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In-flight cLFV conversion: $e - \mu$, $e - \tau$ and $\mu - \tau$ in minimal extensions of the Standard Model with sterile fermions

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Abstract

We revisit charged lepton flavour in-flight conversions, in which a beam of electrons or muons is directed onto a fixed target, $e + N \rightarrow \mu + N$, $e + N \rightarrow \tau + N$ and $\mu + N \rightarrow \tau + N$, focusing on elastic interactions with a nucleus N. After a general discussion of this observable, we carry a full phenomenological analysis in the framework of minimal Standard Model extensions via sterile neutrinos, with a strong emphasis on the rôle of the increasingly more stringent constraints arising from other (low-energy) charged lepton flavour violation observables. Despite the potential interest of this observable, in particular in the light of certain upcoming facilities with the capability of very intense lepton beams, our study suggests that due to current bounds on three-body decays ($\ell_i \rightarrow 3\ell_j$) and $\mu - e$ conversion in Nuclei, the expected number of conversions in such a minimal framework is dramatically reduced. An experimental observation of such a conversion would thus signal the presence of another source of flavour violation, possibly at tree-level.

1 Introduction

The quest for a Standard Model (SM) extension capable of addressing its several observational caveats has fuelled intensive experimental searches, encompassing high-energy colliders, high-intensity facilities, as well as numerous astroparticle and cosmological searches.

So far, no direct evidence for the new states has been unveiled in collider searches, and this has in turn intensified the interest for the so-called indirect searches, in which very rare processes, strongly suppressed or even forbidden in the SM, are looked for. Among the many observables that are being studied and explored, those signaling the violation of lepton flavour are powerful

probes of New Physics (NP), sensitive to new scales often lying well beyond collider reach. Numerous processes are currently being searched for in high-intensity facilities, and these include charged lepton flavour violating (cLFV) radiative decays, three-body decays and nuclear-assisted transitions; likewise, a vast array of rare transitions and decays is being looked for at high-energy colliders[1]. The current bounds are already impressive, and many running and/or upcoming experiments (as is the case of MEG II, Mu3e, Mu2e, COMET and LHCb) should improve them in the near future. In Table 1 we summarise the present experimental bounds and future sensitivities for several radiative and 3-body cLFV decays (which will be relevant for our subsequent discussion).

cLFV Process	Present Bound	Future Sensitivity
$\mu \to e\gamma$	4.2×10^{-13} [1]	6×10^{-14} [2]
$\tau \to e\gamma$	3.3×10^{-8} [3]	$\sim 3 \times 10^{-9}$ [4]
$\tau \to \mu\gamma$	4.4×10^{-8} [3]	$\sim 3 \times 10^{-9}$ [4]
$\mu \to eee$	1.0×10^{-12} [5]	$\sim 10^{-16}$ [6]
$\tau \to \mu\mu\mu$	2.1×10^{-8} [7]	$\sim 10^{-9}$ [4]
$\tau \to eee$	2.7×10^{-8} [7]	$\sim 10^{-9}$ [4]
$\mu - e$	7×10^{-13} (Au) [8]	$\sim 10^{-14}$ (SiC) [9] $\sim 10^{-17}$ (Al) [10–12]

Table 1: Current experimental bounds and future sensitivities for several cLFV processes, which are considered in this study.

The probing power of cLFV has been at the source of an increasing interest for these processes, leading to further explorations of already existing observables, or to the study of new ones. This was the case of the Coulomb enhanced decays of a muonic atom into two electrons [13, 14], or the lepton flavour and lepton number violation $\mu^- - e^+$ conversion in Nuclei [15–18].

In the wake of the discovery of $\nu_\mu - \nu_\tau$ oscillations - and of large mixing in the neutral lepton sector - the study of cLFV τ lepton production in $\mu + N \to \tau + N$ (with N denoting a generic nucleon) at high energies [19] was originally proposed. The experimental signature for the $\mu - \tau$ cLFV in-flight conversion would be that of a final state composed by a single muon (the tau, despite its large energy, rapidly decaying, $\tau \to \mu\nu\nu$), with a dramatic loss in energy when compared to that of the primary muon beam - the energy loss on target corresponding to the production and subsequent decay of the heavier lepton. First studies also focused on the quasi-elastic in-flight conversion, due to the simpler final state topology and to the associated background. The possibility of having high-intensity (and sufficiently energetic) muon beams (for instance at muon and future neutrino factories) further fuelled the interest for such cLFV observables; as argued in [20], a 50 GeV muon beam, with an expected intensity of 10^{20} muons on target per year could lead to a significant number of $\mu + N \to \tau + N$ events. The original estimation was based on an effective approach, and preceeded the recent stringent bounds on cLFV transitions (many of them collected in Table 1).

Other pioneering studies of in-flight cLFV conversion focused on leptoquark models [19, 21], also highlighting the potential of flavour violating constructions such as R-parity (R_P)-violating supersymmetry, or flavour-violating Higgs interactions. Following the model-independent approach of [20], the prospects of supersymmetric extensions of the SM for $\mu + N \to \tau + X_{\text{final}}$ were discussed [22] in the deep-inelastic scattering (DIS) regime, as were those of low-energy electron-nucleus scattering to probe $e \to \mu$ conversion [23]. In Ref. [24], the impact of massive neutrinos

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[1]Other rare processes, such as those violating lepton flavour universality (LFUV), or total lepton number (LNV), can also emerge in relation with cLFV transitions and decays.

historical motivation: Gninenko et al. 0106302 due to the short lifetime of the tau, the neutrinoless conversion τ–N → μ–N was soon discarded

Although kinematically impossible at rest, μ–(E) + (A,Z) → τ–(E′) + Xh may be feasible. Gninenko considered the four-fermi interaction (μ⁻τ)(u⁻c), involving production of a charmed quark

Shear and Turan 0309183: as many as 10^7 events. Bkg may be severe (The major difficulty is identifying a clear signature). Use effective operators. vertex involving valence quarks

was first considered; contributions arising in the framework of a typical type I seesaw (albeit for low right-handed neutrino masses) were found to enhance those emerging from the presence of three light massive Dirac neutrinos, assuming a CKM-like lepton mixing, by as much as twenty orders of magnitude in the case of dominanting photon contributions. Other studies in the DIS regime focused on cLFV conversions induced by "unparticles" [25]. Recent analyses, again based on an effective-Lagrangian approach, included a detailed discussion of the process' kinematics and hadronic contributions [26]. Associated experimental issues (including a brief overview of backgrounds), and future prospects were discussed in [27].

In view of recent phenomenological and experimental developments, which have led to increasingly severe bounds on the scale of NP mediators and to strong constraints on the strenght of possible cLFV couplings, a re-analysis of the in-flight cLFV conversion - and its potential impact on SM extensions - is clearly justified. Expected experimental prospects (such as the capability of high-intensity, high-energy muon beams [22], or a possible electron-ion collider [21]), further motivate revisiting this observable.

Although not necessarily linked to the problem of neutrino masses and mixings (which signal the violation of neutral lepton flavours), cLFV can also emerge in association with SM extensions incorporating a mechanism of neutrino mass generation. Minimal extensions of the SM via additional sterile fermion states are an appealing class of models, in particular those that succeed in explaining oscillation data by the introduction of (not excessively) heavy states. Numerous studies have examined the impact of these models regarding several cLFV observables [28–37], focusing either on specific realisations, or then evaluating the potential contributions of sterile fermions via model-independent, simple constructions (the so-called "3+N" models).

In the present work we thus revisit cLFV in-flight conversions $\ell_i \to \ell_j$, carrying a full phenomenological analysis in the framework of SM extensions via sterile neutrinos. In particular, we focus on flavour violating (FV) Z- and photon-mediated interactions, recomputing their contributions, and comparing our results to previous studies. We consider the three different cLFV channels ($e - \mu$, $e - \tau$ and $\mu - \tau$), and discuss the corresponding experimental prospects, confronting the latter with other cLFV observables. Our work is organised as follows: after describing the underlying theoretical framework in Section 2, we discuss $\ell_i \to \ell_j$ in-flight conversions, including contributions to the differential cross section and general features of the observables. The experimental prospects, as well as a comparative study with other cLFV observables in minimal SM extensions via sterile neutrinos are collected in Section 4; a brief overview as well as further elements of discussion are summarised in the Conclusions. The relevant expressions of the γ- and Z-mediated interactions, together with other relevant form factors, can be found in Appendices A and B.

2 Minimal SM extensions via sterile fermions

Motivated by several cosmological and experimental observations, sterile fermions are present as constituent blocks of many SM extensions which encompass a mechanism of neutrino mass generation. If on the one hand sterile neutrinos can indeed provide an explanation to the problem of neutrino masses and mixings, they can also open the door to a rich phenomenology, with potential effects in a large number of observables. This is a direct consequence of their mixings with the light (mostly active) neutrinos, which - if non-negligible - lead to the violation of lepton flavour in both neutral and charged leptonic currents [29, 38].

In the presence of n_S additional sterile (Majorana) neutrinos, the vector and scalar currents

are modified as follows[2] (working in the physical basis, i.e., for mass eigenstates):

$$\mathcal{L}_{W^{\pm}} = -\frac{g_w}{\sqrt{2}} W_{\mu}^{-} \sum_{\alpha=1}^{3} \sum_{j=1}^{3+n_S} \mathbf{U}_{\alpha j} \bar{\ell}_{\alpha} \gamma^{\mu} P_L \nu_j + \text{H.c.},$$

$$\mathcal{L}_{Z^0} = -\frac{g_w}{2\cos\theta_w} Z_{\mu} \sum_{i,j=1}^{3+n_S} \bar{\nu}_i \gamma^{\mu} \left(P_L \mathbf{C}_{ij} - P_R \mathbf{C}_{ij}^{*} \right) \nu_j - \frac{g_w}{4\cos\theta_w} Z_{\mu} \sum_{\alpha=1}^{3} \bar{\ell}_{\alpha} \gamma^{\mu} \left(\mathbf{C}_V - \mathbf{C}_A \gamma_5 \right) \ell_{\alpha},$$

$$\mathcal{L}_{H^0} = -\frac{g_w}{2M_W} H \sum_{i,j=1}^{3+n_S} \mathbf{C}_{ij} \bar{\nu}_i \left(P_R m_i + P_L m_j \right) \nu_j + \text{H.c.}. \tag{1}$$

In the above, g_w denotes the weak coupling constant, $\cos^2\theta_w = M_W^2/M_Z^2$, $P_{L,R} = (1 \mp \gamma_5)/2$, and m_i are the physical neutrino masses (light and heavy); the indices α denote the flavour of the charged leptons, while $i, j = 1, \ldots, 3 + n_S$ correspond to the physical (massive) neutrino states. In addition $\mathbf{C}_V$ and $\mathbf{C}_A$ are the SM coefficients parametrizing the vector and axial-vector Z-couplings of charged leptons, $\mathbf{C}_V = \frac{1}{2} + 2\sin^2\theta_w$ and $\mathbf{C}_A = \frac{1}{2}$. Finally, a rectangular $3 \times (3 + n_S)$ mixing matrix, $\mathbf{U}_{\alpha j}$, parametrizes the mixing in charged current interactions (corresponding to the (unitary) PMNS matrix, U_{PMNS} in the case of $n_S = 0$); the mixing between the left-handed leptons corresponds to a 3×3 block of $\mathbf{U}$, usually denoted $\tilde{U}_{\text{PMNS}}$. The structure of $\mathbf{U}_{\alpha j}$ is at the source of lepton flavour violation in neutral currents, which, as seen from above, is now parametrized by

$$\mathbf{C}_{ij} = \sum_{\alpha=1}^{3} \mathbf{U}_{\alpha i}^{*} \mathbf{U}_{\alpha j}. \tag{2}$$

2.1 Constraints on sterile fermions

Due to the presence of the additional sterile states, the modified neutral and charged lepton currents might lead to new contributions to a vast array of observables, possibly in conflict with current data. These SM extensions via sterile fermions must be then confronted to all available constraints arising from high-intensity, high-energy and cosmological observations.

In our subsequent phenomenological analysis, and for the theoretical framework considered, we ensure that compatibility with the following constraints - theoretical (such as perturbativity of the active-sterile couplings) and experimental - is verified at all times.

Sterile states, with a mass above the electroweak (EW) scale, can have sizeable decay widths, a consequence of being sufficiently heavy to decay into a $W^{\pm}$ boson and a charged lepton, or into a light (active) neutrino and either a Z or a Higgs boson. One thus imposes the perturbative unitarity condition [39–44], $\frac{\Gamma_{\nu_i}}{m_{\nu_i}} < \frac{1}{2}$ $(i \geq 4)$. Noticing that the leading contribution to Γ_{ν_i} is due to the charged current term, one obtains the following bounds [39–44]:

$$m_{\nu_i}^2 \mathbf{C}_{ii} < 2\frac{M_W^2}{\alpha_w} \qquad (i \geq 4), \tag{3}$$

where $\alpha_w = g_w^2/4\pi$, and $\mathbf{C}_{ii}$ is given in Eq. (2).

Observational constraints on the sterile masses and their mixings with the active states arise from an extensive number of sources. Firstly, and other than requiring compatibility between

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[2]Likewise, the interactions with neutral and charged Goldstone bosons are also modified: $\mathcal{L}_{G^0} = \frac{ig_w}{2M_W} G^0 \sum_{i,j=1}^{3+n_S} \mathbf{C}_{ij} \bar{\nu}_i \left(P_R m_j - P_L m_i \right) \nu_j + \text{H.c.}; \mathcal{L}_{G^{\pm}} = -\frac{g_w}{\sqrt{2}M_W} G^{-} \sum_{\alpha=1}^{3} \sum_{j=1}^{3+n_S} \mathbf{U}_{\alpha j} \bar{\ell}_{\alpha} \left(m_i P_L - m_j P_R \right) \nu_j + \text{H.c.}.$

the left-handed lepton mixing matrix $\tilde{U}_{\rm PMNS}$ and the corresponding best-fit intervals[3] defined from *ν-oscillation data* [45–51], we also impose, when relevant, *unitarity bounds* as arising from non-standard neutrino interactions with matter, on the deviation of $\tilde{U}_{\rm PMNS}$ from unitarity [52–54]. Further constraints on the active-sterile mixings (and on the mass regime of new states) arise from *electroweak precision observables*; these include new contributions to the invisible Z-decay width (addressed in [55–58]), which must comply with LEP results on $\Gamma(Z \to \nu\nu)$ [59]; moreover, any contribution to cLFV Z decay modes should not exceed the present uncertainty on the total Z width [59], $\Gamma(Z \to \ell_1^\mp \ell_2^\pm) < \delta\Gamma_{\rm tot}$. In our study we also take into account current limits on *invisible Higgs* decays (relevant for $m_{\nu_s} < M_H$), following the approach derived in [60–62]. Likewise, negative results from *laboratory searches* for monochromatic lines in the spectrum of muons from $\pi^\pm \to \mu^\pm \nu$ decays are also taken into account [63,64]. As mentioned in the Introduction, the new states (through the modified currents) induce potentially large contributions to cLFV observables; we evaluate the latter [28–35,37] imposing available limits on a wide variety of observables (some of them collected in Table 1). In addition to the cLFV decays and transitions, which can prove instrumental to test and disentangle these extensions of the SM, important constraints arise from rare *leptonic and semileptonic decays of pseudoscalar mesons* decays (including lepton universality violating, cLFV and lepton number violating modes); we include constraints from numerous K, D, D_s, B modes (see [65,66] for kaon decays, [67,68] for D and D_S decay rates, and [69,70] for B-meson observations), stressing that in the framework of the SM extended by sterile neutrinos particularly severe constraints arise from the violation of lepton universality in leptonic kaon decays (parametrized by the observable Δr_K) [58,71]. Finally, we also take into account the recent constraints on *neutrinoless double beta decay* [72]: should the sterile states be Majorana fermions, they can potentially contribute to to the effective mass m_{ee} [73], which we evaluate following [74,75].

A number of *cosmological observations* [63,76–78] put severe constraints on sterile neutrinos with a mass below the GeV (in particular below 200 MeV). In our study we will in general explore regimes associated with heavier sterile states ($m_{\nu_s} \gtrsim 0.5$ GeV) so that these constraints are not expected to play a relevant rôle.

2.2 Theoretical framework: simple "3+1 model"

Several mechanisms of neutrino mass generation, which in addition to accommodating neutrino data, also address in the Baryon asymmetry of the Universe and/or put forward a viable dark matter candidate, call upon sterile fermions. Among such models, one encounters appealing SM extensions such as the Inverse Seesaw (ISS) [79], the neutrino minimal SM (νMSM) [80], or several low-scale type I seesaw variants.

As done in previous studies of cLFV in SM extensions via sterile neutrinos, one can use as a first phenomenological approach a minimal "toy model", consisting in the addition of a single Majorana sterile neutral fermion to the SM field content [36,37]. This ad-hoc construction - which will be adopted in the present study - makes no assumption on the mechanism of neutrino mass generation, relying on the simple hypothesis that the interaction eigenstates and the physical ones are related via a 4×4 unitary mixing matrix, $\mathbf{U}_{ij}$. Other than the masses of the three light (mostly active) neutrinos, and their mixing parameters, the simple "3+1 model" can be parametrized via the heavier (mostly sterile) neutrino mass m_4, three active-sterile mixing angles as well as three new CP violating phases (two Dirac and one Majorana). In the numerical analyses we will in general consider a normal ordering for the light neutrino spectra; in what concerns the

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[3]We do not impose any constraints on the (yet undetermined) value of the CP violating Dirac phase δ.
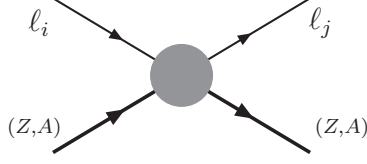
Figure 1: Effective interaction leading to an elastic "on target" cLFV transition, $\ell_i + N \to \ell_j + N$, in which j denotes the heavier charged lepton, and (Z, A) the atomic number and weight of the target nucleus, N.

new degrees of freedom, we will scan over the following range for the mass of the additional heavy state,

$$0.5 \text{ GeV} \lesssim m_4 \lesssim 10^6 \text{ GeV}, \tag{4}$$

while the active-sterile mixing angles are randomly taken to lie in the interval $[0, 2\pi]$ (as are the different CP violating phases).

3 cLFV in flight $\ell_i \to \ell_j$ conversion

In what follows we summarise the most relevant points regarding the computation of the observables associated with the in-flight cLFV conversion; due to the underlying process, in which an intense lepton beam hits a fixed target, the observable is also frequently referred to as an "on target" cLFV transition, $\ell_i + N \to \ell_j + N^{(\prime)}$. As mentioned in the Introduction, there are several possibilities regarding the final state of the nuclei (target) after interaction with the energetic ℓ_i beam: elastic scattering, in which $N = N'$; quasi-elastic scattering, leading to a final state target composed of several bodies (but conserving the total number of nucleons, with no new hadronic states); inelastic processes (including excited nuclear states), and/or nuclear fragmentation with associated pion or other light hadron production (DIS regime). In the present phenomenological analysis we will focus on the case of elastic scatterings; quasi-elastic processes (as well as inelastic ones) were also recently addressed in the study of [26]). The interaction is schematically depicted in Fig. 1.

The kinematics of the in-flight cLFV conversion requires the beam to have a minimal threshold energy (which depends on the nature of the target and on the mass of final state lepton). Denoting the intervening quadri-momenta as

$$\ell_i(k) + T(p) \to \ell_j(k') + T(p'), \quad \text{with} \quad Q^2 = -q^2 = -(k - k')^2 = 2\, M_T\, \Delta E_{\text{beam}}, \tag{5}$$

with ΔE_{beam} the energy loss of the beam, and M_T the target's mass, one thus finds that the (threshold) beam energy is[4]

$$E_{\text{beam}} > m_{\ell_j} \left(1 + \frac{m_{\ell_j}}{2M_T}\right), \tag{6}$$

in which m_{ℓ_j} denotes the mass of the heavier lepton in the final state (muon or tau). Moreover, a non-zero momentum transfer to the nuclear system is unavoidable. Depending on the beam's energy, and the composition of the target, one finds minimal values for the energy transfer -

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[4]While Eq. (6) leads to an effective lower bound to the beam energy, as previously mentioned we will not enter high-energy regimes leading to DIS phenomena.

although these do decrease with increasing beam energy and with the (larger) size of the nuclei, non-zero values of Q^2 are always obtained (see [26] for a comprehensive discussion).

From an effective theory approach, the cLFV conversion depicted in Fig. 1 can be parametrized as [20]

$$\frac{C_{ij}^{\mathrm{NP}}}{\Lambda_{\mathrm{NP}}} (\bar{\ell}_i \Gamma \ell_j)(\bar{q} \Gamma q'),\tag{7}$$

with C_{ij}^{NP} the coefficients of the higher-dimensional effective operators responsible for the cLFV $\ell_i + N \to \ell_j + N$ transition, Λ_{NP} the typical NP scale, and Γ denoting combinations of Dirac matrices. In the framework of NP models in which cLFV occurs via higher-order (loop) transitions (as is the case of R-parity conserving SUSY, seesaw realisations, etc.), the differential cross section for the cLFV conversion of Eq. (5) receives contributions from different processes, depending on the interaction(s) at the source of flavour violation: dipole, Z- and Higgs-penguins, box diagrams, among other contributions. In what follows, we proceed to discuss some of them.

The differential cross section for the on-target conversion of $\ell_i \to \ell_j$, exclusively due to photonic exchanges (i.e., putting to zero all other contributions), can be written as [26]

$$\left.\frac{d\sigma^{i \to j}}{dQ^2}\right|_\gamma = \frac{\pi Z^2 \alpha^2}{Q^4 E_{\mathrm{beam}}^2} H_{\mu\nu}^\gamma L_{ij}^{\gamma\mu\nu},\tag{8}$$

in which Z denotes the target atomic number. The detailed expression for the hadronic tensor $H_{\mu\nu}^\gamma$ can be found in the Appendix A, while the leptonic tensor can be decomposed as

$$L_{ij}^{\gamma\mu\nu} = L_{ij}^\gamma L^{\gamma\mu\nu}(k,q),\tag{9}$$

in which L_{ij}^γ encodes the cLFV (effective) couplings. Important contributions to the on-target cLFV conversion arise from the Z-mediated interaction. Likewise, and in the limiting case in which only Z-interactions are present, one can write

$$\left.\frac{d\sigma^{i \to j}}{dQ^2}\right|_Z = \frac{G_F^2}{32 \pi E_{\mathrm{beam}}^2} H_{\mu\nu}^Z L_{ij}^{Z\mu\nu},\tag{10}$$

with

$$L_{ij}^{Z\mu\nu} = L_{ij}^Z L^{Z\mu\nu}(k,q),\tag{11}$$

where, and as before, the terms L_{ij}^Z encode the cLFV couplings. Other contributions, such as Higgs mediated interactions (as in the case of SUSY models), box diagrams, etc., might be also present and, depending on the given model (and regime), play a relevant rôle.

The $L_{ij}^{\gamma,Z}$ couplings can be interpreted as generic sources of flavour violation at the origin of the cLFV in-flight conversion, in the framework of SM extensions in which cLFV receives important (if not dominant) contributions from penguin loop diagrams; however, and in what follows we focus on a minimal NP model: the SM minimally extended by additional (massive) neutrinos. In such a framework, the most important contributions indeed arise from Z and photon mediated interactions, $W^\pm$ mediated box diagrams, and corrections to the lepton propagators[5], some of

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[5] In our analysis, we will not take into account the contributions arising from the "handbag" (box) diagrams: in the limiting (unrealistic) case of a real quark, these diagrams would correspond to the usual box contributions common to several observables (such as $\mu - e$ conversion in Nuclei, or $\mu \to 3e$ decays). In minimal SM extensions via sterile fermions, and in the large sterile mass regime - which has been shown to be associated with sizeable contributions to the above mentioned decays - the box contributions typically lead to subdominant contributions when compared to the γ and Z penguins [35–37]. Other regimes are known to be associated with important box-diagram contributions [35].
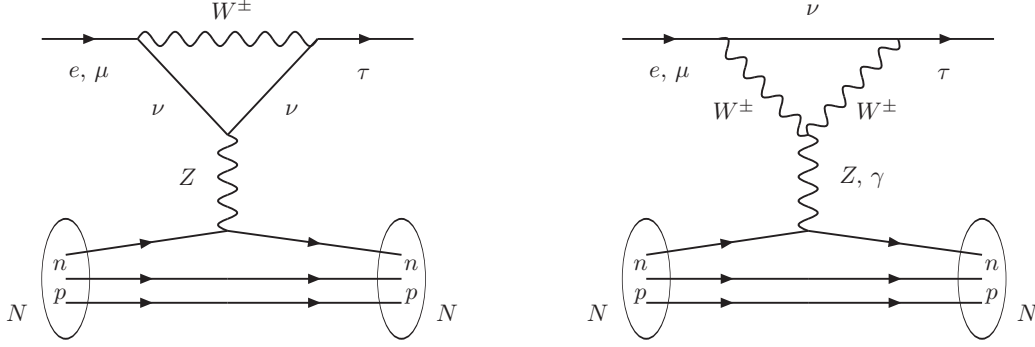
Figure 2: Contributions to the $\ell_i - \tau$ conversion from Z- and γ-penguins.

them schematically depicted in Fig. 2. In this context, L_{ij}^{γ} can be cast as

$$L_{ij}^{\gamma} = \frac{\alpha_w^3 \, s_w^2}{64 \, \pi \, e^2} \frac{m_{\ell_j}^2}{M_W^4} \left| G_{ji}^{\gamma} \right|^2 \, , \tag{12}$$

with G_{ji}^{γ} denoting the photon-lepton dipole coupling, also contributing to other cLFV transitions such as $\ell_j \to \ell_i \gamma$, and which is given in Appendix B; the flavour violating Z-couplings can be written

$$L_{ij}^{Z} = \frac{\alpha_w^4}{G_F^2 \, M_W^4} \frac{2(-1/2 + \sin_w^2)^2 + \sin_w^4}{64} \left| F_{ji}^{Z} \right|^2 \, , \tag{13}$$

in which F_{ji}^{Z} denotes the form factor encoding flavour violating $Z\ell_j\ell_i$ interactions, which is also present in several other cLFV observables (see Appendix B). The full expressions for $d\sigma^{i \to j}/dQ^2|_{\gamma,Z}$, as well as that of full leptonic and hadronic tensors are given in Appendix A.

As is clear from the above discussion concerning the cLFV couplings, current bounds on many low-energy observables (see Table 1) will play a very constraining rôle on the maximal viable values for the in-flight conversion cross section. Particularly relevant will prove to be the bounds from $\ell_j \to 3\ell_i$ decays, radiative decays, as well as $\mu - e$ conversion in Nuclei.

Before entering the study of the prospects for the cLFV on-target conversion in extensions of the SM via sterile fermions, we briefly discuss some issues regarding the nuclear interaction and the beam energy, which can be already understood from the differential cross section, $d\sigma^{i \to j}/dQ^2$. The nuclear tensors - for both photon and Z-mediated interactions - can be computed for either spin 0 and spin 1/2 targets. In our phenomenological study, we consider elastic interactions with individual nucleons, that is with spin 1/2 protons and/or neutrons (which corresponds to setting $M_T = M_{p,n}$ and $Z = 1$ in the relevant equations). The individual differential cross sections, corresponding to the purely Z- or γ-mediated exchanges, for $\mu - \tau$ conversion on a neutron target, are displayed on the left panel of Fig. 3 as a function of the momentum transfer, Q^2, and for two different beam energies, $E = 4, 6$ GeV. These have been evaluated by simply setting by hand, in a model-blind manner, maximal values for the flavour violating terms $L_{ij}^{Z,\gamma}$, see Eqs. (12, 13). (Leading to the results displayed in this section, no observational bounds have been applied.) Although depending on the actual SM extension under consideration (and in the specific case of additional sterile fermions, on the particular mass regime), Z-mediated FV conversions often prove to dominate over the photonic ones (see [35–37]). Unless otherwise stated, in the following numerical discussion, we will in general consider that Z-penguins provide the dominant contributions to the observables under study.
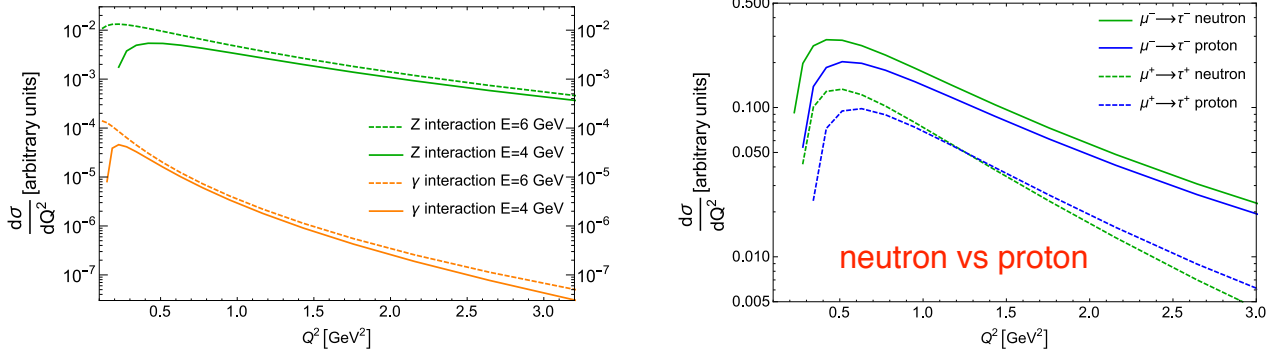
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Figure 3: Differential cross section (arbitrary units) for nucleon targets as a function of the momentum transfer, Q^2. On the left panel, comparison of the individual Z (green) and photon (orange) interactions, for $\mu - \tau$ conversion on a neutron target; full (dashed) lines correspond to $E = 4\,(6)$ GeV beam energy. On the right panel, Z-mediated $\mu - \tau$ conversion for neutron (green) and proton (blue) targets, with full (dashed) lines correspond to lepton (anti-lepton) conversion.

On the right panel of Fig. 3 we compare the Z-mediated contribution for the individual nucleons (proton and neutron). In view of the very similar behaviour for both nucleons, in the following we will for simplicity assume a neutron target (unless otherwise explicitly mentioned). Likewise, and in agreement with the findings of [26], there is only a small difference, typically below 40%, regarding the differential cross section associated with the cLFV conversion of leptons or anti-leptons (cf. Eq. (23), Appendix A); thus in our analysis we will discuss $\ell_i^- n \to \ell_j^- n$. Even though the results displayed in Fig. 3 correspond to $\mu - \tau$ conversion, qualitatively analogous ones have been found for an electron beam (with final state muons or taus).

A second comment concerns the dependency of the differential cross section on the beam's energy, which was already manifest in the results of Fig. 3. Although both photon and Z mediated contributions explicitly scale as E^2_{beam}, the hadronic tensors (see Appendix A) both have non-trivial dependencies (also via Q^2 - cf. Eq. (5)). The left panel of Fig. 4 generalises the choices of beam energy, $E = 4\,(6)$ GeV, presented in Fig. 3; for larger values of the beam energy one enters the strong DIS regime - in the latter case, the behaviour of the differential cross section must be interpreted as only illustrative (the results here computed no longer quantitatively hold). For a fixed value of the momentum transfer Q^2 (which maximises the conversion rate), the dependency of the differential cross section on the beam energy is illustrated on the right panel of Fig. 4. The latter confirms that once the beam energy is sufficiently large to reach the threshold for the in-flight conversion to occur, see Eq. (6), the rate mildly increases until rapidly saturating (in the displayed case at $E_{\text{beam}} \approx 10$ GeV).

4 Experimental prospects

The total expected number of produced leptons ℓ_j for the in-flight $\ell_i \to \ell_j$ conversion can be written as

$$N_{\text{conver.}}(\ell_i \to \ell_j) = N_{\ell_i} \times P(\ell_i \to \ell_j), \tag{14}$$

where N_{ℓ_i} denotes the number of leptons (e, μ) hitting the target, and $P(\ell_i \to \ell_j)$ the conversion probability. For the case of $e \to \mu$ conversion, the total number of signal events can be directly
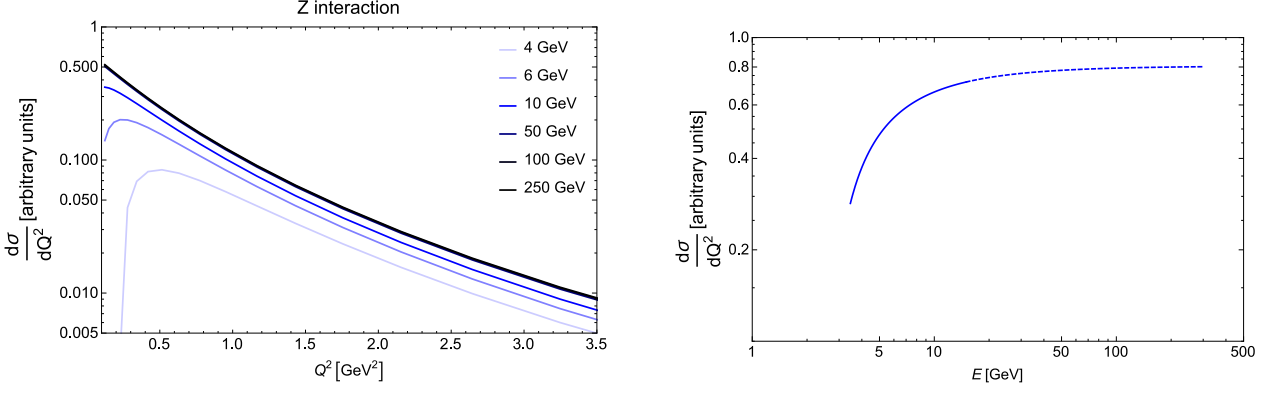
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Figure 4: On the left, differential cross section (arbitrary units) as a function of the momentum transfer, Q^2, for different beam energies. On the right, variation of the differential cross section for $\mu - \tau$ conversion on a neutron target (arbitrary units) with the beam energy for a fixed value of Q^2; full (dashed) lines denote the elastic scattering (naïve extrapolation to DIS regime).

obtained from the above equation, simply rescaling $N_{\text{conver.}}(e \to \mu)$ via parameters associated with the specificity of the target (thickness L and density ρ, or equivalently, the target's mass T_m - expressed in g/cm^2). For the case of final state tau leptons, their average lifetime implies that they will rapidly decay, and hence one has a further correction factor of $\text{BR}(\tau \to \mu\nu\nu)$, which in the SM is approximately 17.4% [59]. Thus, the final number of expected conversions can be cast as [19]

$$N_{\text{signal}}(\ell_i \to \ell_j) = N_{\ell_i} \times \sigma(\ell_i \to \ell_j) \times T_m \times N_{p+n} \left[\times \text{BR}(\tau \to \mu\nu\nu) \right], \quad (15)$$

with $\sigma(\ell_i \to \ell_j)$ the integrated cross section and N_{p+n} the total number of nucleons per gramme of target - assuming for simplicity an average value of the contributions from protons and neutrons to the total cLFV conversion cross section. One thus finds

$$N_{\text{signal}}(\ell_i \to \ell_j) = N_{\ell_i} \times \left(\frac{\sigma(\ell_i \to \ell_j)}{\text{fb}} \right) \times \left(\frac{T_m}{\text{g cm}^{-2}} \right) \times 6 \times 10^{-16} \left[\times \text{BR}(\tau \to \mu\nu\nu) \right]. \quad (16)$$

Recall that in the above two equations, the last term $\text{BR}(\tau \to \mu\nu\nu)$ is only present when the final lepton is a τ. In order to discuss the real expected number of events, one should further take into consideration the detector's intrinsic efficiency, ϵ_{d}, as well as the relevant contributions to the background - which we will not address in the present study.

In Table 2 we collect some operating benchmark values (surface density of the target and intensity of the beam), previously considered in former discussions of this cLFV observable.

Facility	Beam nature	T_m	Intensity (leptons/yr)
Linear Collider	$e^\pm$	10 g/cm^2	10^{22}
Muon Collider (ν-Factory)	$\mu^\pm$	100 g/cm^2	10^{20}
COMET	μ^-	~ 1 g/cm^2 (Al)	10^{19}

Table 2: Illustrative benchmark values for surface density of target (in g/cm^2) as well as nature and intensity of the potential beams used for in-flight cLFV conversion (cf. [11, 22]).

Hereafter focusing on the most minimal "3+1 model", described in Section 2.2, we begin our discussion of the integrated cross section for the several cLFV in-flight conversion modes; as an

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illustrative case, we present the results obtained for a lepton beam energy of 4 GeV (independent of its nature, electron or muon). Prospects for different (higher) energy beams have already been briefly commented in the previous section, and the qualitative outcome holds for the present discussion. Moreover, and although having carried the numerical computation of both Z and photon penguin contributions, we only present the contributions of the former, which in our framework are dominant with respect to those of the latter.

The different panels of Fig. 5 display a general survey of the expected contributions to the different cross sections (arising from Z-mediated cLFV interactions), $\sigma(\ell_i \to \ell_j)$ as a function of the mass of the heavy, mostly sterile state. The left column of Fig. 5 confirms that the cLFV cross section rapidly increases for heavy neutrino masses above the EW scale. Although one could potentially have values for the different observables as large as $\sigma(\ell_i \to \ell_j) \approx \mathcal{O}(10^{-3})$, current experimental bounds - in particular those arising from the violation of several cLFV bounds - exclude these regimes. In terms of expected number of converted leptons, having at least 10 conversions per year lies beyond realistic prospects for beam intensities: even for the least constrained observable, $\mu \to \tau$ conversion, very intense muon beams on a dense target cannot account for more than 0.04 converted tau leptons per year (for $e \to \tau$ one would have at best 0.02 converted τs, and even lower numbers for $e \to \mu$ conversion).

For final state tau leptons, the strongest cLFV constraint arises from the corresponding 3-body decays ($\tau \to 3\ell_i$), while for $e \to \mu$ conversion the current bounds on CR($\mu - e$, Au) further add to the already constraining rôle of $\mu \to 3e$. The right hand side column of Fig. 5 summarises this discussion, displaying $\sigma(\ell_i \to \ell_i)$ as a function of the flavour violation in Z-mediated interactions, $|L_{ij}^Z|^2$ - see Eq. (13), and Appendix B. Horizontal lines denote the cross sections that would account for a minimum of 10 conversions per year (the different line scheme corresponding to the relevant operating benchmarks of Table 2). Other than the coloured points associated with the leading cLFV constraints, grey points are associated with further exclusions arising from many other observables - as described in Section 2.1.

As extensively discussed in the literature, the interplay of distinct cLFV observables (arising from different sectors, and studied at different energies and experimental setups) is a potentially powerful probe to test flavour violating extensions of the SM. For the case of our minimal framework - extending the SM with one sterile fermion - we illustrate in Fig. 6 the potential synergies between the in-flight conversion rate and other cLFV observables, for which Z-penguin exchanges are known to provide important (if not dominant) contributions: BR($\ell_j \to 3\ell_i$), BR($Z \to \ell_i\ell_j$), and - in the case of $e - \mu$ conversion, CR($\mu - e$, N). As could be expected, there is a clear correlation between the in-flight and both high-intensity and high-energy observables. Should one dispose of an unlimited number of leptons in the beam, the in-flight cLFV conversion could simultaneously probe - or even be complementary to other low - and high-energy cLFV observables. Nevertheless, the small expected number of converted leptons, for what are already optimistic beam configurations, dismisses the latter possibilities.

Other theoretical frameworks relying on extensions of the SM via several sterile fermions are expected to lead to similar (or even worse) prospects: studies of observables in which dipole and Z penguin contributions play important rôles [36, 37] allowed to verify that complete models as the (3,3) ISS realisation or the νMSM, systematically lead to smaller predictions to the distinct observables (among them 3-body decays, conversion in Nuclei, or cLFV Z decays) than what is found in the framework of the simple "3+1 model". We have explicitly (numerically) confirmed that this is indeed the case.
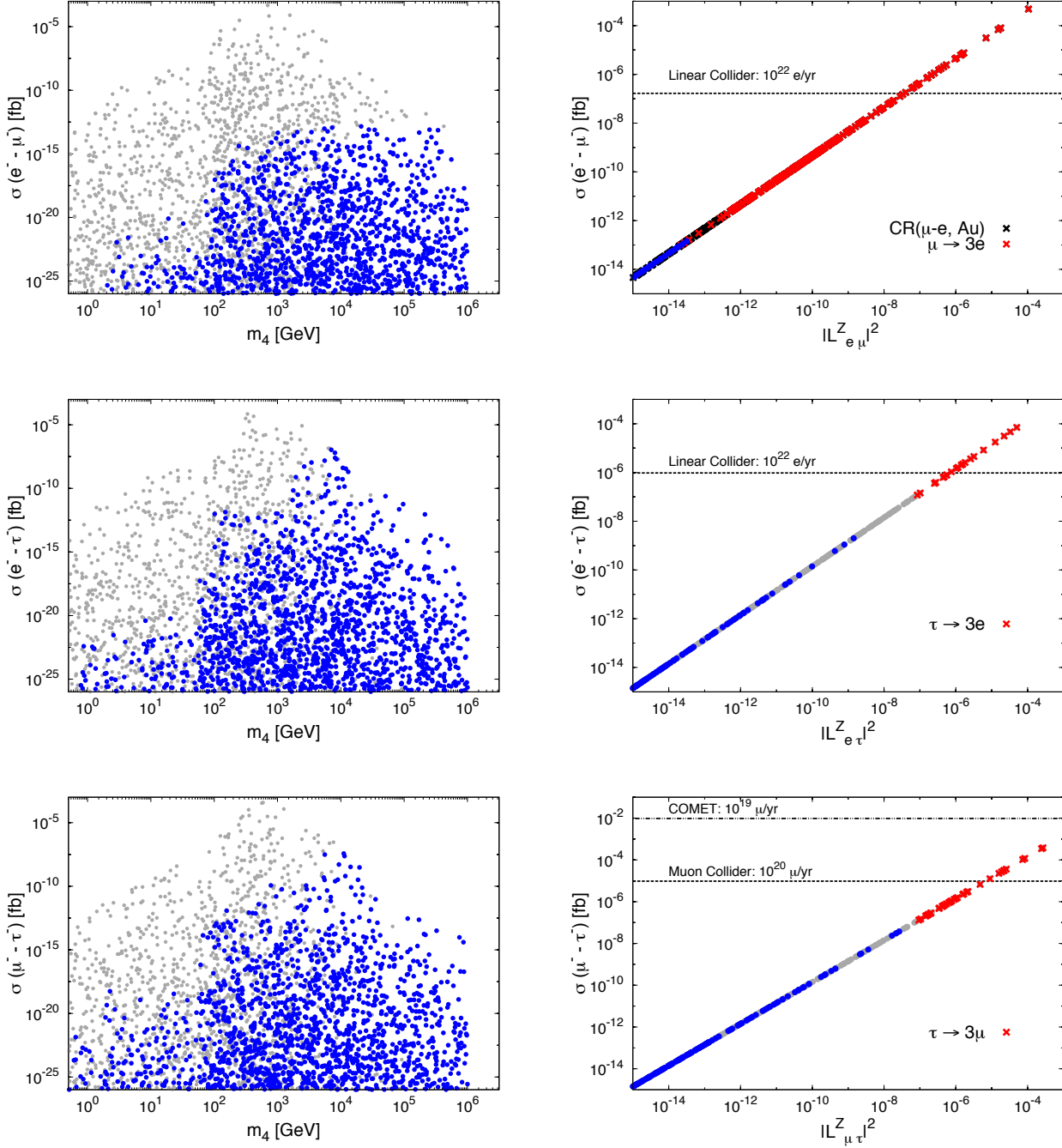
Figure 5: On the left, values of $\sigma(\ell_i \to \ell_i)$ (in fb) as a function of m_4 (in GeV), for a beam energy $E = 4$ GeV. From top to bottom, $e \to \mu$, $e \to \tau$ and $\mu \to \tau$. Blue coloured points comply with the different constraints discussed in Section 2.1; those in grey violate at least one phenomenological and/or experimental bound. On the right column, $\sigma(\ell_i \to \ell_i)$ vs. the amount of flavour violation in the Z-mediated interaction, $|L_{ij}^Z|^2$ - see Eq. (13); the additional colour coding of the points reflects the most stringent cLFV constraints in each case. Horizontal lines further denote the cross sections leading to "observable" in-flight conversions for the appropriate benchmarks of Table 2.
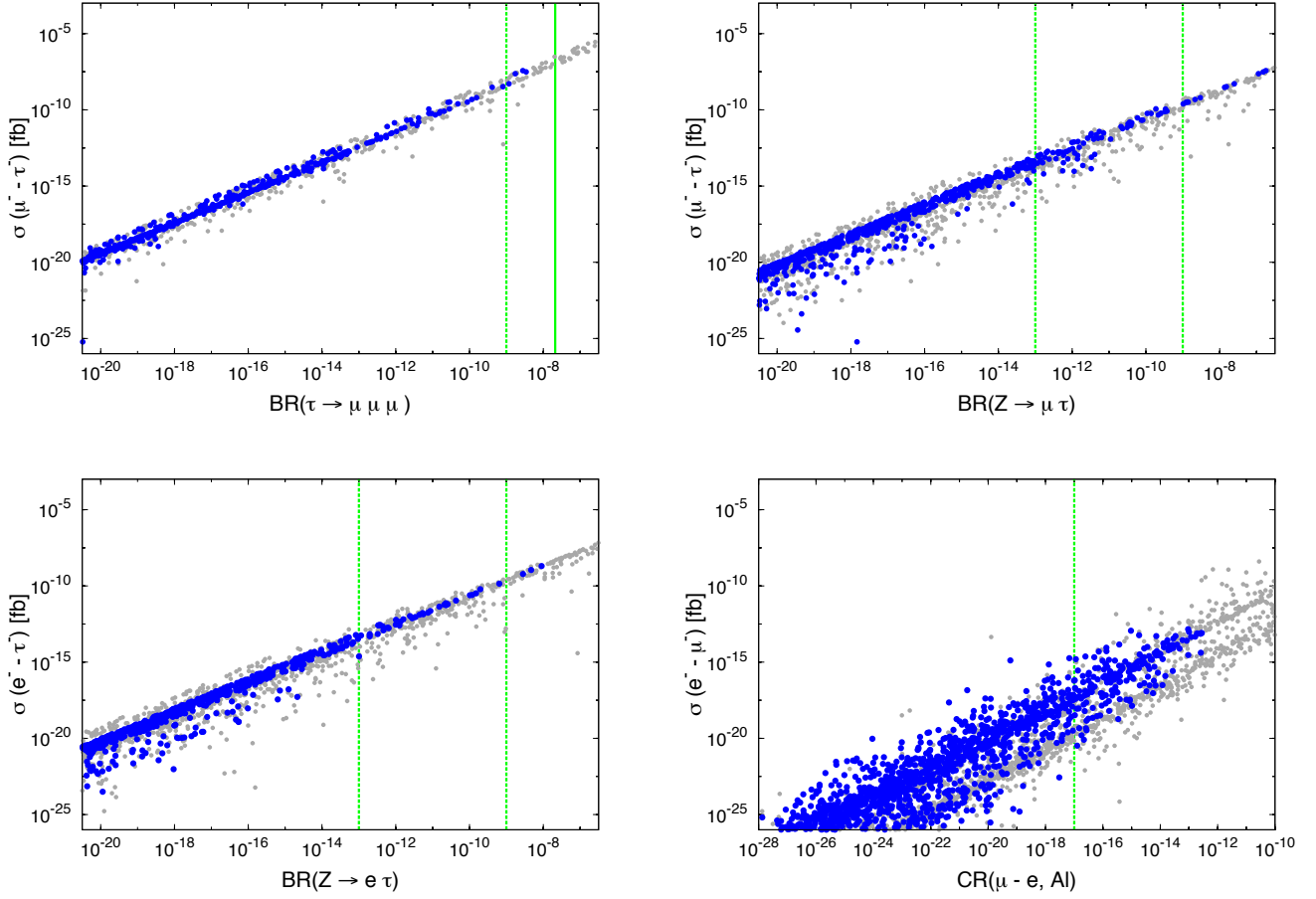
Figure 6: Correlation of cLFV in-flight cross sections with other cLFV observables. Upper panels: $\sigma(\mu \to \tau)$ vs. BR$(\tau \to 3\mu)$ and BR$(Z \to \mu\tau)$. Lower panels: on the right $\sigma(e \to \tau)$ vs. BR$(Z \to e\tau)$; on the left $\sigma(e \to \mu)$ vs. CR$(\mu - e,\, \text{Al})$. Blue coloured points comply with the different constraints discussed in Section 2.1; vertical full (dashed) green lines denote, in each case, the corresponding current bounds (future sensitivities).

5 Concluding remarks

In the past years, charged lepton flavour violating observables have gained an increasing interest stemming from their potential to probe scenarios of New Physics, even those whose typical scales lie beyond collider reach. In view of upcoming facilities, which are expected to operate with intense lepton beams (for example those dedicated to high-intensity cLFV searches as COMET, future neutrino factories, or even a Muon Collider), in-flight lepton flavour conversions occuring when the intense beams hit a fixed target, are potentially interesting cLFV observables.

In this study we have thus revisited cLFV in-flight conversion, $e + N \to \mu + N$, $e + N \to \tau + N$ and $\mu + N \to \tau + N$, focusing on elastic interactions with a nucleus N (considering moderately energetic beams, with an energy not far from the kinematical threshold). We have studied the different contributions to the differential cross sections, and our findings concerning the derivation of the leptonic and hadronic tensors are in agreement with those of Ref. [26]. Motivated by classes of NP models in which cLFV processes occur at higher order, we have moreover focused on the dipole and Z-penguin contributions to the in-flight cLFV conversion.

After a general discussion of the observable, we carried a thorough phenomenological analysis in the framework of minimal SM extensions via sterile neutrinos, in which Z-penguin transitions do indeed dominate over the dipole contributions (and box diagrams as well). Although such minimal frameworks do offer the possibility to have sizeable values for the cross sections, $\sigma(\ell_i \to \ell_j)$, these values are precluded due to the stringent bounds arising from a number of other cLFV observables. Particularly constraining are those observables in which the Z-penguin contributions also play a relevant rôle - among them $\mathrm{BR}(\mu \to 3e)$, $\mathrm{BR}(\tau \to 3e)$, $\mathrm{BR}(\tau \to 3\mu)$, and $\mathrm{CR}(\mu - e, \mathrm{Au})$. Once the latter bounds are taken into account, the distinct cross sections are strongly reduced - at most one expects values of $\mathcal{O}(10^{-8} \text{ fb})$, for the case of $\mu - \tau$ conversion (for which the associated low-energy cLFV constraints are less stringent). Even when assuming the possibility of very intense lepton beams, our study suggests that the expected number of conversions lies beyond experimental sensitivity (below $\mathcal{O}(10^{-2} \text{ events/year})$).

Albeit the results here obtained concern minimal SM extensions via sterile neutrinos, the strong correlation between the in-flight conversion and the cLFV observables which preclude its observability should be common to other NP constructions exhibiting similar features. This is the case of minimal (constrained) SUSY models, where there is typically a strong correlation between radiative decays and the γ-penguins providing the dominant contributions to 3-body decays; in this sense, our findings confirm those of [23] which pointed out that former bounds on $\mu \to e\gamma$ already forbade SUSY contributions to $\sigma(e \to \mu)$ larger than 10^{-8} fb.

It is also worth considering the possibility of having additional sterile states: if on the one hand this might contribute to enhance the $\ell_i \to \ell_j$ cross sections (via a multiplicative factor, thus leading at most to a single order of magnitude enhancement), the additional states would also contribute to the other cLFV observables, so that one does not expect an overall improvement. Likewise, a study in the DIS limit should not qualitatively change the general results here derived.

Should experimental searches for the in-flight cLFV conversion observable be carried in the future, and should an event be observed, then another source of flavour violation, different from - or in addition to - those present in minimal SM extensions via sterile fermions must be necessarily present. Moreover, available (phenomenological) results would suggest that such a NP model would likely exhibit a smaller degree of correlation between different cLFV observables (as is the case of leptoquark models): for example, some transitions occurring at tree-level, while others being mediated via higher order exchanges.

Finally, and as in the case of any cLFV observable, the experimental observation of the in-flight

cLFV conversion would clearly signal the presence of New Physics, and allow selecting classes of models (other than those here discussed) which could account for it.

Acknowledgements

A   Nuclear and leptonic tensors

We describe the most relevant elements leading to the computation of both the photon- and Z-mediated interactions.

A.1   Photonic interaction

The hadronic tensor relevant for the photon mediated on-target conversion, as given in Eq. (8), can be cast as

$$H^{\gamma}_{\mu\nu} = - \left(\eta_{\mu\nu} - q_\mu q_\nu/q^2\right) W_1 \; + \; \frac{1}{M_T^2}(p_\mu - q_\mu\, p.q/q^2)\,(p_\nu - q_\nu\, p.q/q^2)\, W_2\,, \qquad (17)$$

where, and for a spin $1/2$ target, one has

$$W_1 \; = \; \frac{Q^2}{4\,M_T^2}(F_1 + F_2)^2\,, \qquad W_2 \; = \; F_1^2 \; + \; \frac{Q^2}{4\,M_T^2}F_2^2\,, \qquad (18)$$

with $F_{1,2}$ the Dirac and Pauli form factors, which in our analysis refer to the nucleon form factors, $F_i^{p,n}$. In agreement with [26], one can write the latter as:

$$F_1^{p(n)}(Q^2) \; = \; \frac{1}{1 + Q^2/4M_N^2} \left[\frac{1\,(0)}{1 + Q^2/4M_V^2} + \frac{Q^2}{4M_N^2}\frac{\mu_p\,(\mu_n)}{1 + Q^2/4M_V^2} \right]\,,$$

$$F_2^{p(n)}(Q^2) \; = \; \frac{1}{1 + Q^2/4M_N^2} \left[\frac{\mu_p - 1\,(\mu_n)}{1 + Q^2/4M_V^2} \right]\,, \qquad (19)$$

where M_N denotes the nucleon mass $(M_{p,n})$, and M_V the relevant scale for the interaction, $M_V \; = \; M_W$, and $\mu_{p,n}$ the total magnetic moments, respectively $\mu_{p(n)} = 2.79\;(-1.91)\;e/2M_{p(n)}$.

Likewise, the leptonic tensor also present in Eq. (8) can be expressed in terms of momenta as[6]

$$L^{\gamma}_{\mu\nu} \; = \; - 2\left[m_\ell^2\,(m_\ell^2 - q^2)\,(\eta_{\mu\nu} - q_\mu q_\nu/q^2) \; + \; 4\,q^2\,(k^\mu - q^\mu\, k.q/q^2)\,(k^\nu - q^\nu\, k.q/q^2) \right]\,, \qquad (20)$$

in which m_ℓ denotes the mass of the final state (heavier) lepton.

Bringing all the elements together, the final expression for the photon contribution to the differential cross section is given by

$$\left.\frac{d\sigma^{i \to j}}{dQ^2}\right|_\gamma = \frac{2\pi\, Z^2\, \alpha^2}{E^2\, Q^4}\, L^{\gamma}_{ij}\, \left\{ W_1\,(Q^2 + m_{\ell_j}^2)\,(2m_{\ell_j}^2 - Q^2) + \right.$$

$$\left. + \; \frac{W_2}{M_T^2}\left(4\,Q^2\,(p.k)^2 \; + \; (Q^2 + m_{\ell_j}^2)\left[(p.q)^2 \; - \; 4\,p.q\,p.k \; + \; M_T^2\,m_{\ell_j}^2\right]\right) \right\}\,, \qquad (21)$$

---

[6]We adopt a similar notation to that of Ref. [26], the results of which we agree with.

with L_{ij}^{γ} given in Eq. (12).

A.2 Z-mediated interaction

Assuming the case of unpolarised lepton beams, the leptonic tensor entering in the Z-interaction contribution to the differential cross section (see Eq. (10)) can be written as

$$L_{\mu\nu}^{Z} = 16 \left(k^{\mu} k'^{\nu} + k^{\nu} k'^{\mu} - k.k' \eta^{\mu\nu} + i\, \varepsilon^{\mu\nu\rho\sigma} k_{\rho} q_{\sigma} \right) . \tag{22}$$

The hadronic tensor can be in general cast in terms of six dimensionless structure functions as

$$H_{\mu\nu}^{Z} = -\eta_{\mu\nu}\, W_1 + \frac{p_{\mu} p_{\nu}}{M_T^2}\, W_2 \pm i\, \varepsilon_{\mu\nu\rho\sigma} \frac{p^{\rho} q^{\sigma}}{2M_T^2}\, W_3 + \frac{q_{\mu} q_{\nu}}{M_T^2}\, W_4 + \frac{p_{\mu} q_{\nu} + p_{\nu} q_{\mu}}{2M_T^2}\, W_5 + i\, \frac{p_{\mu} q_{\nu} - p_{\nu} q_{\mu}}{2M_T^2}\, W_6 , \tag{23}$$

where in the above equation the $\pm$ corresponds to having a lepton (or antilepton) conversion. The different expressions for the structure functions $W_i(Q^2)$ can be found in Ref. [26], with which we agree after an independent derivation.

As above, the contraction of both leptonic and hadronic tensors leads to the following differential cross section for Z-mediated contribution, which we have used throughout the analysis,

$$\left. \frac{d\sigma^{i \to j}}{dQ^2} \right|_{Z} = \frac{G_F^2}{2\pi\, E^2}\, L_{ij}^{Z} \left\{ \left[(Q^2 + m_{\ell_j}^2)\left(W_1 - \frac{1}{2} W_2\right) + \frac{p.k}{M_N^2}\left(2\, p.k - Q^2\right) W_2 + \right. \right.$$
$$\left. \left. + \frac{1}{2}(Q^2 + m_{\ell_j}^2)\frac{m_{\ell_j}^2}{M_N^2}\, W_4 - (p.k)\frac{m_{\ell_j}^2}{M_N^2}\, W_5 \right] \pm \frac{Q^2}{4M_N^2}\left(4\, p.k - Q^2 - m_{\ell_j}^2\right) W_3 \right\} , \tag{24}$$

with L_{ij}^{Z} has been given in Eq. (13).

B cLFV form factors

The relevant form factors for the computation of the diagrams of Fig. 2 are given by [28–30, 34]:

$$G_{\gamma}^{\mu e} = \sum_{j=1}^{3+n_S} \mathbf{U}_{ej} \mathbf{U}_{\mu j}^{*} G_{\gamma}(x_j), \qquad F_{\gamma}^{\mu e} = \sum_{j=1}^{3+n_S} \mathbf{U}_{ej} \mathbf{U}_{\mu j}^{*} F_{\gamma}(x_j),$$

$$F_{Z}^{\mu e} = \sum_{j,k=1}^{3+n_S} \mathbf{U}_{ej} \mathbf{U}_{\mu k}^{*} \left(\delta_{jk} F_Z(x_j) + \mathbf{C}_{jk} G_Z(x_j, x_k) + \mathbf{C}_{jk}^{*} H_Z(x_j, x_k) \right) , \tag{25}$$

where $x_i = \frac{m_{\nu_i}^2}{m_W^2}$ carries the neutrino mass dependency and $\mathbf{C}$ has been defined in Eq. (2).

The loop functions entering the previous form factors are defined as [28–30, 34]:

$$
\begin{aligned}
F_Z(x) &= -\frac{5x}{2\,(1-x)} - \frac{5x^2}{2\,(1-x)^2}\ln x\,, \\
G_Z(x,y) &= -\frac{1}{2(x-y)}\left[\frac{x^2\,(1-y)}{1-x}\ln x - \frac{y^2\,(1-x)}{1-y}\ln y\right]\,, \\
H_Z(x,y) &= \frac{\sqrt{x\,y}}{4\,(x-y)}\left[\frac{x^2-4x}{1-x}\ln x - \frac{y^2-4y}{1-y}\ln y\right]\,, \\
F_\gamma(x) &= \frac{x\,(7x^2-x-12)}{12\,(1-x)^3} - \frac{x^2\,(x^2-10x+12)}{6\,(1-x)^4}\ln x\,, \\
G_\gamma(x) &= -\frac{x\,(2x^2+5x-1)}{4\,(1-x)^3} - \frac{3x^3}{2\,(1-x)^4}\ln x\,.
\end{aligned}
\tag{26}
$$

The contributions to different cLFV observables such as radiative and 3-body decays, conversion in Nuclei, or FV Z decays, which have been evaluated and analysed in the present work (including the relevant loop functions [28–30, 34]), have been discussed in previous studies (see, for example, [30, 34, 36, 37]), and we will not include them here.

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