

Review on D^0 - \bar{D}^0 mixing

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Abstract

We review the current status of D^0 - \bar{D}^0 mixing, with special emphasis in the most recent results. We begin with a discussion of charm mixing and CP violation phenomenology, the evolution with the decay proper time, and physics processes contributing to these. Then we follow with the summary of the main experimental techniques and the results in the various final states. We make use of the analysis reporting the first evidence of D^0 - \bar{D}^0 mixing by BaBar in $D^0 \rightarrow K^+\pi^-$ decays as a textbook example, and then we discuss the results from other two-body and three-body final states. We conclude with the combination of all experimental results. Time-integrated CP violation measurements are not discussed here.

1. Introduction

Particle-antiparticle oscillation (also referred as mixing) is a well known phenomenon observed in the kaon system in 1956 [1], in the B_d^0 system in 1987 [2], and more recently in 2006 in the B_s^0 system [3]. Mixing and CP violation (CPV) in the charm sector were first discussed over three decades ago [4], but experimental evidence for oscillation has been presented only in the last two years [5, 6, 7, 8, 9, 10], and no evidence for CPV has yet been reported, with upper limits currently at about 1% level.

Charm mixing is the only involving down-type quarks in the mixing loop, since neutral pions do not oscillate and the top quark does not have bound states. Thus D^0 - \bar{D}^0 mixing offers an unique probe for New Physics (NP) via flavor changing neutral currents (FCNC) in the down-quark sector, providing interesting constraints on NP models. The caveat is how to distinguish NP from Standard Model (SM) long-distance (non-perturbative) uncertainties. A possible avenue is correlating charm mixing studies (and possibly also rare charm decays) with a comprehensive account of CP violation in D^0 - \bar{D}^0 mixing (both within the SM and beyond). The two effects are heavily suppressed in the SM (charm mixing is about two orders of magnitude slower than in the neutral-kaon system and CPV is well below the per mille level), which makes these experimentally difficult to observe, although NP can produce significant enhancements.

2. Charm mixing phenomenology

Neutral- D mesons are created as flavor eigenstates of strong interactions, but they mix through weak interactions. The time evolution is obtained by solving the time-dependent Schrödinger equation,

$$i \frac{\partial}{\partial t} \begin{bmatrix} D^0(t) \\ \bar{D}^0(t) \end{bmatrix} = \mathbf{H}_w \begin{bmatrix} D^0(t) \\ \bar{D}^0(t) \end{bmatrix}, \quad (1)$$

with $\mathbf{H}_w = \mathbf{M} - i\mathbf{\Gamma}/2$ the effective Hamiltonian, where \mathbf{M} and $\mathbf{\Gamma}$ are 2×2 matrices that represent transitions via off-shell (dispersive) and on-shell (absorptive) intermediate

states, respectively. Assuming CPT invariance, we have $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. Since these matrices are Hermitian, $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$. If CP in mixing is conserved, $M_{12} = \bar{M}_{21}$ and $\Gamma_{12} = \bar{\Gamma}_{21}$.

The physical (mass) eigenstates are linear combinations of the interaction eigenstates, $|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$ [11], with time evolution $|D_{1,2}(t)\rangle = e^{-i\lambda_{1,2}t}|D_{1,2}\rangle$, where $\lambda_{1,2} = m_{1,2} - i\Gamma_{1,2}/2$ are the eigenvalues. Here, $m_{1,2}$ ($\Gamma_{1,2}$) represent the mass (decay width) of the physical states. The complex mixing parameters p and q obey the normalization condition $|p|^2 + |q|^2 = 1$, and their ratio is

$$\frac{q}{p} = \pm \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}} = \left| \frac{q}{p} \right| e^{-i\phi}, \quad (2)$$

where ϕ is the CP-violating phase in D^0 - \bar{D}^0 mixing.

The time-dependent amplitude for a D^0 or \bar{D}^0 decaying into a final state f after a time t is

$$\langle f|H|D^0(t)\rangle = \frac{1}{2} \left\{ A_f g_{\pm}(t) + \frac{q}{p} \bar{A}_f g_{\mp}(t) \right\}, \quad (3)$$

where $A_f = \langle f|H|D^0\rangle$ and $\bar{A}_f = \langle f|H|\bar{D}^0\rangle$ are the decay amplitudes at $t = 0$, and $g_{\pm}(t) = e^{-i\lambda_1 t} \pm e^{-i\lambda_2 t}$. The corresponding time evolution probability is [12, 13]

$$\Gamma \left(\begin{matrix} D^0 \\ \bar{D}^0 \end{matrix} \rightarrow f \right) (t) = e^{-\Gamma t} |A_f|^2 \left[\frac{C_y}{\bar{C}_y} \cosh(y\Gamma t) + \frac{C_x}{\bar{C}_x} \cos(x\Gamma t) + \frac{S_y}{\bar{S}_y} \sinh(y\Gamma t) + \frac{S_x}{\bar{S}_x} \sin(x\Gamma t) \right], \quad (4)$$

where

$$\begin{aligned} C_y &= \frac{1 + |\lambda_f|}{2}, & C_x &= \frac{1 - |\lambda_f|}{2}, \\ S_y &= -\Re \lambda_f, & S_x &= \Im \lambda_f, \\ (\bar{C}_y, \bar{S}_y, \bar{C}_x, \bar{S}_x) &= \left| \frac{p}{q} \right|^2 (C_y, S_y, -C_x, -S_x), \end{aligned} \quad (5)$$

with the definitions

$$\begin{aligned} x &= \frac{m_1 - m_2}{\Gamma}, & y &= \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, & \Gamma &= \frac{\Gamma_1 + \Gamma_2}{2}, \\ \lambda_f &= \frac{q}{p} \frac{\bar{A}_f}{A_f} = \left| \frac{q}{p} \right| r_d e^{-i(\Delta_f + \phi)}. \end{aligned} \quad (6)$$

Here Δ_f is the relative phase between \bar{A}_f and A_f , and r_d is the magnitude of the ratio between the two amplitudes. Mixing will occur either if x or y is non zero, while CP violation in mixing is signaled by $p \neq q$, which

can occur either if $|q/p| \neq 1$ (CP violation in mixing) or $\phi \neq 0$ (CP violation in the interference between mixing and decay). Direct CP violation is signaled by $A_f \neq \bar{A}_f$.

In the SM, D^0 - \bar{D}^0 mixing arises from $|\Delta C| = 2$ (C is the charm quantum number) short-range box diagrams (see Fig. 1.Left) containing down-type quarks, strongly suppressed either by small b -quark couplings (CKM suppressed) or by the GIM cancellation mechanism [14] for the light d - and s -quarks. As a consequence, non-zero values for x and y are generated in the SM only at second order in $SU(3)_F$ breaking, $x, y \sim \sin^2 \theta_C \times (m_s^2 - m_d^2)/m_c^2$, where θ_C is the Cabibbo angle and m_s , m_d , and m_c are quark masses [15]. Lowest order calculations yield $x \sim \mathcal{O}(10^{-5})$ and $y \sim \mathcal{O}(10^{-7})$, although enhancements due to higher orders in operator product expansion (OPE) up to $\mathcal{O}(10^{-3})$ have been calculated [16]. Models involving NP can greatly increase estimates for both x and y [17], but so can $|\Delta C| = 1$ long-range SM processes with intermediate states accessible to D^0 and \bar{D}^0 [18] (see Fig. 1.Right). While most studies find $|x|, |y| < 10^{-3}$, some estimates for x and y allow for values as large as $\mathcal{O}(10^{-2})$ and suggest they are of opposite sign [19]. Overall, theoretical predictions for x and y within the SM span several orders of magnitude, reflecting the fact that these processes are difficult to calculate [13, 20].

However, it would be a sign of NP if x were to be significantly larger than y or if CPV either in mixing ($p \neq q$) or decay were observed [17] with current data samples. The observation of (large) CP violation as an unambiguous sign for NP is due to the fact that all quarks building up the hadronic states in weak decays of charm mesons belong to the first two generations. Since the 2×2 Cabibbo quark-mixing matrix is real, no CP violation is possible at tree level, and only penguin or box diagrams induced by virtual b -quarks can generate CP-violating amplitudes. However, as stated above, their contributions are strongly CKM suppressed.

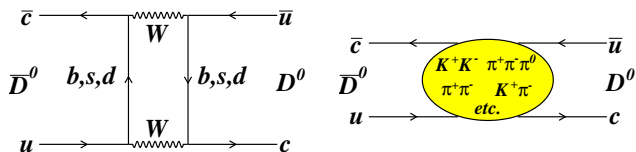


Fig. 1. SM processes contributing to D^0 - \bar{D}^0 mixing: (Left) Short-range box diagram and (Right) long-range interactions with intermediate states.

3. Experimental methods

A generic D^0 - \bar{D}^0 mixing analysis is performed in three steps. First, the D^0 (or \bar{D}^0) flavor at production time ($t = 0$) is identified (“tagged”) using $D^{*+} \rightarrow D^0 \pi^+$ decays [21]. These events are usually selected and characterized using the invariant mass of the exclusively reconstructed D^0 meson, m_{D^0} , and the mass difference between the reconstructed D^{*+} and D^0 mesons, $\Delta m = m(D^{*+}) - m_{D^0}$. The distribution of Δm shows a narrow peak, due to the small Q -value of the $D^{*+} \rightarrow D^0 \pi^+$ decay. Other tools used to improve the event selection and reduce backgrounds are particle identification (for leptons, kaons or pions, depending on the D^0 final state) and cuts on D^0 (high) and soft pion (low) momentum. The charge of the soft pion from the D^* decay unambiguously identifies the D^0 flavor at production. Then, the D^0 flavor at decay time is identified using the charge of the final state particles. For example, if the reconstructed final state is a positive kaon and a negative pion, and the soft pion from the D^* decay has a negative charge, then we have a “Right sign (RS)” combination, $\bar{D}^0 \rightarrow K^+ \pi^-$. On the contrary, if the D meson has been tagged as D^0 , then we have a “Wrong sign (WS)” combination, $D^0 \rightarrow K^+ \pi^-$. When we have a tagged D^0 meson at production, a WS combination can occur either if the D^0 meson decays via a double-Cabibbo-suppressed (DCS) transition or if it oscillates into a \bar{D}^0 meson followed by decay through a Cabibbo favored (CF) transition. Only a time-dependent analysis of the WS rate allows to distinguish between these two effects. Finally, the production and decay vertices of the D^0 meson are reconstructed in order to calculate the decay flight length and hence the decay proper time t and its uncertainty σ_t . At B factories, restricting the production point to the luminous region of the collider (beam spot) greatly improves the precision on the decay time reconstruction, as well as on Δm . At these facilities the average decay length is about $240 \mu\text{m}$, with typical resolution about $100 \mu\text{m}$ (the latter depends on the specific reconstructed final state). Analyses usually apply quality cuts on the proper-time error, in order to reduce effects from wrongly reconstructed vertices.

The tagging can also be performed using coherent D^0 - \bar{D}^0 production at charm factories running slightly above the D^0 - \bar{D}^0 threshold, although in this case only time-integrated mixing related measurements are possible at present facilities [22]. On the other hand, some analyses can be performed using tagged or untagged samples, as it is the case of the lifetime differences between decays to CP eigenstates (like $D^0 \rightarrow K^+ K^-$) and to the CP-mixed state $D^0 \rightarrow K^- \pi^+$, as discussed later.

Four experimental techniques have been used to measure D^0 - \bar{D}^0 mixing, depending on the specific D^0 final state: WS semileptonic decays, WS hadronic decays, decays to CP eigenstates, and self-conjugate three-body final states containing a combination of quasi-zero body flavor and CP eigenstates, particularly $K_S^0 \pi^+ \pi^-$. Quantum-correlated final states at charm factories are also sensitive to D^0 - \bar{D}^0 mixing (mostly y) via time-integrated observables [22], although their sensitivity is not competitive with time-dependent measurements. These are however fundamental to provide information on magnitudes and phases of relevant amplitude ratios, as described later.

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4. Wrong sign hadronic decays

Sensitivity to mixing using WS hadronic decays, for example $D^0 \rightarrow K^+ \pi^-$, is obtained by analyzing their proper-time evolution. Time-dependent studies allow separation of the direct DCS $D^0 \rightarrow K^+ \pi^-$ amplitude from the mixing contribution followed by the CF decay, $D^0 \rightarrow \bar{D}^0 \rightarrow K^+ \pi^-$ [12, 13]. Taking $|\lambda_f| \ll 1$ and assuming small mixing,

$$\frac{\Gamma(D^0 \rightarrow f)(t)}{e^{-\Gamma t}} \propto R_D + (\Gamma t)^2 \frac{R_M}{2} + (\Gamma t) \sqrt{R_D y'}, \quad (7)$$

where f represents the WS final state, $R_D = r_d^2$ is the ratio of DCS to CF decay rates, $R_M = (x^2 + y^2)/2$ is the mixing rate, and $y' = y \cos \delta_f - x \sin \delta_f$, where $\delta_f = -\Delta_f$ is the relative strong phase between the DCS and CF decay amplitudes. The minus sign originates from the sign of V_{us} relative to V_{cd} , where V denotes the

quark-mixing CKM matrix. In Eq. (7), the first term corresponds to the DCS contribution to the WS rate (time independent), the second term is the contribution from mixing, and the third term is the interference between mixing and CF decays. Since x and y are small, $\mathcal{O}(10^{-2})$, it is precisely the interference term (linear in decay time, x and y) which gives the best sensitivity to mixing through y' . Let us note that $R_M \equiv (x'^2 + y'^2)/2$, with $x' = x \cos \delta_f + y \sin \delta_f$. However, a direct extraction of x and y from Eq. (7) is not possible due to the unknown relative phase δ_f .

Searches for charm mixing in WS $D^0 \rightarrow K^+\pi^-$ decays have been performed by the experiments E971 (using 2×10^{10} events from $\pi^- N$ interactions at 500 GeV) and FOCUS (from $10^6 D \rightarrow Kn(\pi)$ events from γN interactions), and by CLEO from 9 fb^{-1} of $e^+e^- \rightarrow \Upsilon(4S)$ data [23]. However, the available statistics from these experiments was not enough to obtain evidence of mixing.

The first evidence for D^0 - \bar{D}^0 mixing in WS $D^0 \rightarrow K^+\pi^-$ decays has been reported by BaBar using 384 fb^{-1} of data [5]. The simultaneous fit to the RS and WS data samples to describe the signal and the random soft pion and misreconstructed D^0 background components yields 1141500 ± 1200 and 4030 ± 90 signal events, respectively. Thus the fraction of WS decays is measured to be $R_{WS} = [0.353 \pm 0.008(\text{stat.}) \pm 0.004(\text{syst.})]\%$. In the presence of mixing, $R_{WS} > R_D$, as can easily be obtained integrating Eq. (7).

The measured proper-time distribution for the WS signal is modeled by Eq. (7) convolved with a resolution function determined using the RS proper-time fit. The proper-time distribution for WS data in the $m_{D^0} - \Delta m$ signal box is shown in Fig. 2., together with the fit results with and without mixing, shown as the overlaid curves. The mixing parameters are $y' = [0.97 \pm 0.44(\text{stat.}) \pm 0.31(\text{syst.})]\%$ and $x'^2 = [-0.022 \pm 0.030(\text{stat.}) \pm 0.021(\text{syst.})]\%$, and a correlation between them of -0.95 . The ratio of DCS to CF decay rates is measured to be $[0.303 \pm 0.016(\text{stat.}) \pm 0.010(\text{syst.})]\%$. The systematic uncertainties are dominated by the signal resolution function as extracted from the RS sample. As expected, $R_{WS} > R_D$, revealing the presence of mixing. As another cross-check of the mixing signal, R_{WS} can also be measured in slices of proper time, repeating the fit to the RS and WS data samples in each of these slices. The fitted and expected WS fractions are shown in Fig. 3. and are seen to increase quadratically with time, as expected according to Eq. (7). The significance of the mixing signal is equivalent to 3.9σ or 9.6×10^{-5} confidence level (CL), where σ denotes one standard deviation. Separate proper-time fits to D^0 and \bar{D}^0 events allow to determine a CP-violating asymmetry $A_D = (R_D^+ - R_D^-)/(R_D^+ + R_D^-) = [-2.1 \pm 5.2(\text{stat.}) \pm 1.5(\text{syst.})]\%$, where $R_D^+(R_D^-)$ is the ratio of DCS and CF decay rates for $D^0(\bar{D}^0)$, thus no evidence for CP violation is observed.

These results have been confirmed by the CDF experiment using a data sample of 1.5 fb^{-1} of $p\bar{p}$ interactions at $\sqrt{s} = 1.96 \text{ TeV}$ [7]. The analysis is similar to that from BaBar, although the different production environment makes the details to differ significantly. The time-integrated fit to the RS and WS data samples yield $(3.044 \pm 0.002) \times 10^6$ and $(12.7 \pm 0.3) \times 10^3$ signal events, respectively. The ratio of WS to RS decays as a function of the decay proper time in the range between 0.75 and 10 D^0 lifetimes shows again an approximately linear dependence, as observed in Fig. 4. The parabolic fit

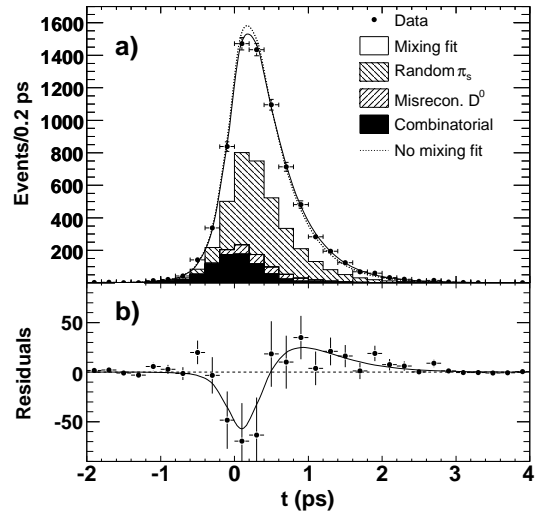


Fig. 2. BaBar WS $D^0 \rightarrow K^+\pi^-$ analysis [5]. (a) Projections of the proper-time distribution of WS candidates and fit results allowing (solid curve) and not allowing (dashed curve) mixing. (b) The points represent the difference between the data and the no-mixing fit, and the curve shows the difference between the fits with and without mixing.

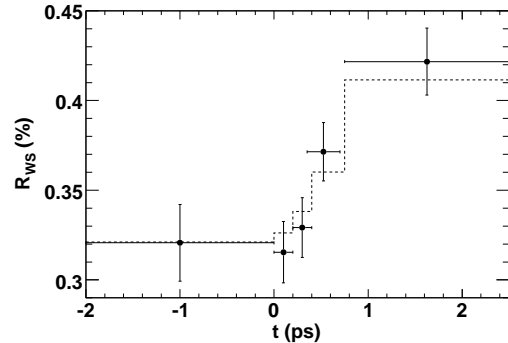


Fig. 3. BaBar WS $D^0 \rightarrow K^+\pi^-$ analysis [5]. The measured WS fraction in slices of measured proper time (points). The dashed line shows the expected WS fraction as determined from the mixing fit. In the absence of mixing, no time dependence would be observed.

of the data in this figure returns $y' = (0.85 \pm 0.76)\%$, $x'^2 = (-0.012 \pm 0.035)\%$, and $R_D = (0.304 \pm 0.055)\%$, where the errors include statistical and systematic uncertainties. The significance of the mixing signal is equivalent to 3.8σ (1.5×10^{-4} CL). These results are essentially identical to those obtained by BaBar, in spite of the very different production environment and sources of systematic uncertainties.

An earlier search by Belle for mixing in this decay mode using 400 fb^{-1} of data did not yield clear evidence for mixing [24]. The time-integrated fit to the RS and WS data samples returns very similar yields as those obtained by BaBar, 1073993 ± 1108 and 4024 ± 88 signal events, respectively, from which $R_{WS} = [0.377 \pm 0.008(\text{stat.}) \pm 0.005(\text{syst.})]\%$. The time-dependent mixing fit yields $y' = (0.06^{0.49}_{0.39})\%$, $x'^2 = (0.018^{0.021}_{0.023})\%$, and $R_D = (0.364 \pm 0.017)\%$, where the errors include statistical and systematic uncertainties. The correlation between y' and x'^2 is -0.91 . The no-mixing hypothesis is excluded at 2.1σ (3.9% CL). This result agrees with those obtained by BaBar and CDF at 2σ level. Separate proper-time fits to D^0 and \bar{D}^0 events show no evidence

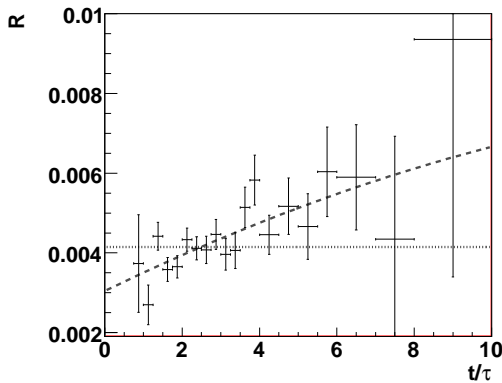


Fig. 4. CDF WS $D^0 \rightarrow K^+\pi^-$ analysis [7]. The measured ratio of WS to RS decays as a function of the decay proper time (points). The dashed curve is the result of the mixing fit, while the dotted line is a fit assuming no mixing.

for CP violation.

Quantum-correlated D^0 - \bar{D}^0 pairs produced in $\Psi(3770)$ decays at charm factories, with definite charge-conjugation eigenvalue $C = -1$, can be exploited to make a determination of the relative strong phase $\delta_{K\pi}$ to translate the measurement of y' into y [25]. At slightly higher energies (above DD^* threshold) one can also produce such pairs with $C = +1$ (additional photons in the final state). One can use the fact that heavy-meson pairs produced in the decays of heavy-quarkonium states have the property that the two mesons are in CP- or flavor-correlated states [22]. For instance, one may tag one of the neutral- D mesons as a CP eigenstate through its decay into CP eigenstates, such $K_S(\pi^0, \rho^0, \omega, \eta, \eta', \phi)$, K^+K^- , and $\pi^+\pi^-$. The other neutral- D meson must then have opposite CP if $C(D^0\bar{D}^0) = -1$ and the same CP if $C(D^0\bar{D}^0) = +1$. Then one measures its decay rate into $K^+\pi^-$, which includes, as discussed, an interference between CF and DCS amplitudes. The measured rate thus depends on the CF and DCS rates and the relative strong phase $\delta_{K\pi}$. More generally, one can measure time-integrated yields of correlated (“double tags”) and uncorrelated (“single tags”) neutral- D meson decays to CP eigenstates (CP-even and CP-odd) and flavor eigenstates (semileptonic and hadronic decays, with leptons and/or kaons as final state particles). The ratio of correlated and uncorrelated decay rates depends on mixing parameters R_D , $\sqrt{R_D}\cos\delta_f$, y , x^2 , and $\sqrt{R_D}x\sin\delta_f$. From 818 pb^{-1} of data recorded at $\Psi(3770)$ (and at slightly higher energies) and using branching ratios from other experiments, CLEOc obtains $\cos\delta_{K\pi} = 1.03_{-0.17}^{+0.31}(\text{stat.}) \pm 0.06(\text{syst.})$ [25]. The extraction of other mixing parameters is not competitive with time-dependent methods.

Further evidence for D^0 - \bar{D}^0 mixing has been reported by BaBar using a time-dependent Dalitz plot analysis of the multi-body WS decay $D^0 \rightarrow K^+\pi^-\pi^0$ [10]. The analysis in such decays is formally similar to the WS $D^0 \rightarrow K^+\pi^-$, but now the decay rate is a function of both the decay proper time and the Dalitz plot variables

$$s_0 = m_{K^+\pi^-}^2 \quad \text{and} \quad s_+ = m_{K^+\pi^0}^2,$$

$$\frac{\Gamma(D^0 \rightarrow f)(s_0, s_+, t)}{e^{-\Gamma t}} = |A_f|^2 + |\bar{A}_f|^2(\Gamma t)^2 \frac{R_M}{2} + (\Gamma t)|A_f||\bar{A}_f|y', \quad (8)$$

where $A_f(s_0, s_+)$ is the DCS amplitude, $\bar{A}_f(s_0, s_+)$ is the CF amplitude, and $y' = y \cos \delta_f(s_0, s_+) - x \sin \delta_f(s_0, s_+)$, with $\delta_f(s_0, s_+) = \arg[A_f^*(s_0, s_+)\bar{A}_f(s_0, s_+)]$ the relative strong phase between the DCS and CF amplitudes, now varying with the Dalitz plot position. As it can be seen in Eq. (8), the sensitivity to mixing comes from the variation of the Dalitz plot distribution with time produced by the CF-mixing interference term, which in turn mainly depends on the interference between the CF $\bar{D}^0 \rightarrow K^+\rho^-$ and DCS $D^0 \rightarrow K^{*+}\pi^-$ amplitudes, since these decays dominate the RS and WS Dalitz plots, respectively.

BaBar determined the CF amplitude \bar{A}_f in a time-integrated Dalitz plot analysis of the RS decay sample, consisting of 658,986 events with a purity of 99%. This amplitude is then used in the analysis of the WS sample, containing 3009 events with a purity of 50%, where the DCS amplitude A_f is extracted along with the mixing parameters. Each of the amplitudes \bar{A}_f and A_f are in turn described as a coherent sum of amplitudes, each describing a separate resonance (the usual isobar approach). Figure 5. shows the RS and WS proper-time distributions as well as the projections on s_0 and s_+ . Since for both \bar{A}_f and A_f one complex amplitude must be fixed arbitrarily and the CF and DCS Dalitz plots are different, the sensitivity to x and y is in the form $y' = y \cos \delta_{K\pi\pi^0} - x \sin \delta_{K\pi\pi^0}$ and $x' = x \cos \delta_{K\pi\pi^0} + y \sin \delta_{K\pi\pi^0}$, where $\delta_{K\pi\pi^0}$ is the strong phase difference between the DCS $D^0 \rightarrow K^+\rho^-$ and the CF $\bar{D}^0 \rightarrow K^+\rho^-$ amplitudes. This phase is unknown and different from $\delta_{K\pi}$. The measured mixing parameters are $x' = [2.61_{-0.68}^{+0.57}(\text{stat.}) \pm 0.39(\text{syst.})]\%$ and $y' = [-0.06_{-0.64}^{+0.55}(\text{stat.}) \pm 0.34(\text{syst.})]\%$, and a correlation between them of -0.75 . The significance of the mixing signal is equivalent to 3.2σ (0.1% CL). No evidence for CP violation is seen.

5. Hadronic decays to CP eigenstates

The lifetime difference between states of different CP content, for example $D^0 \rightarrow K^+K^-$ (CP even) compared to $D^0 \rightarrow K^-\pi^+$ (CP mixed), can also be used to measure D^0 - \bar{D}^0 mixing. For small mixing and taking $\lambda_f \approx 1$ (for CP-even decays, since $\delta_f = 0, \pi$ for CP = $+1, -1$) [12, 13],

$$\Gamma \left(\begin{matrix} D^0 \\ \bar{D}^0 \end{matrix} \rightarrow f_{CP} \right) (t) \propto e^{-\Gamma_{\pm} t}, \quad (9)$$

where $\Gamma_{\pm} = \Gamma(1 + y'_{\pm})$ is the D^0 or \bar{D}^0 effective lifetime, with $y'_{\pm} \approx y \cos \phi \mp x \sin \phi$. For untagged neutral- D mesons,

$$\Gamma (D^0 \text{ or } \bar{D}^0 \rightarrow f_{CP}) (t) \propto e^{-\langle \Gamma_{\pm} \rangle t}, \quad (10)$$

where $\langle \Gamma_{\pm} \rangle = \Gamma(1 + \langle y'_{\pm} \rangle)$ is the average D^0 and \bar{D}^0 effective lifetime. We clearly observe that for $y \neq 0$, the lifetimes to CP eigenstates (Γ_{\pm} , $\langle \Gamma_{\pm} \rangle$) and CP mixed

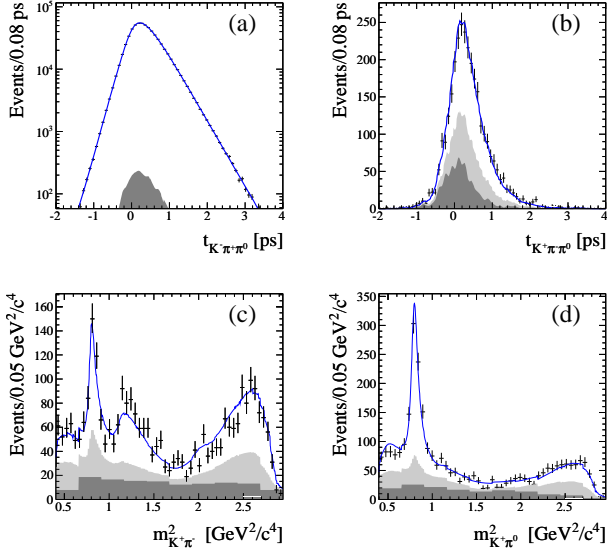


Fig. 5. BaBar WS $D^0 \rightarrow K^+\pi^-\pi^0$ analysis [10]. Proper-time distribution for (a) RS and (b) WS decays with the fit projection overlaid. Projections of the Dalitz plot distributions for (c) s_0 and (d) s_+ . The gray histograms represent the random soft pion background, while the dark histograms show the misreconstructed D^0 background.

states (Γ) differ. The experimentally defined observables are

$$y_{CP} = \frac{\tau_{K^-\pi^+}}{\langle\tau_{h+h^-}\rangle} - 1 = \frac{\tau}{(\tau_+ + \tau_-)/2} - 1 \approx y \cos \phi,$$

$$A_\tau = \frac{\tau_+ - \tau_-}{\tau_+ + \tau_-} \approx x \sin \phi, \quad (11)$$

where $\tau_{K^-\pi^+} \equiv \tau = 1/\Gamma$ and $\langle\tau_{h+h^-}\rangle = (\tau_+ + \tau_-)/2$ with $\tau_\pm = 1/\Gamma_\pm$ is the mean lifetime for neutral- D mesons decaying into CP eigenstates. The observable A_τ is the asymmetry in their lifetimes, sometimes replaced by $\Delta Y = \tau A_\tau / \langle\tau_\pm\rangle = (1 - y_{CP})A_\tau$. In the limit of vanishing CP violation $y_{CP} = y$, and A_τ (or ΔY) is zero. Both y_{CP} and A_τ (or ΔY) vanish if there is no D^0 - \bar{D}^0 mixing. The measurement of y_{CP} requires precise determinations of lifetimes using either tagged or untagged neutral- D mesons, but A_τ can only be measured using tagged D^0 and \bar{D}^0 mesons. The advantage of these observables is that most of the systematic uncertainties related to the signal cancel in the ratios, although background related systematic uncertainties do not.

Searches for D^0 - \bar{D}^0 mixing in hadronic decays to CP eigenstates have been done by the E971, FOCUS, and CLEO experiments [26]. However, the first evidence for charm mixing in these decays has been presented by Belle [6] simultaneously with the BaBar WS $D^0 \rightarrow K^+\pi^-$ [5] evidence, and both constitute the chief analyses reporting the first evidences for D^0 - \bar{D}^0 mixing (quickly confirmed by CDF [7]).

Using 540 fb^{-1} of data, Belle has measured $y_{CP} = [1.31 \pm 0.32(\text{stat.}) \pm 0.25(\text{syst.})]\%$ employing the K^+K^- and $\pi^+\pi^+$ final states (both CP-even), and found no evidence for CP violation in these decays since they obtained for the lifetime asymmetry $A_\tau = [0.01 \pm 0.30(\text{stat.}) \pm 0.15(\text{syst.})]\%$ [6]. The proper-time distribution for the $D^0 \rightarrow K^+K^-, K^-\pi^+, \pi^+\pi^-$ samples, consisting of 111×10^3 , 1.22×10^6 , 49×10^3 signal events with

purities of 98%, 99%, 92%, are shown in Fig. 6.(a,b,c), respectively, together with the projection of the fit. The CL of the no-mixing hypothesis ($y_{CP} = 0$) is 6×10^{-4} , which corresponds to a significance of 3.2σ . The mixing effect can be seen in Fig. 6.(d), which shows the ratio of decay-time distributions for $D^0 \rightarrow K^+K^-, \pi^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ decays, increasing linearly with time, as expected according to Eq. (10) to first order in y , $\Gamma(D^0 \text{ or } \bar{D}^0 \rightarrow f_{CP})(t) \propto 1 - \Gamma y_{CP} t$.

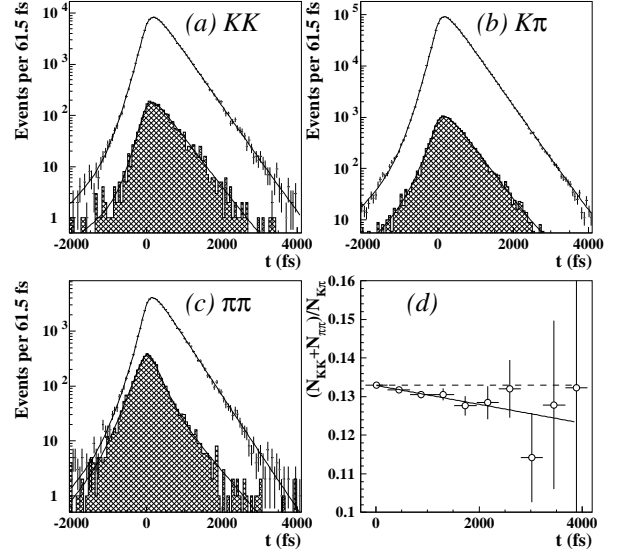


Fig. 6. Belle $D^0 \rightarrow K^+K^-, \pi^+\pi^-$ analysis [6]. Decay time distributions of (a) $D^0 \rightarrow K^+K^-$, (b) $D^0 \rightarrow \pi^+\pi^-$, and (c) $D^0 \rightarrow K^-\pi^+$ decays. The curves are the projections of the mixing fit, and the cross-hatched areas represent the background contribution. (d) Ratio of decay-time distributions between $D^0 \rightarrow K^+K^-, \pi^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$. The solid line is a linear fit to the data points.

A very similar study has also been performed by BaBar using 384 fb^{-1} of data, yielding $y_{CP} = [1.24 \pm 0.39(\text{stat.}) \pm 0.13(\text{syst.})]\%$ and $\Delta Y = [-0.26 \pm 0.36(\text{stat.}) \pm 0.08(\text{syst.})]\%$ [8]. These results are obtained fitting simultaneously the proper-time distributions for the D^* tagged $K^+K^-, \pi^+\pi^-$, and $K^-\pi^+$ samples, consisting of 69, 696, 30, 679 and 730, 880 signal events with purities of 99.6%, 98.0%, 99.9%. The significance of the no-mixing hypothesis is of 3.0σ , reflecting the significance of the difference of lifetimes between $K^+K^-, \pi^+\pi^-$ and $K^-\pi^+$, as summarized in Fig. 7.

Recently, BaBar has presented an untagged analysis using the same data sample of 384 fb^{-1} [9]. The proper-time distributions for the untagged $D^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^+K^-$ samples are shown in Fig. 8., together with the projection of the simultaneous fit to these samples. In this case, the samples contain 2710.2×10^3 and 263.6×10^3 signal events with purities of 94.2% and 80.9%, for $D^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^+K^-$ respectively. The measured lifetimes are $\langle\tau_{K^+K^-}\rangle = 405.85 \pm 1.0(\text{stat.}) \text{ fs}$ and $\tau_{K^-\pi^+} = 410.39 \pm 0.38(\text{stat.}) \text{ fs}$, yielding $y_{CP} = [1.12 \pm 0.26(\text{stat.}) \pm 0.22(\text{syst.})]\%$, which excludes no-mixing with 3.3σ . In this analysis, since the initial flavor of the decaying D^0 does not need to be identified, no D^{*+} reconstruction is required, increasing significantly the reconstruction efficiency but increasing the amount of background. To minimize it, the life-

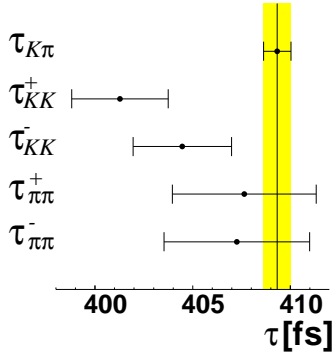


Fig. 7. BaBar $D^0 \rightarrow K^+K^-, \pi^+\pi^-$ analysis [8]. Summary of the measured lifetimes in the 5 D^* tagged K^+K^- , $\pi^+\pi^-$ and $K^-\pi^+$ samples.

time fit is performed in a narrow D^0 mass region around the nominal D^0 mass. The proper-time distribution for the main background component (combinatorial) is estimated from sideband D^0 mass regions, while for the small admixture of misreconstructed charm decays it is obtained from the simulation. Combining the tagged and untagged results taking into account both statistical and systematic uncertainties, BaBar finds $y_{CP} = [1.16 \pm 0.22(\text{stat.}) \pm 0.18(\text{syst.})]\%$. Summing statistical and systematic uncertainties in quadrature, the significance of this measurement is 4.1σ .

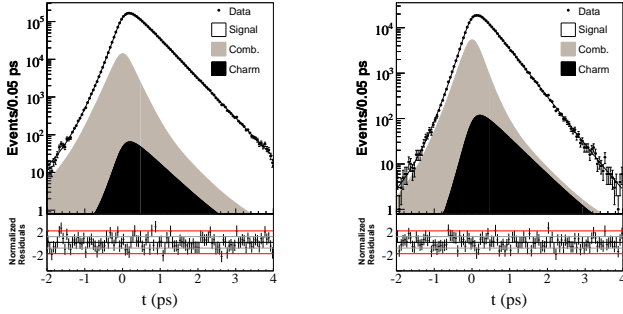


Fig. 8. BaBar $D^0 \rightarrow K^+K^-$ untagged analysis [9]. (Left) $D^0 \rightarrow K^-\pi^+$ and (Right) $D^0 \rightarrow K^-K^+$ decay-time distributions with the data (points) and the projection of the lifetime fit overlaid. The gray and black distributions represent the combinatorial and charm background contributions.

Yet another lifetime-difference analysis uses the $K_S^0 K^+ K^-$ final state, where the ϕK_S^0 region and its sidebands are examined to extract CP-odd and CP-even amplitudes. This is in effect a measurement of the lifetime in the CP-even and CP-odd parts of the $K_S^0 K^+ K^-$ Dalitz plot. Using 673 fb^{-1} , Belle has measured $y_{CP} = [0.11 \pm 0.61(\text{stat.}) \pm 0.52(\text{syst.})]\%$ [27]. This is done by measuring the mean lifetime τ_{ON} in the ϕK_S^0 region (mainly CP-odd) and the mean lifetime τ_{OFF} in the sidebands (mainly CP-even), along with the corresponding fractions f_{ON} and f_{OFF} of CP-even events in these regions. The lifetime asymmetry in these regions can then be related to y_{CP} ,

$$\frac{\tau_{OFF} - \tau_{ON}}{\tau_{OFF} + \tau_{ON}} = y_{CP} \frac{f_{ON} - f_{OFF}}{1 + y_{CP}(1 - f_{ON} - f_{OFF})}, \quad (12)$$

from which relation the latter is then determined. The main systematic uncertainties in this analysis come from

ON-OFF differences in the proper-time resolution function and the selection criteria, while the uncertainty from the Dalitz model assumptions needed to evaluate the CP-even content is negligible (0.01%).

Figure 9. summarizes all the available y_{CP} results. The combination of all these measurements is performed by the Heavy Flavor Averaging Group (HFAG) [28], and yields $y_{CP} = (1.107 \pm 0.217)\%$, which differs significantly (about 5σ) from zero. The combined lifetime asymmetry is $A_\tau = (-0.123 \pm 0.248)$, thus there is no evidence for CPV.

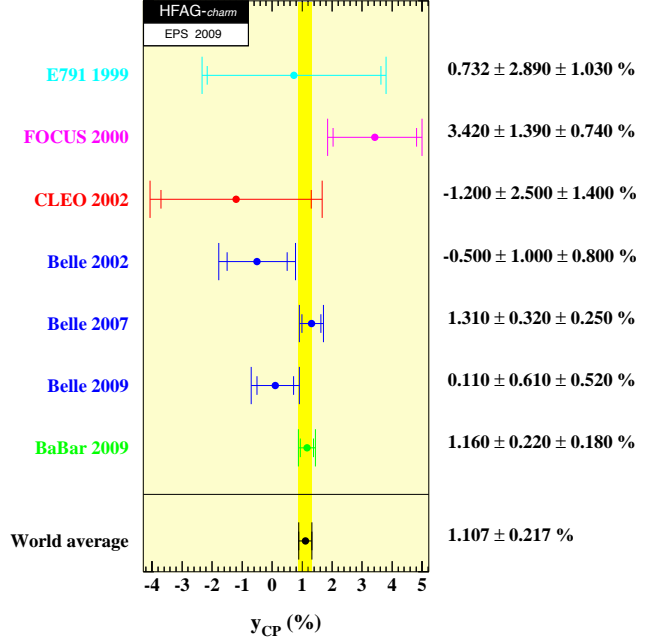


Fig. 9. Summary of y_{CP} measurements. The average $y_{CP} = (1.107 \pm 0.217)\%$ differs significantly from zero [28].

6. $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays

The last employed technique to study D^0 - \bar{D}^0 mixing involves the multi-body final state $K_S^0 \pi^+ \pi^-$. As in the case of the WS $D^0 \rightarrow K^+ \pi^- \pi^0$ analysis, the decay rate is a function of both the Dalitz plot variables $s_+ = m_{K_S^0 \pi^+}^2$ and $s_- = m_{K_S^0 \pi^-}^2$, and the D^0 decay proper time. With the usual approximations,

$$\frac{\Gamma(D^0 \rightarrow f)(s_+, s_-, t)}{e^{-\Gamma t}} = |A_f|^2 + |\bar{A}_f|^2 (\Gamma t)^2 \frac{R_M + |A_f|^2 (y^2 - x^2)/2}{2} + (\Gamma t) |A_f| |\bar{A}_f| y', \quad (13)$$

with $y' = y \cos[\delta_f(s_+, s_-) + \phi] - x \sin[\delta_f(s_+, s_-) + \phi]$, where $\delta_f(s_+, s_-) = \arg[A_f^*(s_+, s_-) \bar{A}_f(s_+, s_-)]$ is the relative strong phase between the D^0 and \bar{D}^0 decay amplitudes to the same final state $f = K_S^0 \pi^+ \pi^-$. Here, and contrary to the WS $D^0 \rightarrow K^+ \pi^- \pi^0$ case, the strong phase δ_f is fixed by the fact that the D^0 and \bar{D}^0 Dalitz plots are identical (the s_+ and s_- axes are just interchanged), assuming CP is not violated in the D^0 decay. Thus the analysis is free of unknown phases, providing a unique method to simultaneously measure the interfering D^0 and \bar{D}^0 amplitudes, the mixing parameters x and y (without rotations, and also their signs), and even the CP-violating parameters ϕ and $|q/p|$.

The CLEO experiment pioneered this analysis using only 9 fb^{-1} of data [29], obtaining the constraints $(-4.5 < x < 9.3)\%$ and $(-6.4 < y < 3.6)\%$ at 95% CL. Using 60 times more data, Belle has also performed this analysis [30], first assuming CP conservation and subsequently allowing for CP violation. The amplitudes A_f and \bar{A}_f are described using a coherent sum of 18 amplitudes, dominated by CF $D^0 \rightarrow K^{*-}\pi^+$, DCS $D^0 \rightarrow K^{*+}\pi^-$ and CP $D^0 \rightarrow K_S^0\rho^0$ decays. Assuming negligible CP violation, $x = [0.80 \pm 0.29(\text{stat.})^{+0.09}_{-0.07}(\text{syst.})^{+0.10}_{-0.14}(\text{model})]\%$ and $y = [0.33 \pm 0.24(\text{stat.})^{+0.08}_{-0.12}(\text{syst.})^{+0.06}_{-0.08}(\text{model})]\%$. This corresponds to a significance of 2.2σ from the no-mixing hypothesis. Figure 10. shows both the statistical-only and overall contours for both the CPV-allowed and the CP-conservation cases. No evidence for CP violation is found.

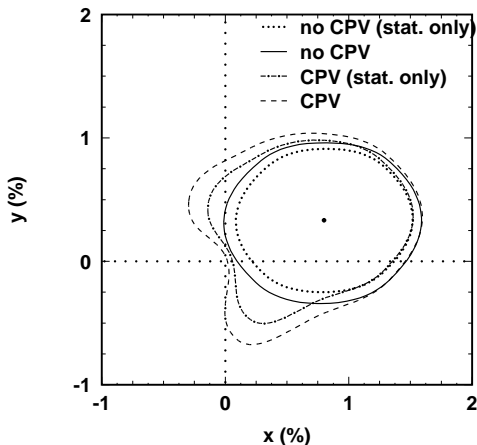


Fig. 10. Belle $D^0 \rightarrow K_S^0\pi^+\pi^-$ analysis [30]. 95% CL contours for (x, y) : dotted (solid) corresponds to statistical (statistical and systematic) contour for no CPV, and dash-dotted (dashed) corresponds to statistical (statistical and systematic) contours for the CPV-allowed case. The point is the best fit result for the no CPV case.

7. Wrong sign semileptonic decays

The most straightforward although not the most sensitive way to search for charm mixing is to use WS semileptonic decays, for instance $D^0 \rightarrow K^{(*)+}\ell^-\bar{\nu}_\ell$ [31, 32, 33]. In this case WS combinations can only occur through mixing,

$$\Gamma(D^0 \rightarrow f)(t) \propto e^{-\Gamma t}(\Gamma t)^2 R_M. \quad (14)$$

Therefore, these final states are only sensitive to $R_M \sim \mathcal{O}(10^{-4})$. Using semileptonic decays for mixing searches involves the measurement of the time-dependent or time-integrated rate for the WS decays. The main experimental challenge in these analyses is the limited mass resolution on Δm due to the presence of neutrinos. Significant improvements on Δm resolution are obtained applying kinematic constraints on the invariant mass of the neutrino and the kaon-lepton-neutrino system. The best current limits are from BaBar, $R_M < 0.12\%$ [32], and Belle, $R_M < 0.06\%$ [33], both at 90% CL, using 344 fb^{-1} and 492 fb^{-1} of data, respectively.

8. Combined results

The task of combining the wide variety of charm mixing results is done by the HFAG [28]. Figure 11. shows

the (x, y) contours of the collective experimental data, for the case of CP conservation. The central values $x = (0.989 \pm 0.241)\%$ and $y = (0.809 \pm 0.160)\%$ exclude the no-mixing point with 10.2σ . Other relevant combined parameters are $R_D = (0.3360 \pm 0.0084)\%$, $\delta_{K\pi} = 0.44 \pm 0.17$ rad, and $\delta_{K\pi\pi^0} = 0.24 \pm 0.37$ rad. When CP violation is allowed, the mixing parameters remain basically unchanged, $x = (0.976 \pm 0.249)\%$, $y = (0.833 \pm 0.160)\%$, $R_D = (0.3367 \pm 0.0086)\%$, $\delta_{K\pi} = 0.46 \pm 0.17$ rad, and $\delta_{K\pi\pi^0} = 0.26 \pm 0.37$ rad, and the following values for the CP-violating parameters are obtained: $|q/p| = 0.866 \pm 0.160$, $\phi = -0.148 \pm 0.126$ rad, and $A_D = (-2.2 \pm 2.4)\%$.

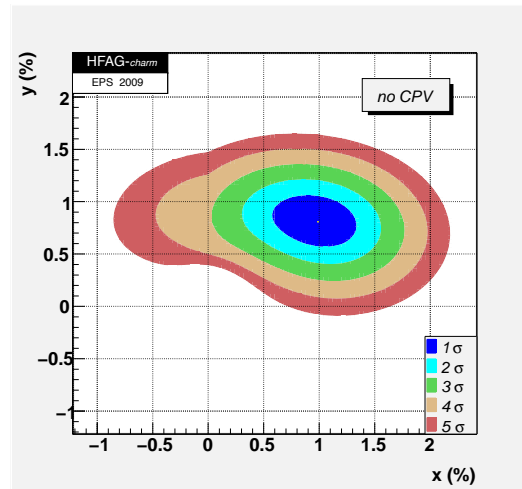


Fig. 11. Two-dimensional contours for the mixing parameters (x, y) from the HFAG combination [28] using all available measurements, for the case of CP conservation. The no-mixing point is excluded with a significance clearly exceeding 5σ .

9. Summary and conclusions

More than thirty years after the discovery of the D^0 meson [34] and the first theoretical discussion on mixing and CPV in the charm sector [4], BaBar, Belle and CDF Collaborations have provided compelling experimental evidence for D^0 - \bar{D}^0 mixing. Collective experimental data favor the mixing hypothesis at 10.2σ level (including systematic uncertainties). The mixing measurement from D^0 lifetime differences y_{CP} is significantly positive (about 5σ), indicating that the $|D_1\rangle$ eigenstate (\approx CP-even) has a shorter lifetime than the $|D_2\rangle$ eigenstate (\approx CP-odd).

However, no observation (more than 5σ) in a single measurement has yet been presented. In addition, to date there is only one direct measurement (from $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays) of x and y free of rotations (thus the only indication of the x and y relative sign), which do not differ significantly from zero but do affect the combination of all mixing measurements due to the large uncertainties that arise from the unknown phases δ_f that are inherent in other determinations. This is especially true for x . Clearly, more such measurements are a high priority and are foreseen in the future.

The measured values of the mixing parameters $x \approx y \approx 1\%$ are about compatible with SM expectations, although with large theoretical uncertainties. There is no evidence for CP violation in D^0 mesons, either in mixing, in decay or in interference.

Significant improvements in precision are foreseen in the short term with the analysis of complete data sets from current facilities (B factories and Tevatron). In the

long term, facilities about to start or just starting (LHCb and BESIII) have the potential to improve the precision on the mixing parameters in about a factor 5 and look deeper into CP violation searches. In the longer term, SuperB factories could be the last opportunity to observe CP violation in D^0 - \bar{D}^0 mixing [35] and search for NP in FCNC in the down-quark sector.

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