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# Neutrino Mixing: $A_4$ Variations

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**Abstract**

In the context of the non-Abelian discrete symmetry $A_4$, the neutrino mass matrix has been studied extensively. A brief update is presented to focus on the conceptual shift from tribimaximal mixing ($\theta_{13} = 0$, $\theta_{23} = \pi/4$, $\tan^2\theta_{12} = 1/2$) to cobimaximal mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$) which agrees well with present data. Three specific realistic examples are proposed, two with three and the third with just two parameters.

The non-Abelian discrete symmetry $A_4$ is the symmetry of the tetrahedron. It has 12 elements and is the smallest group which admits an irreducible $\underline{3}$ representation. It also has three one-dimensional representations $\underline{1}, \underline{1}', \underline{1}''$. The basic multiplication rule is

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \tag{1}$$

Its application to neutrino mixing began with Ref. [1], where the representation matrices were chosen so that

[1] E. Ma and G. Rajasekaran, Phys. Rev. D64, 113012 (2001). "Softly Broken A_4 Symmetry for Nearly Degenerate Neutrino Masses", arXiv:hep-ph/0106291

$$a_1 b_1 + a_2 b_2 + a_3 b_3 \sim \underline{1}, \tag{2}$$

$$a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3 \sim \underline{1}', \tag{3}$$

$$a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3 \sim \underline{1}'', \tag{4}$$

$$(a_2 b_3 \pm a_3 b_2, a_3 b_1 \pm a_1 b_3, a_1 b_2 \pm a_2 b_1) \sim \underline{3}, \tag{5}$$

where $a_i, b_i \sim \underline{3}$ and $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. The three lepton families are assumed to transform as follows:

$$(\nu_i, l_i)_L \sim \underline{3}, \quad l^c_{iL} \sim \underline{1}, \underline{1}', \underline{1}'', \tag{6}$$

with three Higgs doublets $(\phi_i^+, \phi_i^0) \sim \underline{3}$. Hence the charged-lepton mass matrix is given by

$h_{ijk}\overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + h.c.$ After EWSB →

$$h_{i1k} = h_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad h_{i2k} = h_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix}, \quad h_{i3k} = h_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{bmatrix}$$

$$\mathcal{M}_l = \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega^2 v_2^* & f_\tau \omega v_2^* \\ f_e v_3^* & f_\mu \omega v_3^* & f_\tau \omega^2 v_3^* \end{pmatrix} = \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}. \tag{7}$$

For $v_1 = v_2 = v_3$, the $A_4$ symmetry breaks to its residual $Z_3$ and the unitary transformation linking $\mathcal{M}_l$ to $\mathcal{M}_\nu$ is [2, 3]

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{8}$$

In the $(e, \mu, \tau)$ basis, the neutrino mass matrix (assumed Majorana) is

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_\omega \mathcal{M}_A U_\omega^T. \tag{9}$$

For many years, the preferred meutrino mixing matrix is of the tribimaximal form [4], i.e.

TBM:
$$U_B = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (10)$$

It is related to $U_\omega$ through

$$U_A = U_\omega^\dagger U_B = \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \end{pmatrix}. \quad (11)$$

The neutrino mas matrix in the tribimaximal basis is then

$$\mathcal{M}_B = U_A^\dagger \mathcal{M}_A U_A^*. \quad (12)$$

Let [5, 6]

$$\mathcal{M}_B = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}, \quad (13)$$

then for [7]

$$\mathcal{M}_A = \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix}, \quad (14)$$

they are related by

$$m_1 = b + (d+f)/2, \quad m_2 = a, \quad m_3 = b - (d+f)/2, \quad (15)$$

$$m_4 = i(f-d)/2, \quad m_5 = i(e-c)/\sqrt{2}, \quad m_6 = (e+c)/\sqrt{2}. \quad (16)$$

To obtain tribimaximal mixing ($\theta_{13} = 0$, $\theta_{23} = \pi/4$, $\tan^2\theta_{12} = 1/2$), $c = e = 0$ and $f = d$ are required. The remaining three parameters $(a, b, d)$ are in general complex. To obtain cobimaximal mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$) which agrees well with present data [8] with $\delta_{CP} = -\pi/2$ [9], what is required [7] is that $\mathcal{M}_A$ be diagonalized by an orthogonal matrix. To see this, let

$$U_{l\nu} = U_\omega \mathcal{O}, \quad (17)$$



[7] Transformative A 4 mixing of neutrinos with C P violation,
E. Ma, Phys. Rev. D92, 051301(R) (2015)

where $\mathcal{O}$ is a real orthogonal matrix, then it is obvious that $U_{\mu i} = U^*_{\tau i}$ for $i = 1, 2, 3$. Comparing this with the Particle Data Group (PDG) convention of the neutrino mixing matrix, i.e.

$$U^{PDG}_{l\nu} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{18}$$

it is obvious that after rotating the phases of the third column and the second and third rows, the two matrices are identical if and only if $s_{23} = c_{23}$ and $\cos\delta = 0$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$. This important insight is a rediscovery of what was actually known already many years ago [10, 11]. It is guaranteed if $(a, b, c, d, e, f)$ are all real, so that $\mathcal{M}_A$ is both symmetric and Hermitian.

Another way to arrive at cobimaximal mixing is to use Eqs. (9) and (14), i.e.

$$\mathcal{M}^{(e,\mu,\tau)}_\nu = U_\omega \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U^T_\omega = \begin{pmatrix} A & C & E^* \\ C & D^* & B \\ E^* & B & F \end{pmatrix}, \tag{19}$$

where

$$A = (a + 2b + 2c + d + 2e + f)/3, \tag{20}$$

→ m_ee=|A|

$$B = (a - b - c + d - e + f)/3, \tag{21}$$

$$C = (a - b - \omega^2 c + \omega d - \omega e + \omega^2 f)/3, \tag{22}$$

$$D^* = (a + 2b + 2\omega c + \omega^2 d + 2\omega^2 e + \omega f)/3, \tag{23}$$

$$E^* = (a - b - \omega c + \omega^2 d - \omega^2 e + \omega f)/3, \tag{24}$$

$$F = (a + 2b + 2\omega^2 c + \omega d + 2\omega e + \omega^2 f)/3. \tag{25}$$

If again $(a, b, c, d, e, f)$ are real, then $A, B$ are real, whereas $E = C$ and $F = D$. This well-known special form was written down already many years ago [12, 13], and it was pointed out soon afterward [14] that it is protected by a generalized $CP$ transformation

[Grimus & Lavoura]



[12] E. Ma, Phys. Rev. D66, 117301 (2002).
[13] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. B552, 207 (2003)

[14] W. Grimus and L. Lavoura, Phys. Lett. B579, 113 (2004).

$$\mathcal{M}_B = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix},$$

$$\mathcal{M}_A = \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix},$$

under $\mu - \tau$ exchange, and it guarantees cobimaximal mixing. With the knowledge that $\theta_{13} \neq 0$ [15, 16, 17], this extended symmetry is now the subject of many studies, which began with generalized $S_4$ [18]. In fact, remnant residual symmetries may be used [19, 20, 21] to reconstruct the neutrino mixing matrix with cobimaximal mixing.

Since tribimaximal mixing is not what the data show, $\mathcal{M}_B$ cannot be diagonal. Many studies are then centered on looking for small off-diagonal terms, i.e. $m_{4,5,6}$ which may be complex. On the other hand, data are perfectly consistent with $\mathcal{M}_A$ as long as it is real. Of course, $\theta_{13}$ and $\theta_{12}$ are not predicted, but if extra conditions are imposed, they may be correlated. For example, it has been proposed [22] that $c = e = 0$, but $f \neq d$, with $a, b, d, f$ real. This yields cobimaximal mixing together with the prediction that

[22] X.-G. He, arXiv:1504.01560 [hep-ph].
A Model of Neutrino Mass Matrix
With δ=−π/2 and θ23=π/4

$$\tan^2\theta_{12} = \frac{1}{2 - 3\sin^2\theta_{13}} > \frac{1}{2}. \tag{26}$$

Using the 2014 Particle Data Group value [8]

$$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}, \tag{27}$$

the value of $\sin^2(2\theta_{12})$ from Eq. (26) is 0.90 with very little deviation, as compared with the PDG value

$$\sin^2(2\theta_{12}) = 0.846 \pm 0.021. \tag{28}$$

This is a generic result corresponding to choosing $m_5 = m_6 = 0$ in Eq. (13). If $m_4 = m_6 = 0$ is chosen instead, then another generic prediction is

$$\tan^2\theta_{12} = \frac{1}{2}(1 - 3\sin^2\theta_{13}). \tag{29}$$

Again using Eq. (27), $\sin^2(2\theta_{12}) = 0.866 \pm .002$ is obtained, which agrees well with Eq. (28). Note that both generic results hold for arbitrary values of $\delta_{CP}$. In Ref. [6], $e + c = 0$ is assumed so that $m_6 = 0$. In addition, $\delta_{CP} = 0$ and $\theta_{23} = \pi/4$ are assumed, which can be achieved if both $m_4$ and $m_5$ are nonzero. In the case $m_4 = m_6 = 0$, but $m_{1,2,3,5}$ complex,

$$\mathcal{M}_B = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix},$$

$$\mathcal{M}_A = \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix},$$



[21] Origin of Constrained Maximal CP Violation in Flavor Symmetry
Hong-Jian He, Werner Rodejohann, Xun-Jie Xu

[19] Neutrino Mixing from CP Symmetry
Peng Chen, Chang-Yuan Yao, Gui-Jun Ding (Hefei, CUST)
arXiv:1507.03419 [hep-ph]

[20] Generalized μ-τ symmetry and discrete subgroups of O(3)
Anjan S. Joshipura, Ketan M. Patel
arXiv:1507.01235 [hep-ph]

[6] E. Ma and D. Wegman, Phys. Rev. Lett. 107, 061803 (2011).
Nonzero θ13 for neutrino mixing in the context of A4 symmetry,
[arXiv:1106.4269]

[23] H. Ishimori and E. Ma, Phys. Rev. D86, 045030 (2012).
New Simple A4 Neutrino Model for Nonzero θ13 and Large δCP
arXiv:1205.0075 [hep-ph]

an analysis shows [23] that large $\delta_{CP}$ correlates with $\theta_{23} \neq \pi/4$ for a fixed nonzero $\theta_{13}$. Of course, $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$ are now favored by data, and cobimaximal mixing should be chosen as the preferred starting point of any improved model of neutrino mass and mixing.

Starting case:

Consider a real $\mathcal{M}_A$ with $d = f$ and $c = -e$, i.e.

$$\mathcal{M}_A = \begin{pmatrix} a & -e & e \\ -e & d & b \\ e & b & d \end{pmatrix}. \tag{30}$$

In that case,

$$\mathcal{M}_B = \begin{pmatrix} b+d & 0 & 0 \\ 0 & a & i\sqrt{2}e \\ 0 & i\sqrt{2}e & b-d \end{pmatrix}, \tag{31}$$

i.e. $m_4 = m_6 = 0$, hence the desirable condition of Eq. (29) is obtained. Let $\mathcal{M}_B$ be diagonalized by

$$U_E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & is \\ 0 & is & c \end{pmatrix}, \tag{32}$$

so that

$$\mathcal{M}_B = U_E \begin{pmatrix} m_1' & 0 & 0 \\ 0 & m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix} U_E^T, \tag{33}$$

where $s = \sin\theta_E$, $c = \cos\theta_E$. Then

$$\frac{sc}{c^2 - s^2} = \frac{e\sqrt{2}}{a+b-d}, \tag{34}$$

and the three neutrino mass eigenvalues are

$$m_1' = b + d, \tag{35}$$

$$m_2' = \frac{1}{c^2 - s^2}[c^2 a + s^2 (b-d)], \tag{36}$$

$$m_3' = \frac{1}{c^2 - s^2}[s^2 a + c^2 (b-d)]. \tag{37}$$

The neutrino mixing matrix is now $U_B U_E$, from which

$$s = \sqrt{3}\sin\theta_{13} \tag{38}$$

is obtained. As it is, $\mathcal{M}_A$ has four real parameters $(a, , b, d, e)$ to fit three observables $(\theta_{13}, \Delta m^2_{21}, \Delta m^2_{32})$, hence no prediction is possible other than cobimaximal mixing and Eq. (29). → $\tan^2\theta_{12} = \frac{1}{2}(1 - 3\sin^2\theta_{13})$. (Altarelli&Feruglio, E.Ma)

In the case of tribimaximal mixing, i.e. $e = 0$, the simplest $A_4$ model [24, 25] has $d = a$.

$$\mathcal{M}_B = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}, \quad \mathcal{M}_A = \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix}$$

Example: With this condition, but $e \neq 0$, the three neutrino masses are (d=a & d=f, c=-e )

$$\mathcal{M}_A = \begin{pmatrix} a & -e & e \\ -e & a & b \\ e & b & a \end{pmatrix}.$$

$$m'_1 = b + a, \quad m'_2 = a + \frac{s^2 b}{c^2 - s^2}, \quad m'_3 = -a + \frac{c^2 b}{c^2 - s^2}. \tag{39}$$

Using Eq. (27) with the central value $s = 0.2673$, they become

$$m'_1 = b + a, \quad m'_2 = a + 0.08336b, \quad m'_3 = -a + 1.08336b. \tag{40}$$

$$\mathcal{M}_B = \begin{pmatrix} b+a & 0 & 0 \\ 0 & a & i\sqrt{2}e \\ 0 & i\sqrt{2}e & b-a \end{pmatrix}$$

Using the central values of [8]

$$\Delta m^2_{21} = 7.53 \pm 0.18 \times 10^{-5}\ \text{eV}^2, \quad \Delta m^2_{32} = 2.44 \pm 0.06 \times 10^{-3}\ \text{eV}^2, \tag{41}$$

the solution is $b/a = -1.714$ and $a = 0.0183$ eV, with $e/a = -0.3642$. Using Eq. (20), the effective neutrino mass in neutrinoless double beta decay is predicted to be

$$m_{ee} = |A| = |a + 2b/3| = 2.6 \times 10^{-3}\ \text{eV}, \tag{42}$$

which is very small, as expected from a normal ordering of neutrino masses, and beyond the sensitivity of current and planned experiments.

(d=f, c=-e ) &

Example: Another possible three-parameter model is to assume $d = b$, then

$$\mathcal{M}_B = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}, \quad \mathcal{M}_A = \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix}$$

$$m'_1 = 2b, \quad m'_2 = 1.08336a, \quad m'_3 = 0.08336a. \tag{43}$$

This implies inverted ordering of neutrino masses with $a = 0.0465$ eV, $b = 0.0248$ eV, and $e = 0.0099$ eV. Hence $m_{ee} = |(a + 4b)/3| = 0.0486$ eV which is presumably verifiable in the future.

$$\mathcal{M}_A = \begin{pmatrix} a & -e & e \\ -e & b & b \\ e & b & b \end{pmatrix}.$$

$$\mathcal{M}_B = \begin{pmatrix} 2b & 0 & 0 \\ 0 & a & i\sqrt{2}e \\ 0 & i\sqrt{2}e & a-0 \end{pmatrix}$$

$$\mathcal{M}_B = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix},$$

$$\mathcal{M}_A = \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix},$$

↓

$$\mathcal{M}_A = \begin{pmatrix} a & -e & e \\ -e & 2a & -2a \\ e & -2a & 2a \end{pmatrix}.$$

$$\mathcal{M}_B = \begin{pmatrix} 4a & 0 & 0 \\ 0 & a & i\sqrt{2}e \\ 0 & i\sqrt{2}e & -4a \end{pmatrix}$$

Example: As a third example, consider the following new remarkable model of just two parameters, with $d = -b = 2a$: (& d=f, c=-e )

$$m_1' = 0, \quad m_2' = \left(\frac{c^2 - 4s^2}{c^2 - s^2}\right) a = 0.75a, \quad m_3' = \left(\frac{s^2 - 4c^2}{c^2 - s^2}\right) a = -4.25a. \tag{44}$$

As a result, $\Delta m^2_{21}/\Delta m^2_{32}$ is predicted to be 0.032, as compared with the experimental value of 0.031. In this case, $a = 0.0116$ eV and $m_{ee} = |a/3| = 3.9 \times 10^{-3}$ eV, with $e/a = -0.6375$.

In conclusion, it has been pointed out in this paper that $A_4$ is intimately related to cobimaximal mixing ($\theta_{13} \neq 0$, $\theta_{23} = \pi/4$, $\delta_{CP} = \pm\pi/2$) which agrees well with present data. In particular, a model is proposed with just two real parameters, with the following predictions:

$$\theta_{23} = \pi/4, \quad \delta_{CP} = \pm\pi/2, \quad \tan^2\theta_{12} = \frac{1}{2}(1 - 3\sin^2\theta_{13}), \tag{45}$$

$$\frac{\Delta m^2_{21}}{\Delta m^2_{31}} = \left(\frac{1 - 15\sin^2\theta_{13}}{4 - 15\sin^2\theta_{13}}\right)^2, \quad m_{ee} = 3.9 \times 10^{-3} \text{ eV (for } \sin^2 2\theta_{13} = 0.093), \tag{46}$$

which are all well satisfied by present data (except $m_{ee}$ which is yet to be measured).

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