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Renormalizable SU(5) Unification

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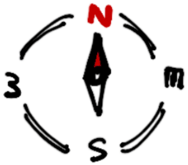
Motivation

Theorem for model builders:

Beauty - **S**implicity - **P**redictability

- Aesthetics: why three gauge groups?
- Simplicity: why three different strengths? why so many representations? are quarks and leptons that different?
- Predictability: why so many inputs? why arbitrary charges? what about the Yukawas? and the Weinberg angle?

Motivation



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Beauty - **S**implicity - **P**redictability

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- Predictability: why so many inputs? why arbitrary charges? what about the Yukawas? and the Weinberg angle?

SU(5) is the only rank 4 candidate able to embed SM!

The simplest GUT: SU(5)

Simplest GUT: SU(5)

- Matter content:

$$\bar{5} \sim \underbrace{(1, \bar{2}, -1/2)}_{l_L} \oplus \underbrace{(\bar{3}, 1, 1/3)}_{(d^c)_L} \qquad 10 \sim \underbrace{(\bar{3}, 1, -2/3)}_{(u^c)_L} \oplus \underbrace{(3, 2, 1/6)}_{q_L} \oplus \underbrace{(1, 1, 1)}_{(e^c)_L}$$

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu \end{pmatrix} \qquad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}$$

Simplest GUT: SU(5)

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$\Rightarrow$ The 15 SM Weyl d.o.f. perfectly fit
in the two simplest SU(5) representations!

Simplest GUT: SU(5)

- What about anomalies? $\mathcal{A}(\mathcal{R})d_{abc} = \mathrm{Tr}\left(\{T_a^R, T_b^R\}, T_c^T\}\right)$

For SU(N), $\mathcal{A}(\text{anti-fund}) + \mathcal{A}(\text{anti-symmetric}) = -\frac{1}{2} + \frac{N-4}{2}$

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for N=5, $\mathcal{A}(\bar{5}) + \mathcal{A}(10) = 0$

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- Hypercharge generator:

$$T_{24} = \frac{1}{\sqrt{15}} \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{2} \end{pmatrix} \propto Y$$

Fixing the normalisation:

$$Y = \sqrt{\frac{5}{3}} T_{24}$$

Simplest GUT: SU(5)

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All hypercharges predicted!

Simplest GUT: SU(5)

- And the gauge bosons?

$$V_{24} \sim \underbrace{(8,1,0)}_{G_\mu} \oplus \underbrace{(1,3,0)}_{W_\mu} \oplus \underbrace{(3,2,-5/6)}_{X_\mu} \oplus \underbrace{(\bar{3},2,5/6)}_{Y_\mu} \oplus \underbrace{(1,1,0)}_{\gamma_\mu}$$

Simplest GUT: SU(5)

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$$V_\mu = \sum_{a=1}^{24} V_\mu^a \frac{\tau^a}{2} =$$

$$= \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} G_{1\mu} + \frac{2B_\mu}{\sqrt{30}} & G^1_{2\mu} & G^1_{3\mu} & X^{C1}_\mu & Y^{C1}_\mu \\ G^2_{1\mu} & G^2_{2\mu} + \frac{2B_\mu}{\sqrt{30}} & G^2_{3\mu} & X^{C2}_\mu & Y^{C2}_\mu \\ G^3_{1\mu} & G^3_{2\mu} & G^3_{3\mu} + \frac{2B_\mu}{\sqrt{30}} & X^{C3}_\mu & Y^{C3}_\mu \\ \hline X^1_\mu & X^2_\mu & X^3_\mu & \frac{W^3_\mu}{\sqrt{2}} - \sqrt{\frac{3}{10}} B_\mu & W^+_\mu \\ Y^1_\mu & Y^2_\mu & Y^3_\mu & W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} - \sqrt{\frac{3}{10}} B_\mu \end{array} \right)$$

SU(5): scalar sector

- Breaking I: $SU(5) \stackrel{24_H}{\rightarrow} SU(3) \otimes SU(2) \otimes U(1)_Y$

$$24_H = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(\bar{3},2)} & \Sigma_3 \end{pmatrix} + \Sigma_S \, \lambda_{24},$$

$$\begin{cases} \langle 24_H \rangle = \frac{\text{v}_{24}}{\sqrt{15}} \text{diag}(2,2,2,-3,-3) : \quad \text{breaks to SM} \\ \\ \langle 24_H \rangle = \text{v}_{24} \, \text{diag}(1,1,1,1,-4) : \quad \text{breaks to } SU(4) \otimes U(1) \\ \\ \langle 24_H \rangle = \text{diag}(0,0,0,0,0) : \quad \text{no breaking} \end{cases}$$

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- Breaking II: $SU(3) \otimes SU(2) \otimes U(1)_Y \stackrel{5_H}{\to} SU(3) \otimes U(1)_Q$

$$5_H = \begin{pmatrix} T^1 \\ T^2 \\ T^3 \\ H_1^+ \\ H_1^0 \end{pmatrix} \stackrel{SSB}{\to} \langle 5_H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mathrm{v}_5/\sqrt{2} \end{pmatrix}$$

But it is too predictive...

Predictability may be its most beautiful feature

but it is also its Achilles' heel

Fermion masses

- Yukawa Lagrangian (i, j, k are SU(3) indices and α, β are SU(2) indices)

$$\mathcal{L}_Y \quad \supset \quad Y_1 \bar{5}\ 10\ 5_H^* \ + \ Y_3 10\ 10\ 5_H\ \epsilon_5$$

Fermion masses

- Yukawa Lagrangian (i, j, k are SU(3) indices and α, β are SU(2) indices)

$$
\begin{aligned}
\mathcal{L}_Y &\supset Y_1 \bar{5}\ 10\ 5_H^* \ +\ Y_3 10\ 10\ 5_H\ \epsilon_5 \\
&\supset Y_1(\bar{5}_i 10^{i\alpha} + \bar{5}_\beta 10^{\beta\alpha}) 5_{H\alpha}^* + Y_3(10^{ij} 10^{k\alpha} + 10^{i\alpha} 10^{jk}) 5_H^\beta \epsilon_{ijk\alpha\beta}
\end{aligned}
$$

- After SSB: $\langle 5_H \rangle = \mathrm{v}_5/\sqrt{2}$,

$$
\mathcal{L}_Y \supset Y_1 \frac{v^*}{\sqrt{2}} (d_i^C d^i + e e^C) + 4(Y_3 + Y_3^T) \frac{v}{\sqrt{2}} u_i^C u^i .
$$

- Fermion masses:

$$
\begin{aligned}
M_d &= M_e^T = Y_1 \frac{v_5^*}{\sqrt{2}} \\
M_u &= 4(Y_3 + Y_3^T) \frac{v_5}{\sqrt{2}}
\end{aligned}
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- Fermion masses:

$$\boxed{M_d = M_e^T = Y_1 \frac{v_5^*}{\sqrt{2}}}$$

$$M_u = 4(Y_3 + Y_3^T)\frac{v_5}{\sqrt{2}}$$

RULED OUT

Unification constraints

- RGEs:

$$\alpha_i^{-1}(M_Z) = \alpha_{GUT}^{-1} + \frac{B_i}{2\pi}\text{Log}\left(\frac{M_{GUT}}{M_Z}\right) \quad \begin{cases} B_i = b_i^{SM} + b_i^I r_I \\ r_I = \dfrac{\text{Log}(M_{GUT}/M_I)}{\text{Log}(M_{GUT}/M_Z)}, \; M_Z < M_I < M_{GUT} \end{cases}$$

- In Yang-Mills theories:

$$b_i = \left[\frac{1}{3}\sum_R S(R) T_i(R) \prod_{j\neq i} \text{dim}_j(R)\right], \quad S(R) \begin{cases} 1 \text{ for } R \text{ scalar}, \\ 2 \text{ for } R \text{ chiral fermion}, \\ -11 \text{ for } R \text{ gauge boson}. \end{cases}$$

- Table of $B_{ij} \equiv B_i - B_j$ contributions to the running:

	$\bar{5}$		10			V_{24}		5_H		24_H	
b_i/B_{ij}	l_L	$(d^c)_L$	$(u^c)_L$	q_L	$(e^c)_L$	G_μ	W_μ	H_1	T	Σ_8	Σ_3
b_1	$\frac{3}{5}$	$\frac{2}{5}$	$\frac{8}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	0	0	$\frac{1}{10}$	$\frac{1}{15} r_T$	0	0
b_2	1	0	0	3	0	0	$-\frac{22}{3}$	$\frac{1}{6}$	0	0	$\frac{1}{3} r_{\Sigma_3}$
b_3	0	1	1	2	0	-11	0	0	$\frac{1}{6} r_T$	$\frac{1}{2} r_{\Sigma_8}$	0
B_{12}	$-\frac{4}{5}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{44}{15}$	$-\frac{2}{5}$	0	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15} r_T$	0	$-\frac{1}{3} r_{\Sigma_3}$
B_{23}	1	-1	-1	1	0	11	$-\frac{22}{3}$	$\frac{1}{6}$	$-\frac{1}{6} r_T$	$-\frac{1}{2} r_{\Sigma_8}$	$\frac{1}{3} r_{\Sigma_3}$

Unification constraints

- Introducing $B_{ij} = B_i - B_j$,

$$\frac{B_{23}}{B_{12}} = \frac{5}{8}\,\frac{\sin^2\theta_W(M_Z) - \alpha(M_Z)/\alpha_3(M_Z)}{3/8 - \sin^2\theta_W(M_Z)}$$

$$\mathrm{Log}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{16\pi}{5\alpha(M_Z)}\,\frac{3/8 - \sin^2\theta_W(M_Z)}{B_{12}}$$

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- Input:

$$\alpha^{-1}(M_Z) = 127.94, \qquad \sin^2\theta_W(M_Z) = 0.231, \qquad \alpha_s(M_Z) = 0.1185$$

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- To achieve unification:

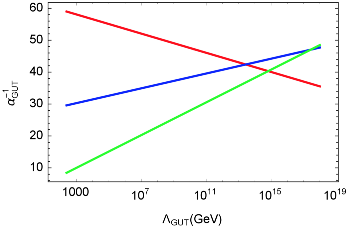
$$\frac{B_{23}}{B_{12}} = 0.718$$

$$\mathrm{Log}\left(\frac{M_{GUT}}{M_Z}\right) = \frac{184.87}{B_{12}}$$

Unification constraints

- SM contribution (assuming only the spitting of 5_H):

$$\frac{B_{23}^{SM}}{B_{12}^{SM}} = 0.53$$



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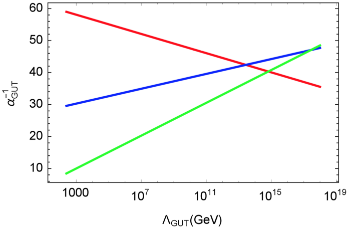

- SU(5) contribution (assuming splitting of 24_H too):

$$\frac{B_{23}^{SU(5)}}{B_{12}^{SU(5)}} = \frac{B_{23}^{SM} + \frac{1}{3}r_{\Sigma_3} - \frac{1}{6}r_T - \frac{1}{2}r_{\Sigma_8}}{B_{12}^{SM} - \frac{1}{3}r_{\Sigma_3} + \frac{1}{15}r_T} \xrightarrow{\text{most optimistic case}} \frac{B_{23}^{SU(5)}}{B_{12}^{SU(5)}} \lesssim 0.6$$

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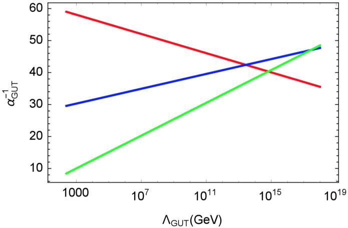

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$\Rightarrow$ Unification cannot be achieved at any scale...

RULED OUT

Too beautiful to be truth...

$$SU(5) \begin{cases} \text{Wrong fermion mass relation :} \qquad M_e(M_{GUT}) = M_d(M_{GUT}) \\ \\ \text{Unification cannot be achieved at any scale :} \qquad \dfrac{B_{23}}{B_{12}} \lesssim 0.6 \\ \\ \text{Massless neutrinos:} \qquad M_\nu = 0 \end{cases}$$

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But it is also too beautiful to give up on it!

Too beautiful to be truth...

What is the simplest realistic renormalizable model based on SU(5)?

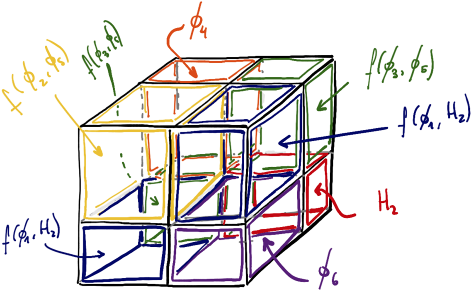
Too beautiful to be truth...

$$SU(5) \begin{cases} \text{Wrong fermion mass relation :} \quad \mathbf{45_H} \\ \text{Unification cannot be achieved at any scale :} \quad \mathbf{45_H} \\ \text{Massless neutrinos:} \quad \mathbf{10_H} \end{cases}$$

The 45_H representation

$$45_H \sim \underbrace{(8,2,1/2)}_{\Phi_1} \oplus \underbrace{(\bar{6},1,-1/3)}_{\Phi_2} \oplus \underbrace{(3,3,-1/3)}_{\Phi_3} \oplus \underbrace{(\bar{3},2,-7/6)}_{\Phi_4} \oplus$$

$$\oplus \underbrace{(3,1,-1/3)}_{\Phi_5} \oplus \underbrace{(\bar{3},1,4/3)}_{\Phi_6} \oplus \underbrace{(1,2,1/2)}_{H_2}$$



The 45_H representation

$(45_H)_i^{jk}$	$\sim f\{\Phi_2, \Phi_5\} = \epsilon^{jkl}\Phi_{2li} + \epsilon^{jkl}\epsilon_{lim}{\Phi_5}^m$	6+3-3= 6 d.o.f
	$\Phi_{2il} = \Phi_{2li}$ and $\Phi_{5il} = -\Phi_{5li}$	6 + 3
$(45_H)_i^{j\alpha}$	$\sim f\{\Phi_1, H_2\} = [\lambda^a]_i^j\, {\Phi_1}_a^\alpha + \delta_i^j H_2^\alpha$	16 + 2 = 18 d.o.f
$(45_H)_\alpha^{ij}$	$\sim \epsilon^{ijk}\Phi_{4\alpha k}$	6 d.o.f.
$(45_H)_\alpha^{i\beta}$	$\sim f\{\Phi_3 \equiv (\Delta_1, \Delta_2, \Delta_3), \Phi_5\} = \frac{1}{\sqrt{2}}{\Phi_3}^a[\sigma^a]_\alpha^\beta + \delta_\alpha^\beta {\Phi_5}^i$	12 d.o.f.
	$45_5^{i4} \sim ({\Delta_1}^i + i{\Delta_2}^i)/\sqrt{2} \equiv ({\phi_3}^{+\frac{2}{3}})^i$	3
	$45_4^{i5} \sim ({\Delta_1}^i - i{\Delta_2}^i)/\sqrt{2} \equiv ({\phi_3}^{-\frac{4}{3}})^i$	3
	$45_4^{i4} \sim {\Delta_3}^i/\sqrt{2} + {\Phi_5}^i \equiv ({\phi_3}^{-\frac{1}{3}})^i/\sqrt{2} + {\Phi_5}^i$	3
	$45_5^{i5} \sim -{\Delta_3}^i/\sqrt{2} + {\Phi_5}^i \equiv -({\phi_3}^{-\frac{1}{3}})^i/\sqrt{2} + {\Phi_5}^i$	3
$(45_H)_i^{\alpha\beta}$	$\sim \epsilon^{\alpha\beta}\Phi_{6i}$	3 d.o.f.

Tackling problem of fermion mass relations

$$\mathcal{L}_Y = \bar{5}\,10\,(Y_1\,5_H^* + Y_2\,45_H^*) + 10\,10\,(Y_3\,5_H + Y_4\,45_H)\epsilon_5 + Y_5\,\bar{5}\,\bar{5}\,10_H\epsilon_2 + \text{h.c.}$$

Tackling problem of fermion mass relations

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- Fermion masses as a function of Yukawas:

$$M_d = M_d(Y_1, Y_2),$$
$$M_u = M_u(Y_3, Y_4),$$
$$M_e = M_e(Y_1, Y_2).$$

- Where do the Higgs doublets live?

$$H_1^\alpha \sim {5_H}^\alpha,$$
$$H_2^\alpha \sim {45_H}_i^{j\alpha}\delta_j^i - \frac{1}{3}\epsilon_{\beta\gamma}\epsilon^{\delta\alpha}{45_H}_\delta^{\beta\gamma},$$

Tackling problem of fermion mass relations

$$\mathcal{L}_Y = \bar{5}\, 10\, (Y_1\, 5^*_H + Y_2\, 45^*_H) + 10\, 10\, (Y_3\, 5_H + Y_4\, 45_H)\epsilon_5 + Y_5\, \bar{5}\, \bar{5}\, 10_H \epsilon_2 + \text{h.c.}$$

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- Where do the Higgs doublets live?

$$H_1^\alpha \sim {5_H}^\alpha,$$
$$H_2^\alpha \sim 45_{H_i}^{j\alpha}\delta_j^i - \frac{1}{3}\epsilon_{\beta\gamma}\epsilon^{\delta\alpha}45_{H_\delta}^{\beta\gamma},$$

$$M_d = Y_1 \frac{v_5^*}{\sqrt{2}} + 2Y_2 \frac{v_{45}^*}{\sqrt{2}}$$
$$M_e = Y_1^T \frac{v_5^*}{\sqrt{2}} - 6Y_2^T \frac{v_{45}^*}{\sqrt{2}}$$
$$M_u = 4(Y_3 + Y_3^T)\frac{v_5}{\sqrt{2}} - 8(Y_4 - Y_4^T)\frac{v_{45}}{\sqrt{2}}$$

Tackling problem of unification

	$\bar{5}$		10			V_{24}		5_H		24_H	
b_i/B_{ij}	l_L	$(d^c)_L$	$(u^c)_L$	q_L	$(e^c)_L$	G_μ	W_μ	H_1	T	Σ_8	Σ_3
B_{12}	$-\frac{4}{5}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{44}{15}$	$-\frac{2}{5}$	0	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15} r_T$	0	$-\frac{1}{3} r_{\Sigma_3}$
B_{23}	1	-1	-1	1	0	11	$-\frac{22}{3}$	$\frac{1}{6}$	$-\frac{1}{6} r_T$	$-\frac{1}{2} r_{\Sigma_8}$	$\frac{1}{3} r_{\Sigma_3}$

	45_H							10_H		
	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2	δ^+	$\delta_{(3,2)}$	δ_T
	$-\frac{8}{15} r_{\Phi_1}$	$\frac{2}{15} r_{\Phi_2}$	$-\frac{9}{5} r_{\Phi_3}$	$\frac{17}{15} r_{\Phi_4}$	$\frac{1}{15} r_{\Phi_5}$	$\frac{16}{15} r_{\Phi_6}$	$-\frac{1}{15} r_{H_2}$	$\frac{1}{5} r_{\delta^+}$	$-\frac{7}{15} r_{\delta_{(3,2)}}$	$\frac{4}{15} r_{\delta_T}$
	$-\frac{2}{3} r_{\Phi_1}$	$-\frac{5}{6} r_{\Phi_2}$	$\frac{3}{2} r_{\Phi_3}$	$\frac{1}{6} r_{\Phi_4}$	$-\frac{1}{6} r_{\Phi_5}$	$-\frac{1}{6} r_{\Phi_6}$	$\frac{1}{6} r_{H_2}$	0	$\frac{1}{6} r_{\delta_{(3,2)}}$	$-\frac{1}{6} r_{\delta_T}$

Tackling problem of unification

	$\bar{5}$		10			V_{24}		5_H		24_H	
b_i/B_{ij}	l_L	$(d^c)_L$	$(u^c)_L$	q_L	$(e^c)_L$	G_μ	W_μ	H_1	T	Σ_8	Σ_3
B_{12}	$-\frac{4}{5}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{44}{15}$	$-\frac{2}{5}$	0	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}r_T$	0	$-\frac{1}{3}r_{\Sigma_3}$
B_{23}	1	-1	-1	1	0	11	$-\frac{22}{3}$	$\frac{1}{6}$	$-\frac{1}{6}r_T$	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$

			45_H				10_H		
Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2	δ^+	$\delta_{(3,2)}$	δ_T
$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{5}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$	$-\frac{1}{15}r_{H_2}$	$\frac{1}{5}r_{\delta^+}$	$-\frac{7}{15}r_{\delta_{(3,2)}}$	$\frac{4}{15}r_{\delta_T}$
$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$	$\frac{1}{6}r_{H_2}$	0	$\frac{1}{6}r_{\delta_{(3,2)}}$	$-\frac{1}{6}r_{\delta_T}$

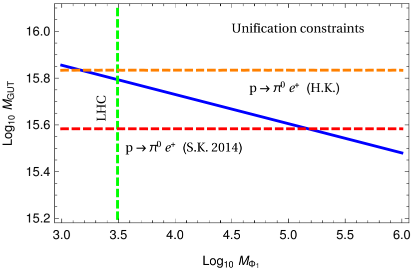
Tackling problem of unification

	$\bar{5}$		10			V_{24}		5_H		24_H	
b_i/B_{ij}	l_L	$(d^c)_L$	$(u^c)_L$	q_L	$(e^c)_L$	G_μ	W_μ	H_1	T	Σ_8	Σ_3
B_{12}	$-\frac{4}{5}$	$\frac{2}{15}$	$\frac{8}{15}$	$-\frac{44}{15}$	$-\frac{2}{5}$	0	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}r_T$	0	$-\frac{1}{3}r_{\Sigma_3}$
B_{23}	1	-1	-1	1	0	11	$-\frac{22}{3}$	$\frac{1}{6}$	$-\frac{1}{6}r_T$	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$

45_H							10_H		
Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	H_2	δ^+	$\delta_{(3,2)}$	δ_T
$-\frac{8}{15}r_{\Phi_1}$	$\frac{2}{15}r_{\Phi_2}$	$-\frac{9}{5}r_{\Phi_3}$	$\frac{17}{15}r_{\Phi_4}$	$\frac{1}{15}r_{\Phi_5}$	$\frac{16}{15}r_{\Phi_6}$	$-\frac{1}{15}r_{H_2}$	$\frac{1}{5}r_{\delta^+}$	$-\frac{7}{15}r_{\delta_{(3,2)}}$	$\frac{4}{15}r_{\delta_T}$
$-\frac{2}{3}r_{\Phi_1}$	$-\frac{5}{6}r_{\Phi_2}$	$\frac{3}{2}r_{\Phi_3}$	$\frac{1}{6}r_{\Phi_4}$	$-\frac{1}{6}r_{\Phi_5}$	$-\frac{1}{6}r_{\Phi_6}$	$\frac{1}{6}r_{H_2}$	0	$\frac{1}{6}r_{\delta_{(3,2)}}$	$-\frac{1}{6}r_{\delta_T}$

Unification constraints

By only assuming the splitting in the 45_H,



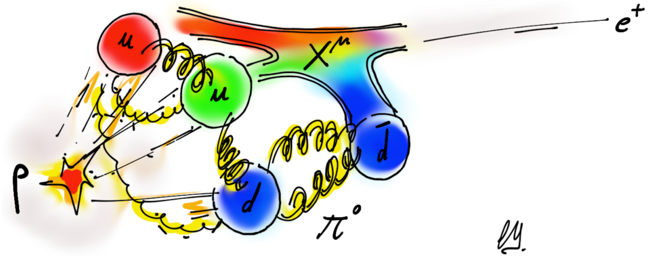
$\tau_p(p \to \pi^0 e^+) > 1{,}29 \times 10^{34}$ years (SK), $\tau_p(p \to \pi^0 e^+) > 1{,}3 \times 10^{35}$ years (HK)

- 45_H alone is enough to achieve unification!
- $\Phi_1 \sim (8, 2, 1/2)$ predicted to be light! i.e. $M_{\phi_1} \sim 10^3 - 10^5$ GeVs

$$\mathcal{L}_Y \supset 2\, d^c Y_2 \Phi_1^\dagger q_L \; + \; 4\, u^c (Y_4 - Y_4^T) q_L \Phi_1 \; + \; \text{h.c.}$$

- $\Phi_3 \sim (3, 3, -1/3)$, $M_{\phi_3} \sim 10^{8,6} - 10^{8,9}$ GeVs

Proton decay



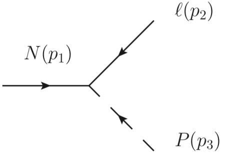
Proton decay mediators

Very naivly:

field	$\mathcal{L}^{p.d}_{d=6} \sim \frac{1}{m^2} QQQL$	decay channel	decay width Γ
$X_\mu\,,\,Y_\mu$	$\mathcal{L}_{X,Y} \supset \frac{g^2_{GUT}}{M^2_X}\,\epsilon_{ijk}\,(\overline{u^C})^i\gamma_\mu q^{\alpha j}\{\overline{e^C}\epsilon_{\alpha\beta}\gamma^\mu q^{k\beta} + (\overline{d^C})^k\gamma^\mu\epsilon_{\alpha\beta}\ell^\beta\}$	$p \to e^+ (\mu^+)\,\pi^0$	$\alpha^2_{GUT}\,\frac{m_p^5}{M_X^4}$
T	$\mathcal{L}_T \supset \frac{1}{m_T^2}\left\{(l_\alpha\,Y_1\,q^\alpha)(q_\beta\,Y_3\,q_\gamma)\epsilon^{\beta\gamma}) + (d^c\,Y_1\,u^c)(u^c\,Y_3\,e^c)\right\}$	$p \to \pi^0\,e^+(\mu^+)$ $p \to \bar{\nu}\,\pi^+$	$(Y_1\,Y_3)^2\,\frac{m_p^5}{m_T^4}$
Φ_3	$\mathcal{L}_{\Phi_3} \supset \frac{1}{m^2_{\Phi_3}}(l_\alpha\,Y_2\,q_\beta)(q_\alpha\,\tilde{Y}_4\,q_\gamma)\epsilon^{\beta\gamma}$	$p \to \bar{\nu}\,K^+$ $p \to e^+\,\pi^0$	$(Y_2\,\tilde{Y}_4)^2\,\frac{m_p^5}{m^4_{\Phi_3}}$
Φ_5	$\mathcal{L}_{\Phi_5} \supset \frac{1}{m^2_{\Phi_5}}(d^c\,Y_2\,u^c)(u^c\,\tilde{Y}_4\,e^c)$	$p \to \pi^0\mu^+(\tau^+)$	$(Y_2\tilde{Y}_4)^2\,\frac{m_p^5}{m^4_{\Phi_5}}$
Φ_6	$\mathcal{L}_{\Phi_6} \supset \frac{1}{m^2_{\Phi_6}}(d^c\,Y_2\,e^c)(u^c\,\tilde{Y}_4\,u^c)$	$p \to \pi^0 e^+(\mu^+)(\tau^+)$	$(Y_2\,\tilde{Y}_4)^2\,\frac{m_p^5}{m^4_{\Phi_6}}$

Proton decay mediated by vector leptoquarks

Decay process:
$N(p_1) \to \bar{\ell}(p_2) + P(p_3)$



Proton decay mediated by vector leptoquarks

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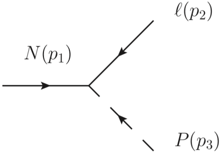
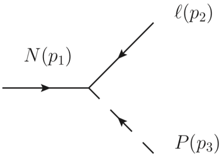


$$\Gamma(N \to P\bar{\ell}) = \frac{m_N}{32\pi}\left(1 - \left(\frac{m_P}{m_N}\right)^2\right)^2 |\left\langle \pi^0 \middle| O_I^{B-L} \middle| p \right\rangle|^2.$$

Proton decay mediated by vector leptoquarks

Decay process:
$N(p_1) \to \bar{\ell}(p_2) + P(p_3)$



$$\Gamma(N \to P\bar{\ell}) = \frac{m_N}{32\pi}\left(1 - \left(\frac{m_P}{m_N}\right)^2\right)^2 |\left\langle \pi^0 \middle| O_I^{B-L} \middle| p \right\rangle|^2.$$

- Integrating out the heavy vector leptoquarks:

$$O_i^{B-L} = \frac{g_{GUT}^2}{2M_X^2}\, \epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{u_{i\alpha}^C}\gamma^\mu Q_{j\alpha a}\overline{e_b^C}\gamma_\mu Q_{k\beta b},$$

$$O_{ii}^{B-L} = \frac{g_{GUT}^2}{2M_X^2}\, \epsilon_{ijk}\, \epsilon_{\alpha\beta}\, \overline{u_{ia}^C}\gamma^\mu Q_{j\alpha a}\overline{d_{kb}^C}\gamma_\mu L_{\beta b}.$$

Proton decay mediated by vector leptoquarks

$$
\begin{aligned}
O(e^c_\alpha, d_\beta) &= C(e^C_\alpha, d_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{e^C_\alpha}\gamma_\mu d_{k\beta}, \\
O(e_\alpha, d^C_\beta) &= C(e_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{d^C_{k\beta}}\gamma_\mu e_\alpha, \\
O(\nu_l, d_\alpha, d^C_\beta) &= C(\nu_l, d_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu d_{j\alpha} \overline{d^C_{k\beta}}\gamma_\mu \nu_l.
\end{aligned}
$$

$$
C = \frac{g^2_{GUT}}{2M^2_X} c
$$

Proton decay mediated by vector leptoquarks

$$
\begin{aligned}
O(e^c_\alpha, d_\beta) &= C(e^C_\alpha, d_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{e^C_\alpha}\gamma_\mu d_{k\beta}, \\
O(e_\alpha, d^C_\beta) &= C(e_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{d^C_{k\beta}}\gamma_\mu e_\alpha, \\
O(\nu_l, d_\alpha, d^C_\beta) &= C(\nu_l, d_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu d_{j\alpha} \overline{d^C_{k\beta}}\gamma_\mu \nu_l.
\end{aligned}
$$

$$
C = \frac{g^2_{GUT}}{2M^2_X} c
$$

$$
\begin{aligned}
&c(e^c_\alpha, d_\beta) = V_1^{11} V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta}(V_2 V_{UD}^\dagger)^{\alpha 1}, \\
&c(e_\alpha, d^c_\beta) = V_1^{11} V_3^{\beta\alpha}, \\
&c(\nu_l, d_\alpha, d^c_\beta) = (V_1 V_{UD})^{1\alpha}(V_3 V_{EN})^{\beta l}.
\end{aligned}
$$

Proton decay mediated by vector leptoquarks

$$
\begin{aligned}
O(e^c_\alpha, d_\beta) &= C(e^C_\alpha, d_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{e^C_\alpha}\gamma_\mu d_{k\beta}, \\
O(e_\alpha, d^C_\beta) &= C(e_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu u_j \overline{d^C_{k\beta}}\gamma_\mu e_\alpha, \\
O(\nu_l, d_\alpha, d^C_\beta) &= C(\nu_l, d_\alpha, d^C_\beta)\epsilon_{ijk}\overline{u^C_i}\gamma^\mu d_{j\alpha} \overline{d^C_{k\beta}}\gamma_\mu \nu_l.
\end{aligned}
$$

$$C = \frac{g^2_{GUT}}{2M^2_X} c$$

$$
\begin{aligned}
& c(e^c_\alpha, d_\beta) = V_1^{11} V_2^{\alpha\beta} + (V_1 V_{UD})^{1\beta} (V_2 V^\dagger_{UD})^{\alpha 1}, \\
& c(e_\alpha, d^c_\beta) = V_1^{11} V_3^{\beta\alpha}, \\
& c(\nu_l, d_\alpha, d^c_\beta) = (V_1 V_{UD})^{1\alpha} (V_3 V_{EN})^{\beta l}.
\end{aligned}
$$

$$V_1 = U^\dagger_C U, V_2 = E^\dagger_C D,\ V_3 = D^\dagger_C E,$$
$$V_{UD} = U^\dagger D \text{ and } V_{EN} = E^\dagger N.$$

$$U^T_C Y_u U = Y_u^{\text{diag}},\ D^T_C Y_d D = Y_d^{\text{diag}},$$
$$E^T_C Y_e E = Y_e^{\text{diag}},\ N^T Y_\nu N = Y_\nu^{\text{diag}}.$$

- Using the identity: $\Psi^T C\gamma^\mu \epsilon = (\Psi^T C\gamma^\mu \epsilon)^T = -\epsilon^T C\gamma^\mu \Psi$

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$$\begin{aligned}
O(e^c_\alpha, d_\beta) &= -2\, C(e^C_\alpha, d_\beta)\, \epsilon_{ijk}\, u^T_{jL} C\gamma^\mu u_{iR}\, e^T_{\alpha R} C\gamma_\mu d_{k\beta L}, \\
O(e_\alpha, d^C_\beta) &= -2\, C(e_\alpha, d^C_\beta)\, \epsilon_{ijk}\, u^T_{jL} C\gamma^\mu u_{iR}\, d^T_{k\beta R} C\gamma_\mu e_{\alpha L}, \\
O(\nu_l, d_\alpha, d^C_\beta) &= -2\, C(\nu_l, d_\alpha, d^C_\beta)\, \epsilon_{ijk}\, d^T_{j\alpha L} C\gamma^\mu u_{iR}\, d^T_{k\beta R} C\gamma_\mu \nu_{lL},
\end{aligned}$$

- Using the identity: $\Psi^T C\gamma^\mu \epsilon = (\Psi^T C\gamma^\mu \epsilon)^T = -\epsilon^T C\gamma^\mu \Psi$

$$
\begin{aligned}
O(e^c_\alpha, d_\beta) &= -2\, C(e^C_\alpha, d_\beta)\, \epsilon_{ijk}\, u^T_{jL} C\gamma^\mu u_{iR}\, e^T_{\alpha R} C\gamma_\mu d_{k\beta L}, \\
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O(\nu_l, d_\alpha, d^C_\beta) &= -2\, C(\nu_l, d_\alpha, d^C_\beta)\, \epsilon_{ijk}\, d^T_{j\alpha L} C\gamma^\mu u_{iR}\, d^T_{k\beta R} C\gamma_\mu \nu_{lL},
\end{aligned}
$$

- Using Fierz relations:

- Using the identity: $\Psi^T C\gamma^\mu \epsilon = (\Psi^T C\gamma^\mu \epsilon)^T = -\epsilon^T C\gamma^\mu \Psi$

$$
\begin{aligned}
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\end{aligned}
$$

- Using Fierz relations:

$$
\begin{aligned}
O(e^c_\alpha, d_\beta) &= 2\, C(e^C_\alpha, d_\beta)\, \epsilon_{ijk}\, (u^T_{jL} C d_{k\beta L})(e^T_{\alpha R} C u_{iR}), \\
O(e_\alpha, d^C_\beta) &= 2\, C(e_\alpha, d^C_\beta)\, \epsilon_{ijk}\, (u^T_{jL} C e_{\alpha L})(d^T_{k\beta R} C u_{iR}), \\
O(\nu_l, d_\alpha, d^C_\beta) &= 2\, C(\nu_l, d_\alpha, d^C_\beta)\, \epsilon_{ijk}\, (d^T_{j\alpha L} C \nu_{lL})(d^T_{k\beta R} C u_{iR}).
\end{aligned}
$$

$$
\boxed{\Gamma(N \to P\bar{\ell}) = A\, \frac{m_N}{8\pi} \left(1 - \left(\frac{m_P}{m_N}\right)^2\right)^2 \left|\sum_I C^I W^I_0(N \to P)\right|^2}
$$

- Using the identity: $\Psi^T C\gamma^\mu \epsilon = (\Psi^T C\gamma^\mu \epsilon)^T = -\epsilon^T C\gamma^\mu \Psi$

$$
\begin{aligned}
O(e^c_\alpha, d_\beta) &= -2\, C(e^C_\alpha, d_\beta)\, \epsilon_{ijk}\, u^T_{jL} C\gamma^\mu u_{iR}\; e^T_{\alpha R} C\gamma_\mu d_{k\beta L}, \\
O(e_\alpha, d^C_\beta) &= -2\, C(e_\alpha, d^C_\beta)\, \epsilon_{ijk}\, u^T_{jL} C\gamma^\mu u_{iR}\; d^T_{k\beta R} C\gamma_\mu e_{\alpha L}, \\
O(\nu_l, d_\alpha, d^C_\beta) &= -2\, C(\nu_l, d_\alpha, d^C_\beta)\, \epsilon_{ijk}\, d^T_{j\alpha L} C\gamma^\mu u_{iR}\; d^T_{k\beta R} C\gamma_\mu \nu_{lL},
\end{aligned}
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$$
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\end{aligned}
$$

$$
\boxed{\Gamma(N \to P\bar{\ell}) = A\, \frac{m_N}{8\pi} \left(1 - \left(\frac{m_P}{m_N}\right)^2\right)^2 \left| \sum_I C^I W^I_0(N \to P) \right|^2}
$$

$$
A = A_{QCD} A_{SR} = \left(\frac{\alpha_3(m_b)}{\alpha_3(M_Z)}\right)^{6/23} \left(\frac{\alpha_3(Q)}{\alpha_3(m_b)}\right)^{6/25} \left(\frac{\alpha_3(M_Z)}{\alpha_3(M_{GUT})}\right)^{2/7}.
$$

Values taken: $\mathcal{A}_{QCD} \sim 1{,}2$ and $\mathcal{A}_{SR} \sim 1{,}5$.

$$
\begin{aligned}
\Gamma(p \to \pi^0 e^+_\beta) &= \frac{m_p}{8\pi} A^2 k_1^4 | \left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle |^2 \left(\left| c(e^c, d) \right|^2 + \left| c(e, d^c) \right|^2 \right), \\
\Gamma(p \to K^+ \bar{\nu}) &= \frac{m_p}{8\pi} \left(1 - \frac{m^2_{K^+}}{m^2_p} \right)^2 A^2 k_1^4 \sum_i \left| c(\nu_i, d, s^c) \left\langle K^+ \right| (us)_R d_L \left| p \right\rangle + \right. \\
&\quad \left. + \, c(\nu_i, s, d^c) \left\langle K^+ \right| (ud)_R s_L \left| p \right\rangle \right|^2 .
\end{aligned}
$$

$$\Gamma(p\to\pi^0 e^+_\beta) = \frac{m_p}{8\pi}A^2 k_1^4 |\left\langle\pi^0\right| (ud)_R u_L \left|p\right\rangle|^2 \left(\left|c(e^c,d)\right|^2 + \left|c(e,d^c)\right|^2\right),$$

$$\Gamma(p\to K^+\bar{\nu}) = \frac{m_p}{8\pi}\left(1-\frac{m_{K^+}^2}{m_p^2}\right)^2 A^2 k_1^4 \sum_i \left|c(\nu_i,d,s^c)\left\langle K^+\right| (us)_R d_L \left|p\right\rangle + \right.$$

$$\left. + \, c(\nu_i,s,d^c)\left\langle K^+\right| (ud)_R s_L \left|p\right\rangle\right|^2.$$

- $k_1 = \dfrac{g_{GUT}}{\sqrt{2}M_X}$

$$
\begin{aligned}
\Gamma(p \to \pi^0 e^+_\beta) &= \frac{m_p}{8\pi} A^2 k_1^4 | \left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle |^2 \left(\left| c(e^c, d) \right|^2 + \left| c(e, d^c) \right|^2 \right), \\
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&\left. + \, c(\nu_i, s, d^c) \left\langle K^+ \right| (ud)_R s_L \left| p \right\rangle \right|^2 .
\end{aligned}
$$

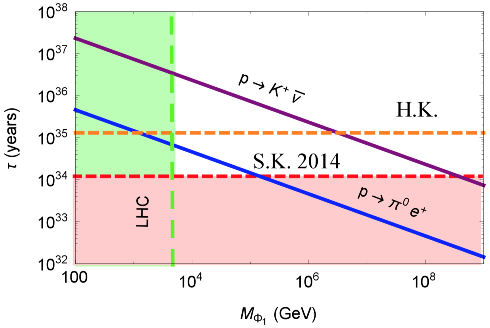
- $k_1 = \dfrac{g_{GUT}}{\sqrt{2} M_X}$
- Matrix elements: Lattice QCD (Aoki et al., 2014)

$$
\begin{aligned}
\Gamma(p \to \pi^0 e^+_\beta) &= \frac{m_p}{8\pi} A^2 k_1^4 |\left\langle \pi^0 \right| (ud)_R u_L \left| p \right\rangle|^2 \left(\left| c(e^c, d) \right|^2 + \left| c(e, d^c) \right|^2 \right), \\
\Gamma(p \to K^+ \bar{\nu}) &= \frac{m_p}{8\pi} \left(1 - \frac{m_{K^+}^2}{m_p^2} \right)^2 A^2 k_1^4 \sum_i \left| c(\nu_i, d, s^c) \left\langle K^+ \right| (us)_R d_L \left| p \right\rangle + \right. \\
&\left. + \, c(\nu_i, s, d^c) \left\langle K^+ \right| (ud)_R s_L \left| p \right\rangle \right|^2 .
\end{aligned}
$$

- $k_1 = \dfrac{g_{GUT}}{\sqrt{2} M_X}$
- Matrix elements: Lattice QCD (Aoki et al., 2014)
- Most conservative scenario chosen:

$$
\begin{aligned}
&\text{for } p \to \pi^0 e^+ &:& \quad c(e, d^c) = 1, & c(e^c, d) = 2 \\
&\text{for } p \to K^+ \bar{\nu} &:& \quad c(\nu_l, d, s^c) = 1 & c(\nu_l, s, d^c) = V_{CKM}^{12}
\end{aligned}
$$

Proton decay constraints



S.K. $\tau_p(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ years (red dashed)
H.K. $\tau_p(p \to \pi^0 e^+) > 1.3 \times 10^{35}$ years (orange dashed)

Massive neutrinos

Massive neutrinos?

$Y_e \neq Y_d^T$	$m_\nu \neq 0$	Unification	Extra field content	number of new d.o.f.
45_H	type-I seesaw	✓	1_F, 1_F	47
	type-II seesaw	✓	15_H	60
	type-III seesaw	✓	24_F	69
	Zee model	✓	10_H	55

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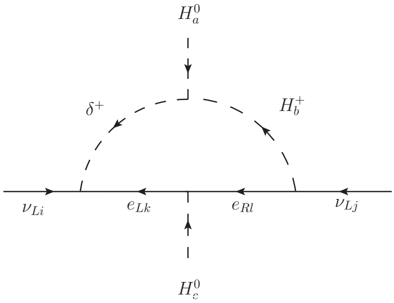
$$
\begin{aligned}
-\mathcal{L}_{Zee} \quad \supset & \quad \bar{5}\, 10 \left(Y_1^* 5_H^* - \frac{1}{6} Y_2^* 45_H^* \right) \\
+ & \quad \lambda \bar{5}\, \bar{5}\, 10_H - \frac{1}{6} \mu\, 5_H 45_H 10_H^* \\
+ & \quad \text{h.c.}
\end{aligned}
$$



Zee mechanism

$$V_{\text{Zee}} = \ell_L \lambda \, \ell_L \delta^+ \; + \; \overline{\ell_L} Y_a H_a e_R \; + \; \mu H_1 H_2 \delta^- \; + \; \text{h.c.},$$

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$$\begin{aligned} M_\nu &= \frac{1}{8\pi^2} \left(\lambda M_e \left(Y_1^\dagger \cos\beta - Y_2^\dagger \sin\beta \right) + \left(Y_1^* \cos\beta - Y_2^* \sin\beta \right) M_e^T \lambda^T \right) \times \\ & \sin 2\theta_+ \text{Log} \left(\frac{\text{m}^2_{\text{h}_2^+}}{\text{m}^2_{\text{h}_1^+}} \right) \end{aligned}$$

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$$\begin{aligned} H_1^\pm &= \cos\beta H^\pm + \sin\beta G^\pm & \delta^\pm &= \cos\theta_+ h_1^\pm + \sin\theta_+ h_2^\pm \\ H_2^\pm &= -\sin\beta H^\pm + \cos\beta G^\pm & H^\pm &= -\sin\theta_+ h_1^\pm + \cos\theta_+ h_2^\pm \end{aligned}$$

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Radiative neutrino masses

$$
\begin{aligned}
Y_1 &= \frac{1}{2\sqrt{2}v_5}(M_e + 3M_d^T), \\
Y_2 &= \frac{3}{2\sqrt{2}v_45}(M_e - M_d^T).
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$$c_e = \frac{(1-4\sin^2\beta)}{8\pi^2\sqrt{2}v\sin 2\beta}\sin 2\theta_+ \mathrm{Log}\left(\frac{\mathrm{m}^2_{\mathrm{h}_2^+}}{\mathrm{m}^2_{\mathrm{h}_1^+}}\right) \quad \mathrm{c_d} = \frac{1}{8\pi^2\sqrt{2}v\sin 2\beta}\sin 2\theta_+ \mathrm{Log}\left(\frac{\mathrm{m}^2_{\mathrm{h}_2^+}}{\mathrm{m}^2_{\mathrm{h}_1^+}}\right)$$

Conclusions

- SU(5) deserves a second chance: simple renormalizable extension
- Consistent fermion masses and unification can be simultaneously achieved with the 45_H
- Neutrinos can get mass at 1-loop level by adding the 10_H together with 45_H
  $\rightarrow$ Relation between charged fermion masses and neutrino mass
- Predicts a light octet which could give rise to exotic signatures at the LHC
- Consistent with proton decay constraints (could be ruled out by HK)

Thanks for your attention!