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Clockwork for Neutrino Masses and Lepton Flavor Violation

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Abstract

We investigate the generation of small neutrino masses in a clockwork framework which includes Dirac mass terms as well as Majorana mass terms for the new fermions. We derive analytic formulas for the masses of the new particles and for their Yukawa couplings to the lepton doublets, in the scenario where the clockwork parameters are universal. When the Majorana masses all vanish, the zero mode of the clockwork sector forms a Dirac pair with the active neutrino, with a mass which is in agreement with oscillations experiments for a sufficiently large number of clockwork gears. On the other hand, when the Majorana masses do not vanish, neutrino masses are generated via the seesaw mechanism. In this case, and due to the fact that the effective Yukawa couplings of the higher modes can be sizable, neutrino masses can only be suppressed by postulating a large Majorana mass for all the gears. Finally, we discuss the constraints on the mass scale of the clockwork fermions from the non-observation of the rare leptonic decay $\mu \to e\gamma$.

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1 Introduction

The smallness of neutrino masses stands as one of the most puzzling open questions in Fundamental Physics. A plausible solution to this puzzle is provided by the seesaw mechanism, in which the smallness of neutrino masses is explained by the breaking of the lepton number at a very high energy scale [1–4]. Models with conserved lepton number, on the other hand, can also reproduce the observations, at the expense of postulating tiny Yukawa couplings of the neutrino to the Standard Model Higgs. Such small parameters are usually regarded as unnatural, however the existence of tiny Yukawa couplings is a phenomenologically viable possibility, and can be accomplished in further extensions of the model (for reviews and recent models, see *e.g.* in [5–18]).

Recently, a new mechanism of generating small couplings in theories coupled to the Standard Model has been introduced [19, 20]. The mechanism, reminiscent of deconstruction models [21, 22], can be summarized as a linear quiver model with no large hierarchies in the theory parameters, that gives rise to site-dependent suppressed couplings to the zero-mode [23]. Originally, introduced for a quiver of Abelian Goldstone bosons (axions), it has been generalized to fermions, vectors and other fields [23, 24] (See also [25]). Applications and generalizations of this mechanism can be found in [26–42].

In this work we explore the application of the fermionic clockwork to the generation of small neutrino masses. Concretely, we identify the right-handed neutrinos with the zero modes of a clockwork sector [23], such that small couplings can be naturally generated and therefore small neutrino masses. We generalize the clockwork framework for the right handed neutrinos by including also Majorana mass terms. We show that the clockwork mechanism, *i.e.*, the suppression of the Yukawa couplings by site dependent power factors, is not affected by the presence of the Majorana mass terms. In fact, the combination of the clockwork "suppression" and the Majorana "seesaw" sets now the neutrino mass scale. When all the Majorana terms are set to zero, the clockwork provides an interesting alternative to the existing models of Dirac neutrinos, which we investigate in this paper. Furthermore, while the clockwork mechanism suppresses the couplings of the zero mode, the couplings of the higher modes can be sizable and induce, via loops, potentially large rates for the leptonic rare decays.

The rest of the paper is organized as follows. In section 2, we present the most general framework for clockwork neutrinos with Dirac and Majorana mass terms, and we discuss their phenomenology in subsections 2.1 and 2.2, respectively. In section 3, we discuss lepton flavour violation in the clockwork scenario and calculate limits on the gear masses. We close with a summary.

2 Neutrinos in Clockwork

We extend the Standard Model with n left-handed and $n+1$ right-handed chiral fermions, singlets under the Standard Model gauge group, which we denote as $\psi_{Li}(i = 0, ..., n-1)$ and $\psi_{Ri}(i = 0, ..., n)$ respectively. The Lagrangian of the model reads:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm Clockwork} + \mathcal{L}_{\rm int} \ , \tag{1}$$

where $\mathcal{L}_{\rm SM}$ is the Standard Model Lagrangian, $\mathcal{L}_{\rm Clockwork}$ is the part of the Lagrangian involving only the new fermion singlets, and $\mathcal{L}_{\rm int}$ is the interaction term of the new fields with the Standard Model fields. Following [23], we assume that the Standard Model only couples to the last site of the fermionic clockwork, therefore,

$$\mathcal{L}_{\rm int} = -Y \widetilde{H} \overline{L}_L \psi_{Rn} \ , \tag{2}$$

with $\widetilde{H} = i\tau_2 H^*$, H the Standard Model Higgs doublet and L_L the left handed lepton fields (we assume only one generation of fermions; the generalization to more than one generation will be discussed below).

In full generality, the clockwork Lagrangian can be cast as:

$$\mathcal{L}_{\rm Clockwork} = \mathcal{L}_{\rm kin} - \sum_{i=0}^{n-1} \left(m_i \overline{\psi}_{Li} \psi_{Ri} - m_i' \overline{\psi}_{Li} \psi_{Ri+1} + \text{h.c.} \right) - \sum_{i=0}^{n-1} \frac{1}{2} M_{Li} \overline{\psi^c_{Li}} \psi_{Li} - \sum_{i=0}^{n} \frac{1}{2} M_{Ri} \overline{\psi^c_{Ri}} \psi_{Ri} \ , \tag{3}$$

where $\mathcal{L}_{\rm kin}$ denotes the kinetic term for all fermions, and m, m' and $M_{L,R}$ are mass parameters. Denoting $\Psi = (\psi_{L0}, \psi_{L1}, ...\psi_{Ln-1}, \psi^c_{R0}, \psi^c_{R1}, ..., \psi^c_{Rn})$, the clockwork Lagrangian can be written in the compact form:

$$\mathcal{L}_{\rm Clockwork} = \mathcal{L}_{\rm Kin} - \frac{1}{2} (\overline{\Psi^c} \mathcal{M} \Psi + \text{h.c.}) \tag{4}$$

with $\mathcal{M}$ a $(2n+1)\times(2n+1)$ mass matrix. We note that $\mathcal{L}_{\rm kin}$ is invariant under the global group $U(n)_L\times U(n+1)_R$. The mass terms m_i break the global group $U(n)_L\times U(n+1)_R\to\prod_{i=0}^{n-1}U(1)_i$, where $U(1)_i$ acts as $\psi_{L,i}\to e^{i\alpha_i}\psi_{L,i}$, $\psi_{Ri}\to e^{i\alpha_i}\psi_{Ri}$, and combined with the mass terms m_i', break the global symmetry $U(n)_L\times U(n+1)_R\to U(1)_{\rm CW}$, where $U(1)_{\rm CW}$ acts as $\psi_{L,i}\to e^{i\alpha}\psi_{L,i}$, $\psi_{R,i}\to e^{i\alpha}\psi_{R,i}$ for all i. Finally, M_{Li} and M_{Ri} are Majorana masses for the left and right handed singlet fields. It is sufficient that M_{Li} or M_{Ri} is non-vanishing for one i to break the symmetry group $U(n)_L\times U(n+1)_R\to$ nothing.

We assume for simplicity universal Dirac masses, Majorana masses and nearest neighbor interactions, namely $m_i=m$, $m_i'=mq$ $M_{Ri}=M_{Li}=m\widetilde{q}$ for all i. Under this assumption, the mass matrix reads:

$$\mathcal{M}=m\begin{pmatrix} \widetilde{q} & 0 & \cdots & 0 & 1 & -q & \cdots & 0\\ 0 & \widetilde{q} & \cdots & 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & \widetilde{q} & 0 & 0 & 0 & -q\\ 1 & 0 & \cdots & 0 & \widetilde{q} & 0 & \cdots & 0\\ -q & 1 & \cdots & 0 & 0 & \widetilde{q} & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & -q & 0 & 0 & 0 & \widetilde{q} \end{pmatrix}, \tag{5}$$

which has eigenvalues M_k given by:

$$\begin{aligned} M_0 &= m\widetilde{q}\,,\\ M_k &= m\widetilde{q}-m\sqrt{\lambda_k}\,, \qquad k=1,\dots,n\,,\\ M_{n+k} &= m\widetilde{q}+m\sqrt{\lambda_k}\,, \qquad k=1,\dots,n\,, \end{aligned} \tag{6}$$

with λ_k defined as

$$\lambda_k\equiv q^2+1-2q\cos\frac{k\pi}{n+1}\,. \tag{7}$$

The mass eigenstates, which we denote as χ_k, are related to the interaction eigenstates Ψ_j by the unitary transformation $\mathcal{U}$, namely $\Psi_j=\sum_j\mathcal{U}_{jk}\chi_k$. The matrix $\mathcal{U}$ can be explicitly calculated, the result being:

$$\mathcal{U}=\begin{pmatrix} \vec{0} & \frac{1}{\sqrt{2}}U_L & -\frac{1}{\sqrt{2}}U_L\\ \vec{u}_R & \frac{1}{\sqrt{2}}U_R & \frac{1}{\sqrt{2}}U_R \end{pmatrix}. \tag{8}$$

where $\vec{0}$ and $\vec{u}_R$ are n-dimensional vectors, with entries:

$$\vec{0}_j=0\,, \qquad j=1,...,n\,, \tag{9}$$

$$(u_R)_j=\frac{1}{q^j}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}}\,, \qquad j=1,...,n\,, \tag{10}$$

while U_L and U_R are, respectively, $n\times n$ and $(n+1)\times n$ matrices with elements

$$\begin{aligned} (U_L)_{jk} &= \sqrt{\frac{2}{n+1}}\sin\frac{jk\pi}{n+1}\,, \qquad j,k=1,...,n\,,\\ (U_R)_{jk} &= \sqrt{\frac{2}{(n+1)\lambda_k}}\left[q\sin\frac{jk\pi}{n+1}-\sin\frac{(j+1)k\pi}{n+1}\right]\,, \quad j=0,..,n, \quad k=1,...,n\,, \end{aligned} \tag{11}$$

We note that the mixing matrix $\mathcal{U}$ does not depend on the parameter $\widetilde{q}$, which is a consequence of our assumption of universality of the Majorana masses $M_{Ri}=M_{Li}=m\widetilde{q}$ for all i.

The interaction Lagrangian of the clockwork fields to the Standard Model fields, Eq. (4), can now be recast in terms of mass eigenstates:

$$\mathcal{L}_{\rm int}=-Y\,\overline{L}_L\widetilde{H}\mathcal{U}_{nk}\chi_k\equiv-\sum_{k=0}^{2n}Y_k\,\overline{L}_L\widetilde{H}\chi_k\,, \tag{12}$$

where

$$Y_0 \equiv Y(u_R)_n = \frac{Y}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}}\ , \tag{13}$$

$$Y_k = Y_{k+n} \equiv \frac{1}{\sqrt{2}}Y(U_R)_{nk} = Y\sqrt{\frac{1}{(n+1)\lambda_k}}\left[q\sin\frac{nk\pi}{n+1}\right]\ , \qquad k=1,...,n\ . \tag{14}$$

The components $(u_R)_n$ and $(U_R)_{np}$, which describe the fraction of the n_{th} "gear" in the zero mode, will play a major role in the phenomenology, as they parametrize the portal strength between the Standard Model sector and the clockwork sector.

After electroweak symmetry breaking new mass terms arise which mix the Standard Model neutrino with the clockwork fermions. The mass matrix of the $2n+2$ electrically neutral fermion fields of the model reads:

$$m_\nu = \begin{array}{c} \\ \nu_L \\ \chi_0 \\ \chi_1 \\ \chi_2 \\ \vdots \\ \chi_{2n} \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} \nu_L & \chi_0 & \chi_1 & \chi_2 & \cdots & \chi_{2n} \end{array} \\ \begin{pmatrix} 0 & vY_0 & vY_1 & vY_2 & \cdots & vY_{2n} \\ vY_0 & M_0 & 0 & 0 & \cdots & 0 \\ vY_1 & 0 & M_1 & 0 & & \\ vY_2 & 0 & 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ vY_{2n} & 0 & 0 & 0 & \cdots & M_{2n} \end{pmatrix} \end{array}\ , \tag{15}$$

where $v = 246/\sqrt{2}$ GeV is the Higgs vacuum expectation value. Upon diagonalizing this mass matrix, one finds a mass for the active neutrino. Furthermore, the off-diagonal entries in the mass matrix translate into charged current interactions between the charged lepton and the k-th mode, as well as neutral-current and Higgs interactions of the light neutrino, proportional to $\sim vY_k/M_k$, and which can be sizable.

In order to accommodate the leptonic mixing observed in Nature it is necessary to introduce three generations of lepton doublets, as well as N generation of clockwork fermions, each consisting of n_α left-handed and $n_\alpha+1$ right-handed gears, where $\alpha = 1,\dots,N$ (phenomenologically, $N \geq 2$, in order to account for the two observed oscillation frequencies). Furthermore, the Yukawa coupling in Eq. (2) and all the mass parameters in Eq. (3) must be promoted to matrices in flavor space. In this work we will assume for simplicity $m_i^{\alpha\beta} = m\delta^{\alpha\beta}$, ${m'}_i^{\alpha\beta} = mq_\alpha\delta^{\alpha\beta}$ $M_{Ri}^{\alpha\beta} = M_{Li}^{\alpha\beta} = m\widetilde{q}_\alpha\delta^{\alpha\beta}$ for all i. Namely, the mass parameter m is universal for all gears and all generations, while the mass parameters m', M_R and M_L are common for all gears within one generation, but in principle different among generations.

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Denoting $\Psi^\alpha = (\psi^\alpha_{L0}, \psi^\alpha_{L1}, ..., \psi^\alpha_{Ln-1}, \psi^{\alpha\,c}_{R0}, \psi^{\alpha\,c}_{R1}, ..., \psi^{\alpha\,c}_{Rn})$ as the fermion field which has as component all the clockwork fields within the generation α, the clockwork and interaction Lagrangian can be written as:

$$\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{Kin}} - \frac{1}{2}(\overline{\Psi^{\alpha\,c}}\mathcal{M}^{\alpha\beta}\Psi^\beta + \text{h.c.})\ , \tag{16}$$

$$\mathcal{L}_{\text{int}} = -Y^{a\alpha}\,\overline{L^a_L}\widetilde{H}\psi^\alpha_{R,n}\ , \tag{17}$$

where $a = 1,2,3$ and $\alpha,\beta = 1,...,N$. As for the one generation case, we assumed that the Standard Model lepton doublets only couple to the n-th sites of the N clockwork generations.

The Lagrangian expressed in the mass eigenstate basis, $\Psi^\alpha_k = \mathcal{U}^{\alpha\beta}_{kj}\chi^\beta_j$, read:

$$\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{Kin}} - \frac{1}{2}(\overline{\chi^{\alpha\,c}_k}M^\alpha_k\chi^\alpha_k + \text{h.c.})\ , \tag{18}$$

$$\mathcal{L}_{\text{int}} = -\sum_{k=0}^{2n} Y^{a\beta}_k\,\overline{L}^a_L\chi^\beta_k\ , \tag{19}$$

where $Y^{a\beta}_k \equiv Y^{a\alpha}\mathcal{U}^{\alpha\beta}_{nk}$ with $\mathcal{U}^{\alpha\beta}_{nk}$ the matrix that mixes fermions of different clockwork gears and different generations. Finally, after electroweak symmetry breaking, the mass matrix of the $N(2n+1)+3$ electrically neutral fermions of the model reads:

$$m_\nu = \begin{array}{c} \\ \nu_L^a \\ \chi_0^\beta \\ \chi_1^\beta \\ \chi_2^\beta \\ \vdots \\ \chi_{2n}^\beta \end{array} \begin{array}{c} \begin{array}{cccccc} \nu_L^a & \chi_0^\beta & \chi_1^\beta & \chi_2^\beta & \cdots & \chi_{2n}^\beta \end{array} \\ \begin{pmatrix} 0 & vY_0^{a\beta} & vY_1^{a\beta} & vY_2^{a\beta} & \cdots & vY_{2n}^{a\beta} \\ vY_0^{\beta a} & M_0^\beta & 0 & 0 & \cdots & 0 \\ vY_1^{\beta a} & 0 & M_1^\beta & 0 & & \\ vY_2^{\beta a} & 0 & 0 & M_2^\beta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ vY_{2n}^{\beta a} & 0 & 0 & 0 & \cdots & M_{2n}^\beta \end{pmatrix} \end{array} . \tag{20}$$

This matrix has in general a non-trivial flavor structure and leads not only to mixing among the three active neutrinos, but also to potentially large lepton flavour violating charged current, neutral current and Higgs interactions, thus providing a possible test of this framework, as will be discussed in Section 3.

We consider in what follows two cases: $M_{Li}, M_{Ri} = 0$, for all i, such that the Clockwork Lagrangian has a residual $U(1)_{\rm CW}$ global symmetry, and $M_{Li}, M_{Ri} \neq 0$ for some i, such that the Clockwork Lagrangian has no global symmetry.

2.1 $M_{Li}, M_{Ri} = 0$, for all i Lepton number conservation: Dirac neutrinos

We consider first the case where all Majorana masses are equal to zero. In this case, the global symmetry of the Lagrangian is broken as $U(n)_L \times U(n+1)_R \to U(1)_{\rm CW}$, which will be identified with total lepton number. The eigenstates and eigenvalues of the mass matrix can be determined using the results of Section 2, by setting $\widetilde{q} = 0$.

It is useful to recast the clockwork Lagrangian as

$$\mathcal{L}_{\rm clockwork} = \mathcal{L}_{\rm kin} - \overline{N_L} m_\nu^D N_R + \text{h.c.} \tag{21}$$

where we have defined new fields $N_L = (\nu_L, N_{L1}, ..., N_{Ln})$ and $N_R = (N_{R0}, N_{R1}, ..., N_{Rn})$, with

$$N_{Rk} = \frac{1}{\sqrt{2}}(\chi_k + \chi_{k+n}) \ , \qquad k = 0, ...n \ , \tag{22}$$

$$N_{Lk} = \frac{1}{\sqrt{2}}(-\chi_k + \chi_{k+n}) \ , \qquad k = 1, ...n \ . \tag{23}$$

In this basis, the mass matrix has the form:

$$m_\nu^D = \begin{array}{c} \\ \nu_L \\ N_{L1} \\ N_{L2} \\ \vdots \\ N_{Ln} \end{array} \begin{array}{c} \begin{array}{ccccc} N_{R0} & N_{R1} & N_{R2} & \cdots & N_{Rn} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & M_1 & 0 & \cdots & 0 \\ 0 & 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_n \end{pmatrix} \end{array} . \tag{24}$$

where $M_k = m\sqrt{\lambda_k}$, with λ_k defined in Eq. (7). Namely, the fields ν_L and N_{R0} form a massless Dirac pair, while the fields N_{Rk} and N_{Lk} form, for $k = 1, ..., n$, Dirac pairs with mass M_k. The overall scale of the massive pairs is determined by the parameter m, and the mass difference between pairs depends on q and n. Assuming $q > 1$, one obtains that the masses of the modes with $k > 0$ increase monotonically with n, from $M_1 \approx m(q-1)$ to $M_n \approx m(q+1)$. In Fig. 1, left panel, we show for illustration the mass spectrum of the particles of the clockwork sector, labeled by k, taking for concreteness $n = 10$ and $q = 2$. The mass spectrum has been normalized to m.

The mass spectrum is modified after electroweak symmetry breaking by the interactions with the Higgs field. Expressed in terms of N_{Rk}, the interaction Lagrangian reads:

$$\mathcal{L}_{\rm int} = \sum_{k=0}^{n} Y_k \overline{L}_L \widetilde{H} N_{Rk} + \text{h.c.} \tag{25}$$

Y0 is suppresed w.r.t. the other Y's

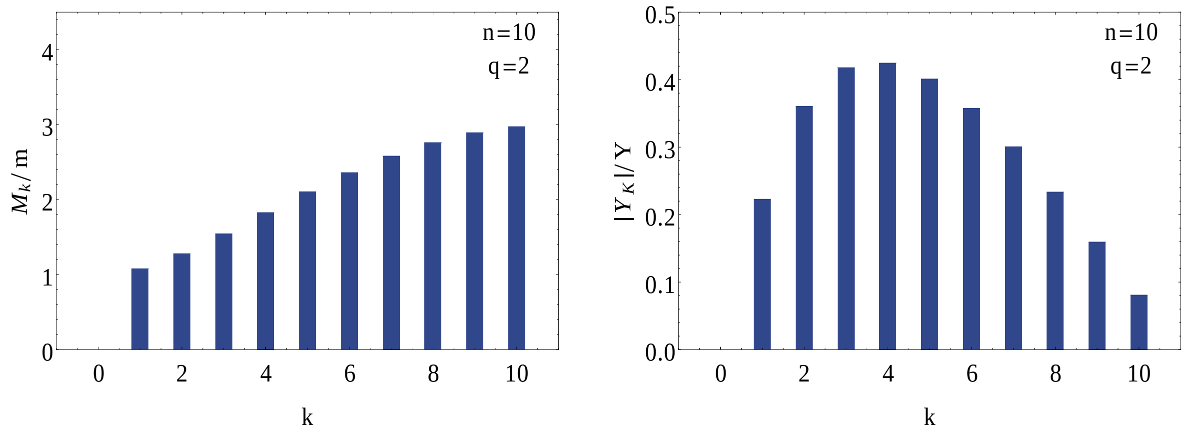

Figure 1: Dirac masses (left panel) and Yukawa couplings (right panel) of the singlet fermions of the clockwork sector, normalized respectively to m and Y, for the specific case $n = 10$ and $q = 2$.

with

$$Y_0 \equiv Y(u_R)_n = \frac{Y}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}}\;, \tag{26}$$

$$Y_k \equiv Y(U_R)_{nk} = Y\sqrt{\frac{2}{(n+1)\lambda_k}}\left[q\sin\frac{nk\pi}{n+1}\right]\;, \qquad k = 1,...,n\;. \tag{27}$$

The Yukawa coupling of the massless mode Y_0 is suppressed by q^n, provided $q > 1$, whereas the couplings of the kth-mode are of the same order as Y. This is illustrated in Fig. 1, right panel, which shows the Yukawa couplings of the clockwork fermions to the Standard Model lepton doublets, normalized to Y, for the same values of n and q as in the left panel (in this case, $|Y_0|/Y \approx 8 \times 10^{-4}$ and is not visible from the figure.)

The mass matrix of the electrically neutral fermion fields now reads:

$$m_\nu^D = \begin{array}{c} \\ \nu_L \\ N_{L1} \\ N_{L2} \\ \vdots \\ N_{Ln} \end{array} \begin{array}{c} \begin{array}{ccccc} N_{R0} & N_{R1} & N_{R2} & \cdots & N_{Rn} \end{array} \\ \begin{pmatrix} vY_0 & vY_1 & vY_2 & \cdots & vY_n \\ 0 & M_1 & 0 & \cdots & 0 \\ 0 & 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_n \end{pmatrix} \end{array}\;. \tag{28}$$

Concretely, a mass term for the active neutrinos is generated. Assuming that $M_k \gg Y_0 v$, which as we will see below is justified from the current limits on rare leptonic decays, one can approximate the active neutrino mass by

$$m_\nu \approx vY_0 \tag{29}$$

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and can be made small by choosing appropriate values of Y, q and n. For instance, assuming $Y = \mathcal{O}(1)$, $q = 2$, one obtains $m_\nu = \mathcal{O}(0.1)$ eV for $n \approx 40$.

The generalization of the above setup to three leptonic generations and N clockwork generations is straightforward. The clockwork Lagrangian is:

$$\mathcal{L}_{\text{clockwork}} = \mathcal{L}_{\text{kin}} - \overline{N_L^\alpha} m_\nu^\alpha N_R^\alpha + \text{h.c.} \tag{30}$$

with $N_L^\alpha = (\nu_L^\alpha, N_{L1}^\alpha, ..., N_{Ln}^\alpha)$ and $N_R^\alpha = (N_{R0}^\alpha, N_{R1}^\alpha, ..., N_{Rn}^\alpha)$, where

$$N_{Rk}^\alpha = \frac{1}{\sqrt{2}}(\chi_k^\alpha + \chi_{k+n}^\alpha)\;, \qquad k = 0,...,n \quad \alpha = 1...,N\;, \tag{31}$$

$$N_{Lk}^\alpha = \frac{1}{\sqrt{2}}(-\chi_k^\alpha + \chi_{k+n}^\alpha)\;, \qquad k = 1,...,n, \quad \alpha = 1...,N\;, \tag{32}$$

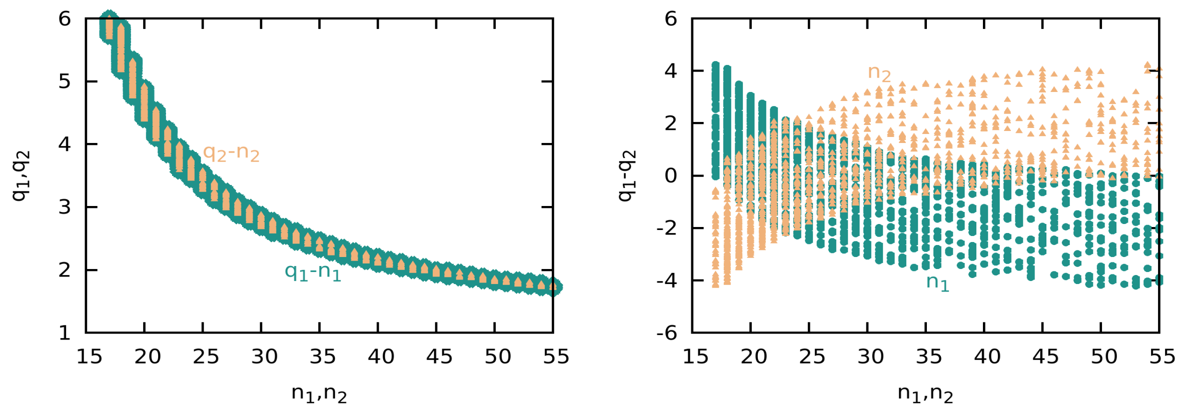

Figure 2: Values of q_1 and q_2 (left panel) and difference between them (right panel), as a function of n_1 and n_2, compatible with the measured values of the neutrino mass splittings and mixing angles within 1σ, for a scenario with two clockwork generations.

and the interaction Lagrangian,

$$\mathcal{L}_{\text{int}} = -\sum_{k=0}^{n} Y_k^{a\beta} \overline{L_L^a} \widetilde{H}_0 N_{Rk}^{\beta} \,, \tag{33}$$

with $Y_k^{a\beta} = Y^{a\alpha} \mathcal{U}_{nk}^{\alpha\beta}$.

After electroweak symmetry breaking the neutrino mass matrix reads:

$$m_\nu^D = \begin{matrix} \\ \nu_L^a \\ N_{L1}^\beta \\ N_{L2}^\beta \\ \vdots \\ N_{Ln}^\beta \end{matrix} \begin{matrix} \begin{matrix} N_{R0}^\beta & N_{R1}^\beta & N_{R2}^\beta & \cdots & N_{Rn}^\beta \end{matrix} \\ \begin{pmatrix} vY_0^{a\beta} & vY_1^{a\beta} & vY_2^{a\beta} & \cdots & vY_n^{a\beta} \\ 0 & M_1^\beta & 0 & \cdots & 0 \\ 0 & 0 & M_2^\beta & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_n^\beta \end{pmatrix} \end{matrix} . \tag{34}$$

where M_k^β is the mass of k-th clockwork gear for the Dirac pair N_L^β,N_R^β.

We analyze in detail the case where the clockwork consists of two generations with n_1 and n_2 gears, respectively. We scan $Y^{a\alpha}$ within the ranges $\frac{1}{4} < |Y^{a\alpha}| < 4$, q_α between 1.5 and 6 and n_α between 15 and 55, and we select the points that reproduce the observed values of the solar and atmospheric mass splitting and mixing angles within 1σ, as determined in Ref. [43]. In Fig. 2 (left panel) we show as green circles (yellow triangles) the values of n_1 (n_2) as a function of q_1 (q_2) that satisfy the experimental constraints. As apparent from the plot, larger q_α require a smaller number of gears to reproduce the small neutrino Yukawa coupling. Furthermore, the allowed values for n_1 and n_2 have a big overlap, which is a consequence of our assumption of comparable elements in the coupling $Y^{a\alpha}$ and the necessity of producing a mild hierarchy between the solar and the atmospheric neutrino mass scales. In particular, we find that the scenario with $q_1 = q_2$ and $n_1 = n_2$, namely the scenario where the clockwork parameters are universal also among generations, is allowed by observations. This is illustrated in Fig. 2 (right panel), which shows the allowed values of $q_1 - q_2$ as a function of n_1(green circle) and n_2(yellow triangle); the scenario with $n_1 = n_2$ and $q_1 = q_2$ corresponds to the region where the green circles and the yellow triangles overlap.

2.2 $M_{Li}, M_{Ri} \neq 0$ for some i Lepton number violation: Majorana neutrinos

In this case the mass matrix of the model is given by Eq. (15) and the Yukawa couplings by Eq. (27). Identifying $\widetilde{q}$ as the order parameter of the $U(1)_{\text{CW}}$ symmetry breaking, one can consider two limits of interest: $\widetilde{q} \ll q, 1$ and $\widetilde{q} \gg q, 1$.

Fig. 3 shows the masses of the singlet fermions (left panel) and their corresponding Yukawa couplings (right panel) for the specific case $n = 10$, $q = 2$, and $\widetilde{q} = 0.1$ (dark blue) or $\widetilde{q} = 10$ (light blue); the

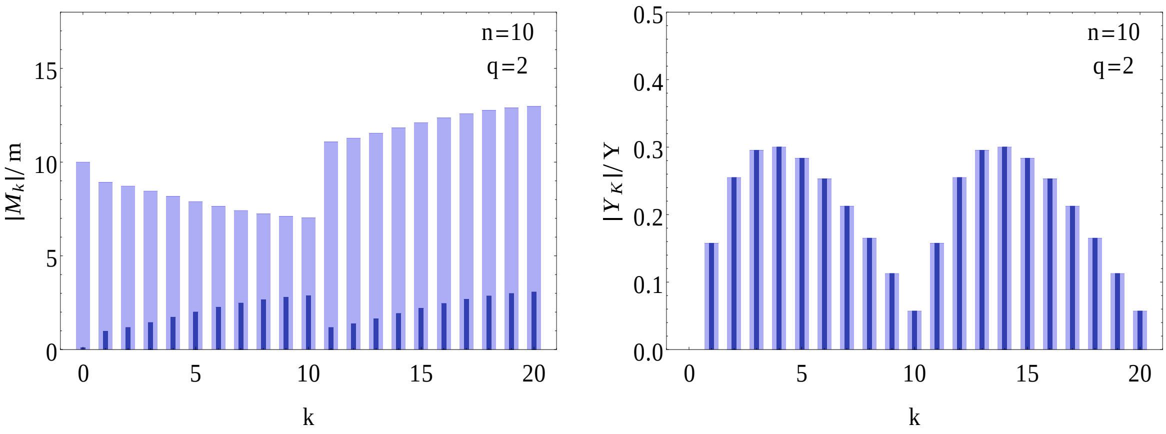

Figure 3: Majorana masses (left panel) and Yukawa couplings (right panel) of the singlet fermions of the clockwork sector, normalized respectively to m and Y, for the specific case $n = 10$, $q = 2$ and $\widetilde{q} = 0.1$ (dark blue) or $\widetilde{q} = 10$ (light blue).

former case corresponds to a mild breaking of the $U(1)_{\rm CW}$ symmetry and the latter to a strong breaking. For $\widetilde{q} = 0.1$ one notices that the mode k and the mode $n + k$ have very similar masses and suggest a pseudo-Dirac structure, which results from the mild $U(1)_{\rm CW}$ breaking; in the limit $\widetilde{q} \to 0$, they would form an exact Dirac pair and have identical masses. For $\widetilde{q} = 10$, however, the masses of all the modes are markedly different.

On the other hand, the Yukawa couplings of the singlet fermions to the left-handed leptons, shown in the right panel, do not depend on the value of $\widetilde{q}$, as demonstrated in subsection 2.1. The phenomenology of the scenario $\widetilde{q} \ll q, 1$ is then very similar to the one already discussed in subsection 2.1, while the phenomenology of the scenario $\widetilde{q} \gg q, 1$ can be rather distinct from the one in the (pseudo-)Dirac case. Indeed, in this scenario one obtains a mass for the active neutrino through the seesaw mechanism given by:

$$m_\nu \approx \sum_k \frac{Y_k^2 v^2}{M_k} \, . \tag{35}$$

Then, since the couplings for the higher modes are expected to be $\mathcal{O}(Y)$, the resulting neutrino mass can be orders of magnitude larger than the value inferred from oscillation experiments, unless $Y \ll 1$ and/or the gear masses are very large, in the same spirit as in the standard seesaw mechanism. A similar conclusion was also reached in [38].

3 Lepton Flavor Violation

The clockwork mechanism suppresses the Yukawa couplings for the zero mode, hence explaining the smallness of neutrino masses. However the Yukawa couplings for the higher modes are in general unsuppressed and can lead to observable effects at low energies. In particular, the lepton flavor violation generically present in the Yukawa couplings of the higher modes contributes, through quantum effects induced by clockwork fermions, to generate rare leptonic decays (such as $l_i \to l_j \gamma$) or μ-e conversion in nuclei, with rates that could be at the reach of current or future experiments if the gear masses are sufficiently low.

We calculate the rate for $l_i \to l_j \gamma$ following [44–46]. For N clockwork generations, we obtain:

$$B(\mu \to e\gamma) \simeq \frac{3\alpha_{\rm em} v^4}{8\pi} \left| \sum_{\alpha=1}^{N} \sum_{k=1}^{n_\alpha} \frac{Y_k^{e\alpha} Y_k^{\mu\alpha}}{M_k^{\alpha\,2}} F(x_k^\alpha) \right|^2 ,$$

where $\alpha_{\rm em}$ is the fine structure constant, n_α is the number of gears in the α-th generation, M_k^α is the mass of the k-th mode in the α-th generation ($k = 1, ..., n_\alpha$), and $x_k^\alpha \equiv M_k^{\alpha\,2}/M_W^2$. The loop function $F(x)$ is defined as

$$F(x) \equiv \frac{1}{6(1-x)^4}(10 - 43x + 78x^2 - 49x^3 + 4x^4 - 18x^3 \log x) \, , \tag{36}$$

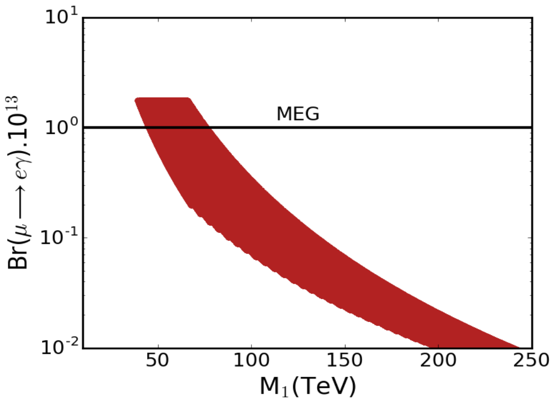

Figure 4: Predicted value of $Br(\mu \to e\gamma)$ for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear. The black solid line shows the current upper limit from the MEG experiment.

and has limits $F(0) = 5/3$ and $F(\infty) = 2/3$.

The current upper bound $\mathrm{Br}(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$ from the MEG experiment [47] poses stringent constraints on the mass scale of the clockwork. In Fig.4 we show the branching ratio expected for points reproducing the measured neutrino parameters, assuming two clockwork generations, as obtained in the scan presented in section 2.1, as a function of the mass of the first clockwork gear. It follows from the figure that the clockwork gears must be larger than ~ 40 TeV in order to evade the experimental constraints, unless very fine cancellations among all contributions to this process exist. For a larger number of clockwork generations we expect even stronger lower limits on the lightest gear mass, due to the larger number of particles in the loop.

4 Summary

The origin of small neutrino masses remains a mystery to this day. The recently proposed clockwork mechanism provides new insights into this puzzle, as it naturally generates small parameters in the effective Lagrangian. In the present work, we have scrutinized the mechanism of neutrino mass generation within the clockwork framework. We have generalized the clockwork formalism to include, in addition to Dirac masses and nearest neighbor interactions, also Majorana mass terms in the clockwork sector; and we have derived analytical expressions for the masses and couplings of the new singlet fermions for the specific case where the Dirac masses, Majorana masses and nearest neighbor interactions are universal among all clockwork "gears".

We have investigated in detail the impact of the Majorana masses in the clockwork sector in the generation of small neutrino masses. When the Majorana masses vanish, the zero mode of the clockwork sector is strictly massless and forms a Dirac pair with the active neutrino. In this framework, small Dirac neutrino masses can be generated for a sufficiently large number of gears, depending on the hierarchy between the mass scales in the clockwork sector. On the other hand, when the Majorana masses are non-vanishing, the zero mode is no longer massless. However, the corresponding Yukawa coupling still has the clockwork structure. In this case, small neutrino masses are the result of the interplay between the standard seesaw mechanism and the "clockworked" Yukawa couplings, and typically require very large Majorana masses in order to reproduce the small neutrino mass scale inferred from oscillation experiments.

The Standard Model leptons couple to the fermions of the clockwork sector with a site dependent strength, giving rise to (possibly lepton flavour violating) charged current, neutral current and Higgs boson interactions. We have investigated the constraints on this framework from the non-observation of the rare leptonic decay $\mu \to e\gamma$. Our results indicate that the lightest particle of the clockwork sector must have a mass $\gtrsim 40$ TeV, if the Yukawa couplings of the fundamental theory are $\mathcal{O}(1)$.

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