

# The Many Guises of a Neutral Fermion Singlet

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## Abstract

The addition of a neutral fermion singlet to the standard model of particle interactions leads to many diverse possibilities. It is not necessarily a right-handed neutrino. I discuss many of the simplest and most interesting scenarios of possible new physics with this approach. In particular I propose the possible spontaneous breaking of baryon number, resulting in the massless 'sakharon'.

*Introduction :*

In the standard model (SM) of particle interactions, there are three families of quarks and leptons. Under its  $SU(3)_C \times SU(2)_L \times U(1)_Y$  gauge symmetry, they transform as follows:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \sim (3, 2, 1/6), \quad u_R \sim (3, 1, 2/3), \quad d_R \sim (3, 1, -1/3), \quad (1)$$

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L \sim (1, 2, -1/2), \quad l_R \sim (1, 1, -1), \quad (2)$$

where electric charge is given by

$$Q = I_{3L} + Y. \quad (3)$$

There is also one scalar Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, 1/2), \quad (4)$$

which breaks the electroweak  $SU(2)_L \times U(1)_Y$  symmetry spontaneously to electromagnetic  $U(1)_Q$  through the vacuum expectation value  $\langle \phi^0 \rangle = v$ . As a result, three vector gauge bosons  $W^\pm, Z$  become massive, whereas the eight  $SU(3)_C$  gluons and the one  $U(1)_Q$  photon remain massless. There is also just one physical real scalar, i.e. the Higgs boson, presumably the 125 GeV discovered in 2012 at the Large Hadron Collider (LHC) [1, 2].

As it stands, the standard model has the following automatic conserved global symmetries: baryon number  $B = 1/3$  for each quark, lepton number  $L_e = 1$  for the electron and its neutrino  $\nu_e$ ,  $L_\mu = 1$  for  $\mu$  and  $\nu_\mu$ , and  $L_\tau = 1$  for  $\tau$  and  $\nu_\tau$ . As such, all neutrinos are massless.

Because of the observation of neutrino oscillations, we know that at least two neutrinos are massive, and that  $\nu_e, \nu_\mu, \nu_\tau$  are not mass eigenstates. The simplest theoretical implementation of this fact is to add one singlet neutral fermion  $N_R$  to each family:

$$N_R \sim (1, 1, 0). \quad (5)$$

Because of Eq. (3),  $N_R$  has no gauge interaction and it couples to the SM only through the Yukawa terms

$$f_{ij}^\nu \bar{N}_{iR} (\nu_{jL} \phi^0 - l_{jL} \phi^+) + H.c. \quad (6)$$

Hence  $N_R$  is commonly called the right-handed neutrino and assigned lepton number  $L = 1$  with its Dirac mass matrix given by  $f_{ij}^\nu v$ . The separate conservation of  $L_e, L_\mu, L_\tau$  is no longer valid, replaced now by  $L = L_e + L_\mu + L_\tau$ .

If the Majorana mass terms

$$\frac{1}{2} M_{ij} N_{iR} N_{jR} + H.c. \quad (7)$$

are added, then  $L$  is broken to  $(-1)^L$ , and for large  $M_{ij}$ , the light neutrino mass matrix is given by the famous seesaw formula [3, 4, 5, 6]

$$\mathcal{M}_{ij}^\nu = -(f_{ik}^\nu v) M_{kl}^{-1} (f_{lj}^\nu v). \quad (8)$$

Lepton number extensions :

Suppose  $N_R$  is still assumed to have  $L = 1$  as implied by Eq. (6), and that Eq. (7) is forbidden, then another interesting possibility exists if a scalar singlet  $\sigma$  with  $L = -2$  is added. Now the terms

$$\frac{1}{2} f_{ij}^N \sigma N_{iR} N_{jR} + H.c. \quad (9)$$

would generate a mass matrix  $f_{ij}^N \langle \sigma \rangle$ . However, the spontaneous breaking of  $L$  implies a massless Goldstone boson, i.e. the singlet majoron [7].

The above model can be interpreted another way if we add a heavy singlet quark [8, 9] which also transforms under  $L$ . However, whereas  $Q_L$  has  $L = 1$ , its chiral partner  $Q_R$  has  $L = -1$ . In that case, the Yukawa term

$$f^Q \sigma \bar{Q}_R Q_L + H.c. \quad (10)$$

exists and  $L$  becomes an anomalous global symmetry which is identifiable with the Peccei-Quinn symmetry [10] which solves the strong CP problem, and a very light axion [11, 12] appears instead of the massless majoron. This idea [13] also connects the axion scale with the neutrino mass seesaw scale, and may be extended [14] to include supersymmetry.

Since  $N_R$  is a new addition to the SM, we are free to assign it whatever symmetry we desire. Suppose it has  $L = 0$ . This means that Eq. (7) is allowed, but Eq. (6) is forbidden, and there is no connection between  $N_R$  and the SM. However, suppose we now add a second scalar doublet

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} \sim (2, 1, 1/2) \quad (11)$$

with  $L = -1$ , then the terms

$$f_{ij}^\eta \bar{N}_{iR} (\nu_{jL} \eta^0 - l_{jL} \eta^+) + H.c. \quad (12)$$

are allowed, and if  $L$  is spontaneously broken by  $\langle \eta^0 \rangle = u$ , neutrinos would become massive, but a massless doublet majoron would also appear. It would contribute to the invisible decay width of the  $Z$  boson, which is known to be consistent with exactly three neutrinos. This scenario is thus ruled out.

Suppose now that  $L$  is also broken explicitly but softly by the bilinear term

$$\mu^2 \eta^\dagger \Phi + H.c., \quad (13)$$

then it can be shown [15] that  $u \ll v$  naturally, and the smallness of the neutrino Dirac mass  $f^\eta u$  is understood. Together with the seesaw mechanism of Eq. (8), this implies that the mass of  $N_R$  may be reduced to below 1 TeV, lending hope that the seesaw mechanism may be verifiable experimentally.

Another possible lepton number assignment for  $N_R$  is  $L = -1$ . Now both Eqs. (6) and (7) are forbidden. Suppose we add  $\eta$  but with  $L = -2$  instead, then Eq. (12) is allowed.

Using again the dimension-two term of Eq. (13) to break  $L$  softly, we obtain Dirac masses for the light neutrinos [16, 17] without the dimension-three Majorana mass terms of Eq. (7). The resulting Lagrangian actually conserves the usual  $L$ . What has been gained is the understanding of how Dirac neutrino masses may be small at the expense of a second scalar doublet with a suppressed vacuum expectation value.

*Dark matter extensions* : Another possible identity for  $N_R$  is dark matter (DM). Suppose it is odd under an exactly conserved  $Z_2$  discrete symmetry under which all SM particles are even. This scenario is actually identical to the case  $L = 0$  discussed in the previous section, i.e. Eq. (6) is forbidden but Eq. (7) is allowed. They are related by defining dark matter parity

$$D = (-1)^{L+2j}, \quad (14)$$

as pointed out in Ref. [20]. However,  $N_R$  decouples entirely from the SM and may not be relevant as a DM candidate.

To connect  $N_R$  to the SM, the scalar doublet  $\eta$  of Eq. (11) may again be added, and the Yukawa couplings of Eq. (12) be allowed by assigning  $\eta$  to be odd under dark  $Z_2$ . Now the concept of lepton number for Eq. (12) becomes ambiguous. It could be assigned to  $N_R$  or  $\eta$ . However, if we abandon  $L$  and just consider lepton parity, i.e.  $(-1)^L$ , with  $L = 0$  for  $N_R$  and  $L = -1$  for  $\eta$ , then this term conserves both lepton parity and dark parity. In fact the latter is derivable [18] from the former as shown in Eq. (14).

With both  $N_R$  and  $\eta$ , it is now possible to obtain a radiative seesaw neutrino mass [19], as shown in Fig. 1. Since this mechanism uses dark matter to generate a nonzero neutrino mass, it has been called 'scotogenic' from the Greek 'scotos' meaning darkness. The concept of lepton parity may be promoted to matter parity

$$M = (-1)^{3B+L}, \quad (15)$$

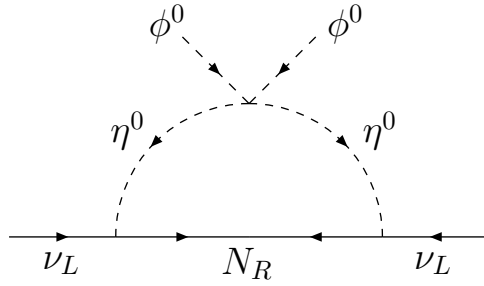


Figure 1: One-loop  $Z_2$  scotogenic seesaw neutrino mass.

so that dark matter parity becomes

$$D = (-1)^{3B+L+2j}, \quad (16)$$

which is identical to that of  $R$  parity in supersymmetry, but without having to extend the SM to include supersymmetry itself. In that case, with the addition of scalar fermion doublets and singlets of odd  $D$  parity, all quark and lepton masses may be generated [20] from the heavy  $3 \times 3$   $N_R$  Majorana mass matrix. This idea connects SM masses to those of the dark sector, and offers an explanation for the light fermion masses as being scotogenic.

Gauge  $U(1)$  extensions : Whereas  $N_R$  has no SM gauge interactions, it may transform nontrivially under an extra gauge  $U(1)$  symmetry. The most commonly studied scenario is gauge  $B - L$  from the observation that [21]

$$Q = I_{3L} + I_{3R} + \frac{1}{2}(B - L) \quad (17)$$

for the known quarks and leptons so that the SM may be embedded into  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . On the other hand, if we consider  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_F$  with one  $N_R$  added to each family quarks and leptons as shown in Table 1, then many possible different models [22] may be obtained. To constrain  $n_{1,2,3}$  and  $n'_{1,2,3}$ , the requirement of gauge anomaly cancellation is imposed. The contributions of color triplets to the  $[SU(3)]^2 U(1)_F$

Table 1: Fermion assignments under  $U(1)_F$ .

Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_F$
$Q_{iL} = (u, d)_{iL}$	3	2	1/6	$n_i$
$u_{iR}$	3	1	2/3	$n_i$
$d_{iR}$	3	1	-1/3	$n_i$
$L_{iL} = (\nu, l)_{iL}$	1	2	-1/2	$n'_i$
$l_{iR}$	1	1	-1	$n'_i$
$N_{iR}$	1	1	0	$n'_i$

anomaly sum up to

$$[SU(3)]^2 U(1)_F : \frac{1}{2} \sum_{i=1}^3 (2n_i - n_i - n_i); \quad (18)$$

and the contributions of  $Q_{iL}, u_{iR}, d_{iR}, L_{iL}, l_{iR}$  to the  $U(1)_Y [U(1)_F]^2$  anomaly sum up to

$$U(1)_Y [U(1)_F]^2 : \sum_{i=1}^3 \left[ 6 \left( \frac{1}{6} \right) - 3 \left( \frac{2}{3} \right) - 3 \left( -\frac{1}{3} \right) \right] n_i^2 + \left[ 2 \left( -\frac{1}{2} \right) - (-1) \right] n_i'^2. \quad (19)$$

Both are automatically zero, as well as the  $[U(1)_F]^3$  anomaly because all fermions couple to  $U(1)_F$  vectorially. The contributions of the  $SU(2)_L$  doublets to the  $[SU(2)]^2 U(1)_F$  anomaly sum up to

$$[SU(2)]^2 U(1)_F : \frac{1}{2} \sum_{i=1}^3 (3n_i + n'_i); \quad (20)$$

and the contributions to the  $[U(1)_Y]^2 U(1)_F$  anomaly sum up to

$$\begin{aligned} [U(1)_Y]^2 U(1)_F &: \sum_{i=1}^3 \left[ 6 \left( \frac{1}{6} \right)^2 - 3 \left( \frac{2}{3} \right)^2 - 3 \left( -\frac{1}{3} \right)^2 \right] n_i + \left[ 2 \left( -\frac{1}{2} \right)^2 - (-1)^2 \right] n'_i \\ &= \sum_{i=1}^3 \left( -\frac{3}{2} n_i - \frac{1}{2} n'_i \right). \end{aligned} \quad (21)$$

Both are zero if

$$\sum_{i=1}^3 (3n_i + n'_i) = 0. \quad (22)$$

There are many specific examples of models which satisfy this condition as discussed in Ref. [22].

The neutral vector gauge boson  $Z_F$  associated with  $U(1)_F$  couples in general to the  $u$  and  $d$  quarks, so it may be produced at the LHC if kinematically allowed. Its branching fractions to  $e^-e^+$  and  $\mu^-\mu^+$  are given by

$$B(Z_F \rightarrow e^-e^+, \mu^-\mu^+) = \frac{2n'_{1,2}{}^2}{12 \sum n_i^2 + 3 \sum n_i'^2}. \quad (23)$$

The  $c_{u,d}$  coefficients used in the experimental search [23, 24] of  $Z_F$  are then

$$c_u = c_d = 2n_1^2 g_F^2 (2n_1'^2 + 2n_2'^2) / (12 \sum n_i^2 + 3 \sum n_i'^2). \quad (24)$$

With current LHC data, a typical bound [22] on  $Z_F$  is about 4 TeV.

**Baryon number extensions**: An interesting but seldom explored possibility is to make  $N_R$  a baryon. Suppose a scalar quark

$$\zeta \sim (3, 1, -1/3) \quad (25)$$

is added with  $B = -2/3$  so that the Yukawa terms

$$f_{Lij}^\zeta \zeta (d_{iL} u_{jL} - u_{iL} d_{jL}) + f_{Rij}^\zeta \zeta d_{iR} u_{jR} + f_{ij}^N N_{iR} d_{jR} \zeta^* + H.c. \quad (26)$$

are allowed, then  $N_R$  has  $B = -1$ . This assignment was first proposed [25] in the context of superstring-inspired  $E_6$  models. Note that both Eqs. (6) and (7) are forbidden by  $B$ , but if the latter is allowed, then  $B$  is broken softly to  $B$  parity. In that case, whereas proton decay is still forbidden, neutron-antineutron ( $n - \bar{n}$ ) oscillation is possible.

**See the other paper**

**Model 1** The scotogenic mechanism may also be extended to accommodate (26)  $n - \bar{n}$  oscillation.

The idea is very simple. Replace  $\Phi$  by the singlet scalar quark  $\delta \sim (3, 1, -1/3; +)$  and  $\eta$  by  $\xi \sim (3, 1, -1/3; -)$ , which is distinguished from  $\delta$  by having odd  $B$  parity. Together with  $N_R$  having even  $B$  parity, Fig. 1 becomes Fig. 2. Since  $\delta$  acts as a diquark because it couples to  $u_{L,R} d_{L,R}$ , this diagram generates  $n - \bar{n}$  oscillation. The idea of combining Figs. 1 and 2 means that neutrino mass,  $n - \bar{n}$  oscillation are possible only through their connection to



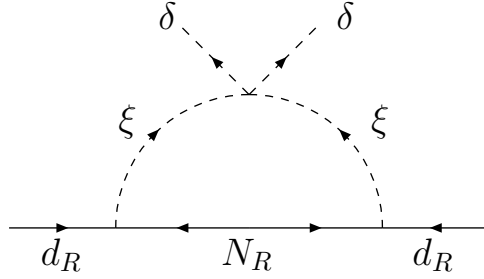


Figure 2: One-loop  $Z_2$  scotogenic  $n - \bar{n}$  oscillation.

dark matter. Proton decay is forbidden at this stage by the separate conservation of  $L$  parity and  $B$  parity, but if only the product is conserved, then it also becomes possible [26].

Another use of  $N_R$  having  $B = 1$  is in the context of supersymmetry. Since the fermionic component of the  $N_R$  superfield has even  $R$  parity, whereas the bosonic component has odd  $R$  parity, the latter may be dark matter. The addition of a pair of color triplet superfields with weak hypercharge  $= \pm 2/3$  or  $\mp 1/3$  may also facilitate baryogenesis from the decay of the  $N_R$  fermion at the TeV scale [27].

Instead of having an allowed mass term for  $N_R$  as in Eq. (7) so that  $B$  is broken to  $B$  parity in the above, we can generalize the idea of the spontaneous breaking of lepton number to that of baryon number. The resulting massless Goldstone boson may be called the 'sakharon', after Andrei Sakharov. To implement this idea in a renormalizable extension of the Standard Model, the simplest solution is to use Eq. (26) and add Eq. (9) instead of Eq. (7). Hence  $\zeta$  decays into  $\bar{u}\bar{d}$  and  $\zeta^*$  decays into  $ud$ . As  $\sigma$  acquires a large vacuum expectation value,  $B$  is broken to  $(-)^{3B}$  and  $N$  may decay into  $udd$  or  $\bar{u}\bar{d}\bar{d}$ . If there are two or more  $N$  fields, this is a mechanism for generating a baryon asymmetry [28] in the early Universe, which gets converted into a  $B - L$  asymmetry through the electroweak sphalerons [29].

In this scenario where  $B$  is spontaneously broken at a very high scale, the sakharon  $S$  couples directly only to  $N$ , just as in the case of the singlet majoron. However, whereas  $N$

would mix with  $\nu$  in the presence of electroweak symmetry breaking, it stands alone in this scenario. This means that there is no tree-level sakharon interaction with ordinary matter, and its presence is even more elusive than that of the singlet majoron [30].

**Model 2** Consider now the extreme opposite scenario of a very low energy scale for the spontaneous breakdown of baryon number. This is somewhat akin to the case of the triplet majoron model [31] where lepton number is spontaneously broken at a very low scale, i.e. that of neutrino mass. That is however experimentally ruled out because the triplet majoron interacts with the  $Z$  boson, which decays into it and its necessarily light scalar partner so that the  $Z$  invisible width is increased by twice that of a single neutrino species. Here the sakharon will be a singlet as detailed below.

Table 2: Particle content of model with low-scale sakharon.

Particle	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$B$	$L$
$Q = (u, d)_L$	$(3, 2, 1/6)$	$1/3$	$0$
$u_R$	$(3, 1, 2/3)$	$1/3$	$0$
$d_R$	$(3, 1, -1/3)$	$1/3$	$0$
$h_L$	$(3, 1, -1/3)$	$-2/3$	$-1$
$h_R$	$(3, 1, -1/3)$	$-2/3$	$-1$
$L = (\nu, l)_L$	$(1, 2, -1/2)$	$0$	$1$
$l_R$	$(1, 1, -1)$	$0$	$1$
$N_R$	$(1, 1, 0)$	$0$	$1$
$(\phi^+, \phi^0)$	$(1, 2, 1/2)$	$0$	$0$
$\zeta$	$(3, 1, -1/3)$	$-2/3$	$0$
$\sigma$	$(1, 1, 0)$	$1$	$1$

**A non-zero VEV breaks B spontaneously**

To implement this extreme scenario, the Standard Model of quarks and leptons is extended to include three heavy Majorana singlet neutral fermions  $N_R$  (for obtaining small neutrino masses through the canonical seesaw mechanism) together with singlet quarks  $h_{L,R}$ , as well as a color-triplet scalar  $\zeta$  and a complex singlet scalar  $\sigma$ . Their baryon and lepton

numbers are listed in Table 2. The interaction Lagrangian is then given by

$$\mathcal{L}_{int} = f_{Lij}^{\zeta} \zeta (d_{iL} u_{jL} - u_{iL} d_{jL}) + f_{Rij}^{\zeta} \zeta d_{iR} u_{jR} + f_k^N \zeta^* N_{kR} h_R + f^{\sigma} \bar{d}_{kR} h_L \sigma + H.c. \quad (27)$$

Allowing  $N_R$  to have a large Majorana mass breaks  $L$  to  $(-)^L$ , under which  $\mathcal{L}_{int}$  of Eq. (27) is still invariant. Consider now the possibility of  $\langle \sigma \rangle \neq 0$ , thereby breaking both  $B$  and  $(-)^L$ . Although  $(-)^{3B+L}$  remains unbroken in this case, it does not impose any extra condition because all fermions are odd and all bosons are even under it. There are many consequences of this scenario. Proton decay is now possible, but is suppressed by two factors: the smallness of  $\langle \sigma \rangle$  and the smallness of  $m_{\nu}$ . Details will be reported elsewhere [32].

Conclusion : In this Brief Review, some of the many guises of a neutral fermion singlet are exposed. In its simplest form, it is used as a right-handed neutrino, but many other options are available. I have discussed lepton and baryon number extensions, axion and dark matter applications, as well as gauge  $U(1)$  family symmetries. The lesson is that for any new particle added to the SM, its lepton or baryon number assignment has to be understood in context, and not an automatic entry.

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