



Proposed New Dalitz model for ICHEP 06 analysis

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$D \rightarrow K_s \pi^+ \pi^-$ ICHEP 04 model

This is the first DK dalitz plot analysis for ICHEP 04.

We followed exactly the CLEO model (2001) and requires adhoc σ scalars, to describe the fit well.

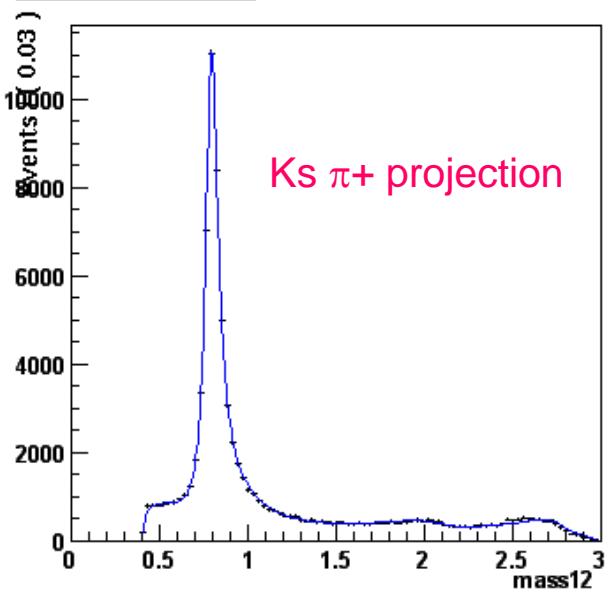
The first DK dalitz model

$K\pi$: $K^*(892), K^*_0(1430), K^*_0(1430)DCS, K^*_1(1410), K^*_2(1430), K^*_2(1430)DCS, K^*(1680)$

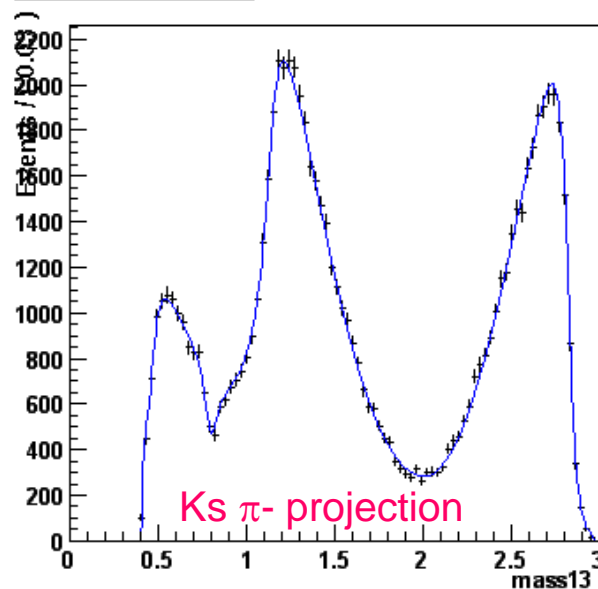
$\pi\pi$: $\sigma(500), \sigma_2, \rho(770), \omega(782), f_0(980), f_2(1270), f_0(1370), \rho(1450)$

We exhausted ALL the $K\pi$ and $\pi\pi$ resonances list in the PDG and still not get a good fit unless adhoc scalar, σ , are added.

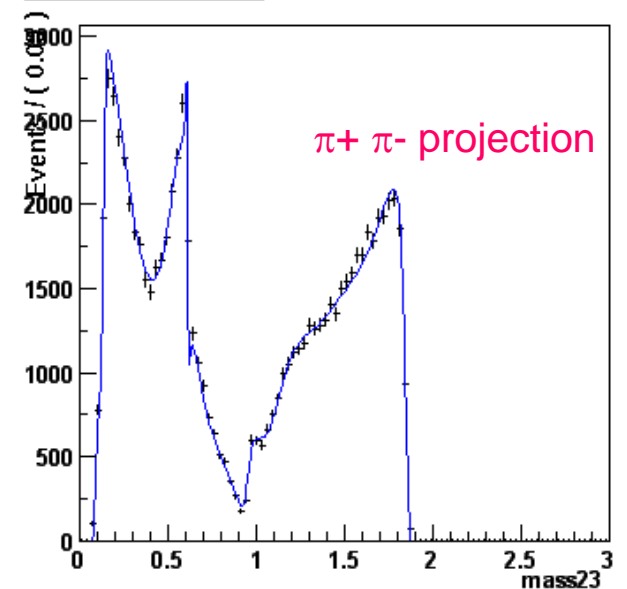
A RooPlot of "mass12"



A RooPlot of "mass13"



A RooPlot of "mass23"





$D \rightarrow K_s \pi^+ \pi^-$ EPS 05 model

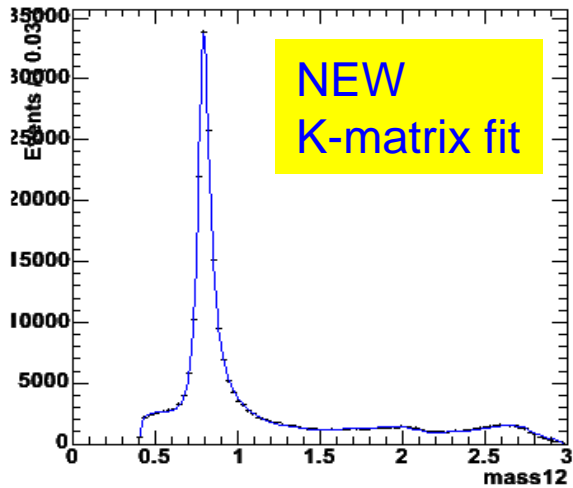
This analysis was done in 2005.

Motivated by the fact that Isobar model cannot treat the $l=0$ S-wave overlapping resonances. We proposed to use K-matrix method to parameterize $\pi^+ \pi^-$ S-wave

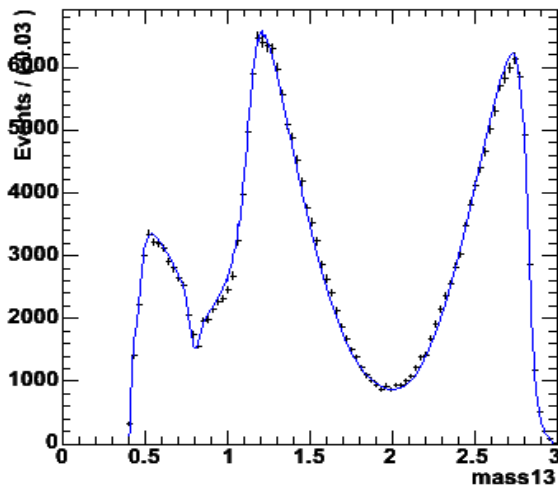
EPS 05 model

253991 events!

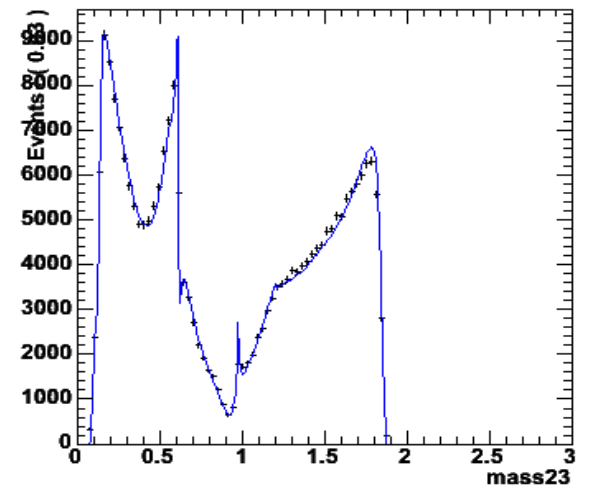
A RooPlot of "mass12"



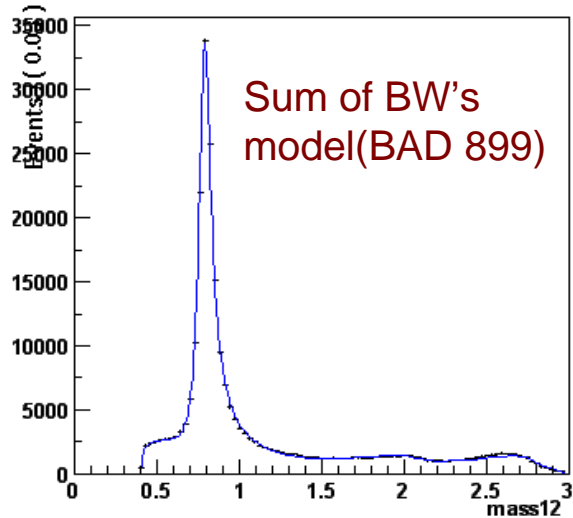
A RooPlot of "mass13"



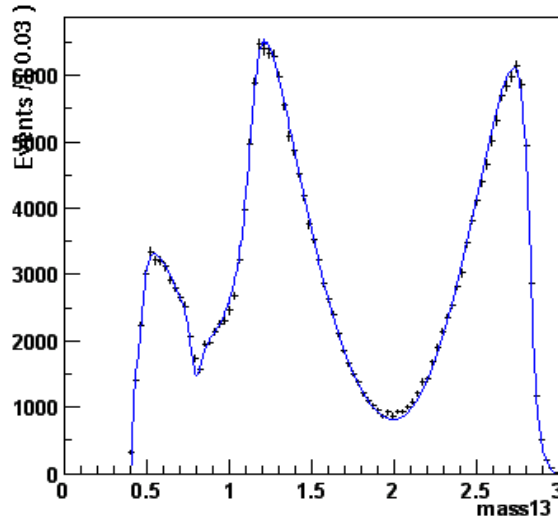
A RooPlot of "mass23"



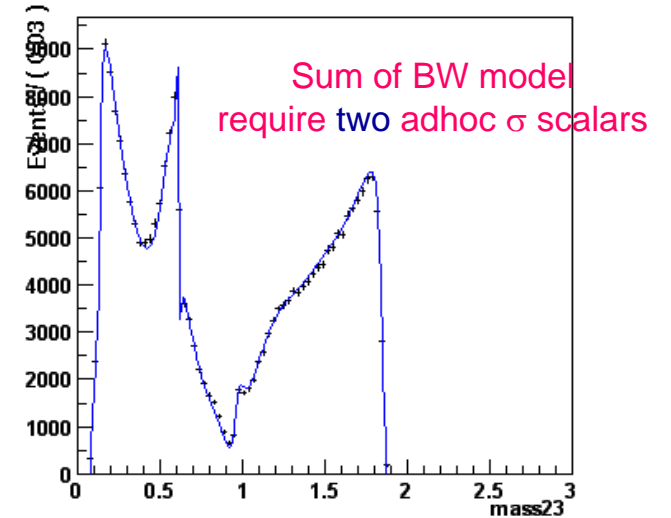
A RooPlot of "mass12"



A RooPlot of "mass13"



A RooPlot of "mass23"

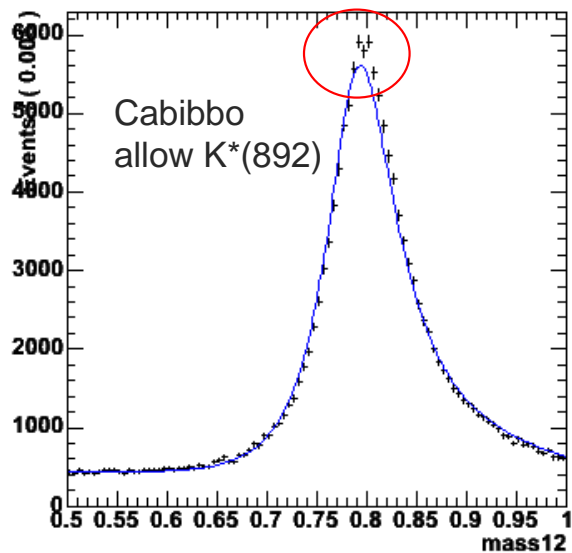


253991 events

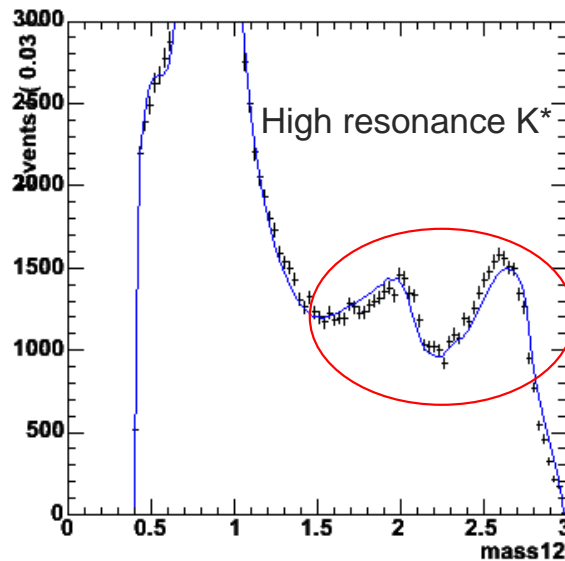
K-matrix model

Notice the fit is not good in the $K\pi$ channel

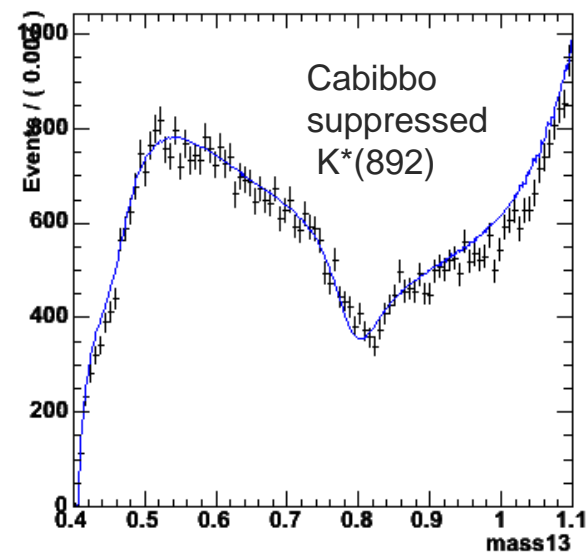
A RooPlot of "mass12"



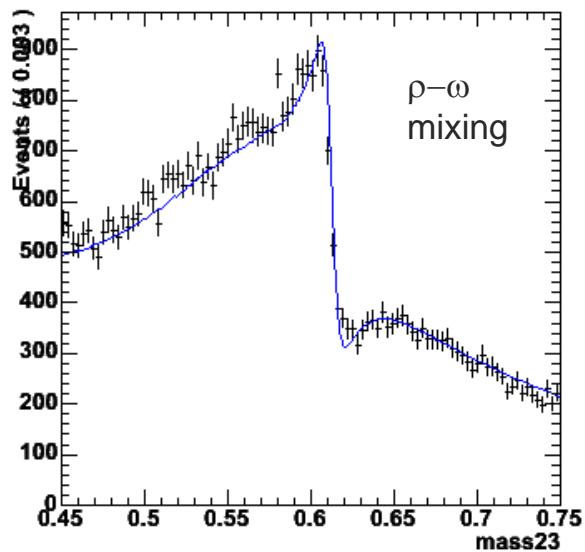
A RooPlot of "mass12"



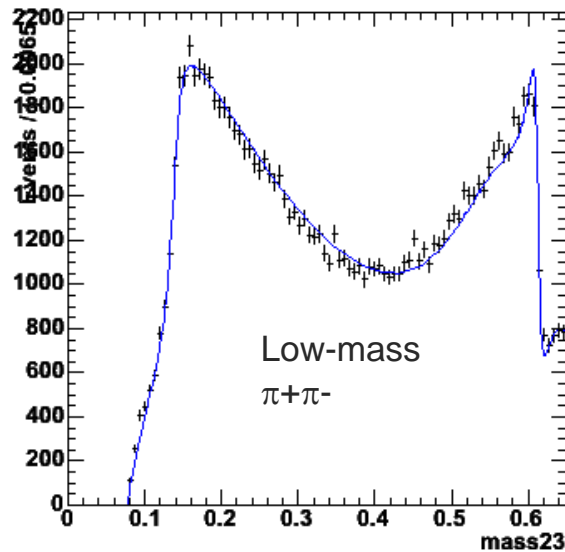
A RooPlot of "mass13"



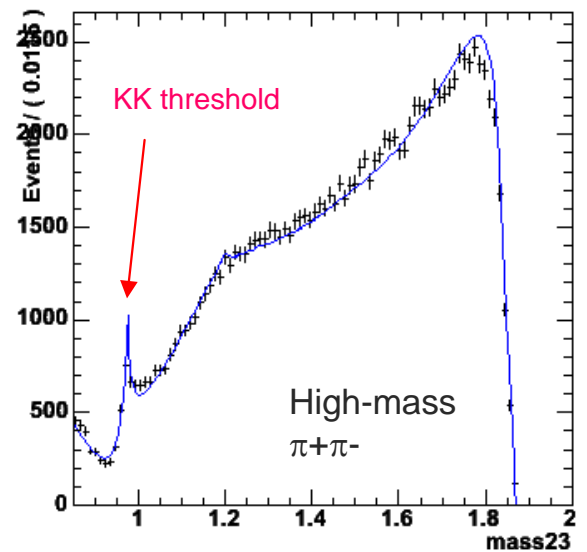
A RooPlot of "mass23"



A RooPlot of "mass23"



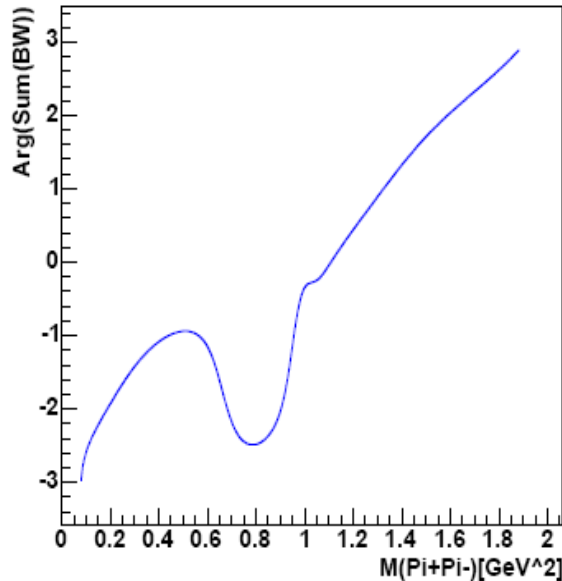
A RooPlot of "mass23"



First Problem: weird bump at low mass $\pi\pi$!

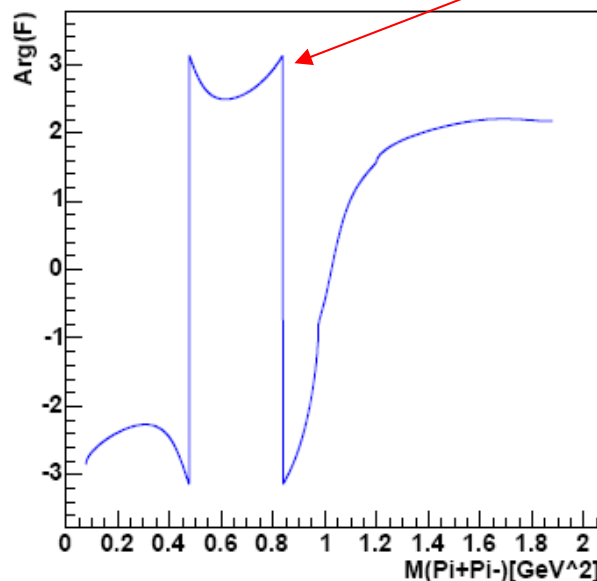
Phase from $\pi+\pi-$ scattering

Isobar model

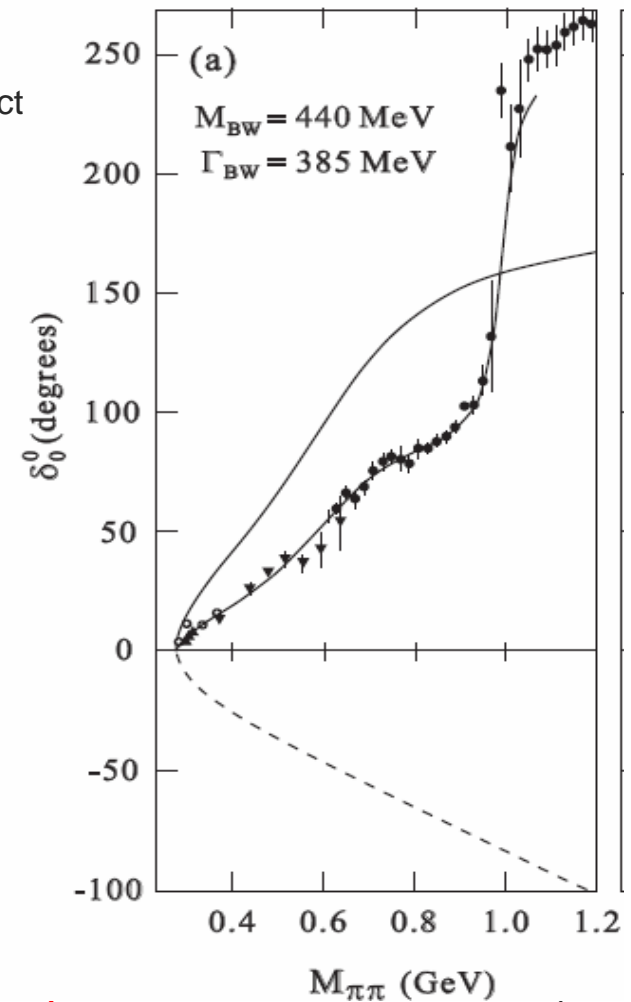


(d) BW fit: phase of the $\pi\pi$ S-wave [rad.]

K-matrix model



(b) KM fit: phase of the F vector [rad.]

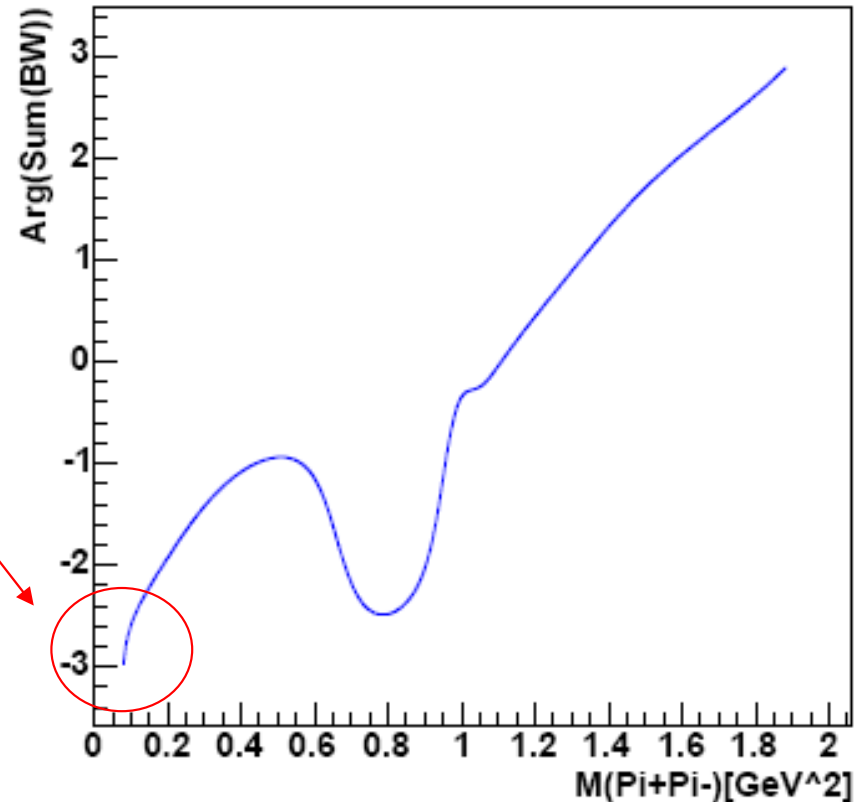


We need to understand the weird bump in the low mass $\pi+\pi-$ region

Second Problem: The initial phase value problem

In our old fit the phase of the S-wave does not start from zero.

Start from -180 degrees!



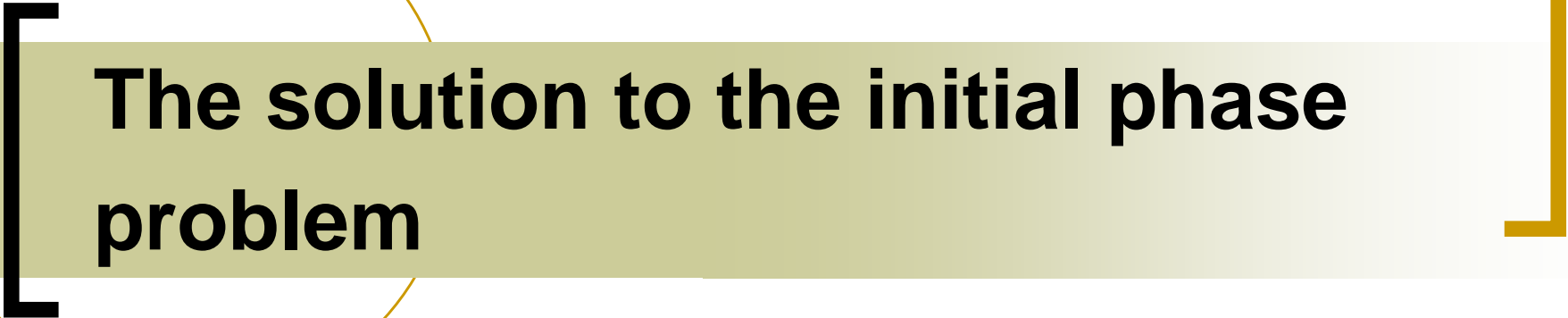
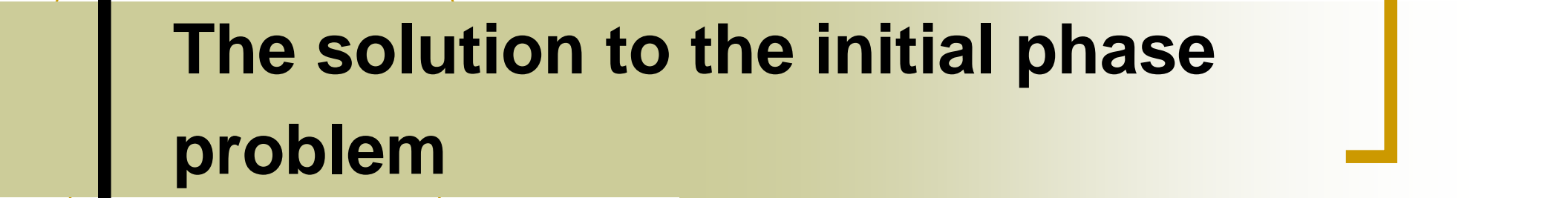
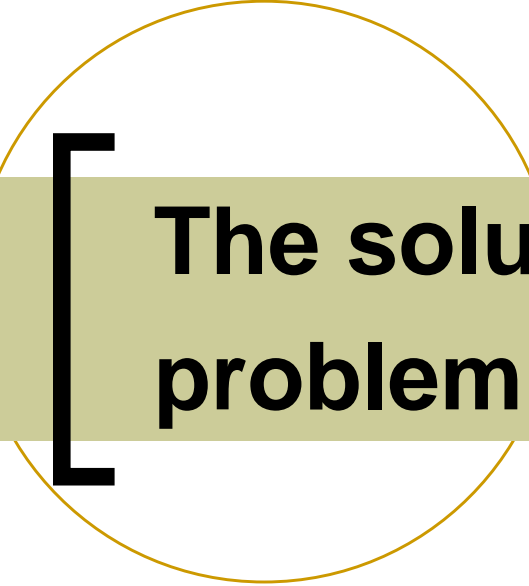
(recall $-\pi$ radian = -3.1416)
Or we had an minus sign problem
 $e^{i\pi} = -1$

(d) BW fit: phase of the $\pi\pi$ S-wave [rad.]

Summary of the problems:

- $K\pi$ {
 - Classics $K^*(892)$ mass problem
 - Higher $K\pi$ S-wave no good
- $\pi+\pi-$ {
 - $\pi+\pi-$ phase shift no good. (has weird bump on low mass $\pi\pi$)
 - Initial phase shift problem. (Phase does not start from zero)
- Both {
 - Choices between Zemach and Helicity model

In today's talk, these issues will be addressed



The solution to the initial phase problem

It turns out that, the problem is coming a typo in CLEO paper

We should fix this

Typo in CLEO paper

Classics CLEO paper
Phys.Rev. D63 (2001) 092001

There is a typo in the spin 1 formula

$$\mathcal{A}_1(ABC|r) = F_D F_r \frac{M_{AC}^2 - M_{BC}^2 + \frac{(M_D^2 - M_C^2)(M_B^2 - M_A^2)}{M_r^2}}{M_r^2 - M_{AB}^2 - iM_r \Gamma_{AB}}.$$

The label A & B are swapped and caused an extra minus sign on P-wave.

Correct Formula

$$\mathcal{A}_1(ABC|r) = F_D F_r \frac{M_{BC}^2 - M_{AC}^2 + \frac{(M_D^2 - M_C^2)(M_A^2 - M_B^2)}{M_r^2}}{M_r^2 - M_{AB}^2 - iM_r \Gamma_{AB}}.$$

A very difficult problem!



The solution to the $\pi\pi$ phase shift

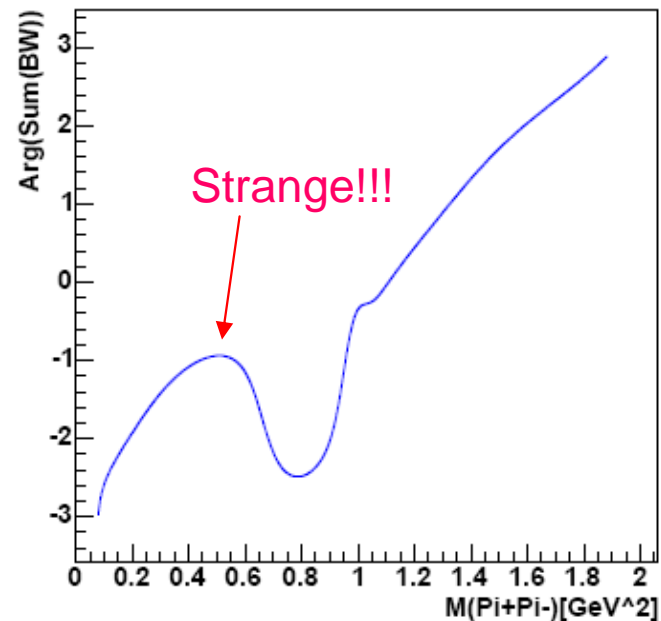
It turns out that, the problem is coming from $K\pi$ S-wave

[Fact: It's not the problem of $\pi^+\pi^-$]

Over the years, a lot of different parameterization on $\pi^+\pi^-$ is tested.
[Isobar, K-matrix model] However, the phase motion is not understood at all

For all a sudden I realized that, this is not the problem of the $\pi\pi$ S-wave

It's the problem of the $K\pi$ S-wave



(d) BW fit: phase of the $\pi\pi$ S-wave [rad.]

[Re-analysis of $K\pi$ S-wave]

For some reason I tried the full $D \rightarrow K_s \pi \pi$ dalitz fit with the mass and width floating:

I noticed, all in the sudden, the width of $K^{*0}(1430)$ shift significantly. and this number, are extremely closed to the E791 value using Isobar model method.

E791 abstract
On $D^+ \rightarrow K^- \pi^+ \pi^+$

We study the Dalitz plot of the decay $D^+ \rightarrow K^- \pi^+ \pi^+$ with a sample of 15090 events from Fermilab experiment E791. Modeling the decay amplitude as the coherent sum of known $K\pi$ resonances and a uniform nonresonant term, we do not obtain an acceptable fit. If we allow the mass and width of the $K_0^*(1430)$ to float, we obtain values consistent with those from PDG but the χ^2 per degree of freedom of the fit is still unsatisfactory. A good fit is found when we allow for the presence of an additional scalar resonance, with mass $797 \pm 19 \pm 43 \text{ MeV}/c^2$ and width $410 \pm 43 \pm 87 \text{ MeV}/c^2$. The mass and width of the $K_0^*(1430)$ become $1459 \pm 7 \pm 5 \text{ MeV}/c^2$ and $175 \pm 12 \pm 12 \text{ MeV}/c^2$, respectively. Our results provide new information on the scalar sector in hadron spectroscopy.

E791 Isobar model for $K^{*0}(1430)$:	Mass = 1.459 GeV Width = 175 MeV
BaBar Isobar model for $K^{*0}(1430)$:	Mass = 1.495 GeV Width = 183 MeV

[$K^*(1430)$ mass and width]

In fact: In Fernando 2004 ICHEP 04 model(81000) events, his fitted value for $K^*(1430)$ mass and width are very consistent with the full statistics:

E791 Isobar model for $K^{*0}(1430)$:
Mass = 1.459 GeV
Width = 175 MeV

250K Isobar model for $K^{*0}(1430)$:
Mass = 1.495 GeV
Width = 183 MeV

81K Isobar model for $K^{*0}(1430)$:
Mass = 1.478 GeV
Width = 193 MeV

Fernando
ICHEP 2004



But! width of $K^{*0}(1430)$ in PDG = 294 MeV (almost 40% smaller than PDG)

The Isobar model, the fit prefer small value of $K^*(1430)$, both seen in E791 and BaBar

[What's the main problem?]

The problem is, the so-called Breit-Wigner mass and width, are highly model dependent. [depends on which parameterization]

In the original ANTON 88 (LASS) analysis, they are using the so called effective range (LASS) parameterization.

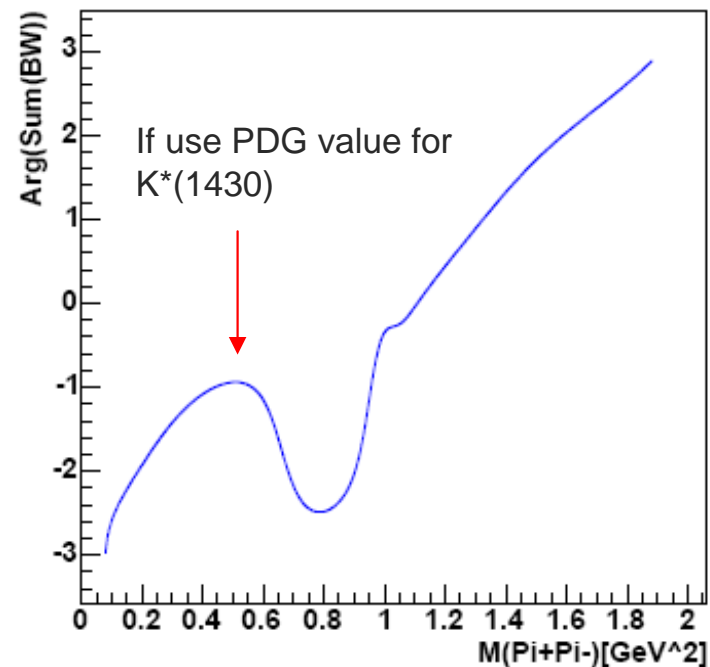
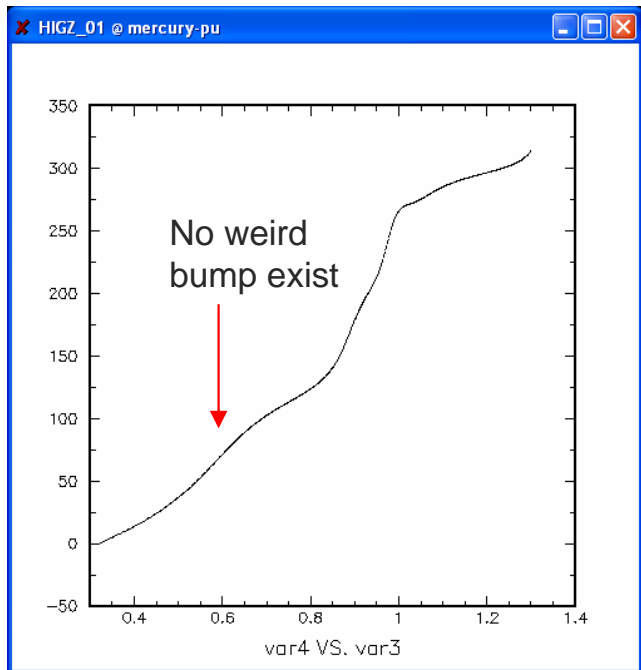
You fitted the mass and width using LASS parameterization, and It is not **SAME** if one using a Breit-Wigner parameterization.

The main problem was: We put 300 MeV for $K^*(1430)$ as the width, But using Breit-Wigner parameterization. This is inconsistent.

Note that there is a comment on PDG about $K^(1430)$, they said LASS assume a background model

The new phase shift:

I did a test, if I use the E791 mass and width for $K^{*0}(1430)$, [others remains unchanged]. The weird bump on $\pi\pi$ phase shift longer exist

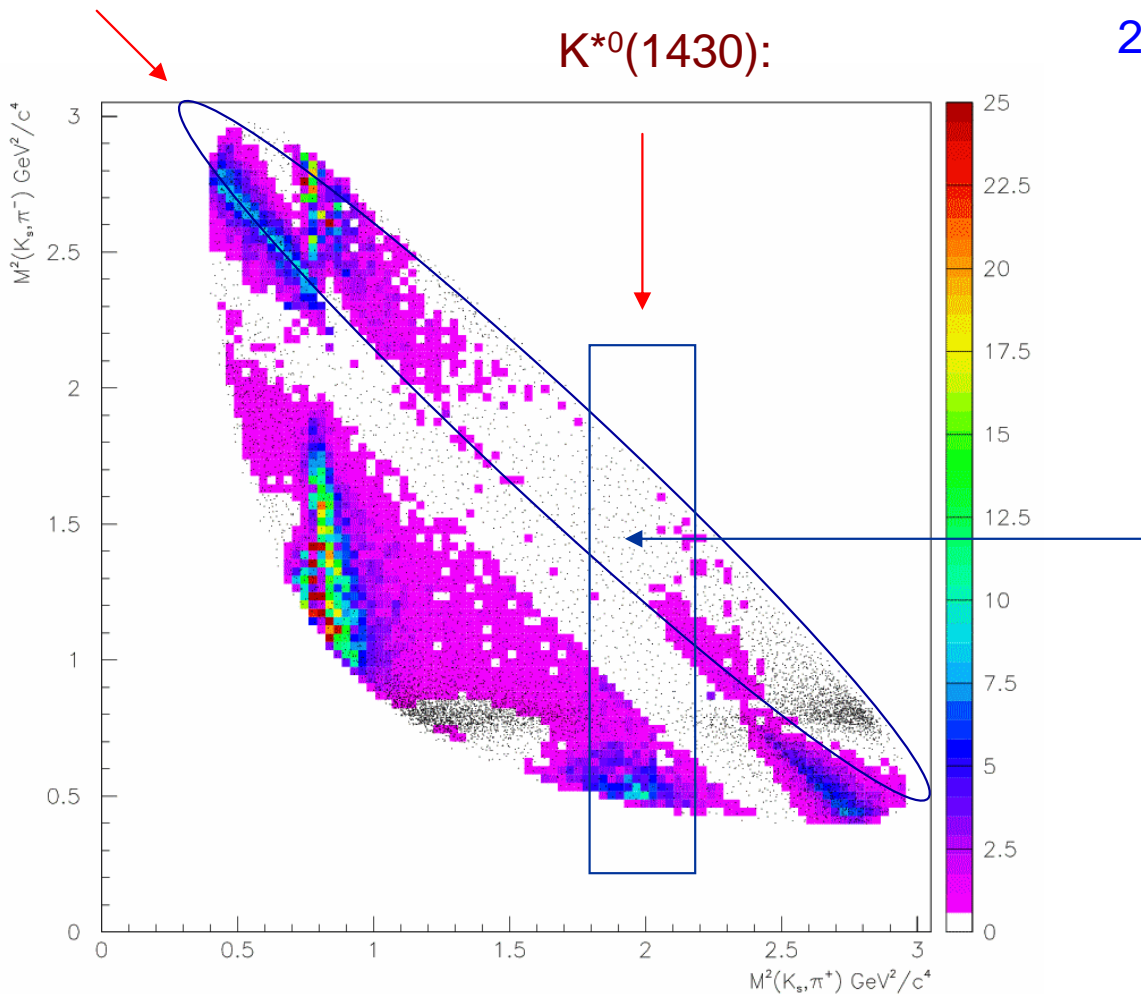


(d) BW fit: phase of the $\pi\pi$ S-wave [rad.]

A Strong indication that the $K\pi$ S-wave can distort the $\pi\pi$ S-wave.

How $K\pi$ S-wave affects $\pi\pi$?

low mass $\pi\pi$



1. The problem appears the low mass $\pi\pi$
2. However, the $K\pi$ S-wave directly interfere the $\pi\pi$ S-wave

The region where the $\pi\pi$ and $K\pi$ S-wave overlap!

[Extra cross-check]

In order to confirm that the problem is coming
From $K\pi$ S-wave. I did a test.

I write the formula for $K^*(1430)$ as LASS parameterization
And allow the parameters to float. I found the following value:

Width of $K^*(1430)$ using LASS formula:

LASS: 279 MeV +/- 6 MeV [note even not same as PDG!]

E791 : 266 MeV +/- 28 MeV

BaBar: 280 MeV +/- 12 MeV

Once I use the consistent formula, the width of
 $K^*(1430)$ is restored to the scattering value.

Why this problem take so long to solve?

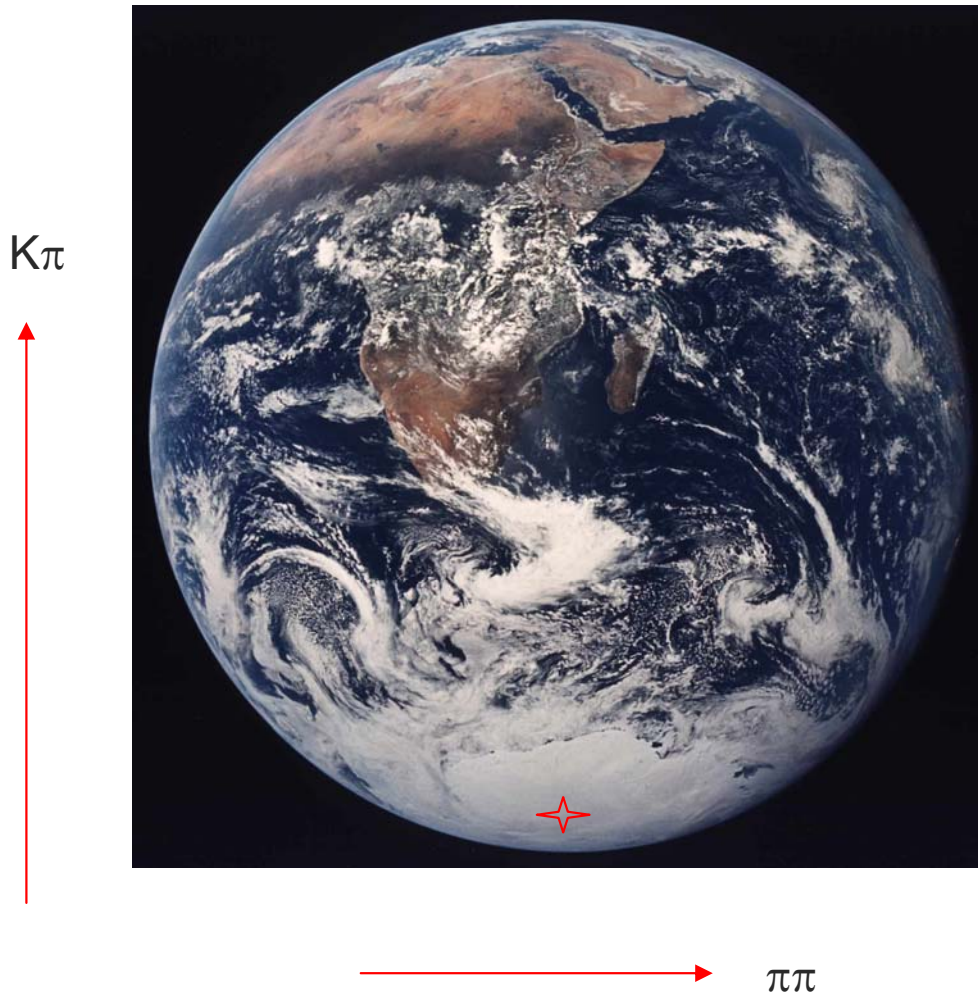
Because it was the 3D problem!
[we have s,t,u channel]

when I first saw the weird
phase shift plot in $\pi\pi$, I thought, ha,
There must be something wrong with $\pi\pi$

Therefore, I thought K-matrix can solve the
problem

And we spent almost 1.5 years on $\pi\pi$ channel
to check what's going wrong.

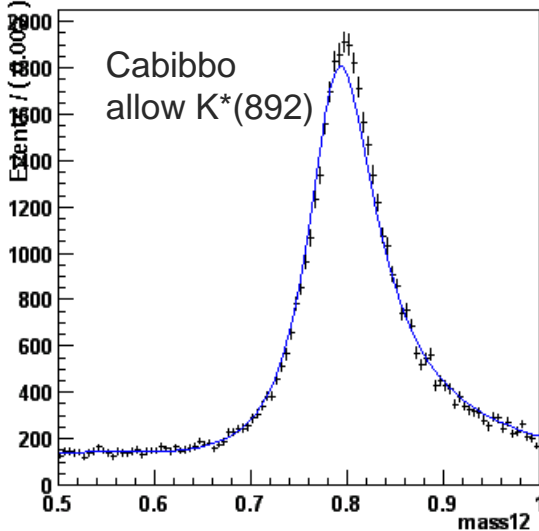
However, the problem is on $K\pi$, looking at
the east-west direction won't get you to south
Pole!



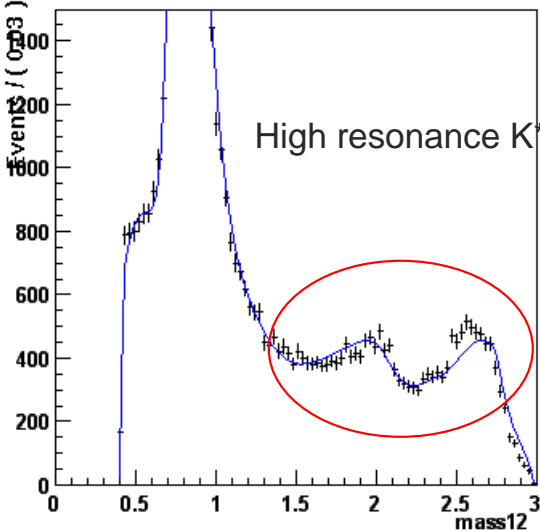
Notice the fit is not good in the $K\pi$ channel

Old Isobar model

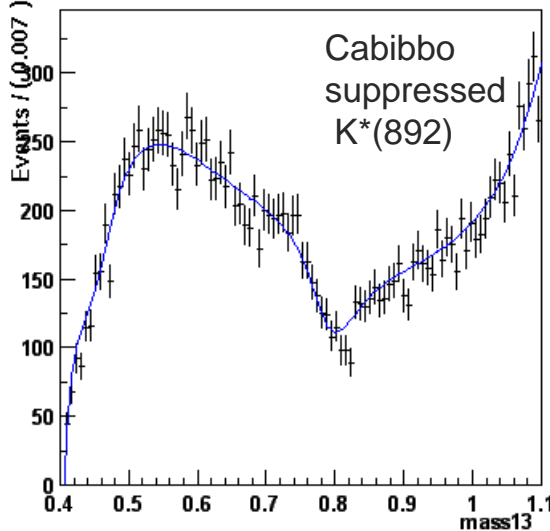
A RooPlot of "mass12"



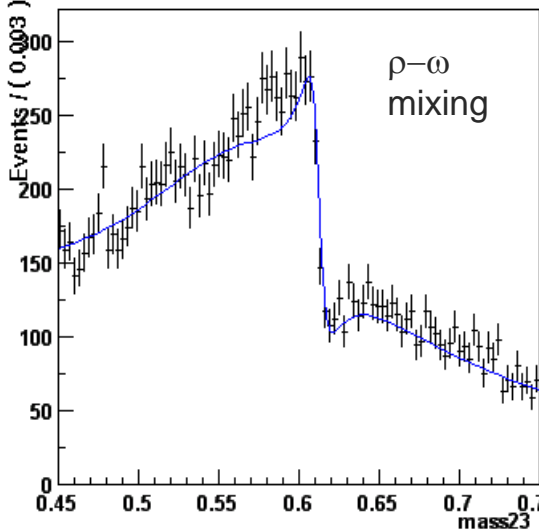
A RooPlot of "mass12"



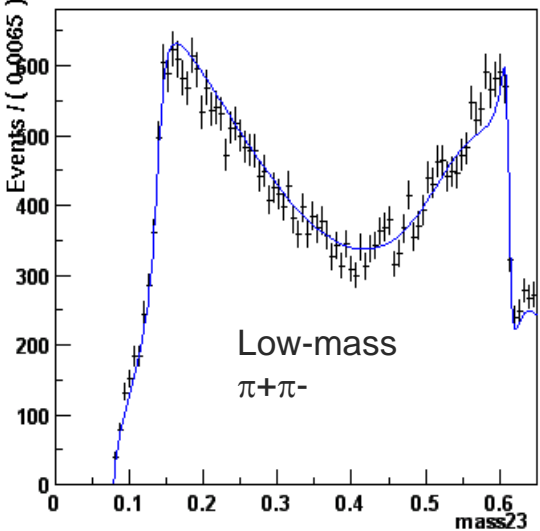
A RooPlot of "mass13"



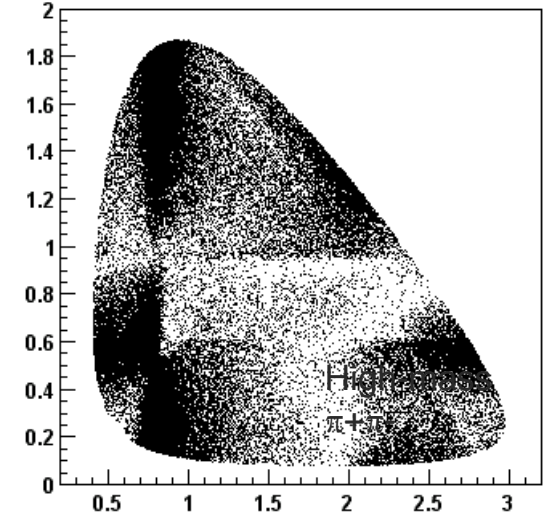
A RooPlot of "mass23"



A RooPlot of "mass23"



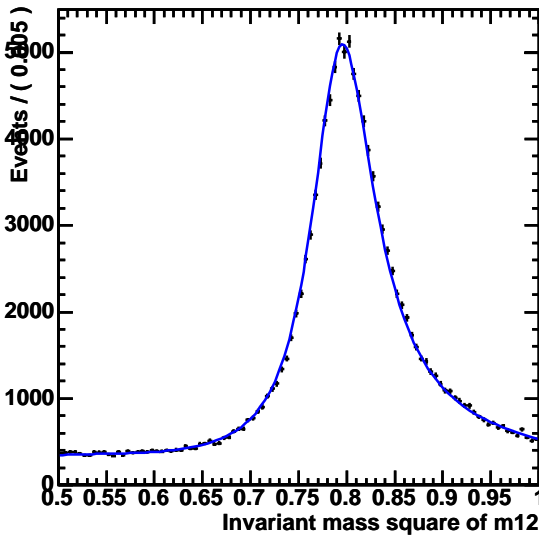
mass23:mass12



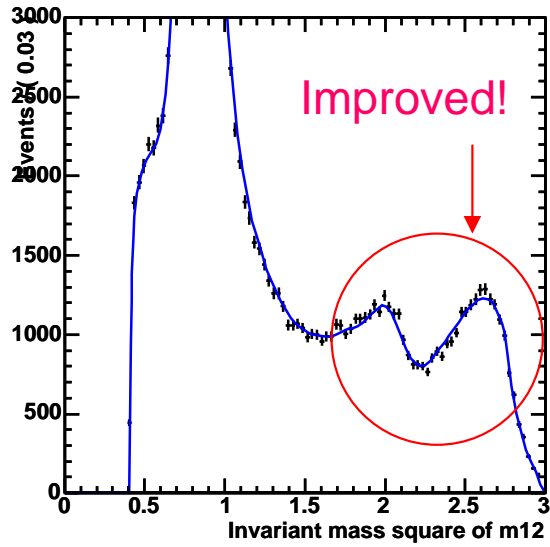
New $K\pi$ S-wave parameterization



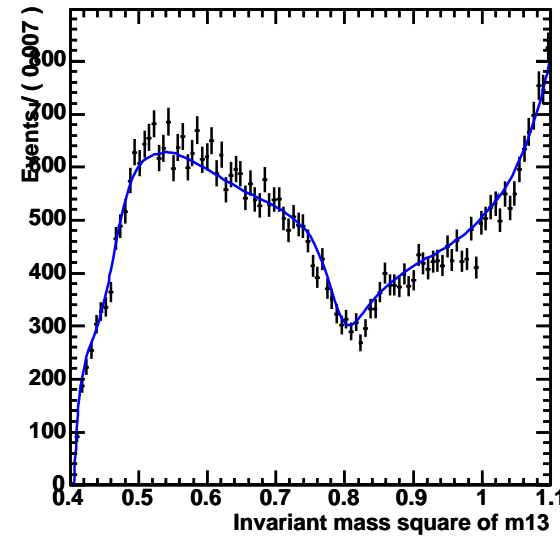
A RooPlot of "Invariant mass square of m12"



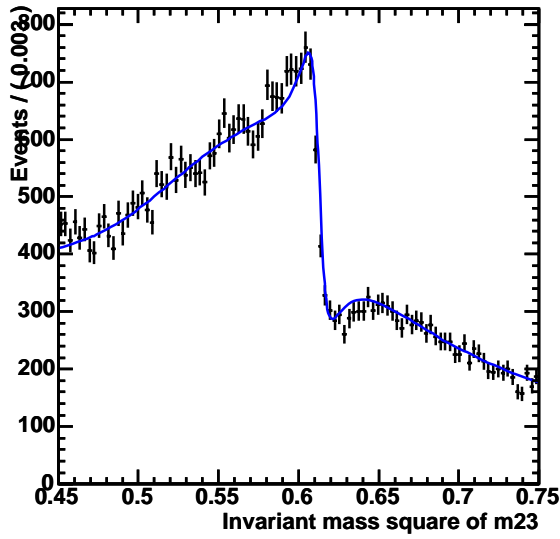
A RooPlot of "Invariant mass square of m12"



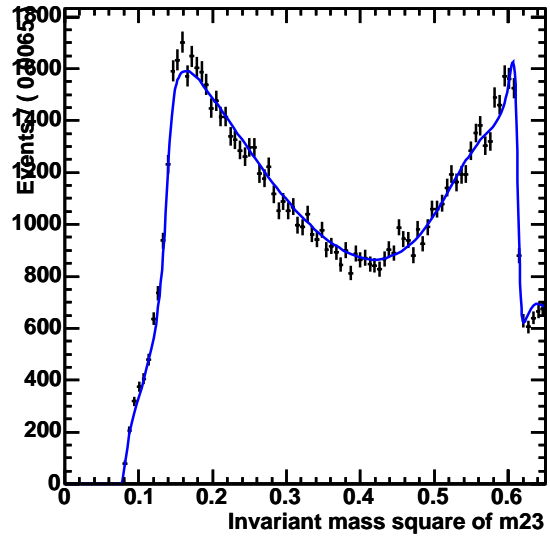
A RooPlot of "Invariant mass square of m13"



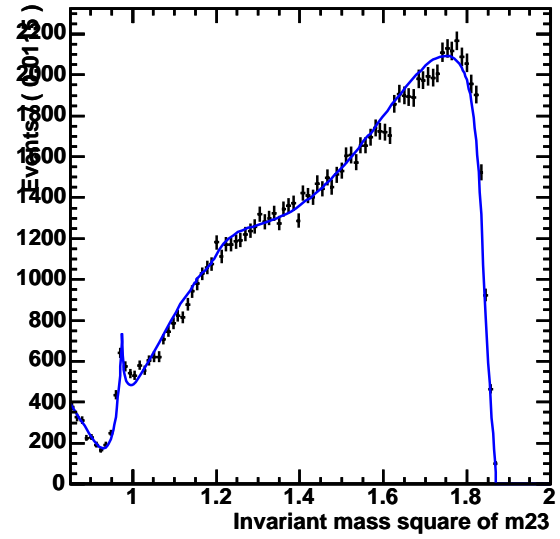
A RooPlot of "Invariant mass square of m23"

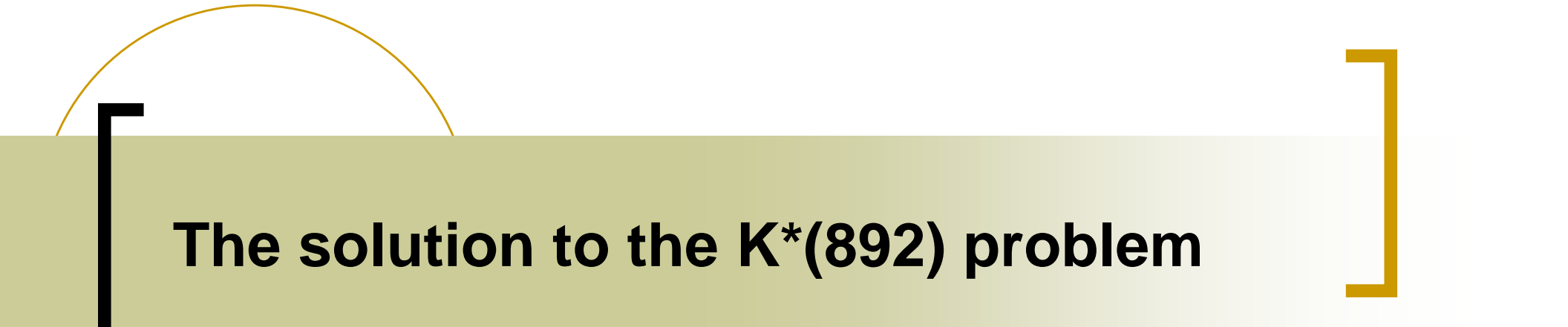


A RooPlot of "Invariant mass square of m23"



A RooPlot of "Invariant mass square of m23"





The solution to the $K^*(892)$ problem

Shall we shift the mass and width?

[Introduction]

The $K^*(892)$ problem last from the beginning to the end.

With many studies, the $K^*(892)$ region can fit well if we allow shift +1MeV in mass, -5 MeV in width

PDG value:

Mass = 891.66 ± 0.26 MeV

Width = 50.8 ± 0.9 MeV

Proposed new value:


Mass = 892.93 MeV

Width = 46.6 MeV

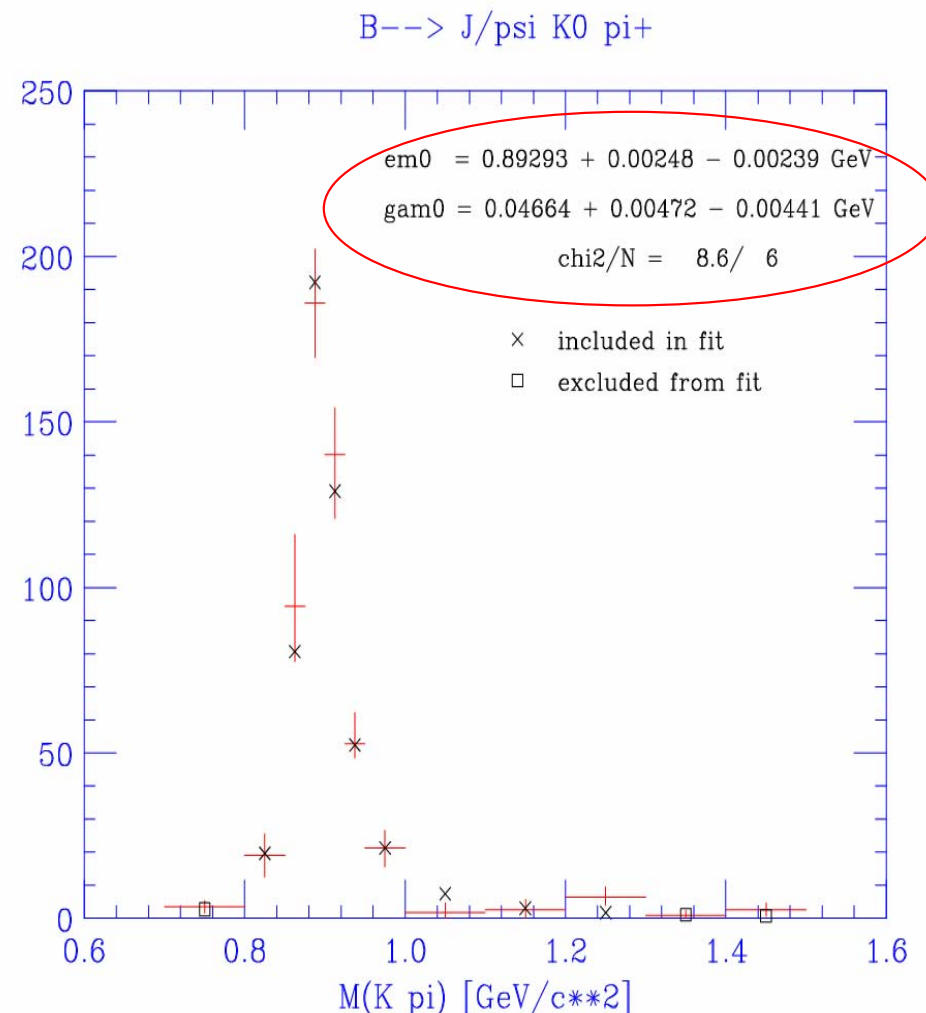
Why shift the width by -5MeV?

- ✓ In PDG those measurements are from 1970's. Very low statistics ~5000 events
- ✓ we have ~200000 $K^*(892)$ events
- ✓ If we allow mass and width to float, we always get the width 46 ± 0.5 MeV, mass 893 ± 0.2 MeV
- ✓ As we noted before, different Breit-Wigner can lead to different mass and width value
- ✓ The most precise measurement (I think) Partial wave analysis of $B \rightarrow J/\psi K \pi$ decay (BaBar)
- ✓ Their value are 100% consistent with our floated value

No background!
No S-wave!
Very clean measurement



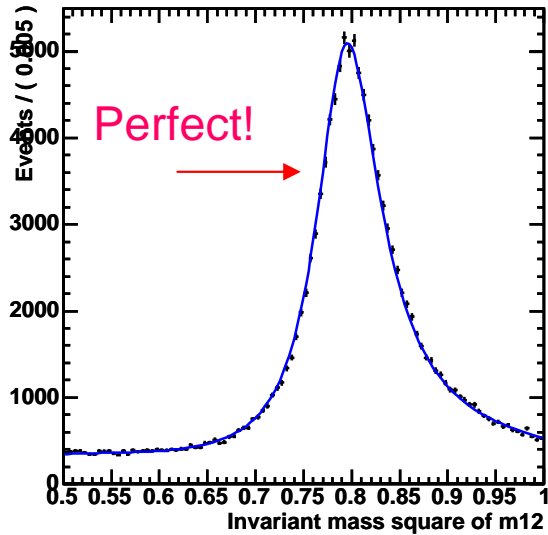
Events/25 MeV/c**2



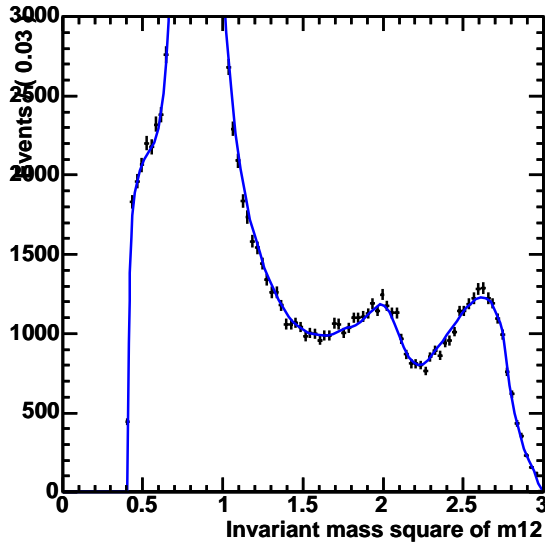
New $K^*(892)$ with new parameters



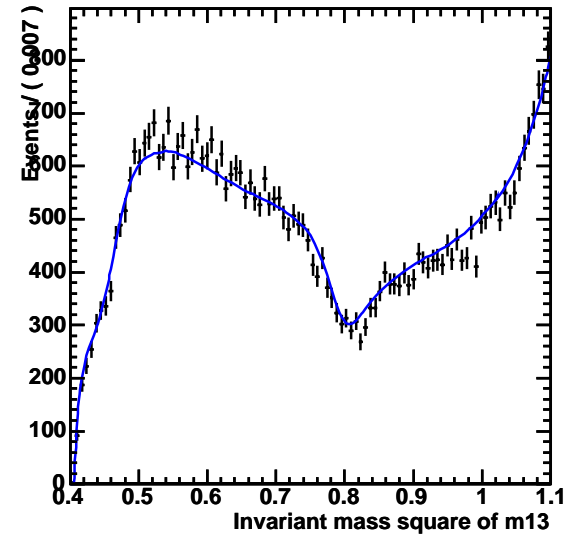
A RooPlot of "Invariant mass square of m12"



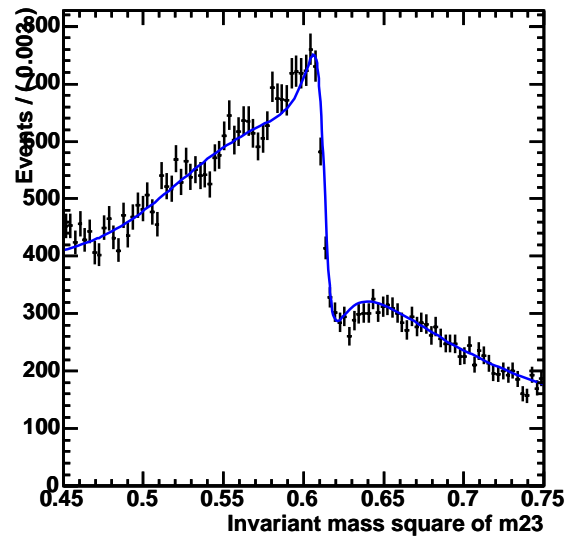
A RooPlot of "Invariant mass square of m12"



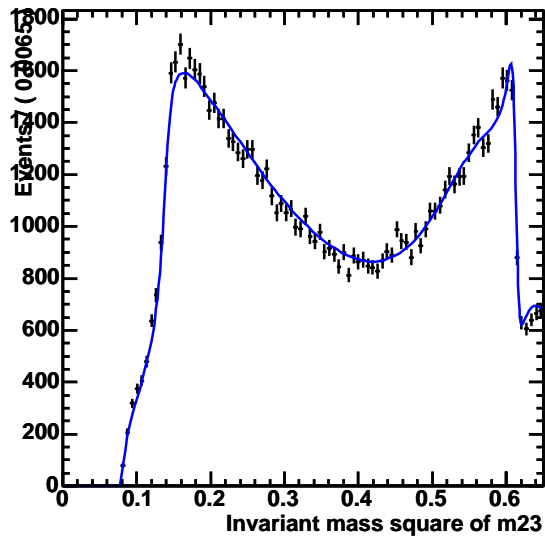
A RooPlot of "Invariant mass square of m13"



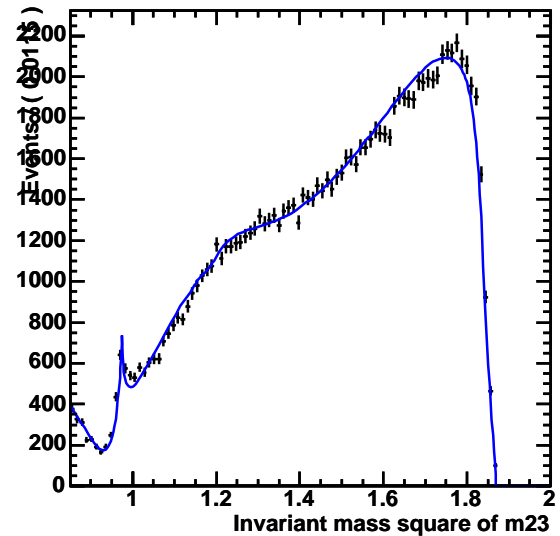
A RooPlot of "Invariant mass square of m23"



A RooPlot of "Invariant mass square of m23"



A RooPlot of "Invariant mass square of m23"



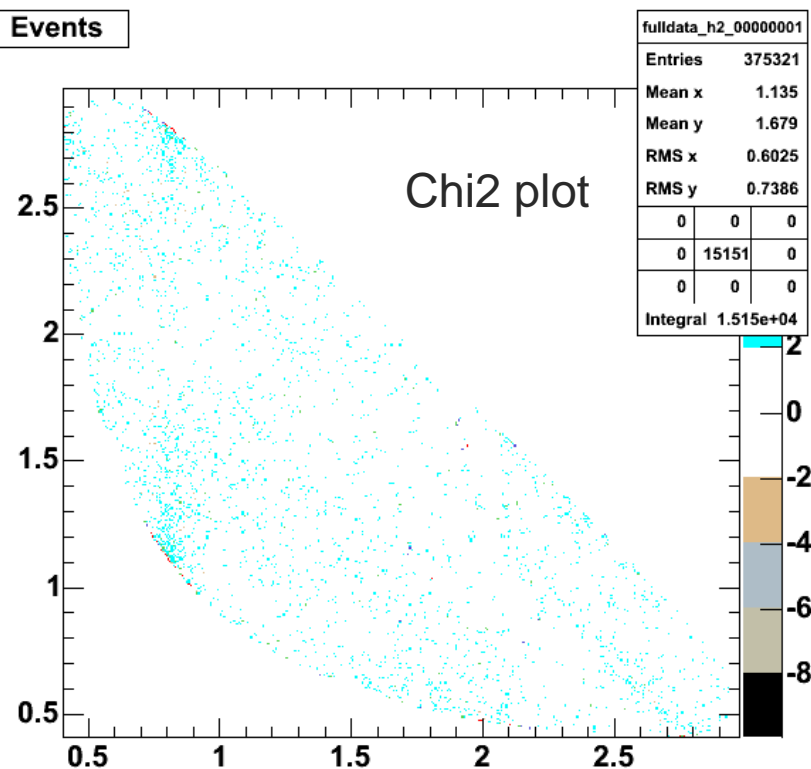
Both are
250K dataset

New chi2 evaluation

The new fit has been evaluated by new chi2 program. (suggested by Fernando)
Binned the whole dalitz plot 400 x 400 grid. Zero bins are neglected.

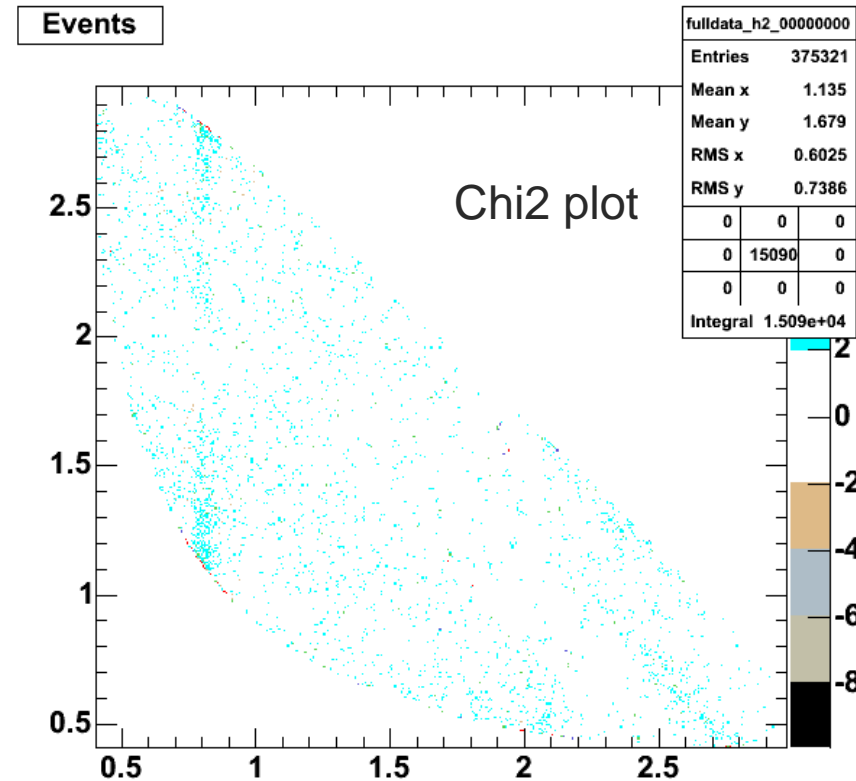
ICHEP 06 model

Chi2 = 52429/49926 = 1.05/dof



Traditional Isobar model

Chi2 = 53608/49926 = 1.073/dof





The solution to Zemach/Helicity model

Not completely resolved, but we made progress here.

Theorist can't solve this problem.

Must be answer experimentally

The choice affects heavily on spin 2

Monte Carlo simulation using $f_2(1270) \rightarrow \pi\pi$

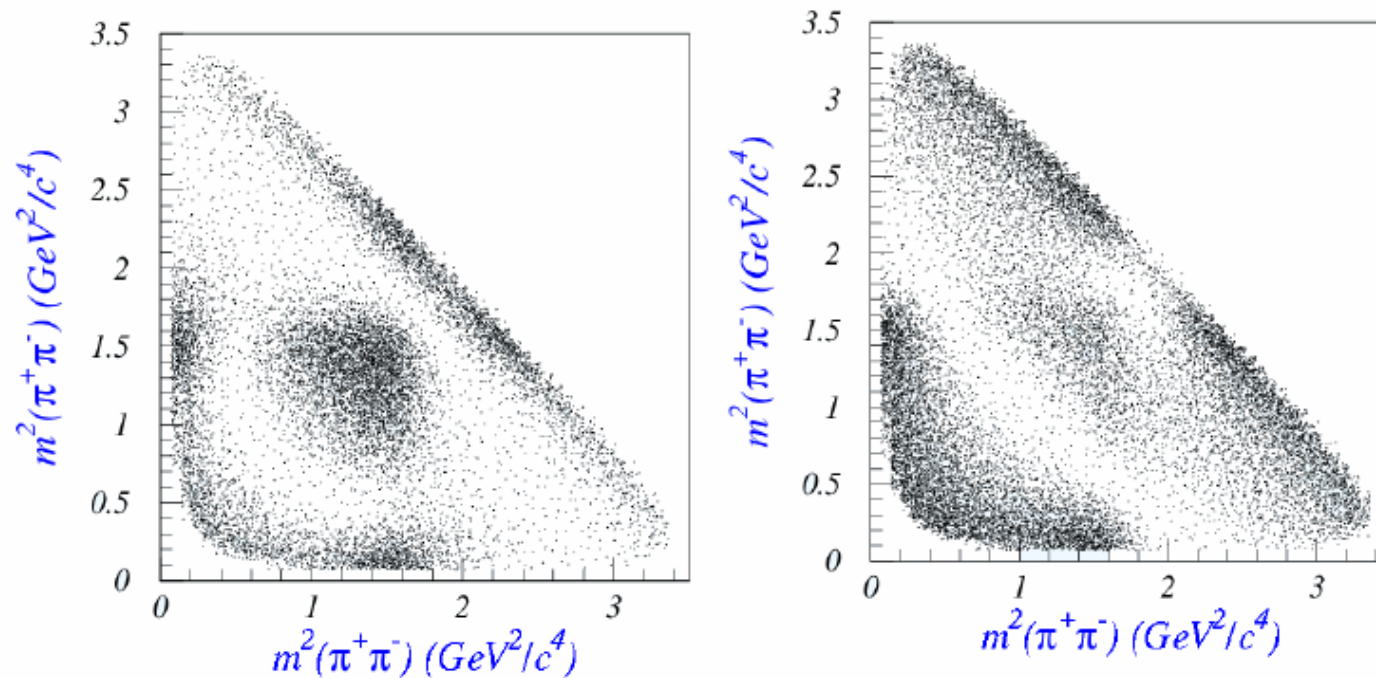


Figure 4.3: Monte Carlo simulation on $f_2(1270) \rightarrow \pi\pi$. Left: Zemach Tensor. Right: Helicity model. The difference between two is clearly visible.

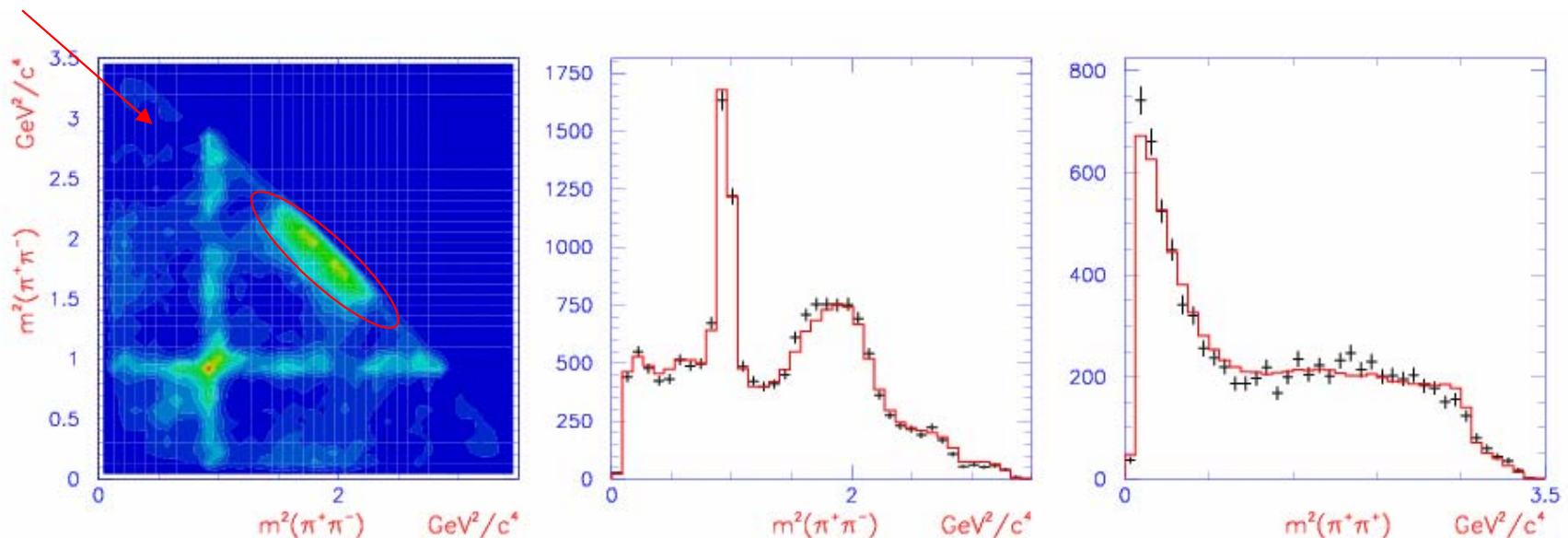
Macro analysis on $D_s \rightarrow 3\pi$

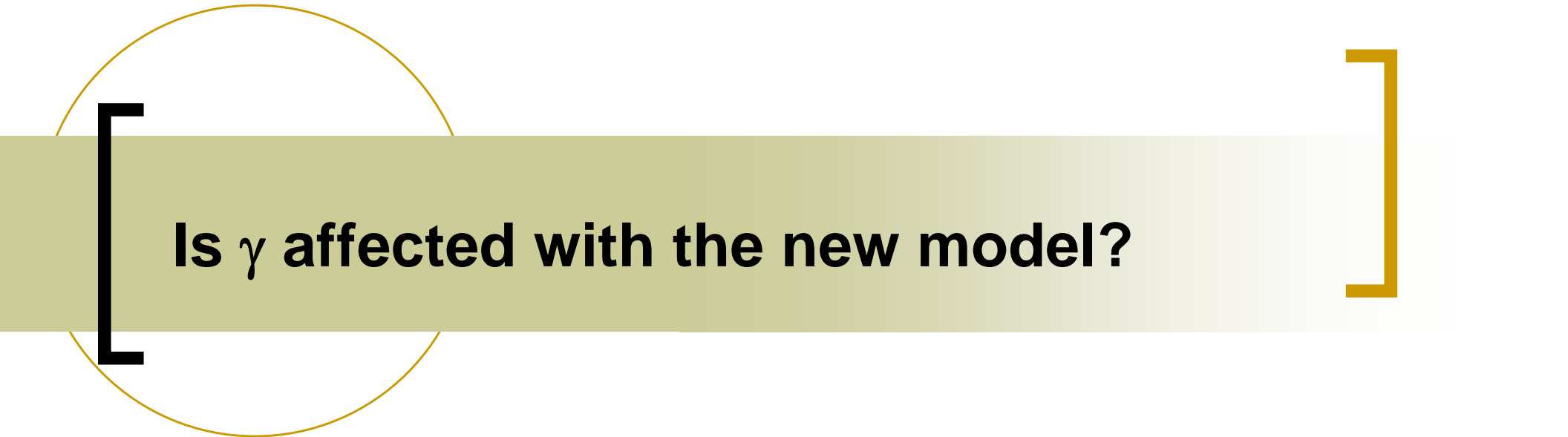
Macro performed dalitz analysis on $D_s \rightarrow 3\pi$.
This is a good channel since we have $\sim 30\%$ $f_2(1270)$

He found using Zemach Tensor the non-resonant term is very small ($\sim 5\%$)
If using helicity model the non-resonant term is very large ($\sim 20 - 25\%$)

Zemach Tensor are more flavorable

actual data!





Is γ affected with the new model?

[Run 1-4 data fit using new (ICHEP06) model]

New ICHEP06
model

CKM γ (DK mode) 63.79 ± 24 degrees
 δ (DK mode) 120.16 ± 25 degrees
 R_b (DK) 0.1328 ± 0.052

EPS 05
K-matrix model

CKM γ (DK mode) 63.88 ± 24.7 degrees
 δ (DK mode) 116.01 ± 24.8 degrees
 R_b (DK) 0.12828 ± 0.052

The CKM γ has NO effect.

The strong phase δ has shift by +4 degrees

The r_b is shifted by $\sim +0.04$

Conclusion: CKM γ is indifferent what model you used, even we made so many changes

[Conclusion]

- The $\pi\pi$ and $K\pi$ S-wave are highly correlated (because no spin term), incorrect parameterization on one channel can heavily affect on another
- In the traditional Isobar model, the $K^*(1430)$ width is only ~ 180 MeV, way too small to the PDG value
- BW mass and width are not **unique** parameters, depends on the parameterization and background model.
- Using **consistent** model (LASS) we got ~ 280 MeV.
- Same argument apply for $K^*(892)$. Our fitted values agrees very well on the clean PWA of $B \rightarrow J/\psi K \pi$
- In $D_s \rightarrow 3\pi$, Zemach Tensor is more favourable than helicity model.



The End!

Let's discuss.

A decorative graphic at the top of the slide consists of a horizontal line with a gradient from light yellow to white. A large black left square bracket is positioned on the left side of the line, and a large yellow right square bracket is on the right side.

The Appendix section

[The CLEO typo]

Because of this typo, all the spin 1 animal has an extra minus sign

But, remember, the $\rho(770)$, is a spin 1 animal, and has fixed the amplitude to one and phase to zero

Therefore, all the S-wave and D-wave has 180 phase change (to account the extra -1 sign from P-wave).

The new formula is now consistent with PDG “Charm Dalitz Formalism”
David Asner, hep-ex/0410014

$$\begin{aligned} Z &= (P_D + P_c)_\mu (-g^{\mu\nu} + q^\mu q^\nu / m_{ab}^2) (P_a - P_b)_\nu \\ &= (m_{bc}^2 - m_{ac}^2) + (m_D^2 - m_c^2) (m_a^2 - m_b^2) / m_{ab}^2 \\ &= -2\vec{p} \cdot \vec{q}, \end{aligned}$$

Formula from PDG
book

[But his spin 2 formula still has typo!]

Both LASS and K-matrix are under the same name : “Unitary model”

- Now consider K-matrix with background, I can write:

$$K = \tan(\delta_r + \delta_b)$$

where the delta_b is the background phase

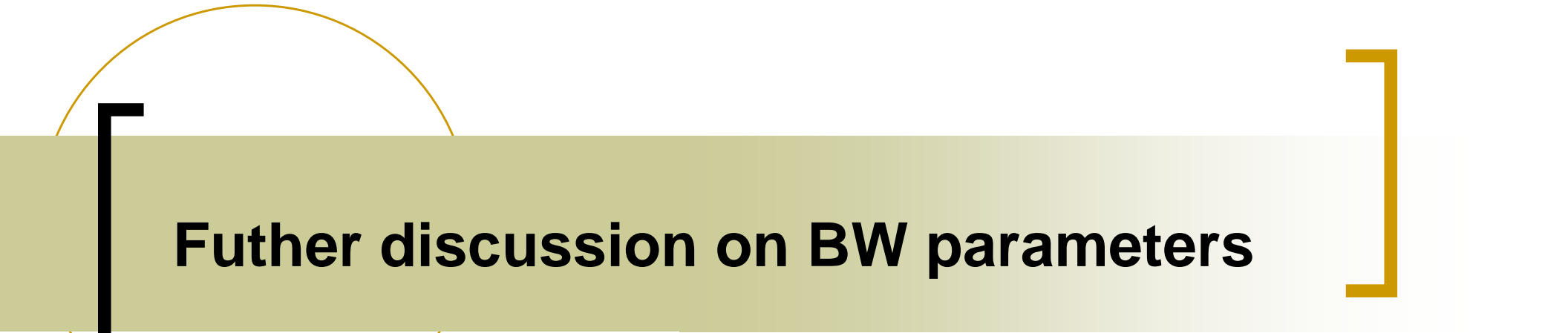
Since:

$$T = K(I - iK)^{-1}$$

We got:

$$T = \sin(\delta_b)e^{i\delta_b} + e^{2i\delta_b} \sin(\delta_r)e^{i\delta_r}$$

Exactly the LASS parameterization



Futher discussion on BW parameters

Are Breit Wigner mass and width
unique parameters?

Is Breit-Wigner mass and width model independent?

- Not really, people in the past already noticed that the “Breit-Wigner” parameters are heavily depends on which parameterization you used.
- Example: Delta(1236)
- In 1972 PDG, it said:

In the last edition of Review of Particle Properties, we indicated some difficulties with the determination of the mass and width of delta(1236). We present her some further efforts towards the resolution of these difficulties

As indicated, three different Breit-Wigner parametrization and a polynomial fit were tried. They are:

1. Standard Breit Wigner
2. Layson Formula
3. Chew-Low
4. Polynomial

Just different formula to parameterize lineshape

[1972 PDG]

Mass and width of $\delta(1236)$ on real axis: The reason for this large spread in values is rather obvious, Since both type of fits describe the data equally well, we have the apparent problem of choosing between equally valid parameterization

It has been suggested by Coleman and recently by Ball et al, this problem is removed if we take the mass and width to be given by the **actual pole position of the $\delta(1236)$ in the complex energy plane.**

Concluding remark:

1. All parametrization of the data on the real axis are equally good provided:
2. A) They fit the data
3. B) they are unitary and have sensible cut feature (goes to zero at threshold)
4. C) For good fits to the same data, the resonance mass and width on the real axis depend upon the parametrization used.
5. **D). For good fit to the same data, the pole position will be essentially independent of the parametrization.**

Breit-Wigner mass and width are model dependent parameters

Three-Body Charmless B Decays Workshop, Paris, 2006
 See: Bill Dunwoodie's Talk:

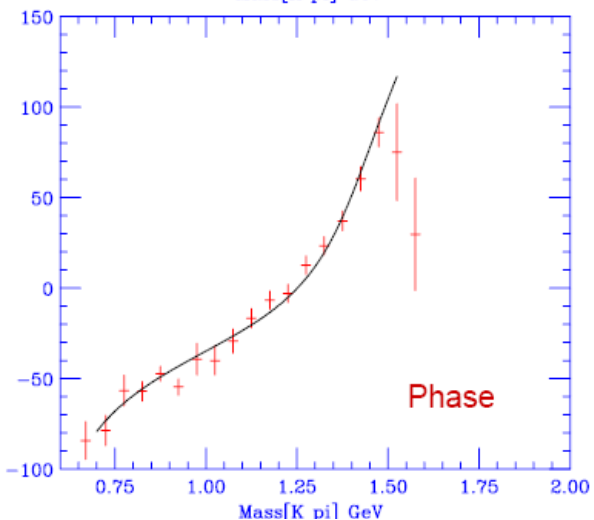
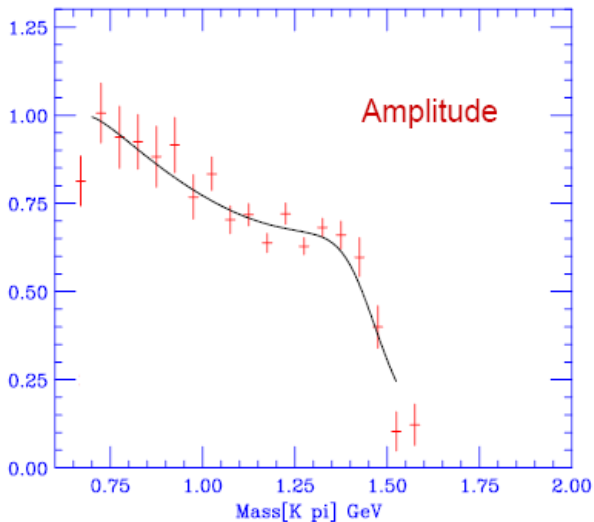
Generalize LASS fit to E791 data 17

{http://www.slac.stanford.edu/~wmd/kpi_swave/kpi_swave.note}

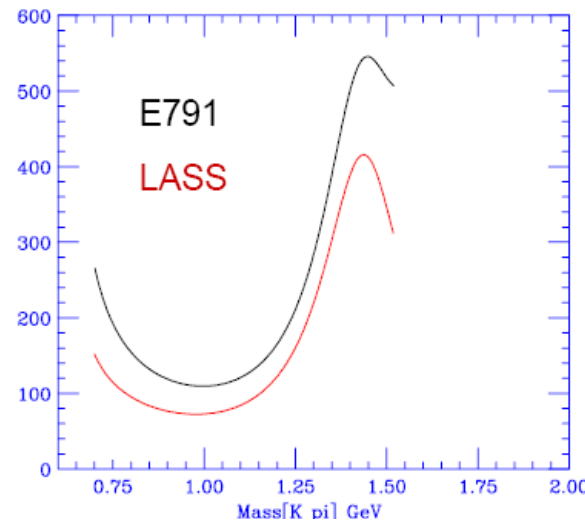
Resonance parameters agree well

	E791	LASS
Mass (MeV)	1428 +/- 16	1435 +/- 5
Width (MeV)	266 +/- 28	279 +/- 6

E791(Breit Wigner)
 1459 +/- 7 (MeV)
 173 +/- 15 (MeV)



$d\phi_s / dm$ (deg./GeV)



Difference choice of model (LASS, BW) can lead to fitted different mass and width value

In this example we want to show that background model can lead different Breit-Wigner mass and width

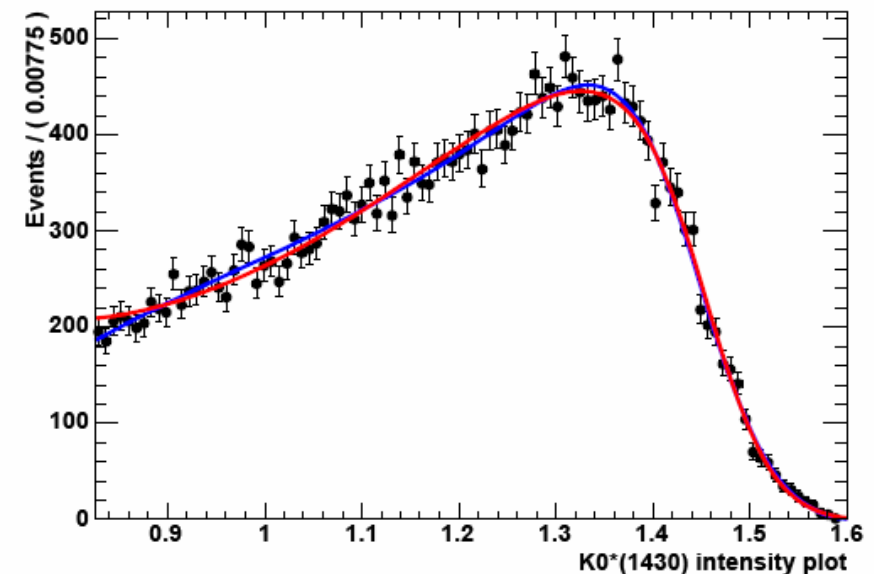
BW mass(generated)	1430(fixed) MeV
BW width (generated)	279(fixed) MeV
BW mass(LASS)	1431 ± 4 MeV
BW width(LASS)	271 ± 6 MeV
BW mass(alternative)	1453 ± 5 MeV
BW width(alternative)	241 ± 7 MeV

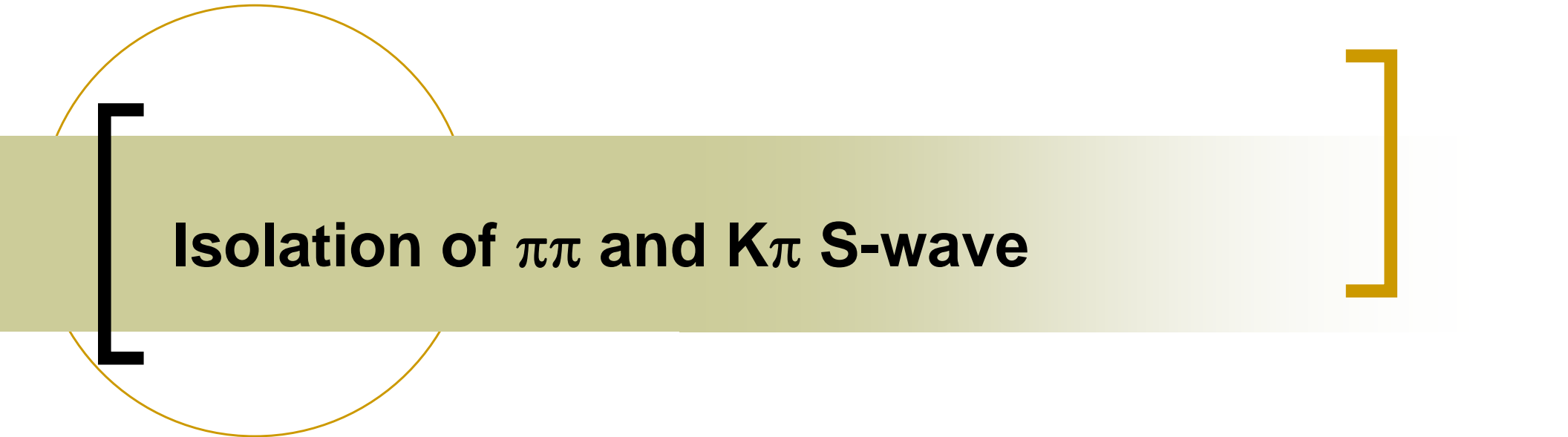
Blue line: Effective range expansion
 Red line: linear Polynomial expansion

Table 4.2: Measured $K_0^*(1430)$ Breit-Wigner parameters using simulated events generated by LASS model. The BW mass and width parameters are highly depends on the choice of background model.

Different choice of background model can fit the “simulated” data well, and the BW mass and width are heavily affected

A RooPlot of "K0*(1430) intensity plot"





Isolation of $\pi\pi$ and $K\pi$ S-wave

[How to isolated?]

According to the conventional isobar model, the amplitude of the decay (1) is written [3, 5] as a sum¹

$$A(s, t) = A_s(s, t) + \underbrace{[A_t(t, s) + (t \leftrightarrow u)]}_{\substack{\pi\pi \\ K\pi \\ K\pi \text{ (DCS)}}}, \quad (3)$$

where the s -channel and t (u)-channel amplitudes are expanded as

$$\underbrace{A_s(s, t) = \sum a_l(s) P_l(\cos \theta_s)}_{\pi+\pi^- \text{ system}}, \quad \underbrace{A_t(t, s) = \sum b_l(t) P_l(\cos \theta_t)}_{K\pi^- \text{ system}}, \quad (4)$$

In the context of the Isobar model, We summed over all the $\pi\pi$ and $K\pi$ final state.

However, when spin is zero (S-wave). They don't have any angular dependence.

Therefore, if the parameterization on $K\pi$ is not right, it will affect on $\pi\pi$ ~~~~

[Isolation of $\pi\pi$ and $K\pi$ S-wave]

This is a particularly difficult problem in the S-wave system, where there is no angular dependence, the S-wave $\pi\pi$ and S-wave $K\pi$ are highly correlated. [In other words, the fit does not have much handle to separate $\pi\pi$ and $K\pi$ S-wave]

In P-wave, the angular dependence terms can separate $K\pi$ and $\pi\pi$ easily.

Quote from CERN MIGARD Manual

Correlation coefficients very close to one (greater than 0.99).
**This indicates both an exceptionally difficult problem,
and one which has been badly parameterized so that individual errors
are not very meaningful because they are so highly correlated.**