

Flavour alignment in multi-Higgs-doublets models

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- **Standard Model:** Great success but still
 - Strong CP problem
 - CP-violation
 - Dark Matter
 - (...)
- Doublets under the $SU(2)_L$ group $\longrightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \sum_i \frac{v_i^2 [T_i(T_i+1) - Y_i^2]}{2v_i^2 Y_i^2} = 1$

- Adding N Higgs doublets \longrightarrow

<p>3 Goldstone bosons + N - 1 charged scalars $\phi_a^\pm (H_a^\pm)$ + 2N - 1 neutral scalars $\rho_a, \eta_a (S_i^0, P_i^0)$</p>

$$\phi_a \equiv e^{i\theta_a} \left[\frac{1}{\sqrt{2}} (v_a + \rho_a + i\eta_a) \right], \quad v = \left(\sum_a v_a^2 \right)^{1/2} > 0.$$

$$\mathcal{L}_Y = - \sum_{a=1}^N \left\{ \bar{Q}'_L \left(\Gamma_a \phi_a d'_R + \Delta_a \tilde{\phi}_a u'_R \right) + \bar{L}'_L \Pi_a \phi_a \ell'_R + \text{h.c.} \right\},$$

↓
 $SU(N)$ transformation

Higgs basis

$$\Phi_a = \sum_{b=1}^N \Omega_{ab} e^{-i\tilde{\theta}_b} \phi_b, \quad \Omega^T \cdot \Omega = 1, \quad \sum_{a=1}^N \Omega_{1a} = v_a/v.$$

$$\Phi_1 = \left[\frac{1}{\sqrt{2}} (v + S_1^0 + i G^0) \right], \quad \Phi_{a>1} = \left[\frac{1}{\sqrt{2}} (S_a^+ + i P_a^0) \right].$$

FCNC at tree level (very constrained phenomenologically)

Yukawa Lagrangian

$$\begin{aligned} \mathcal{L}_Y = & -\frac{1}{v} \{ \bar{d}_L M_d d_R + \bar{u}_L M_u u_R + \bar{\ell}_L M_l \ell_R \} \\ & - \sum_{a=2}^N \frac{1}{v} \left\{ \sum_i^{2N-1} \mathcal{R}_{ia} \varphi_a^0 \left(\bar{d}_L Y_d^{(a)} d_R + \bar{u}_R Y_u^{(a)\dagger} u_L + \bar{\ell}_L Y_\ell^{(a)} \ell_R \right) \right. \\ & \left. + \frac{\sqrt{2}}{v} H_a^+ \left(\bar{u}_L V_{\text{CKM}} Y_d^{(a)} d_R - \bar{u}_R Y_u^{(a)\dagger} V_{\text{CKM}} d_L + \bar{\nu}_L Y_\ell^{(a)} \ell_R \right) \right\} + \text{h.c.}, \end{aligned}$$

Natural flavour conservation

Flavour alignment

Natural flavour conservation

- Discrete $\mathcal{Z}_2^d \otimes \mathcal{Z}_2^u \otimes \mathcal{Z}_2^\ell$ symmetry

$$\mathcal{Z}_2^f : f'_R \rightarrow -f'_R, \quad \phi_{af} \rightarrow -\phi_{af},$$



- Family universality
- Absence of CP-violation
- **Stable** under quantum corrections

$N = 2$: 2HDM (\mathcal{Z}_2 symmetry)

Type : $\{a_u, a_d, a_l\}$

Type I : $\{2, 2, 2\}$,

Type II : $\{1, 2, 1\}$,

Type X : $\{2, 2, 1\}$,

Type Y : $\{1, 2, 2\}$,

$$\varsigma_d = \varsigma_u = \varsigma_\ell = \cot \beta,$$

$$\varsigma_d = \varsigma_\ell = -\tan \beta, \quad \varsigma_u = \cot \beta,$$

$$\varsigma_d = \varsigma_u = \cot \beta, \quad \varsigma_\ell = -\tan \beta,$$

$$\varsigma_d = -\tan \beta, \quad \varsigma_u = \varsigma_\ell = \cot \beta.$$

Flavour alignment

- Alignment in flavour space

$$Y_{d,\ell}^{(a)} = \varsigma_{d,\ell}^{(a)} M_{d,\ell}, \quad Y_u^{(a)} = \varsigma_u^{(a)*} M_u,$$



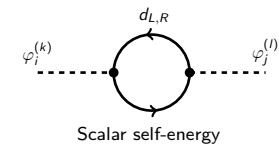
- Family universality
- Sources of CP-violation
- No FCNC at tree level

Most general structure (Family universality)

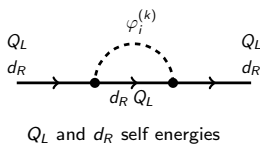
$$\varsigma_{d,\ell}^{(a)} \equiv Y_{d,\ell}^{(a)} M_{d,\ell}^{-1}, \quad \varsigma_u^{(a)\dagger} \equiv Y_u^{(a)} M_u^{-1}.$$

$$\varsigma_d^{(a)} = \text{diag}(\varsigma_d^{(a)}, \varsigma_s^{(a)}, \varsigma_b^{(a)}), \quad \varsigma_u^{(a)} = \text{diag}(\varsigma_u^{(a)}, \varsigma_c^{(a)}, \varsigma_t^{(a)}), \quad \varsigma_\ell^{(a)} = \text{diag}(\varsigma_e^{(a)}, \varsigma_\mu^{(a)}, \varsigma_\tau^{(a)}).$$

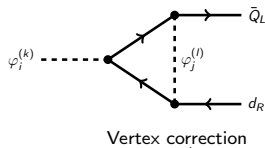
Renormalization group equations



$$\text{Tr} \left(Y_d^{(a)} Y_d^{(b)\dagger} + Y_u^{(a)} Y_u^{(b)\dagger} \right) Y_d^{(b)}$$



$$\begin{aligned} Q_L : & \left(Y_d^{(b)} Y_d^{(b)\dagger} + Y_u^{(b)} Y_u^{(b)\dagger} \right) Y_d^{(a)} \\ d_L : & Y_d^{(a)} Y_d^{(a)\dagger} Y_d^{(b)} \end{aligned}$$



$$V_{\text{CKM}}^\dagger Y_u^{(b)} Y_u^{(a)\dagger} V_{\text{CKM}} Y_d^{(b)} \rightarrow \text{FCNC}$$

FCNC Lagrangian, $N = 2$

$$\begin{aligned} \mathcal{L}_{\text{FCNC}} = & \frac{1}{4\pi^2 v^3} (1 + \zeta_u^* \zeta_d) \sum_{k=1}^3 \varphi_k^0 \left\{ C_d(\mu) (\mathcal{R}_{k2} + i\mathcal{R}_{k3}) (\zeta_d - \zeta_u) \bar{d}_L V_{\text{CKM}}^\dagger M_u M_u^\dagger V_{\text{CKM}} M_d d_R \right. \\ & \left. - C_u(\mu) (\mathcal{R}_{k2} - i\mathcal{R}_{k3}) (\zeta_d^* - \zeta_u^*) \bar{u}_L V_{\text{CKM}} M_d M_d^\dagger V_{\text{CKM}}^\dagger M_u u_R \right\} + \text{h.c.} \end{aligned}$$

$$C_f(\mu) = C_f(\mu_0) - \log \frac{\mu}{\mu_0}, \quad \mu_0 = \Lambda_A \leq M_{\text{Planck}} = 10^{19} \text{ GeV} \rightarrow C_f(M_W) \leq 40$$

Symmetries under flavour-phase transformations

$$\begin{aligned}
 f_X^i &\rightarrow e^{i\alpha_i^{f,X}} f_X^i, & Y_f^{(a),ij} &\rightarrow e^{i\alpha_i^{f,L}} Y_f^{(a),ij} e^{-i\alpha_j^{f,R}}, \\
 M_f^{ij} &\rightarrow e^{i\alpha_i^{f,L}} M_f^{ij} e^{-i\alpha_j^{f,R}}, & V_{\text{CKM}}^{ij} &\rightarrow e^{i\alpha_i^{u,L}} V_{\text{CKM}}^{ij} e^{-i\alpha_j^{d,L}}.
 \end{aligned}$$

$f = u, d, \ell$, $X = L, R$, $i, j \rightarrow$ fermion families

FCNC operators

$$\mathcal{O}_d^{n,m} = \bar{d}_L (\zeta_d)^{p_1} V_{\text{CKM}}^\dagger (\zeta_u^\dagger)^{p_n} (M_u M_u^\dagger)^n (\zeta_u)^{p'_n} V_{\text{CKM}} (\zeta_d)^{p_m} (M_d M_d^\dagger)^m (\zeta_d^\dagger)^{p'_m} (\zeta_d)^{p'_1} M_d d_R,$$

$$\mathcal{O}_u^{n,m} = \bar{u}_L (\zeta_u)^{p_1} V_{\text{CKM}} (\zeta_d)^{p_n} (M_d M_d^\dagger)^n (\zeta_d^\dagger)^{p'_n} V_{\text{CKM}}^\dagger (\zeta_u^\dagger)^{p_m} (M_u M_u^\dagger)^m (\zeta_u)^{p'_m} (\zeta_u^\dagger)^{p'_1} M_u u_R,$$

$$p_k, p'_k \leq k$$

Phenomenology of the A2HDM

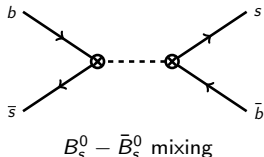
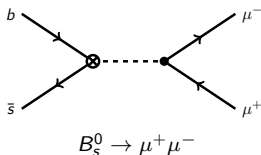
Aligned Two-Higgs Doublet Model (A2HDM)

Two doublets aligned in flavour space, in the CP-conserving limit,

$$Y_{d,\ell} = \underbrace{S_{d,\ell}}_{\text{real}} M_{d,\ell},$$

$$Y_u = \underbrace{S_u}_{\text{real}} M_u.$$

$$\mathcal{R}_{11} = \mathcal{R}_{22} = \cos \tilde{\alpha}, \quad \mathcal{R}_{21} = -\mathcal{R}_{12} = \sin \tilde{\alpha}, \quad \mathcal{R}_{33} = 1, \quad \mathcal{R}_{i3} = \mathcal{R}_{3i} = 0 \quad (i \neq 3)$$



- FCNC vertices at **one loop**
- Also scalar penguins and boxes (H^\pm) (needed to cancel divergences)

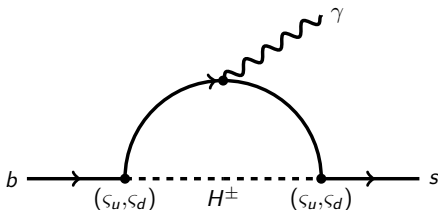
- FCNC vertices at **two loops**
- Box contributions (H^\pm) finite by themselves (GIM mechanism)

Phenomenology of the A2HDM

Mass configurations

Red :	$M_{H^\pm} = 100 \text{ GeV}$,	$M_H = 50 \text{ GeV}$,	$M_A = 50 \text{ GeV}$,
Green :	$M_{H^\pm} = 100 \text{ GeV}$,	$M_H = 200 \text{ GeV}$,	$M_A = 200 \text{ GeV}$,
Blue :	$M_{H^\pm} = 500 \text{ GeV}$,	$M_H = 500 \text{ GeV}$,	$M_A = 200 \text{ GeV}$,
Orange :	$M_{H^\pm} = 500 \text{ GeV}$,	$M_H = 200 \text{ GeV}$,	$M_A = 500 \text{ GeV}$.

+ constraints from $\bar{B} \rightarrow X_s \gamma$



Observable

$$\frac{\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)_{\text{A2HDM}}}{\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)_{\text{SM}}} = \left\{ |P|^2 + \left(1 - \frac{\Delta\Gamma_s}{\Gamma_L^s} \right) |S|^2 \right\},$$

$$P \subset C_{10}^{\text{A2HDM}}, C_P^{\varphi_i^0, \text{A2HDM}}$$

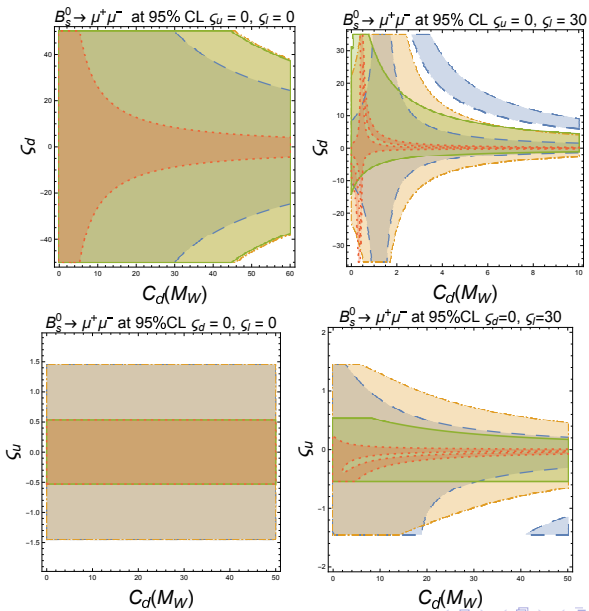
$$S \subset C_S^{\varphi_i^0, \text{A2HDM}}$$

$$\Delta C_{10}^{\text{A2HDM}} = |s_u|^2 \frac{x_t^2}{8} \left[\frac{1}{x_{H^+} - x_t} + \frac{x_{H^+}}{(x_{H^+} - x_t)^2} (\ln x_t - \ln x_{H^+}) \right].$$

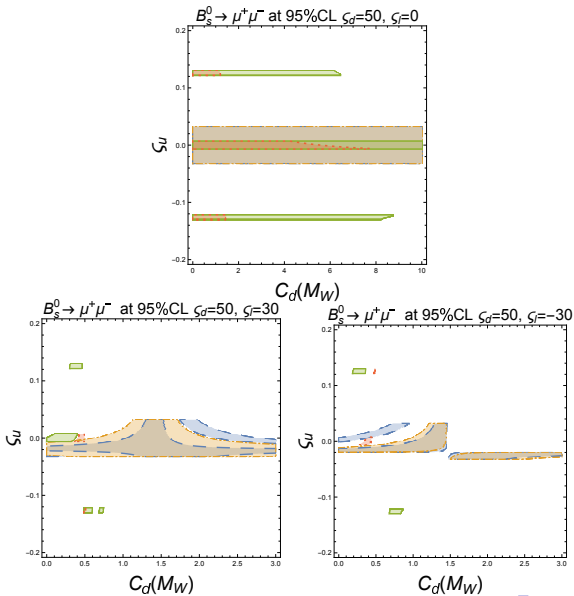
$$\Delta C_S^{\varphi_i^0, \text{A2HDM}} = \frac{x_t}{2x_h} (c_{\tilde{\alpha}} + s_{\tilde{\alpha}} s_\ell) \left\{ s_{\tilde{\alpha}} (s_u - s_d) (1 + s_u s_d) C_d(M_W) + \dots \right\},$$

$$\Delta C_P^{\varphi_i^0, \text{A2HDM}} = -s_\ell \frac{x_t}{2x_A} [(s_u - s_d) (1 + s_u s_d) C_d(M_W) + \dots],$$

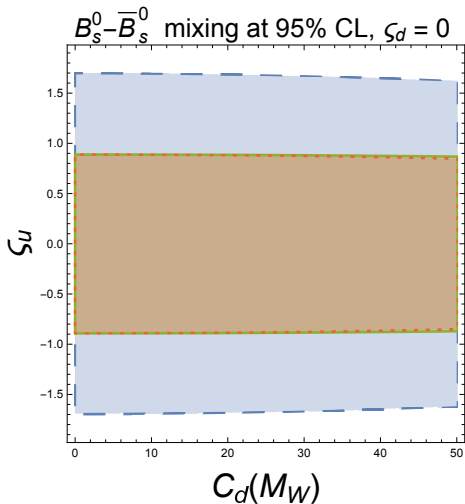
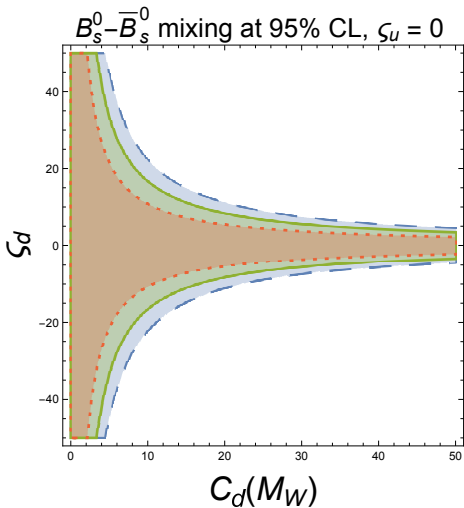
Phenomenology of the A2HDM. $B_s^0 \rightarrow \mu^+ \mu^-$



Phenomenology of the A2HDM. $B_s^0 \rightarrow \mu^+ \mu^-$



Phenomenology of the A2HDM. $B_s^0 - \bar{B}_s^0$ mixing



Summary

- Motivation for the study of extensions \rightarrow aspects of nature not explained only by the SM + freedom to extend the scalar sector
- NHDM: N doublets with the same quantum numbers as the Higgs doublet \rightarrow $2N-1$ scalar neutral particles (φ_i^0) + $N-1$ charged particles (H_i^\pm)
- In general FCNC at tree level, to avoid them:
 - Natural flavour conservation: $Z_2^d \otimes Z_2^u \otimes Z_2^\ell$ symmetry, stable under quantum corrections
 - Flavour alignment: one loop-misalignment
 - small quantum corrections (flavour-phase symmetry)
- Phenomenological constraints for the A2HDM
 - $B_s^0 \rightarrow \mu^+ \mu^-$
 - Meson mixing
- $\Lambda_A \leq M_{\text{Planck}} \sim 10^{19} \text{ GeV} \rightarrow C_d(M_W) = \log \frac{\Lambda_A}{M_W} \leq 40$ ✓

Back up

Multi-Higgs-doublets models: Original and Higgs basis

Original basis:

$$\phi_a = e^{i\theta_a} \left[\frac{1}{\sqrt{2}} (\nu_a + \rho_a + i\eta_a) \right].$$

$$\mathcal{L}_Y = - \sum_{a=1}^N \left\{ \bar{Q}'_L \left(\Gamma_a \phi_a d'_R + \Delta_a \tilde{\phi}_a u'_R \right) + \bar{L}'_L \Pi_a \phi_a \ell'_R + \text{h.c.} \right\},$$

$SU(N)$ transformation, so that just one doublet acquires a vev

$$\Phi_a = \sum_{b=1}^N \Omega_{ab} e^{-i\tilde{\theta}_b} \phi_b, \quad \phi_b = e^{i\tilde{\theta}_b} \sum_{a=1}^N \Omega_{ab} \Phi_a, \quad \Omega \cdot \Omega^T = \Omega^T \cdot \Omega = 1,$$

Multi-Higgs-doublets models: Original and Higgs basis

Higgs basis:

$$\Phi_1 = \left[\frac{1}{\sqrt{2}} (v + S_1^0 + i G^0) \right], \quad \Phi_{a>1} = \left[\frac{1}{\sqrt{2}} (S_a^0 + i P_a^0) \right].$$

In the Higgs basis the Yukawa structures:

$$\sum_{a=1}^N \Gamma_a \phi_a = \sum_{b=1}^N \hat{\Gamma}_b \Phi_b, \quad \sum_{a=1}^N \Delta_a \tilde{\phi}_a = \sum_{b=1}^N \hat{\Delta}_b \tilde{\Phi}_b, \quad \sum_{a=1}^N \Pi_a \phi_a = \sum_{b=1}^N \hat{\Pi}_b \Phi_b,$$

with

$$\hat{\Gamma}_b = \sum_{a=1}^N \Omega_{ba} e^{i\tilde{\theta}_a} \Gamma_a, \quad \hat{\Delta}_b = \sum_{a=1}^N \Omega_{ba} e^{-i\tilde{\theta}_a} \Delta_a, \quad \hat{\Pi}_b = \sum_{a=1}^N \Omega_{ba} e^{i\tilde{\theta}_a} \Pi_a.$$

$\mathbb{Z} \in$ symmetries for N doublets

$N \geq 3$: $3N-1$ inert doublets

$$\begin{aligned} \text{Type A : } & \{1, 1, 1\}, & \zeta_d^{(a)} = \zeta_u^{(a)} = \zeta_\ell^{(a)} = \Omega_{a1}/\Omega_{11} \\ \text{Type B : } & \{1, 2, 1\}, & \zeta_d^{(a)} = \zeta_\ell^{(a)} = \Omega_{a1}/\Omega_{11}, \zeta_u^{(a)} = \Omega_{a2}/\Omega_{12}, \\ \text{Type C : } & \{1, 1, 2\}, & \zeta_d^{(a)} = \zeta_u^{(a)} = \Omega_{a1}/\Omega_{11}, \zeta_\ell^{(a)} = \Omega_{a2}/\Omega_{12}, \\ \text{Type D : } & \{1, 2, 2\}, & \zeta_d^{(a)} = \Omega_{a1}/\Omega_{11}, \zeta_u^{(a)} = \zeta_\ell^{(a)} = \Omega_{a2}/\Omega_{12}, \\ \text{Type E : } & \{1, 2, 3\}, & \zeta_d^{(a)} = \Omega_{a1}/\Omega_{11}, \zeta_u^{(a)} = \Omega_{a2}/\Omega_{12}, \zeta_\ell^{(a)} = \Omega_{a3}/\Omega_{13}. \end{aligned}$$

Renormalization Group Equations

$$\begin{aligned}
 \mathcal{D}\Gamma_a &= a_\Gamma \Gamma_a + \sum_{b=1}^N \left[N_C \text{Tr} \left(\Gamma_a \Gamma_b^\dagger + \Delta_a^\dagger \Delta_b \right) + \text{Tr} \left(\Pi_a \Pi_b^\dagger \right) \right] \Gamma_b \\
 &\quad + \sum_{b=1}^N \left(-2 \Delta_b \Delta_a^\dagger \Gamma_b + \Gamma_a \Gamma_b^\dagger \Gamma_b + \frac{1}{2} \Delta_b \Delta_b^\dagger \Gamma_a + \frac{1}{2} \Gamma_b \Gamma_b^\dagger \Gamma_a \right), \\
 \mathcal{D}\Delta_a &= a_\Delta \Delta_a + \sum_{b=1}^N \left[N_C \text{Tr} \left(\Delta_a \Delta_b^\dagger + \Gamma_a^\dagger \Gamma_b \right) + \text{Tr} \left(\Pi_a^\dagger \Pi_b \right) \right] \Delta_b \\
 &\quad + \sum_{l=1}^N \left(-2 \Gamma_b \Gamma_a^\dagger \Delta_b + \Delta_a \Delta_b^\dagger \Delta_b + \frac{1}{2} \Gamma_b \Gamma_b^\dagger \Delta_a + \frac{1}{2} \Delta_b \Delta_b^\dagger \Delta_a \right), \\
 \mathcal{D}\Pi_a &= a_\Pi \Pi_a + \sum_{b=1}^N \left[N_C \text{Tr} \left(\Gamma_a \Gamma_b^\dagger + \Delta_a^\dagger \Delta_b \right) + \text{Tr} \left(\Pi_a \Pi_b^\dagger \right) \right] \Pi_b \\
 &\quad + \sum_{l=1}^N \left(\Pi_a \Pi_b^\dagger \Pi_b + \frac{1}{2} \Pi_b \Pi_b^\dagger \Pi_a \right),
 \end{aligned}$$

where $\mathcal{D} \equiv 16\pi^2 \mu (d/d\mu)$, being μ the renormalization scale, and $N_C = 3$ is the number of quark colours.

Renormalization Group Equations

$$\mathcal{L}_{\text{FCNC}} = \frac{1}{4\pi^2 v^3} \sum_{a=1}^N \sum_{k=1}^{N-1} \varphi_a^0 \left\{ C_d^{(a)} (\mathcal{R}_{a,2k} + \mathcal{R}_{a,2k+1}) \bar{d}_L \tilde{\Theta}_d^{(a)} M_d d_R \right. \\ \left. - C_u^{(a)} (\mathcal{R}_{a,2k} - \mathcal{R}_{a,2k+1}) \bar{u}_L \tilde{\Theta}_u^{(a)} M_u u_R \right\} + \text{h.c.},$$

$$\tilde{\Theta}_d^{(a)} = -V_{\text{CKM}}^\dagger \sum_{b=1}^N \zeta_u^{(b)\dagger} M_u M_u^\dagger \zeta_u^{(a)} V_{\text{CKM}} \zeta_d^{(b)} + \zeta_d^{(a)} V_{\text{CKM}}^\dagger \sum_{b=1}^N \zeta_u^{(b)\dagger} M_u M_u^\dagger V_{\text{CKM}} \zeta_d^{(b)} + \Delta \tilde{\Theta}_d^{(a)},$$

$$\tilde{\Theta}_u^{(a)} = -V_{\text{CKM}} \sum_{b=1}^N \zeta_d^{(b)} M_d M_d^\dagger \zeta_d^{(a)\dagger} V_{\text{CKM}}^\dagger \zeta_u^{(b)\dagger} + \zeta_u^{(a)\dagger} V_{\text{CKM}} \sum_{b=1}^N \zeta_d^{(b)} M_d M_d^\dagger V_{\text{CKM}}^\dagger \zeta_u^{(b)\dagger} + \Delta \tilde{\Theta}_u^{(a)},$$

$$\Delta \tilde{\Theta}_d^{(a)} = \frac{1}{4} \left[V_{\text{CKM}}^\dagger \left(\sum_{b=1}^N \zeta_u^{(b)\dagger} M_u M_u^\dagger \zeta_u^{(b)} \right) V_{\text{CKM}}, \zeta_d^{(a)} \right] = \frac{N}{4} \left[V_{\text{CKM}}^\dagger M_u M_u^\dagger V_{\text{CKM}}, \zeta_d^{(a)} \right]$$

$$\Delta \tilde{\Theta}_u^{(a)} = \frac{1}{4} \left[V_{\text{CKM}} \left(\sum_{b=1}^N \zeta_d^{(b)} M_d M_d^\dagger \zeta_d^{(b)\dagger} \right) V_{\text{CKM}}^\dagger, \zeta_u^{(a)\dagger} \right] = \frac{N}{4} \left[V_{\text{CKM}} M_d M_d^\dagger V_{\text{CKM}}^\dagger, \zeta_u^{(a)\dagger} \right]$$

Renormalization Group Equations

$$\tilde{\Theta}_d^{(a)} = \left(\zeta_d^{(a)} - \zeta_u^{(a)} \right) \left(\sum_{b=1}^N \zeta_u^{(b)\dagger} \zeta_d^{(b)} \right) V_{\text{CKM}}^\dagger M_u M_u^\dagger V_{\text{CKM}},$$

$$\tilde{\Theta}_u^{(a)} = \left(\zeta_u^{(a)\dagger} - \zeta_d^{(a)\dagger} \right) \left(\sum_{b=1}^N \zeta_u^{(b)\dagger} \zeta_d^{(b)} \right) V_{\text{CKM}} M_d M_d^\dagger V_{\text{CKM}}^\dagger.$$

Flavour symmetries

- In the absence of Yukawa couplings \rightarrow huge $SU(3)^5$ flavour symmetry
- $f_X \rightarrow S_{f_X} f_X$, $S_{f_X} \in SU(3)_{f_X}$

$$\begin{aligned} f_X^i &\rightarrow e^{i\alpha_i^{f,X}} f_X^i, & Y_f^{(a),ij} &\rightarrow e^{i\alpha_i^{f,L}} Y_f^{(a),ij} e^{-i\alpha_j^{f,R}}. \\ M_f^{ij} &\rightarrow e^{i\alpha_i^{f,L}} M_f^{ij} e^{-i\alpha_j^{f,R}}, & V_{\text{CKM}}^{ij} &\rightarrow e^{i\alpha_i^{u,L}} V_{\text{CKM}}^{ij} e^{-i\alpha_j^{d,L}}. \end{aligned}$$

The generalized alignment condition implies then

$$\zeta_f^{(a),ij} \rightarrow e^{i\alpha_i^{f,L}} \zeta_f^{(a),ij} e^{-i\alpha_j^{f,L}}.$$

FCNC operators of the form

$$\begin{aligned} \mathcal{O}_d^{n,m} &= \bar{d}_L (\zeta_d)^{p_1} V_{\text{CKM}}^\dagger (\zeta_u^\dagger)^{p_n} (M_u M_u^\dagger)^n (\zeta_u)^{p_{n'}} V_{\text{CKM}} (\zeta_d)^{p_m} (M_d M_d^\dagger)^m (\zeta_d^\dagger)^{p_{m'}} (\zeta_d)^{p_2} M_d d_R \\ \mathcal{O}_u^{n,m} &= \bar{u}_L (\zeta_u)^{p_1} V_{\text{CKM}} (\zeta_d)^{p_n} (M_d M_d^\dagger)^n (\zeta_d^\dagger)^{p_{n'}} V_{\text{CKM}}^\dagger (\zeta_u^\dagger)^{p_m} (M_u M_u^\dagger)^m (\zeta_u)^{p_{m'}} (\zeta_u^\dagger)^{p_2} M_u u_R \end{aligned}$$

or similar structures with additional factors of V_{CKM} , V_{CKM}^\dagger , $(M_f M_f^\dagger)$ and alignment matrices.

Example of 2HDM as a solution of DM

- A. Drozd, B.Grzadkowski1, J.F. Gunion and Y. Jiang (arXiv: 1408.2106)
- 2HDM type I or II + real gauge singlet S (2HDMS)
- S does not acquire a vev
- Extra symmetry $Z'_2 : S \rightarrow -S$ (remaining particles unchanged)