HEP'99 # 6\_212 Submitted to Pa 6 Pl 6, 8

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Preliminary

**DELPHI** Collaboration

Paper submitted to the HEP'99 Conference Tampere, Finland, July 15-21

## A search for FCNC in $Z \rightarrow b\bar{d}, b\bar{s}$ decays

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#### Abstract

From 1992 to 1995 the DELPHI detector at LEP has collected about 3.5 million hadronic Z decays from which over a half were recorded with a double-sided microvertex detector. The accurate and efficient tracking devices of DELPHI enabled an efficient hadronic flavour tag with high purity allowing the present search for flavour violating Z decays in the process  $e^+e^- \rightarrow b\bar{q}$ , q = d, s at the  $M_Z$  energy scale. No signal for such events was found on the data sample and an upper limit of 2.6 × 10<sup>-3</sup> at 90% CL has been derived to the quantity

$$R_{\mathrm{b}\ell} = \frac{\sum_{\mathrm{q=d,s}} \sigma(\mathrm{e^+e^-} \to \mathrm{b}\bar{\mathrm{q}})}{\sigma(\mathrm{e^+e^-} \to \mathrm{hadrons})} \; .$$

#### 1 Introduction

Flavor Changing Neutral Current processes (FCNC) are governed in the Standard Model (SM) by the Glashow-Iliopoulos-Maiani (GIM) mechanism [1]. In this scenario these transitions are forbidden at tree level and the leading contributions which can produce these processes only result from the one-loop diagrams known as the penguin and box diagrams which then contain suppression factors in the order of  $10^{-6} - 10^{-9}$  with respect to the allowed tree level SM reactions [2]. The experimental search of such processes represents an important test of the validity of the SM [3], either by confirming its prediction or by indicating the need for physics beyond the SM if observed at larger probabilities [4].

In this paper we investigate the possible presence of events originated by FCNC processes in a sample of 3.5 million hadronic events collected by DELPHI through the reaction  $e^+e^- \rightarrow b\bar{q}$ , q = d, s at the energy scale of the Z mass  $(M_Z)^1$ . The branching ratio of this process in the SM is expected to be of the order of  $10^{-7}$  [5] and therefore any observation of such events in the data sample would imply the existence of new physics. The method used to look for these events is based on the precise measurement of the ratio of cross-sections  $R_b = \sigma(e^+e^- \rightarrow b\bar{b})/\sigma(e^+e^- \rightarrow hadrons)$  performed by DELPHI [6]. The implicit SM constraint used in that analysis,  $R_b + R_c + R_\ell = 1$  (where  $R_c$  and  $R_\ell$  are the ratios of cross-sections for charm and light quarks respectively), is here relaxed to allow for FCNC processes in the form  $R'_b + R'_c + R'_\ell + R_{b\ell} = 1$ . The parameters  $R'_b$ ,  $R'_c$  and  $R'_\ell$ are here the renormalized ratios of cross-sections to  $b\bar{b}$ ,  $c\bar{c}$  and  $\ell\bar{\ell}$  events and the flavour violating parameter  $R_{b\ell}$  is defined as<sup>2</sup>

$$R_{\mathrm{b}\ell} = \frac{\sum_{\mathrm{q=d,s}} \sigma(\mathrm{e^+e^-} \to \mathrm{b}\bar{\mathrm{q}})}{\sigma(\mathrm{e^+e^-} \to \mathrm{hadrons})} \ . \tag{1}$$

The analysis compares the rates of events where only one b or one light quark has been identified to those where two b's, two light quarks and one b and one light quark have been tagged, from which  $R_{b\ell}$  can be measured together with the b and uds tagging efficiencies. Systematic uncertainties are then due only to the charm and light quark backgrounds of the highly efficient and pure b tag and to the evaluation of hemisphere tagging correlations in SM and FCNC events. Additional tags for b- and c-quarks are used. All efficiencies apart from the background efficiencies of the primary b tag are measured from data, so that the additional tags reduce the statistical error without increasing the systematic uncertainties. Systematic errors coming from hemisphere correlations are kept well under control due to a separated reconstruction of the primary vertex for each hemisphere. Finally, to account for any possible contribution of FCNC events to  $R_b$  itself, as well as  $e^+e^- \rightarrow c\bar{u}$  processes, the result will be given as a function of the assumed values of  $R_b$  and  $R_c$ .

Recent searches for processes related to the above reaction have been performed by the L3 and CLEO Collaborations [7, 8]. The L3 experiment has set a preliminary limit at 90% CL of  $6.0 \times 10^{-3}$ , by searching for the same process  $e^+e^- \rightarrow b\bar{q}$ , q = d, s at same energy scale,  $M_Z$ . On the other hand, CLEO has produced a limit of  $5.7 \times 10^{-5}$  at 90% CL by looking for the inclusive decay  $b \rightarrow se^+e^-$  at the  $M_{\Upsilon}$  energy scale.

<sup>&</sup>lt;sup>1</sup>In this paper, by  $e^+e^- \rightarrow b\bar{q}$ , q = d, s we mean also its conjugate process  $e^+e^- \rightarrow \bar{b}q$ , q = d, s.

<sup>&</sup>lt;sup> $^{2}$ </sup>If charge conservation is violated the sum would run over u-, d- and s-quarks.

This paper is organised as follows. After a brief description of the detector and of the track and event selections, sections 3 and 4 describe the analysis and tagging techniques. Section 5 gives the results, some consistency checks and the systematic uncertainties. The last section presents our final results and conclusions.

#### 2 Detector description and event selection

The DELPHI detector, surrounding one of the interaction regions at the Large Electron Positron facility LEP at CERN, has been used to record the samples of events considered in this analysis. It provides both tracking and calorimetric information over almost the full solid angle. A detailed description of the detector and its performance, including the exact geometry as well as the trigger conditions and the event processing chain, appear in references [9, 10]. Especially relevant to this analysis is the double-sided microvertex detector (VD) [11], installed in spring 1994, that allowed high values of purity and efficiency in the identification of b- and uds-quarks.

The criteria to select charged tracks and to identify hadronic Z decays are identical to those described in [6]. Charged particles were accepted if:

- their polar angle was between  $20^{\circ}$  and  $160^{\circ}$ ,
- their track length was larger than 30 cm,
- their impact parameter relative to the interaction point was less than 5 cm in the plane perpendicular to the beam direction and less than 8 cm along the beam direction,
- their momentum was larger than 200 MeV/c with relative error less than 100%.

Neutral particles detected in the HPC were required to have measured energy larger than 700 MeV and those detected in the EMF greater than 400 MeV.

Events were then selected by requiring:

- at least 6 reconstructed charged particles,
- the summed energy of the charged particles had to be larger than 15% of the centre of mass energy, with at least 3% of it in each of the forward and backward hemispheres with respect to the beam axis.

The efficiency to find hadronic Z decays with these cuts was about 95% and all backgrounds were below 0.1%. About 1.3 million hadronic Z decays were selected with the two dimensional VD in 1992 and 1993, and 2.1 million hadronic Z decays from 1994 and 1995 data samples with the three dimensional VD. The ratio of hadronic cross-section  $e^+e^- \rightarrow \bar{b}q$ , q = d, s to the total hadronic cross-section is expected to vary very little at centre-of-mass energies close to the Z resonance. Thus no selection on the centre-of-mass energy was made in 1993 and 1995. As the VD is essential for this analysis, the data were limited to events that have most of the tracks inside the acceptance of the VD. For this reason a cut of  $|\cos \theta_{thrust}| < 0.65$  was applied. The hadronic cuts. The bias towards b events in the selected sample was found to be small,  $(1.51 \pm 0.09) \times 10^{-3}$ , and was corrected for; its uncertainty is dominated by Monte Carlo statistics. The bias towards light quarks events was found to be of the same size of the one towards b events, but of opposite sign and almost 100% correlated. For c-quarks the bias and its uncertainty was found to be negligible.

A sample about twice the data statistics of  $Z \rightarrow q\bar{q}$  events was simulated using the Lund parton shower Monte Carlo JETSET 7.3 [12] (with parameters optimised by DELPHI) and the DELPHI detector simulation [10]. In addition dedicated samples of  $Z \rightarrow b\bar{b}$  events were generated. As in this analysis the efficiencies for FCNC events are measured from data, dedicated samples of FCNC events were not required. The simulated events were passed through the same analysis chain as the real ones.

For this analysis a reasonable description of the data by the simulation for the light and charm quarks is required. For this reason a fine tuning of the  $R\phi$  and Rz impact parameter distributions in the simulation was developed and applied [13]. This led to substantially smaller uncertainties due to the understanding of the detector resolution.

#### 3 The experimental method

The method used to investigate the existence of the process  $e^+e^- \rightarrow b\bar{q}$ , q = d, s at  $M_Z$  energy scale relies on the  $R_b$  analysis performed by DELPHI and previously published [6]. Hence only the particular aspects of the analysis are described in the following.

Events are first divided into hemispheres using the plane perpendicular to the thrust axis. The event topology of a Z boson decaying into a b quark and a light quark can then be identified by applying b tagging (B tag) to one of the hemispheres and an inclusive light quark tagging (L tag) to the opposite one.

The fraction of hadronic FCNC events,  $f_E^{BL}$ , corresponding to those events with two tagged hemispheres, one with a B tag and the other with a L tag, can be parameterized as

$$f_E^{\rm BL} = \epsilon_{\rm b\ell}^{\rm BL} R_{\rm b\ell} + \epsilon_{\rm bb}^{\rm BL} R_{\rm b}' + \epsilon_{\rm cc}^{\rm BL} R_{\rm c}' + \epsilon_{\ell\ell}^{\rm BL} R_{\ell}' , \qquad (2)$$

where  $\epsilon_{\alpha}^{\text{BL}}$  is the efficiency to be classified as FCNC an event originating from primary quarks  $\alpha = b\ell$ , bb, cc,  $\ell\ell$  and  $R_{b\ell} + R'_b + R'_c + R'_\ell = 1$ . As said above, the parameters  $R'_b$ ,  $R'_c$  and  $R'_\ell$  are the renormalized ratios of cross-sections to  $b\bar{b}$ ,  $c\bar{c}$  and  $\ell\bar{\ell}$  events with the presence of flavour violating Z decays. Nevertheless, they can be related to their equivalent SM parameters  $R_b$ ,  $R_c$  and  $R_\ell$  verifying  $R_b + R_c + R_\ell = 1$  through the relation

$$R'_{q} = R_{q}(1 - R_{b\ell}), \quad q = b, c, \ell$$
(3)

hence allowing the following relation

$$f_E^{\rm BL} = \epsilon_{\rm back}^{\rm BL} + \left(\epsilon_{\rm b\ell}^{\rm BL} - \epsilon_{\rm back}^{\rm BL}\right) R_{\rm b\ell} \tag{4}$$

with

$$\epsilon_{\text{back}}^{\text{BL}} = \epsilon_{\text{bb}}^{\text{BL}} R_{\text{b}} + \epsilon_{\text{cc}}^{\text{BL}} R_{\text{c}} + \epsilon_{\ell\ell}^{\text{BL}} R_{\ell} .$$
(5)

Each of the double hemisphere efficiencies,  $\epsilon_{\alpha}^{\text{BL}}$ , can then be written in terms of the single hemisphere efficiencies and their correlation. For SM-like events they can be written as

$$\epsilon_{\rm qq}^{\rm BL} = \epsilon_{\rm q}^{\rm B} \epsilon_{\rm q}^{\rm L} (1 + \rho_{\rm qq}^{\rm BL}), \quad {\rm q} = {\rm b, c, \ell} , \qquad (6)$$

where  $\epsilon_q^B$  and  $\epsilon_q^L$  are the hemisphere efficiencies that a quark (or anti-quark) of flavour q to be tagged as B and L respectively. The factors  $\rho_{qq}^{BL}$  account for hemisphere-hemisphere correlations of the tagging efficiencies, and they are exactly the same as those defined and used in [6].

The FCNC event tagging efficiencies can be similarly defined as

$$\epsilon_{b\ell}^{\rm BL} = \frac{\epsilon_b^{\rm B} \epsilon_\ell^{\rm L} + \epsilon_b^{\rm L} \epsilon_\ell^{\rm B}}{2} (1 + \rho_{b\ell}^{\rm BL}) , \qquad (7)$$

where the hemisphere tagging efficiencies are the same as those used in (6) and the factors  $\rho_{b\ell}^{BL}$  account for potential hemisphere correlations in events with flavour changing topology.

Tags B and L give rise to a total of six mutually exclusive combinations of hemispheres: BL, BB, BX, LL, LX and XX, where X denotes here a category of hemispheres nor tagged as B neither as L. All previous equations, written for BL events, apply for each one of these combinations.

Using the above equations,  $R_{b\ell}$  can be extracted together with the tagging efficiencies  $\epsilon_b^B$ ,  $\epsilon_\ell^L$ ,  $\epsilon_c^L$  and  $\epsilon_b^L$  from  $f_E^{BL}$ ,  $f_E^{BB}$ ,  $f_E^{BX}$ ,  $f_E^{LL}$ ,  $f_E^{LX}$  and  $f_E^{XX}$ , provided that the charm and light quark brackgrounds of the B tag ( $\epsilon_c^B$  and  $\epsilon_\ell^B$ ) and the correlations are estimated from the simulation of the experiment, and  $R_b$  and  $R_c$  are known. As the measurement of  $R_{b\ell}$  is in fact the detection of any offset of  $R_b$  from its electroweak prediction, measured now within the FCNC model framework,  $R_b$  can be fixed to its theoretical SM prediction. On the other hand, in order to account for any possible contribution of  $e^+e^- \rightarrow c\bar{u}$  processes, for  $R_c$  we also impose its value from theory and then we give the explicit dependence of the result with this parameter. Finally, to account for any possible contribution of  $R_b$ . In this way precise knowledge of the details of the B-hadron as well as of  $e^+e^- \rightarrow b\bar{q}$ , q = d, s decays, is not required.

If more than the B and L hemisphere tags were available, this 2-tag scheme could be generalized to a multiple tag scheme. In that case equations (4), (5), (6) and (7) read, respectively,

$$f_E^{IJ} = \epsilon_{\text{back}}^{IJ} + \left(\epsilon_{b\ell}^{IJ} - \epsilon_{\text{back}}^{IJ}\right) R_{b\ell} , \qquad (8)$$

$$\epsilon_{\rm back}^{\rm IJ} = \epsilon_{\rm bb}^{\rm IJ} R_{\rm b} + \epsilon_{\rm cc}^{\rm IJ} R_{\rm c} + \epsilon_{\ell\ell}^{\rm IJ} R_{\ell} , \qquad (9)$$

$$\epsilon_{qq}^{IJ} = \epsilon_{q}^{I} \epsilon_{q}^{J} (1 + \rho_{qq}^{IJ}) , \quad q = b, c, \ell , \text{ and}$$
(10)

$$\epsilon_{\mathrm{b}\ell}^{\mathrm{IJ}} = \frac{\epsilon_{\mathrm{b}}^{\mathrm{I}}\epsilon_{\ell}^{\mathrm{J}} + \epsilon_{\mathrm{b}}^{\mathrm{J}}\epsilon_{\ell}^{\mathrm{I}}}{2} (1 + \rho_{\mathrm{b}\ell}^{\mathrm{IJ}}) \tag{11}$$

All the hadronic hemispheres are classified as one of the tags, so that the conditions

$$\sum_{\mathbf{I}} \epsilon_{\mathbf{q}}^{\mathbf{I}} = 1 \tag{12}$$

and

$$\sum_{\mathrm{I}} \epsilon_{\mathrm{q}}^{\mathrm{I}} \rho_{\mathrm{qq}}^{\mathrm{IJ}} = 0 \quad , \tag{13}$$

with  $q = b, c, \ell$ , must be satisfied. It should be noticed here that only (9) contributes to (8) in the  $R_b$  fit [6]. The term given in (11) is the genuine flavour changing contribution.

Three more tags have been added to the B and L tags. Two of them, hereafter called B1 and B2 are designed to identify b quarks, and the third one c quarks (C tag). The B tag has the maximal b purity and it is used as primary b tag. In such an analysis scheme there are 20 independent fractions  $f_E^{IJ}$ . The extra B1 and B2 tags will allow to accept more events without introducing additional systematics because all tagging efficiencies for them are determined from data, resulting in a smaller error on the measurement. Only the uds and c backgrounds of the B tag category and the hemisphere correlations for SM and FCNC events will contribute to systematic errors. Compared with classical methods in searches, where all efficiencies and backgrounds are estimated from the simulation, this measurement has a reduced dependence on our understanding of the underlying physics and detector response. This will result in a strong reduction of systematic errors.

#### 4 Flavour tagging

To provide the six hemisphere tags, three flavour tagging algorithms have been used.

The first technique is the enhanced impact parameter  $b \, tag \, [6]$  which combines several properties of the B hadrons into a single variable to identify b-quarks. They are the long lifetime, the large mass, the high decay multiplicity and the high B hadron energy taken from the initial quark. All discriminating variables are defined for jets (using JADE with  $y_{min}=0.01$ ) with reconstructed secondary vertices. The hemisphere is then identified by the tagged jet. The lifetime information is extracted from the positively signed impact parameters of the tracks included in a jet. The large mass and high decay multiplicity of B hadrons is exploited using as tagging variables the effective invariant mass and the rapidity (computed with respect to the jet direction) of particles included in the secondary vertex. Finally, the fraction of the charged energy carried out by the particles of the secondary vertex is added. Figure 1 shows the data/simulation comparison of distributions of the enhanced tagging variable  $-\log_{10} y$ , for background (u,d,s,c) jets and jets with b-quarks. The contribution of background jets is obtained from clean and almost uncontaminated samples of b-quarks opposite to hemispheres with a high purity of about 99%. Figure 2a) shows the hemisphere b tagging efficiency versus purity obtained with this technique as predicted by the simulation.

The likelihood flavour tagging algorithm [14] is similarly based on the large mass and relatively long lifetime of the b-quark and some event shape properties of its decays. All the available information is combined using multivariate techniques. As before, the lifetime information exploits the large impact parameters of tracks coming from B decays together with a search for secondary vertices and their invariant masses. Then the lifetime information is combined with event shape properties of the B decays like large transverse momentum of the tracks with respect to the jet axis, rapidity distributions and the boosted sphericity. A total of 13 variables is finally adopted.

The third technique, called *flavour confidences* [15], is based on track impact parameters and on two other kinematic variables: the track momentum and the angle with respect to the jet axis. The method uses the simulation to build a function which gives the fraction of tracks which come from uds-, c- and b-quarks in a bin of the three particle characteristics. There are kinematic effects in the decay of B hadrons which produce



Figure 1: Distribution of the tagging variable  $-\log_{10} y$  of the enhanced impact parameter method, for a) background (u,d,s,c) jets and b) jets with b-quarks in the 1994 data sample. The points with errors are from the data and the histogram is the simulation prediction. The contribution of uds-quark jets is shown as the filled histogram in the upper figure. In a), the data have been obtained by subtracting the b-enhanced distribution from the overall one; in b) it is the b-enhanced data sample which is shown.



Figure 2: a) Hemisphere b tagging efficiency versus purity for the enhanced impact parameter b tagging and for the multivariate flavour tagging. b) Hemisphere uds tagging efficiency versus purity for the multivariate flavour tagging. The arrow marks the position of the working points for the enhanced impact parameter b tag and the multivariate flavour uds tag, as used for the measurement presented in this paper.

correlations between the three quantities, but they are automatically taken into account by the three-dimensional binning. The individual flavour confidences are finally combined to make a hemisphere tag.

The likelihood flavour tagging and the flavour confidences can be combined using a simple linear combination for each flavour [6]. There is finally a global multivariate estimator  $\Delta_q$  for each flavour. Figure 3 shows the distributions of the uds and b flavour multivariate discriminators for data and simulation where the level of agreement can be seen over three orders of magnitude. The analysis is insensitive to small disagreements as they would affect only the tagging efficiencies, which are fitted from data. The effects on correlations are discussed latter. Figures 2a) and 2b) show the hemisphere b and uds tagging efficiencies versus purities obtained with this procedure, as it is predicted by the simulation of the experiment.

The definition of the hemisphere tags in terms of the different tagging techniques is identical to that used in [6]. The B (b-tight) and L (uds) tags have the strongest influence on the measurement of  $R_{b\ell}$ , but also the B1 (b-standard) and B2 (b-loose) are used in an attempt to improve the statistical error without increasing systematics. The Monte Carlo expectations for all efficiencies are given separately for 1993 and 1994 in table 1. This table is a measurement of the performance of the tags and tagging techniques all together. In this analysis, only the charm and light quark backgrounds of the B tag are taken from simulation. All the other efficiencies are measured directly from the data and can be used as a cross-check of the analysis (see tables 1, 4 and 5).

		1993			1994	
Tag I	$\epsilon^{\mathrm{I}}_\ell$	$\epsilon^{\mathrm{I}}_{\mathrm{c}}$	$\epsilon^{\mathrm{I}}_{\mathrm{b}}$	$\epsilon^{\mathrm{I}}_\ell$	$\epsilon^{\mathrm{I}}_{\mathrm{c}}$	$\epsilon^{\mathrm{I}}_{\mathrm{b}}$
В	0.00050	0.00381	0.23003	0.00052	0.00376	0.28236
B1	0.00188	0.02631	0.17051	0.00126	0.02692	0.15578
B2	0.01446	0.07754	0.16043	0.01219	0.07858	0.15158
$\mathbf{C}$	0.05814	0.16428	0.05704	0.04942	0.15617	0.04963
L	0.11977	0.03579	0.00548	0.11819	0.03025	0.00471
Х	0.80530	0.69226	0.37649	0.81856	0.70431	0.35591

Table 1: Simulation results for the tagging efficiencies at the nominal cuts for 1993 and 1994.

The  $\rho_{\alpha}^{\text{IJ}}$  hemisphere correlation factors as estimated from simulation for the 1994 analysis together with their sensitivities to  $R_{\text{b}\ell}$  are given in table 2, where the errors are due only to simulation statistics. Only the relevant correlations with a sensitivity to  $R_{\text{b}\ell}$ higher than 0.001 are shown. The sensitivity s is defined as the relative change on  $R_{\text{b}\ell}$ due to a change of a given correlation,  $\frac{\Delta R_{\text{b}\ell}}{R_{\text{b}\ell}} = s\Delta \rho_{\alpha}^{\text{IJ}}$ .

#### 5 Results and systematics

The experimentally measured numbers for the different categories of doubly tagged events which passed the  $|\cos \theta_{\text{thrust}}|$  cut are given in table 3 for the 1993 and 1994 samples.

The fit of  $R_{b\ell}$  and the efficiencies to these numbers gives the following results for each year of operation:



Figure 3: Distribution of the multivariate discriminator  $\Delta_q$  in the uds and b tags for 1994-1995 data and simulation. The different flavour contributions to the simulated event sample are also shown. The simulation distributions are normalised to the data statistics.

Correlation	Sensitivity to $R_{\rm b\ell} \times 10^2$	Value				
b correlations						
$ ho_{ m b}^{ m B,B}$	1.440	$0.0198 \pm 0.0020$				
$ ho_{ m b}^{ m B,B1}$	0.435	$0.0034 \pm 0.0020$				
$ ho_{ m b}^{ m B,B2}$	0.248	$0.0031 \pm 0.0020$				
$ ho_{ m b}^{ m B,C}$	-0.069	$0.0047 \pm 0.0039$				
$ ho_{\rm b}^{\rm B1,B1}$	-0.145	$0.0073 \pm 0.0037$				
$ ho_{ m b}^{ m B1,B2}$	-0.140	$0.0034 \pm 0.0031$				
$ ho_{\rm b}^{\rm B1,C}$	0.048	$0.0042 \pm 0.0058$				
$ ho_{ m b}^{ m B1,L}$	0.001	$0.0564 \pm 0.0199$				
$ ho_{ m b}^{ m B2,B2}$	-0.072	$0.0095 \pm 0.0038$				
$\rho_{\rm b}^{\rm B2,C}$	0.028	$-0.0079 \pm 0.0059$				
$ ho_{ m b}^{ m C,C}$	-0.003	$-0.0171 \pm 0.0196$				
	c correlations					
$ ho_{ m c}^{ m B,B2}$	0.001	$0.0426 \pm 0.0698$				
$ ho_{ m c}^{ m B,C}$	-0.002	$0.0522 \pm 0.0474$				
$ ho_{ m c}^{ m B1,B1}$	-0.002	$0.0882 \pm 0.0482$				
$ ho_{ m c}^{ m B1,B2}$	-0.007	$-0.0019 \pm 0.0255$				
$ ho_{ m c}^{ m B1,C}$	0.023	$0.0015 \pm 0.0173$				
$ ho_{ m c}^{ m B1,L}$	0.002	$-0.0024 \pm 0.0422$				
$ ho_{ m c}^{ m B2,B2}$	-0.011	$0.0447 \pm 0.0161$				
$ ho_{\rm c}^{\rm B2,C}$	0.039	$0.0028 \pm 0.0097$				
$\rho_{\rm c}^{\rm C,C}$	-0.028	$0.0434 \pm 0.0080$				
$ ho_{ m c}^{ m C,L}$	0.006	$0.0323 \pm 0.0164$				
$\ell$ correlations						
$\rho_{\rm c}^{\rm B1,C}$	0.002	$-0.1587 \pm 0.0789$				
$ ho_{c}^{B2,B2}$	-0.001	$0.1427 \pm 0.0583$				
$ ho_{c}^{B2,C}$	0.008	$0.0428 \pm 0.0266$				
$ ho_{ m c}^{ m D2, L}$	-0.004	$0.0114 \pm 0.0163$				
$\rho_{\rm c}^{\circ,\circ}$	-0.014	$0.0315 \pm 0.0135$				
$ ho_{ m c}^{\circ, \perp}$	0.024	$0.0134 \pm 0.078$				
$ ho_{ m c}$ ,		$0.0758 \pm 0.0570$				
B,L	bℓ correlations	0.0220 + 0.1147				
$ ho_{\mathrm{b}\ell}_{\mathrm{B,X}}$	0.003	$0.0338 \pm 0.1147$				
$ ho_{\mathrm{b}\ell}_{\mathrm{B1,L}}$	-0.015	$-0.0196 \pm 0.0250$				
$ ho_{\mathrm{b}\ell}^{,-}$ B1.X	-0.003	$0.0605 \pm 0.1091$				
$ ho_{\mathrm{b}\ell}$	0.004	$-0.0084 \pm 0.0127$				
$ ho_{\mathrm{b}\ell}^{\mathrm{b}\ell,\mathrm{L}}$	-0.002	$-0.0002 \pm 0.0382$				
$\rho_{\mathrm{b}\ell}^{\mathrm{b}2,\mathrm{A}}$	0.003	$-0.0065 \pm 0.0074$				
$ ho_{\mathrm{b}\ell}^{\Lambda,\Lambda}$	0.001	$0.0756 \pm 0.1529$				

Table 2: Hemisphere-hemisphere correlation coefficients  $\rho_{\alpha}^{IJ}$  with major sensitivity (> 0.001) on  $R_{b\ell}$  for the 1994 data sample. Errors are only due to the limited Monte Carlo statistics.

			1	993		
Tag	В	B1	B2	С	$\mathbf{L}$	Х
В	5,158					
B1	7,405	2,762				
B2	$6,\!839$	$5,\!070$	2,764			
$\mathbf{C}$	2,568	2,388	$4,\!196$	4,026		
$\mathbf{L}$	268	416	$1,\!408$	5,504	$4,\!068$	
Х	15,224	$14,\!204$	22,719	$47,\!804$	$51,\!151$	$194,\!345$
			1	994		
Tag	В	B1	B2	С	$\mathbf{L}$	Х
В	16,078					
B1	$17,\!049$	4,564				
B2	16,261	9,017	$5,\!025$			
С	5,737	$4,\!150$	$7,\!386$	6,757		
$\mathbf{L}$	662	766	$2,\!583$	$9,\!877$	$^{9,210}$	
Х	36,764	$25,\!527$	43,749	88,319	109,031	411,116

Table 3: Measured numbers of doubly tagged events passing the  $|\cos \theta_{\text{thrust}}|$  cut in 1993 and 1994.

$$\begin{aligned} R_{\rm b\ell} &= [-3.68 \pm 2.13({\rm stat}) \pm 1.47({\rm syst})] \times 10^{-3}, \quad \chi^2/{\rm ndof} = 8.6/6 \quad (1992), \\ R_{\rm b\ell} &= [2.50 \pm 2.22({\rm stat}) \pm 1.33({\rm syst})] \times 10^{-3}, \quad \chi^2/{\rm ndof} = 7.3/6 \quad (1993), \\ R_{\rm b\ell} &= [1.56 \pm 1.28({\rm stat}) \pm 0.91({\rm syst})] \times 10^{-3}, \quad \chi^2/{\rm ndof} = 7.7/6 \quad (1994), \\ R_{\rm b\ell} &= [0.48 \pm 1.88({\rm stat}) \pm 1.29({\rm syst})] \times 10^{-3}, \quad \chi^2/{\rm ndof} = 3.4/6 \quad (1995). \end{aligned}$$

The first errors are statistical and the second ones systematics. The efficiencies obtained from these fits for 1993 and 1994 are shown in table 4. They can be compared with the simulation predictions of table 1. For a complete comparison, an estimate of the systematic errors must be included. The good values of  $\chi^2$ /ndof for all years indicate consistency between the different tags. For comparison, table 5 gives the same efficiencies but now for the Standard Model  $R_b$  analysis [6] where  $R_{b\ell}$  is imposed to be vanishing and the fitted fraction is  $R_b$  instead of  $R_{b\ell}$ .

The results for the four years are compatible and can be combined using the same assumptions as in [6]. The combined  $R_{b\ell}$  result for the full 1992-1995 data is:

$$R_{\rm b\ell} = [0.67 \pm 0.87(\rm stat) \pm 0.78(\rm syst)] \times 10^{-3}, \tag{14}$$

where the  $\chi^2$ /ndof of the combination is 4.1/3. The mean b and uds purity at the working point for this measurement is about 98.5% and 92.0% respectively. The dependences of this measurement with the assumed values of  $R_b$  and  $R_c$  are, respectively,  $-1.277 \times (R_b -$ 0.21584) and  $-0.030 \times (R_c - 0.172)$ . The central values for  $R_b$  and  $R_c$  were estimated from [16], assuming a mass for the top quark of  $m_t = 173.8 \pm 5.2 \text{ GeV/c}^2$  [17].

The systematic errors are due to the quantities estimated from simulation. In this analysis only light and charm quark backgrounds in the B tag and the correlation of hemisphere tagging efficiencies are taken from simulation. Table 6 reports the breakdown of the systematic uncertainties on this measurement of  $R_{b\ell}$ . As stated before, the method

		1993	
Tag I	$\epsilon^{\mathrm{I}}_\ell$	$\epsilon^{\mathrm{I}}_{\mathrm{c}}$	$\epsilon^{\mathrm{I}}_{\mathrm{b}}$
В	0.00050	0.00381	$0.2399 \pm 0.0010$
B1	$0.0024 \pm 0.0005$	$0.0236 \pm 0.0025$	$0.1755 \pm 0.0012$
B2	$0.0136 \pm 0.0007$	$0.0806 \pm 0.0035$	$0.1612 \pm 0.0015$
С	$0.0730 \pm 0.0008$	$0.1801 \pm 0.0025$	$0.0574 \pm 0.0010$
L	$0.1265 \pm 0.0011$	$0.0333 \pm 0.0034$	$0.0046 \pm 0.0007$
		1994	
Tag I	$\epsilon^{\mathrm{I}}_\ell$	$\frac{1994}{\epsilon_{\rm c}^{\rm I}}$	$\epsilon^{\mathrm{I}}_{\mathrm{b}}$
Tag I B	$\frac{\epsilon_{\ell}^{\mathrm{I}}}{0.00052}$	$\frac{1994}{\epsilon_{\rm c}^{\rm I}} \\ 0.00376$	$\frac{\epsilon_{\rm b}^{\rm I}}{0.2970\pm0.0007}$
Tag I B B1	$\frac{\epsilon_{\ell}^{\rm I}}{0.00052}$ $0.0019 \pm 0.0003$	$\begin{array}{c} 1994 \\ \epsilon_{\rm c}^{\rm I} \\ 0.00376 \\ 0.0240 \pm 0.0014 \end{array}$	$\frac{\epsilon_{\rm b}^{\rm I}}{0.2970\pm0.0007}$ $0.1579\pm0.0008$
Tag I B B1 B2	$\begin{aligned} \epsilon_{\ell}^{\rm I} \\ 0.00052 \\ 0.0019 \pm 0.0003 \\ 0.0124 \pm 0.0004 \end{aligned}$	$\begin{array}{c} 1994 \\ \epsilon_{\rm c}^{\rm I} \\ 0.00376 \\ 0.0240 \pm 0.0014 \\ 0.0790 \pm 0.0020 \end{array}$	$\begin{aligned} \epsilon_{\rm b}^{\rm I} \\ 0.2970 \pm 0.0007 \\ 0.1579 \pm 0.0008 \\ 0.1497 \pm 0.0009 \end{aligned}$
Tag I B B1 B2 C	$\begin{array}{c} \epsilon_{\ell}^{\rm I} \\ 0.00052 \\ 0.0019 \pm 0.0003 \\ 0.0124 \pm 0.0004 \\ 0.0615 \pm 0.0005 \end{array}$	$\begin{array}{c} 1994 \\ \epsilon_{\rm c}^{\rm I} \\ 0.00376 \\ 0.0240 \pm 0.0014 \\ 0.0790 \pm 0.0020 \\ 0.1693 \pm 0.0016 \end{array}$	$\begin{aligned} & \epsilon_{\rm b}^{\rm I} \\ & 0.2970 \pm 0.0007 \\ & 0.1579 \pm 0.0008 \\ & 0.1497 \pm 0.0009 \\ & 0.0511 \pm 0.0006 \end{aligned}$

Table 4: Tagging efficiencies with statistical errors for data as measured from the FCNC  $R_{b\ell}$  fit at the nominal cuts for 1993 and 1994.

		1993	
Tag I	$\epsilon^{\mathrm{I}}_\ell$	$\epsilon^{\mathrm{I}}_{\mathrm{c}}$	$\epsilon^{\mathrm{I}}_{\mathrm{b}}$
В	0.00050	0.00381	$0.2387 \pm 0.0017$
Β1	$0.0024 \pm 0.0005$	$0.0235 \pm 0.0025$	$0.1746 \pm 0.0011$
B2	$0.0135 \pm 0.0007$	$0.0805 \pm 0.0035$	$0.1604 \pm 0.0012$
С	$0.0730 \pm 0.0008$	$0.1800 \pm 0.0025$	$0.0575 \pm 0.0010$
$\mathbf{L}$	$0.1265 \pm 0.0009$	$0.0334 \pm 0.0034$	$0.0052 \pm 0.0005$
		1994	
Tag I	$\epsilon^{\mathrm{I}}_\ell$	$\frac{1994}{\epsilon_{\rm c}^{\rm I}}$	$\epsilon^{\mathrm{I}}_{\mathrm{b}}$
Tag I B	$\frac{\epsilon_{\ell}^{\mathrm{I}}}{0.00052}$	$\frac{1994}{\epsilon_{\rm c}^{\rm I}} \\ 0.00376$	$\frac{\epsilon_{\rm b}^{\rm I}}{0.2959\pm0.0012}$
Tag I B B1	$\frac{\epsilon_{\ell}^{\rm I}}{0.00052} \\ 0.0019 \pm 0.0003$	$\begin{array}{c} 1994 \\ \epsilon_{\rm c}^{\rm I} \\ 0.00376 \\ 0.0239 \pm 0.0014 \end{array}$	$\begin{aligned} & \epsilon_{\rm b}^{\rm I} \\ & 0.2959 \pm 0.0012 \\ & 0.1574 \pm 0.0007 \end{aligned}$
Tag I B B1 B2	$\begin{aligned} \epsilon_{\ell}^{\rm I} \\ 0.00052 \\ 0.0019 \pm 0.0003 \\ 0.0124 \pm 0.0004 \end{aligned}$	$\begin{array}{c} 1994 \\ \epsilon_{\rm c}^{\rm I} \\ 0.00376 \\ 0.0239 \pm 0.0014 \\ 0.0790 \pm 0.0020 \end{array}$	$\begin{aligned} \epsilon_{\rm b}^{\rm I} \\ 0.2959 \pm 0.0012 \\ 0.1574 \pm 0.0007 \\ 0.1492 \pm 0.0008 \end{aligned}$
Tag I B B1 B2 C	$\frac{\epsilon_{\ell}^{\rm I}}{0.00052}$ $0.0019 \pm 0.0003$ $0.0124 \pm 0.0004$ $0.0615 \pm 0.0005$	$\begin{array}{c} 1994 \\ \epsilon_{\rm c}^{\rm I} \\ 0.00376 \\ 0.0239 \pm 0.0014 \\ 0.0790 \pm 0.0020 \\ 0.1692 \pm 0.0015 \end{array}$	$\begin{aligned} \epsilon_{\rm b}^{\rm I} \\ 0.2959 \pm 0.0012 \\ 0.1574 \pm 0.0007 \\ 0.1492 \pm 0.0008 \\ 0.0512 \pm 0.0006 \end{aligned}$

Table 5: Tagging efficiencies with statistical errors for data as measured from the SM  $R_{\rm b}$  fit at the nominal cuts for 1993 and 1994.

has strongly reduced systematic errors because the signal efficiency for flavour changing events is directly estimated from the data. As a consequence, the total uncertainty is dominated by the data and Monte Carlo statistical errors. All sources of systematic uncertainties have been estimated as in the  $R_{\rm b}$  analysis. See reference [6] for a detailed description on how they are determined.

The background is due to light and charm quark events which are tagged as b-quarks. There are 4 different sources of such events: detector resolution, production of  $K^0$ ,  $\Lambda$  and other long lived baryons, production of charm particles and gluon splitting  $g \rightarrow c\bar{c}$ ,  $g \rightarrow b\bar{b}$ . The contribution of the detector resolution and of  $K^0$ ,  $\Lambda$  is almost negligible. The lifetime and production rate of D mesons is assumed from the values measured at LEP [18]. The splitting of gluons to  $c\bar{c}$  and  $b\bar{b}$  gives one of the main contributions to the systematics. Direct measurements of the gluon splitting are used here as input parameters [18].

The hemisphere-hemisphere tagging efficiency correlation is the unavoidable consequence of deriving the efficiencies from the data. However the corresponding systematics is much less than the systematics which would be generated by estimating the efficiencies from the simulation. As the flight directions of the two b-quarks are correlated, and the vertex of primary interaction is measured independently in each hemisphere, the origin of the correlation is well understood to be induced by the geometrical acceptance of the detector (any detector response inhomogeneity generates an efficiency correlation) and the hard gluon emission. The hard gluon emission produces two different effects. First, it takes a part of the event energy so that the momentum of both b-quarks is reduced. Such reduction induces a positive correlation since the tagging efficiency strongly depends on the energy of B hadrons. In some cases the energy of the emitted gluon is so high that both b-quarks are boosted into the same hemisphere of the event. Such effect produces a negative correlation because only the hemisphere containing the two b-quarks can be selected by the b-tagging. The contribution of each source of correlation can be isolated both in the data and in simulation using the distribution of the relevant variables. For the detector acceptance, it can be the direction of the thrust axis. For the hard gluon emission the momentum of the jet can be used. See [6] for details.

Compared with the SM  $R_{\rm b}$  analysis, the only additional source of systematics which is not estimated there is due to hemisphere-hemisphere correlations in flavour violating events,  $\rho_{\rm b\ell}^{\rm IJ}$ . To take into account this contribution properly, a modified JETSET Monte Carlo of FCNC events with full DELPHI detector simulation is required. Nevertheless, the correlation  $\rho_{\rm b\ell}^{\rm IJ}$  can be estimated to be inside the interval defined by the maximal and minimal values between  $\rho_{\rm bb}^{\rm IJ}$ ,  $\rho_{\ell\ell}^{\rm IJ}$  and vanishing correlation, within statistical errors. Moreover, as shown in table 2, compared with the bb correlations, the sensitivity of  $R_{\rm b\ell}$ to the correlations  $\rho_{\rm b\ell}^{\rm IJ}$  is strongly supressed. Therefore, the central value of  $R_{\rm b\ell}$  was computed assuming for  $\rho_{\rm b\ell}^{\rm IJ}$  the weighted average of  $\rho_{\rm bb}^{\rm IJ}$  and  $\rho_{\ell\ell}^{\rm IJ}$ . The systematic error was estimated as the quadratic sum of: i) the sum of simulation statistical errors on  $\rho_{\rm bb}^{\rm IJ}$  and  $\rho_{\ell\ell}^{\rm IJ}$ ; ii) the maximal difference between  $\rho_{\rm bb}^{\rm IJ}$ ,  $\rho_{\ell\ell}^{\rm IJ}$  and zero. The values obtained from this approximation for the 1994 data sample are those given in table 2.

As a cross-check of the measurement,  $R_{b\ell}$  was measured at several values of the B and L tagging efficiencies. Figures 4 and 5 show the stability of the final  $R_{b\ell}$  result as a function of the B tag efficiency for the 1994–1995 and 1992–1993 combinations respectively, together with the contributions to the total error. It can be seen that the minimum error is obtained at a B tag efficiency of 29.6% (i.e. for a cut  $-\log_{10} y \ge 1.2$ ) in 1994–1995, and of 27.1% (cut  $-\log_{10} y \ge 0.4$ ) in 1992–1993. However, to have similar purities in all

Source of error	Range	$\Delta R_{\rm b\ell} \times 10^4$
Data statistics		$\pm$ 8.7
Simulation statistics		$\pm 4.2$
Event selection		$\pm 1.1$
$K^0$ , $\Lambda^0$ , photons, etc.	$\pm 20\%$	$\mp 0.5$
Tracking	See [6]	$\pm 1.6$
Gluon splitting $g \to c\bar{c}$	$(2.33 \pm 0.50)\%$	$\mp 1.0$
Gluon splitting $g \to b\bar{b}$	$(0.269 \pm 0.067)\%$	$\mp 3.4$
$D^+$ fraction in $c\bar{c}$ events	$0.233 \pm 0.027$	$\mp 1.6$
$D_s$ fraction in $c\bar{c}$ events	$0.103 \pm 0.029$	$\pm 0.3$
c-baryon fraction in $c\bar{c}$ events	$0.063 \pm 0.028$	$\pm 1.5$
$BR(D^0 \rightarrow no neutrals)$	$(14.1 \pm 1.1)\%$	$\mp 0.8$
$BR(D^0 \rightarrow 1 \text{ neut.}, \geq 2 \text{ charged})$	$(37.7 \pm 1.7)\%$	$\mp 0.3$
$BR(D^+ \rightarrow no neutrals)$	$(11.2 \pm 0.6)\%$	$\mp 0.6$
$BR(D^+ \rightarrow 1 \text{ neut.}) \geq 2 \text{ charged})$	$(26.1 \pm 2.3)\%$	$\mp 0.2$
$BR(D_s \to K^0X)$	$(33 \pm 18)\%$	$\mp 1.6$
$D^0$ lifetime	$0.415\pm0.004~\mathrm{ps}$	$\mp 0.3$
$D^+$ lifetime	$1.057\pm0.015~\mathrm{ps}$	$\mp 0.3$
$D_s$ lifetime	$0.447\pm0.017~\mathrm{ps}$	$\mp 0.3$
$\Lambda_{\rm c} \ {\rm lifetime}$	$0.206 \pm 0.012 \text{ ps}$	$\mp 0.0$
D decay multiplicity	$2.13 \pm 0.14$	$\mp 1.0$
$\langle x_E({ m c}) angle$	$0.484 \pm 0.008$	$\mp 0.6$
Two b's same hemisphere	$\pm 30\%$	$\pm 0.6$
$\langle x_E(\mathrm{b}) \rangle$	$0.702 \pm 0.008$	$\pm 1.5$
B decay multiplicity	$4.97\pm0.07$	$\mp 1.2$
Average B lifetime	$1.55\pm0.05~\mathrm{ps}$	$\mp 0.0$
c-physics correlations		$\pm 0.6$
Angular effects	See [6]	$\pm 1.2$
Gluon radiation	See [6]	$\pm$ 3.1
FCNC correlations	See text	$\pm 0.6$
Total systematic error	$\pm$ 7.8	
Total error		±11.7

Table 6: Detailed error breakdown for the measurement of  $R_{\mathrm{b}\ell}$  for the combined result.

years and to minimise the combined error, the cut  $-\log_{10} y \ge 0.6$  was used for 1992–1993, which corresponds to a B tag efficiency of 23.9%.

Figure 6 shows the stability of  $R_{b\ell}$  as a function of the other hemisphere B tag category efficiencies (i.e. B1, B2) as well as the L tag efficienciency for 1994–1995. The stability was found consistent in all cases, taking into account the point to point correlation. The stability of the result as as function of the C tag efficiency, not shown here, was also found to be consistent.

#### 6 Conclusions

The existence of events produced by the FCNC process  $e^+e^- \rightarrow b\bar{q}$ , q = d, s at the  $M_Z$  energy scale has been investigated. The powerful tagging and self-data-calibration techniques developed by DELPHI for the  $R_b$  analysis [6] have been used to perform this study and the result obtained has been

$$R_{\rm b\ell} = [0.67 \pm 0.87({\rm stat}) \pm 0.78({\rm syst})] \times 10^{-3}$$
,

which is compatible with no experimental observation of this type of events. The dependences of this measurement with the assumed values of  $R_{\rm b}$  and  $R_{\rm c}$  are, respectively,  $-1.277 \times (R_{\rm b} - 0.21584)$  and  $-0.030 \times (R_{\rm c} - 0.172)$ .

The exclusion limit thus derived is

$$R_{\rm b\ell} = \frac{\sum_{\rm q=d,s} \sigma({\rm e^+e^-} \to {\rm b}\bar{\rm q})}{\sigma({\rm e^+e^-} \to {\rm hadrons})} \le 2.6 \times 10^{-3}$$

at 90% CL, were  $R_{\rm b}$  and  $R_{\rm c}$  are fixed to their electroweak theory predictions [16], assuming a mass for the top quark of  $m_t = 173.8 \pm 5.2 \text{ GeV/c}^2$  [17].

#### Acknowledgments

We are greatly indebted to our technical collaborators and to the funding agencies for their support in building and operating the DELPHI detector, and to the members of the CERN-SL Division for the excellent performance of the LEP collider.



Figure 4: Stability of the 1994–1995  $R_{b\ell}$  result as a function of the B tag efficiency, together with the contributions to the total error. The minimum error is obtained at an efficiency of 29.6%, where the b-purity is 98.5%. In the upper plot the thick error bar represents the statistical uncertainty and the narrow one is the total error. All errors are correlated from point to point. The arrow marks the position of the working point and the dotted line shows the value at that cut.



Figure 5: Stability of the 1992–1993  $R_{b\ell}$  result as a function of the B tag efficiency, together with the contributions to the total error. The working point is chosen to have a similar purity to that at the working point of the 1994–1995 analysis. It results in an efficiency of 23.9% with a b-purity of 98.2%. All errors are correlated from point to point. The arrow marks the position of the working point and the dotted line shows the value at that cut.



Figure 6: Stability of the multivariate  $R_{b\ell}$  result as a function of the efficiencies of the b-standard, b-loose, charm and uds hemisphere tags for 1994–1995. Only the statistical errors are shown. Errors are correlated from point to point. The arrow marks the position of the working point and the dotted line shows the value at that cut.

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