3.8 CP, T and CPT Asymmetries with CP Eigenstates at the $\Upsilon(4S)$

Instead of the oscillation $B^0 \to \overline{B}^0$, the transition of $B^0 \to B_+$ can be time-reversed as $B_+ \to B^0$, and likewise $B^0 \to B_-$, $\overline{B}^0 \to B_+$, and $\overline{B}^0 \to B_-$. Here, B_+ is the CP $= +1$ eigenstate

$$|B_+\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle + e^{i\phi_{\mathrm{CP}B}}|\overline{B}^0\rangle\right)$$

which can be detected by the decay to a CP $= +1$ final state and B_- is the CP $= -1$ eigenstate

$$|B_-\rangle = \frac{1}{\sqrt{2}}\left(|B^0\rangle - e^{i\phi_{\mathrm{CP}B}}|\overline{B}^0\rangle\right)$$

which can be detected by the decay to a CP $= -1$ final state.

The transitions are observed in a measurement of time-dependent CP violation into CP $= +1$ and CP $= -1$ eigenstates at the $\Upsilon(4S)$ with the same weak phase.

However, for this measurement to be used for T and CPT tests, we have to assume (and verify) the following facts:

- CP conservation in the decay is necessary to identify B_+ and B_-. This can be tested through a measurement of the $\cos xT$ coefficient Θ.
- The decays $B_+ \to$ CP + eigenstate and $B_- \to$ CP − eigenstate proceed through the same diagrams (e.g. $B \to J/\psi K^0_S$ and $B \to J/\psi K^0_L$).

Table 3.20 Data samples for T, CP and CPT asymmetry measurements at the $\Upsilon(4S)$. The rates have to be normalized, scaling with the inverse branching fraction and detection efficiency for the CP + and CP − final states.

sample	tag, t_1	final state, t_2	$\Delta t = t_2 - t_1$	process
a	$\overline{B}^0$	CP +	$\Delta t > 0$	$B^0 \to B_+$
b	$\overline{B}^0$	CP +	$\Delta t < 0$	$B_- \to \overline{B}^0$
c	B^0	CP +	$\Delta t > 0$	$\overline{B}^0 \to B_+$
d	B^0	CP +	$\Delta t < 0$	$B_- \to B^0$
e	$\overline{B}^0$	CP −	$\Delta t > 0$	$B^0 \to B_-$
f	$\overline{B}^0$	CP −	$\Delta t < 0$	$B_+ \to \overline{B}^0$
g	B^0	CP −	$\Delta t > 0$	$\overline{B}^0 \to B_-$
h	B^0	CP −	$\Delta t < 0$	$B_+ \to B^0$

The rates measured are defined in table 3.20. The CP asymmetries are calculated from the ratios

$$\frac{a}{c}, \quad \frac{b}{d}, \quad \frac{e}{g}, \quad \frac{f}{h} \tag{3.123}$$

as the asymmetries

$$\frac{a-c}{a+c}, \quad -\frac{b-d}{b+d}, \quad -\frac{e-g}{e+g}, \quad \frac{f-h}{f+h}$$

where the signs reflect the expected sign change if CPT is conserved. If these asymmetries are equal, an unbiased estimator of the combined measurement is the ratio

$$\frac{a+d+f+g}{b+c+e+h} \tag{3.124}$$

and the corresponding asymmetry

$$\frac{a-b-c+d-e+f+g-h}{a+b+c+d+e+f+h}$$

The T asymmetries are calculated from the ratios

$$\frac{a}{h}, \quad \frac{b}{g}, \quad \frac{c}{f}, \quad \frac{d}{e} \tag{3.125}$$

as the asymmetries

$$\frac{a-h}{a+h}, \quad -\frac{b-g}{b+g}, \quad -\frac{c-f}{c+f}, \quad \frac{d-e}{d+e}$$

where again the signs reflect the expected sign change if CPT is conserved. If these asymmetries are equal, an unbiased estimator of the combined measurement is the ratio

$$\frac{a+d+f+g}{b+c+e+h}$$

which is identical to (3.124)!

If we do not assume CPT, we can also calculate four CPT asymmetries from the ratios

$$\frac{a}{f}, \quad \frac{b}{e}, \quad \frac{c}{h}, \quad \frac{d}{g} \tag{3.126}$$

as the asymmetries

$$\frac{a-f}{a+f}, \quad \frac{b-e}{b+e}, \quad \frac{c-h}{c+h}, \quad \frac{d-g}{d+g}$$

There is no way to predict the relative sign, hence there is no combined estimator of the CPT asymmetry.

All asymmetries can be calculated using the 8 samples given in table 3.20, which will imply the dilution factor (3.51) for time-integrated measurements. The sensitivity may be improved if the asymmetry is investigated as a function of $|\Delta t|$.