}^{\dagger} \bar{d}^{\dagger} / \Lambda^{\prime}$. Similarly, the vectors $U_{1}, U_{3}$ also generate operators like $\bar{d}^{\dagger} \sigma_{\mu} H^{\dagger} Q U_{1,3}^{\mu}$. Therefore $\tilde{R}_{2}, U_{1}, U_{3}$ induce nucleon decays such as $n \rightarrow$ $\pi^{+} e^{-}$[68, 69] with decay width $\Gamma\left(n \rightarrow \pi^{+} e^{-}\right) \simeq$ $y^{2} \Lambda_{\mathrm{QCD}}^{5} v^{2} /\left(8 \pi \Lambda^{\prime 2} M^{4}\right)$, where $\Lambda_{\mathrm{QCD}} \sim 1 \mathrm{GeV}$. Using $\Gamma\left(n \rightarrow \pi^{-} e^{+}\right)^{-1} \gtrsim 5.3 \times 10^{33} y$ [70] and $M \leq \Lambda^{\prime}$, we ob-
tain a lower limit $\Lambda^{\prime} \geq 10^{11} \mathrm{GeV}$ for order one couplings, in tension with the neutrino mass bound. Alternatively the scale $\Lambda^{\prime}$ can be taken to be the Planck scale $M_{p}$, which leads to the lower bound $M \gtrsim 10^{7} \mathrm{GeV}$ [59, 71]. In the case of $\tilde{R}_{2}$, the antisymmetry makes it decay predominantly via the channel $p \rightarrow K^{*} \nu$ [63], and the bounds are similar to those above.

Due to its large hypercharge $\tilde{U}_{1}$ will only mediate nucleon decays at larger dimension than 5 , involving multiple mesons and leptons, and thus currently it is not constrained.

Direct searches. There are no competitive constraints for sterile neutrinos $\bar{N}$ yet, but the constraints on the particle masses of the other particles ranges from $M \gtrsim 100 \mathrm{GeV}$ from measurements at LEP to $M>$ 1530 GeV 72 for leptoquarks coupling to muons and light quarks.

## CONCLUSIONS

We have derived general robust upper bounds on the mass of new particles contributing to neutrino masses. Our main results are summarized in Fig. 1. We have also compared our limits with those from Higgs naturalness, which are much stronger, but less robust. The lower bounds are generally model-dependent. Among these nucleon decays provide the most stringent limits. If $V_{2}, \tilde{V}_{2}$, or $U_{1}$ correspond to $\mathrm{SU}(5) / \mathrm{SO}(10)$ gauge bosons, they can not be the dominant source of neutrino masses. The most promising particles to search for are new doublycharged scalars which have masses below $\mathcal{O}\left(10^{6}\right) \mathrm{GeV}$, if they contribute to neutrino masses.

This work is intended to serve not only as an indication of the most promising particles to directly search for at colliders, but also as a simple way of organizing the plethora of neutrino mass models.

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[^0]:    ${ }^{1}$ Similarly the upper bound on neutrino masses can be translated in a lower limit on $\Lambda$ (but not on $M$ ) which is of similar size.

