

Dalitz plot analysis of $D^0 \rightarrow K_S \pi^- \pi^+$ using K-matrix formalism

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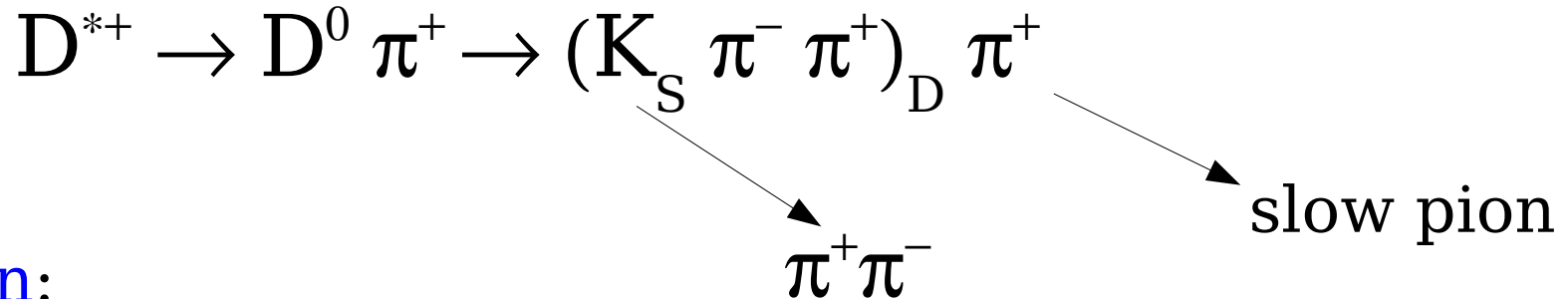
Università di Roma “La Sapienza” & INFN Roma

for the “ $D^0 K$ Dalitz” group

Charm Working Group Meeting

2005 July 21

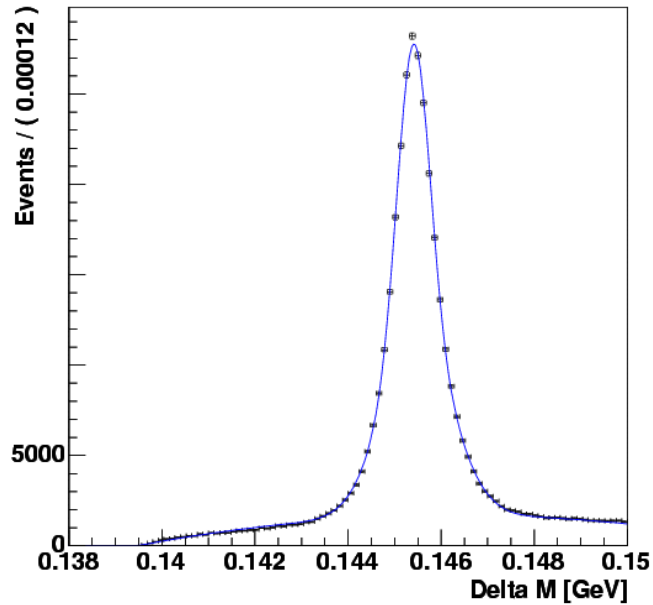
RUN 1 – 4 selection (I)



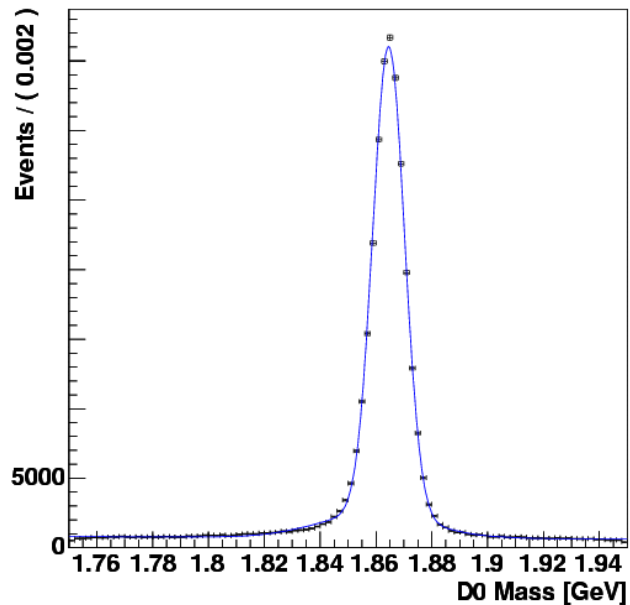
Selection:

- $M(K_S) = [0.35, 0.65] \text{ GeV}/c^2$
- K_S vertex fit: $\chi^2/\text{ndof} > 0.001$
- K_S mass constraint
- $D^0 K_S$ vtx. distance $> 0.4 \text{ cm}$
- $\cos(\alpha(K_S)) > 0.7$
- D^0 vertex fit: $\chi^2/\text{ndof} > 0.001$
- D^0 momentum $> 2.2 \text{ GeV}/c$
- D^0 mass constraint
- D^* vertex fit: $\chi^2/\text{ndof} > 0.001$
- slow π momentum $< 0.6 \text{ GeV}/c$
- ChargedTracks list used for the 2 π from the K_S and for the slow pion
- GoodTracksLoose used for the 2 π from the D^0
- TreeFitter used in all vertex fits

RUN 1 – 4 selection (II)



- Δm fitted with 2 Gaussians (signal) + threshold function (background)
- Cut $1.4 \text{ MeV}/c^2$ ($\sim 2\sigma$) around the mean value
- $M(D^0)$ fitted with 2 Gaussians (signal) + first order polynomial (background)
- Cut 2σ ($\sim 11 \text{ MeV}/c^2$) around the mean value

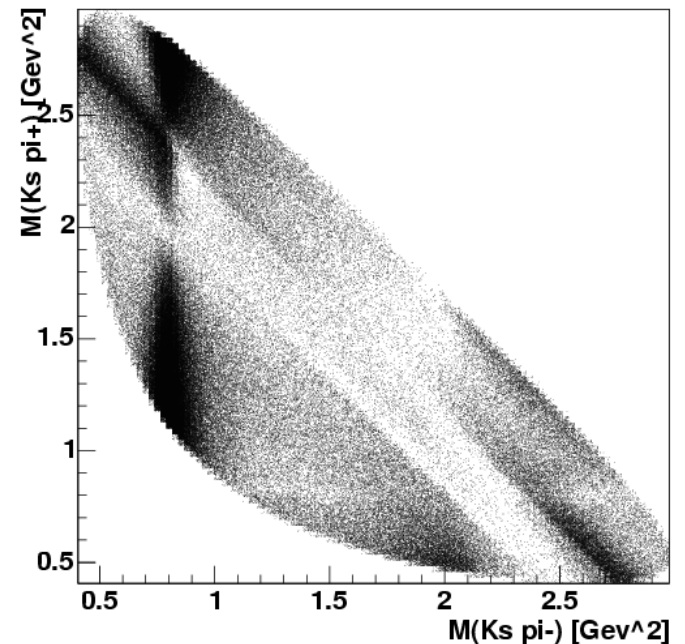


$M(D^0) = 1.8646 \pm 1 \cdot 10^{-5} \text{ GeV}/c^2$

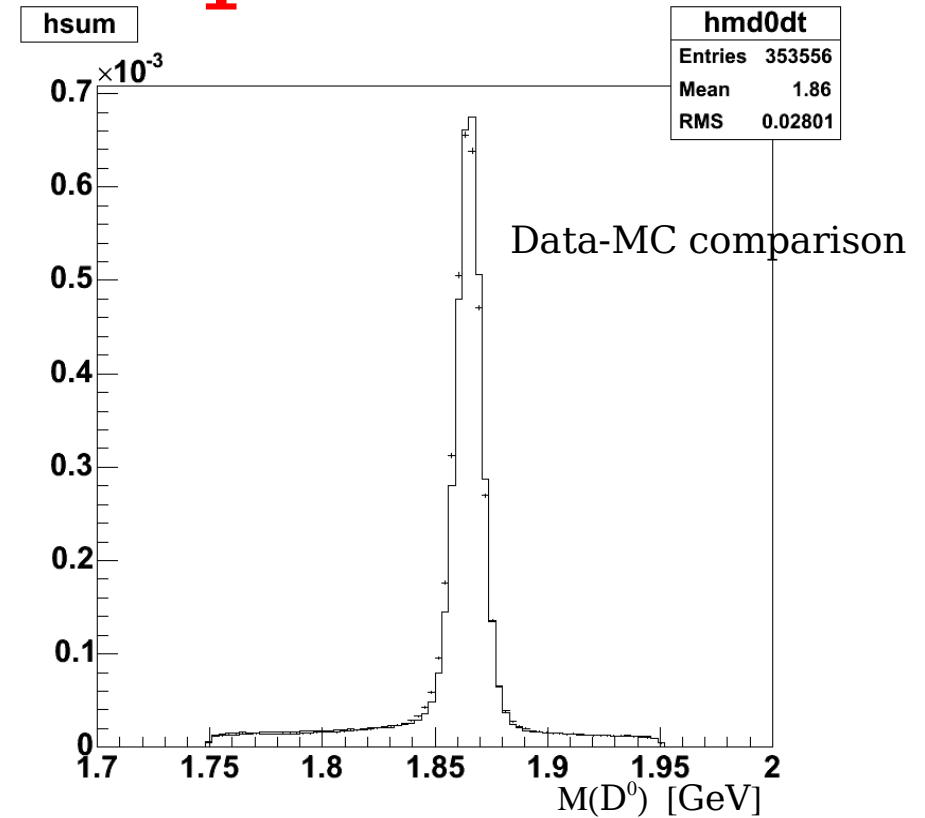
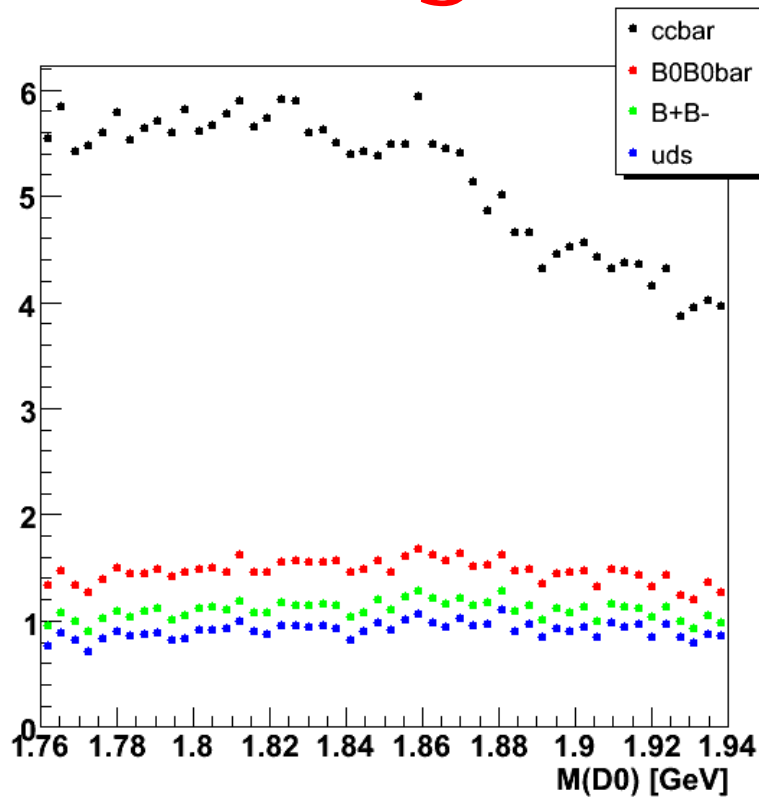
253991 events

From $M(D^0)$ fit

$$\frac{S}{S+B} = 0.977$$



Background composition



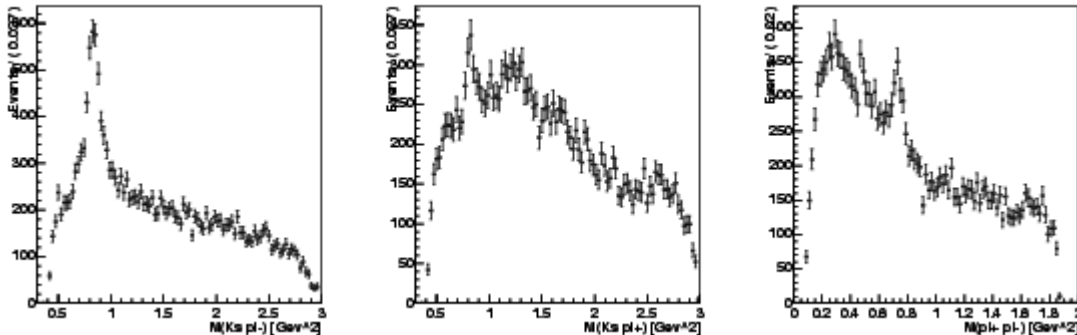
Sample	Selected M_D region	% of events
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	264228	96.7
$c\bar{c}$	5576	2.0
uds	1513	0.6
$B^0 \bar{B}^0$	1484 (895)	0.5 (0.3)
$B^+ B^-$	521 (209)	0.2 (0.1)
$D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$	628	0.2

() = $D^0 \rightarrow K_S \pi^- \pi^+$

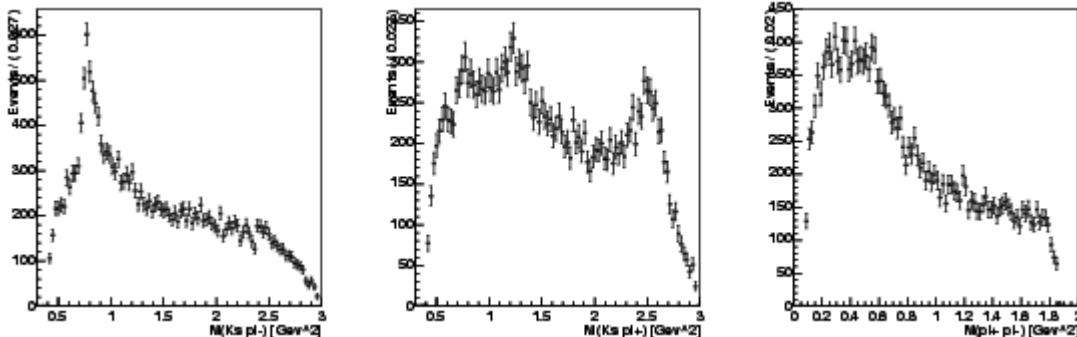
- good Data–MC agreement
- no dominant background
- from MC $S/(S+B)=97.1\%$ close to the one obtained from data (97.7%)
- we use the value from data

Background parametrization (I)

Use of the $M(D^0)$ sidebands for the background parametrization



“left” sideband
 $M(D^0) = [1.7, 1.8] \text{ GeV}/c^2$

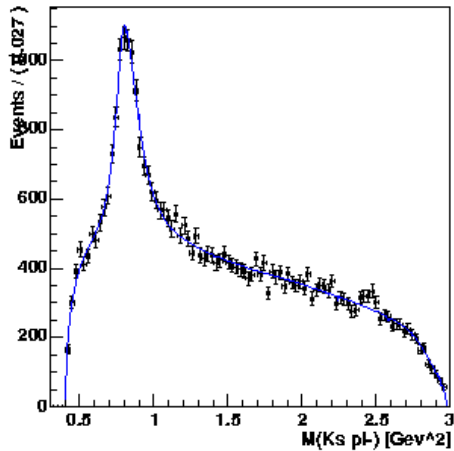


“right” sideband
 $M(D^0) = [1.8925, 1.9] \text{ GeV}/c^2$

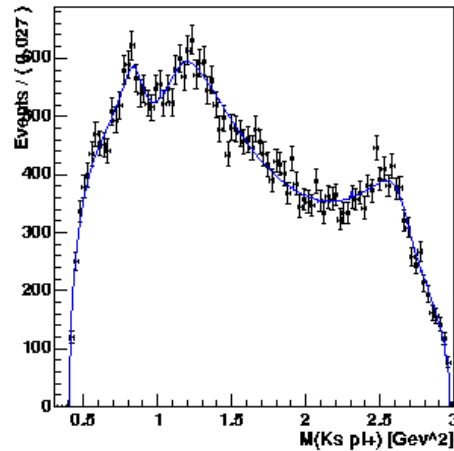
The two distributions are a bit different.

Bkg parametrization and relative systematic still not decided

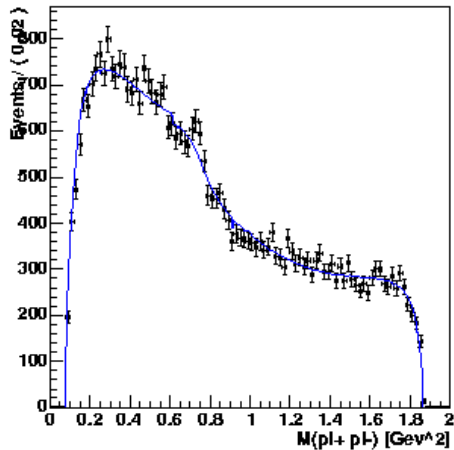
Background parametrization (II)



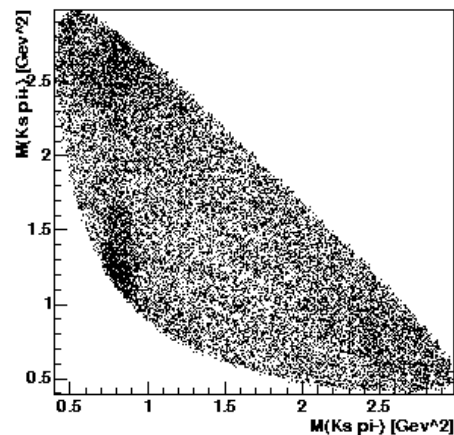
(a) $M_{K_S^0 \pi^-}^2$



(b) $M_{K_S^0 \pi^+}^2$



(c) $M_{\pi^+ \pi^-}^2$



(d) Dalitz plot of the sideband events

Test the effect on the fit of the bkg parametrization using the sum of the two sidebands

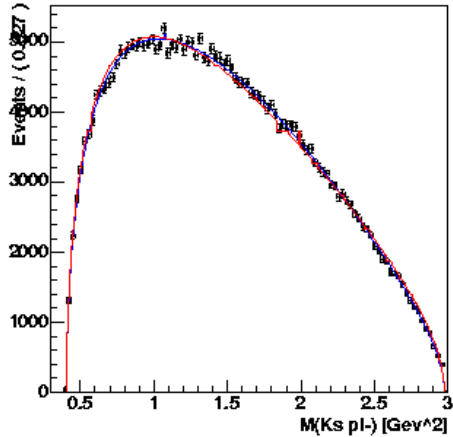
3.5 % of signal events in the sidebands

parametrization of the sidebands with a 3rd order polynomial plus

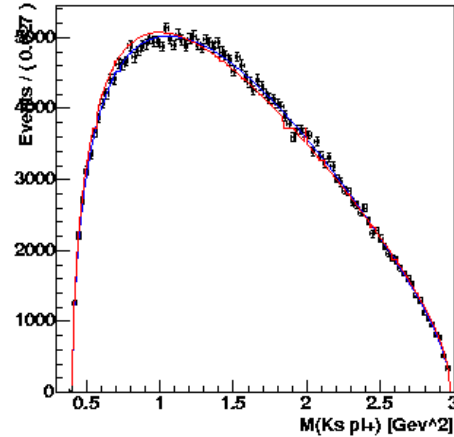
$|\sum_1^6 BW_i|^2$ for background and with the B-W fit (see later) for the signal events

since $B/(S+B)=2.3\%$ we expect a small systematic

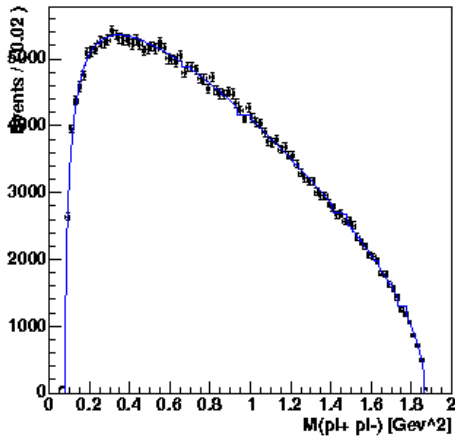
Efficiency map



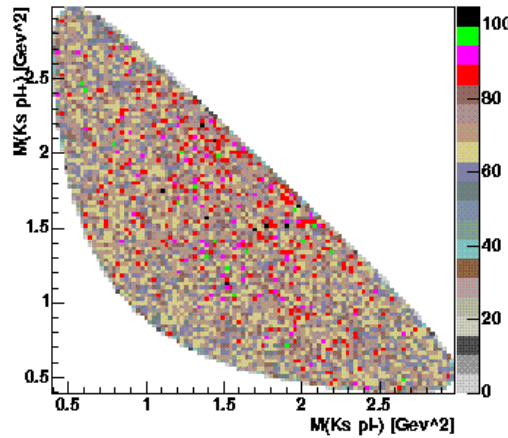
(a) $M_{K_s^0 \pi^\pm}^2$



(b) $M_{K_s^0 \pi^+}^2$



(c) $M_{\pi^+ \pi^-}^2$



(d) Dalitz distribution

— fit result
— flat distribution

Phase space signal MC used

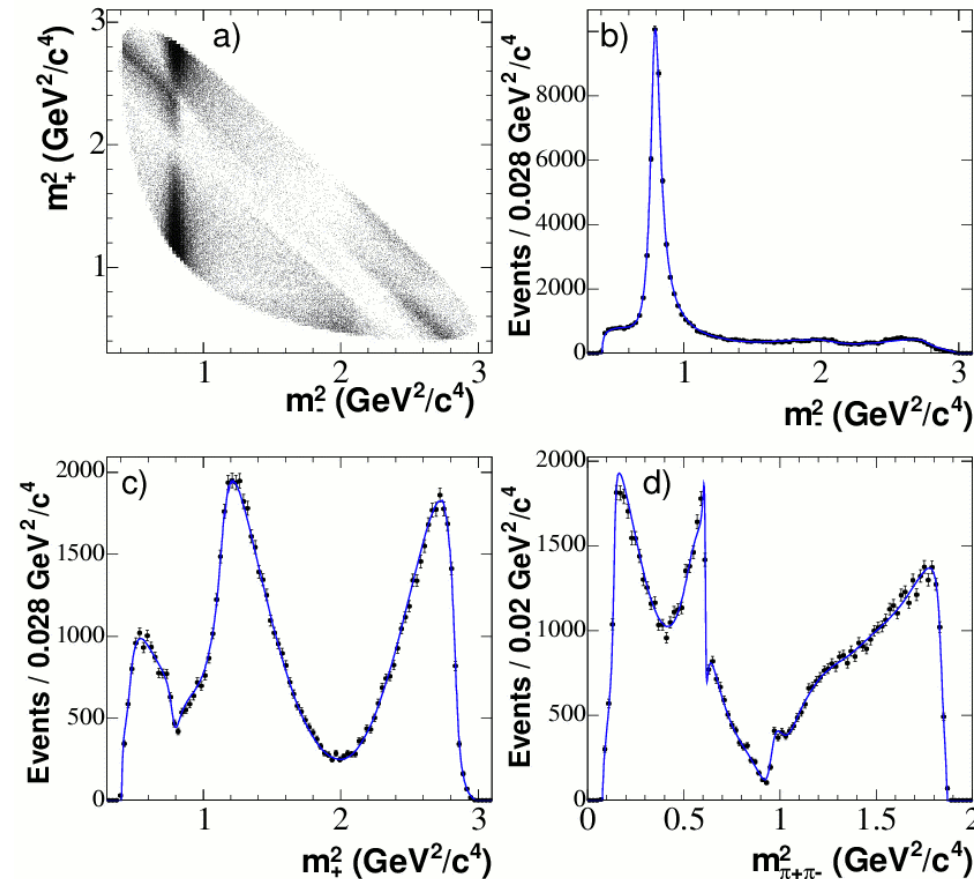
parametrization with a 3rd order polynomial

the efficiency map is very close to a flat distribution

systematic with a flat distribution

Breit Wigner Fit

hep-ex/0504039



Resonance	Amplitude	Phase (deg)	Fit fraction
$K^*(892)^-$	1.781 ± 0.018	131.0 ± 0.82	0.586
$K_0^*(1430)^-$	2.447 ± 0.076	-8.3 ± 2.5	0.083
$K_2^*(1430)^-$	1.054 ± 0.056	-54.3 ± 2.6	0.027
$K^*(1410)^-$	0.515 ± 0.087	154 ± 20	0.004
$K^*(1680)^-$	0.89 ± 0.30	-139 ± 14	0.003
$K^*(892)^+$	0.1796 ± 0.0079	-44.1 ± 2.5	0.006
$K_0^*(1430)^+$	0.368 ± 0.071	-342 ± 8.5	0.002
$K_2^*(1430)^+$	0.075 ± 0.038	-104 ± 23	0.000
$\rho(770)$	1 (fixed)	0 (fixed)	0.224
$\omega(782)$	0.0391 ± 0.0016	115.3 ± 2.5	0.006
$f_0(980)$	0.4817 ± 0.012	-141.8 ± 2.2	0.061
$f_0(1370)$	2.25 ± 0.30	113.2 ± 3.7	0.032
$f_2(1270)$	0.922 ± 0.041	-21.3 ± 3.1	0.030
$\rho(1450)$	0.516 ± 0.092	38 ± 13	0.002
σ	1.358 ± 0.050	-177.9 ± 2.7	0.093
σ'	0.340 ± 0.026	153.0 ± 3.8	0.013
Non Resonant	3.53 ± 0.44	127.6 ± 6.4	0.073

Run1-2 selection
(Run1-4 fit ongoing)

$\Sigma(\text{fit fraction})=124.5 \%$

$\sigma(m=484 \pm 9 \text{ MeV}, \Gamma=383 \pm 14 \text{ MeV})$ $\sigma'(m=1014 \pm 7 \text{ MeV}, \Gamma=88 \pm 13 \text{ MeV})$

K matrix formalism

K-matrix describe the unitarity of the S matrix in processes $a b \rightarrow c d$

$$S = 1 + 2i \sqrt{\rho} T \sqrt{\rho}$$

T = transition operator

ρ = phase space matrix (diagonal matrix)

$$S^+ S = S S^+ = 1 \Rightarrow (T^{-1} + i\rho)^+ = (T^{-1} + i\rho)$$

defining $K^{-1} = (T^{-1} + i\rho)$ $K = K^+$ (**K hermitian**)

Use of the **isobar model** to apply the K-matrix to **Dalitz amplitude**

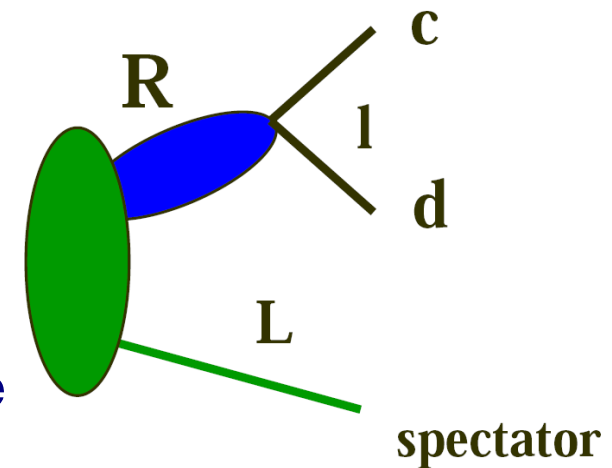
The Lorentz invariant **amplitude**:

Aitchison, Nucl. Phys. A189, 417(1972)

$$F = (1 - i K \rho)^{-1} P$$

resonance propagation

coupling to the initial state
(production vector)



K matrix: $\pi\pi$ S-wave

Parametrize the S-wave component using K matrix formalism

$$F_l = (I - i K \rho)^{-1} \rho_{lj} P_j$$

$j, l = 1 \dots N$ $N =$ number of modes considered
 modes(i, j): $\pi\pi$, KK , 4π , $\eta\eta$, $\eta\eta'$

F = amplitude vector

ρ = phase space matrix (diagonal matrix)

P = "initial" production vector

Following the parametrization of:

V.V. Anisovich, A.V. Sarantsev ([Eur.Phys.J.A16:229-258,2003](#))

$$K_{ij}(s) = \left\{ \sum_{\alpha} \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_{\alpha}^2 - s} + f_{ij}^{scatt.} \frac{1 \text{ GeV}^2 - s_0^{scatt.}}{s - s_0^{scatt.}} \right\} \frac{1 - s_{A0}}{s - s_{A0}} \left(s - s_A m_{\pi}^2 / 2 \right)$$

Pole term

Smooth part

Alder zero term

K matrix poles(α): $f_0(980)$, $f_0(1300)$, $f_0(1200 - 1600)$, $f_0(1500)$, $f_0(1750)$

F-vector

The $\pi\pi$ component of the **F** vector:

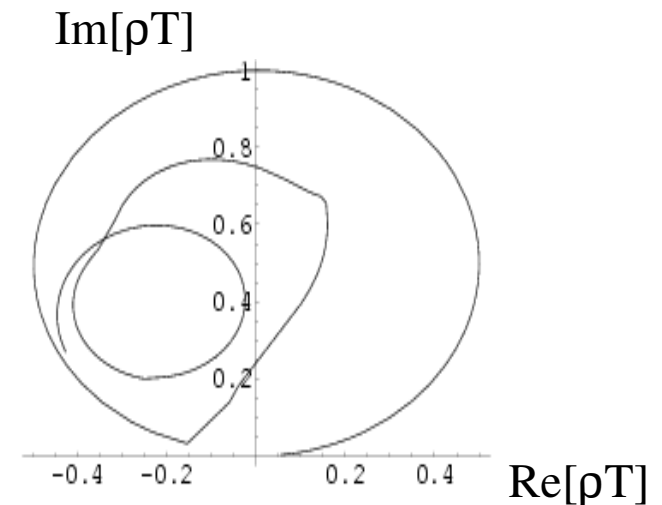
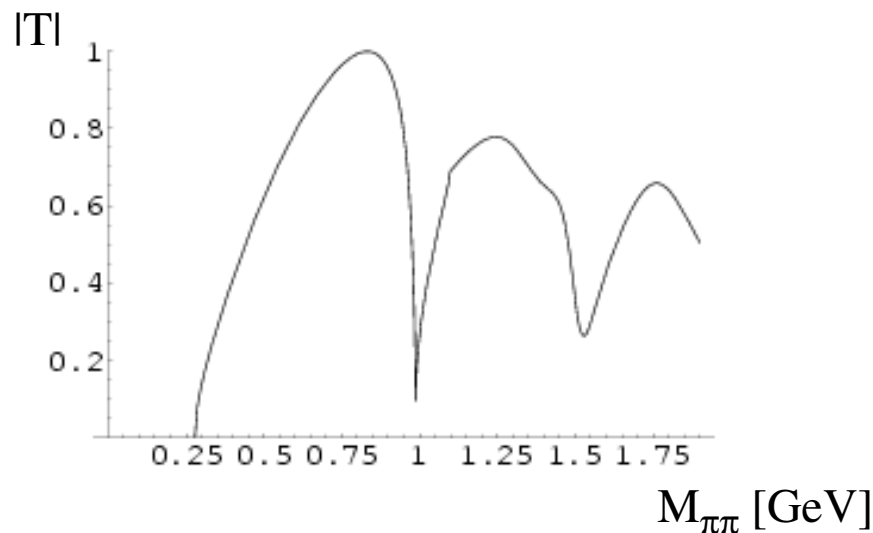
$$F_1 = (I - iK\rho)_{1j}^{-1} \left\{ \sum_{\alpha} \frac{\beta_{\alpha} g_j^{(\alpha)}}{m_{\alpha}^2 - s} + f_{1j}^{prod.} \frac{1 \text{ GeV}^2 - s_0^{prod.}}{s - s_0^{prod.}} \right\} \frac{1 - s_{A0}}{s - s_{A0}} (s - s_A m_{\pi}^2 / 2)$$

β_{α} and $f_{1j}^{prod.}$ are **free parameters** in our fit

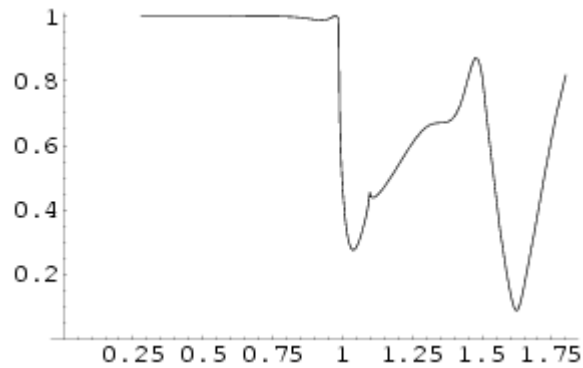
$s_0^{prod.} = s_0^{scatt}$ so far

the “zero term” can be omitted in the P vector

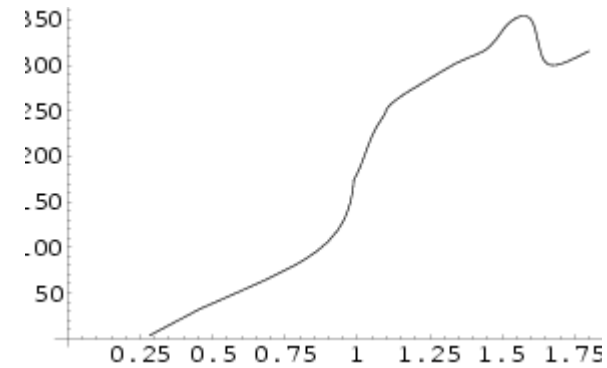
The **transition matrix**: $T = (I - iK\rho)^{-1} K$



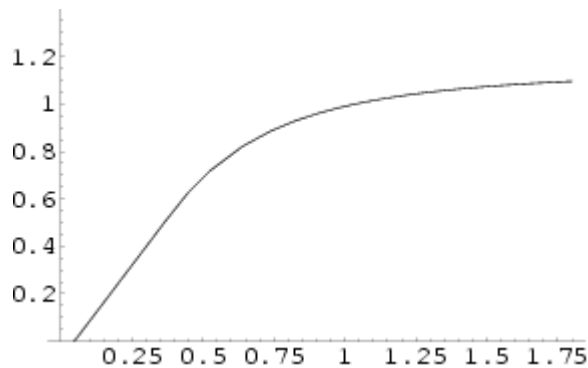
K-Matrix parametrization



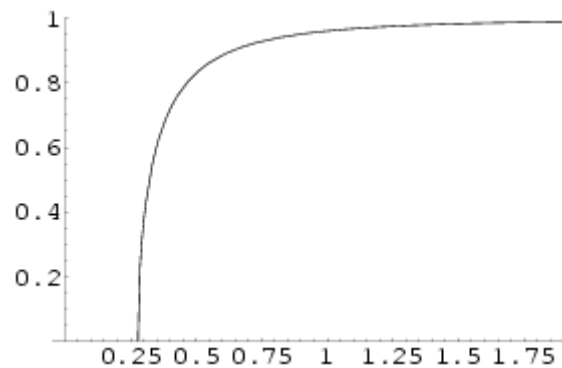
elasticity



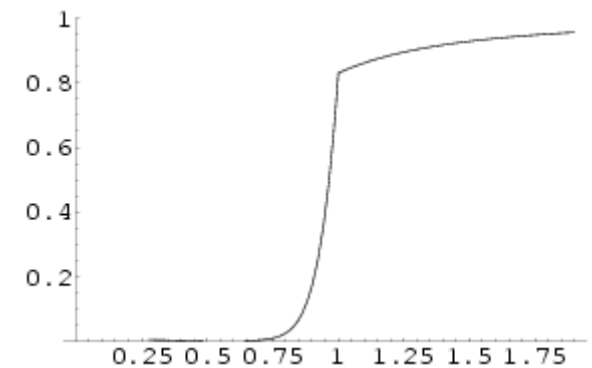
phase shift



Alder zero term
suppression near
 $\pi\pi$ threshold



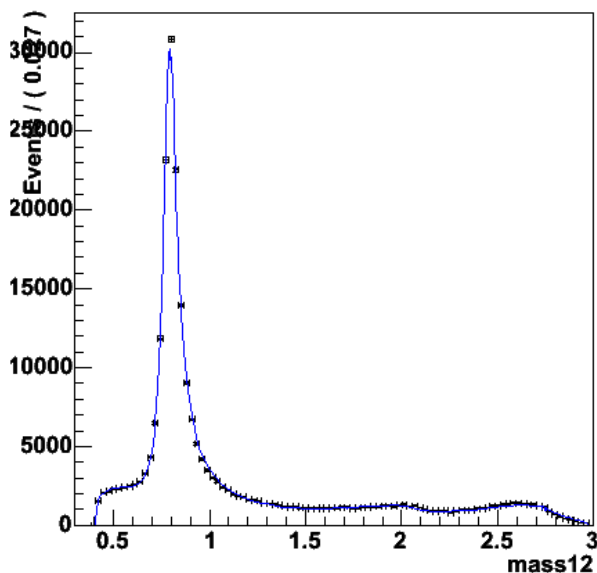
$\pi\pi$ phase space



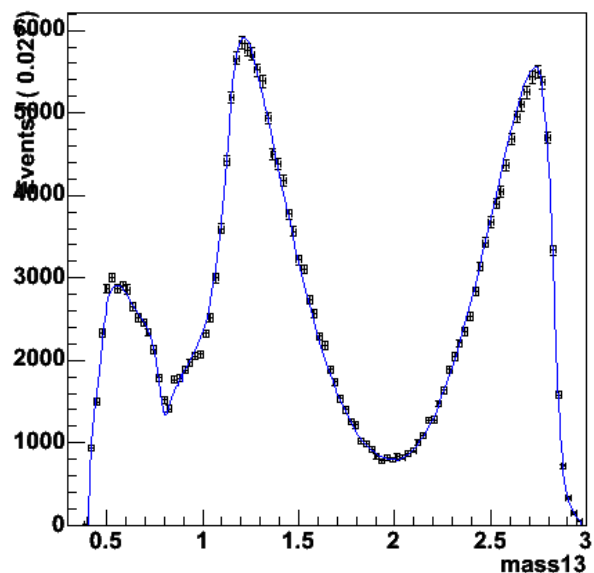
4π phase space

Fit result (I)

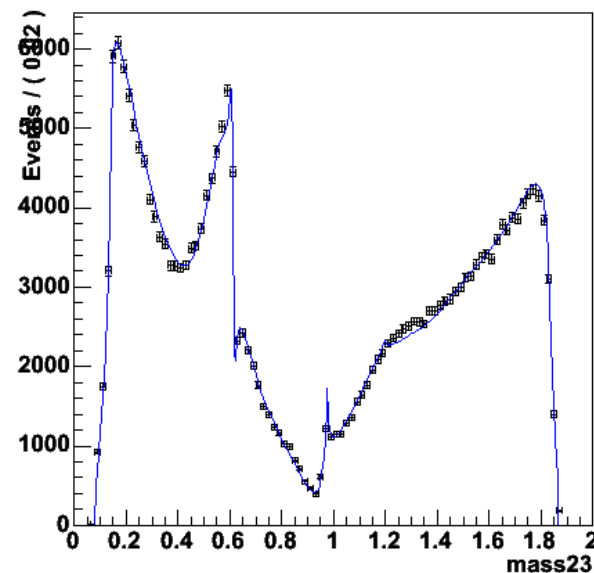
A RooPlot of "mass12"



A RooPlot of "mass13"



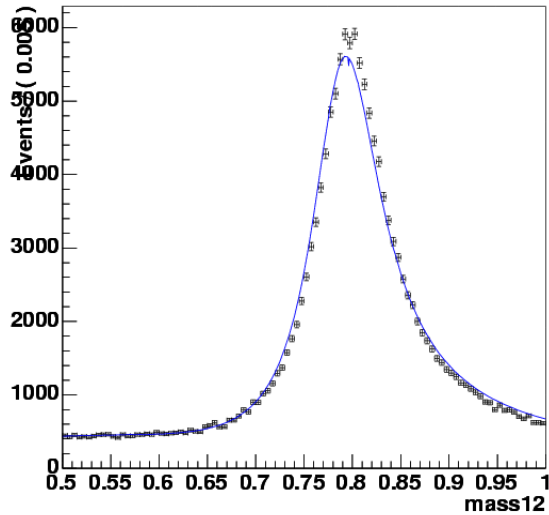
A RooPlot of "mass23"



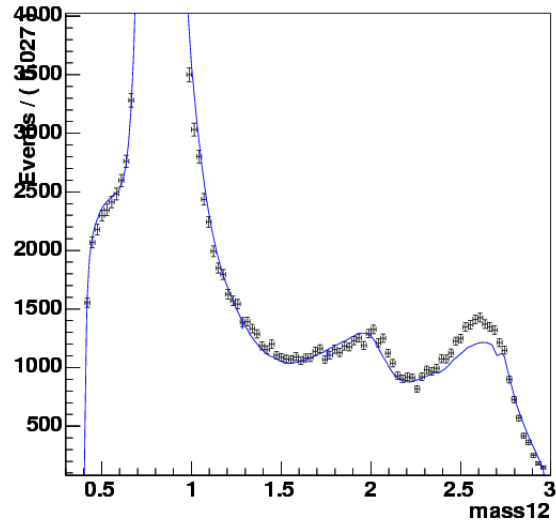
- Values of **mass** and **wight** of **B.-W.** terms fixed to the **P.D.G. 2004** values.
- **Amplitudes** and **phases** of **B.-W.** terms floated.
- Parameter of the **Production vector** floated
- Parameter on the **K-matrix** fixed.

Fit result (II)

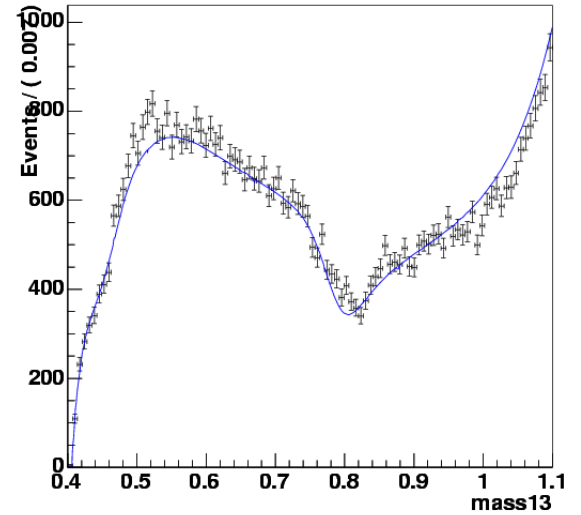
A RooPlot of "mass12"



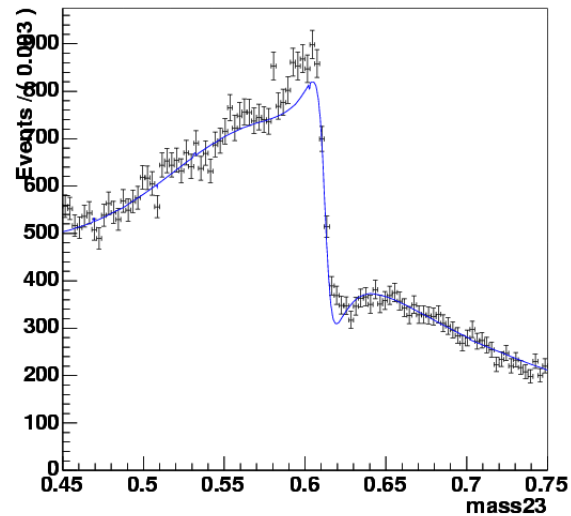
A RooPlot of "mass12"



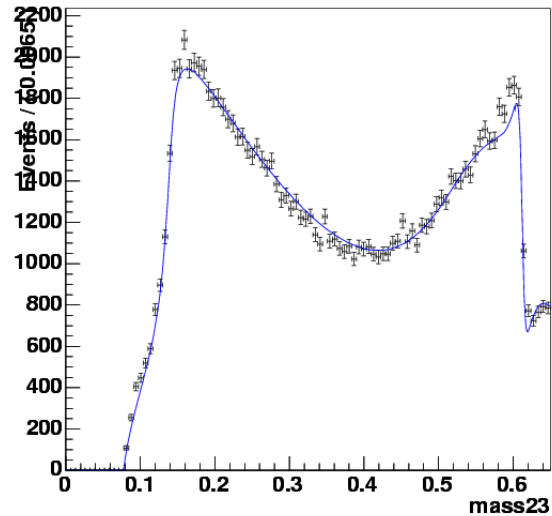
A RooPlot of "mass13"



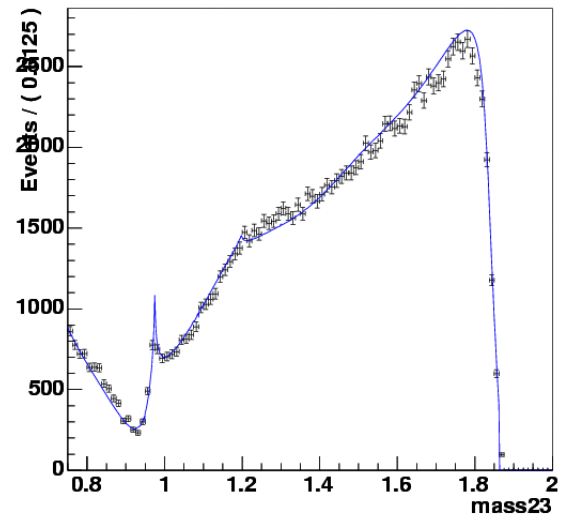
A RooPlot of "mass23"



A RooPlot of "mass23"



A RooPlot of "mass23"



Background effect

	flat bkg val.	flat bkg err	fit bkg val	fit-bkg err	pull
Kst1410_Im	-0.08	0.045	-0.084	0.045	0.130
Kst1410_Re	-0.07	0.050	-0.067	0.049	0.092
K2*1430_DCS_Im	-0.13	0.021	-0.126	0.021	0.115
K2*1430_DCS_Re	0.03	0.022	0.035	0.022	0.118
K2*1430_Im	-0.55	0.029	-0.559	0.029	0.174
K2*1430_Re	0.84	0.022	0.846	0.022	0.301
Kst1430_DCS_Im	-0.17	0.037	-0.190	0.037	0.362
Kst1430_DCS_Re	0.41	0.034	0.422	0.034	0.150
Kst1430_Im	-0.5	0.046	-0.467	0.047	0.433
Kst1430_Re	2.81	0.042	2.808	0.042	0.057
Kst1680_Im	1.13	0.245	1.129	0.244	0.012
Kst1680_Re	-5.04	0.174	-5.023	0.173	0.054
Kstminus_Im	1.33	0.011	1.326	0.011	0.330
Kstminus_Re	-1.1	0.012	-1.101	0.012	0.314
Kstplus_Im	-0.12	0.004	-0.125	0.004	0.154
Kstplus_Re	0.11	0.004	0.110	0.004	0.442
f2_1270_Im	-0.6	0.032	-0.604	0.032	0.197
f2_1270_Re	0.76	0.024	0.769	0.023	0.180
omega782_Im	0.03	0.001	0.031	0.001	0.203
omega782_Re	-0.02	0.001	-0.020	0.001	0.227
rho1450_Im	-0.19	0.087	-0.201	0.086	0.101
rho1450_Re	1.22	0.048	1.194	0.049	0.404
beta1_Im	0.29	0.141	0.32	0.14	0.110
beta1_Re	-2.82	0.119	-2.89	0.12	0.417
beta2_Im	4.22	0.226	4.23	0.23	0.023
beta2_Re	8.29	0.148	8.43	0.15	0.639
beta3_Im	5.05	1.901	5.64	1.91	0.222
beta3_Re	2.69	1.333	3.26	1.34	0.303
beta4_Im	1.44	0.217	1.35	0.22	0.270
beta4_Re	7.5	0.236	7.68	0.24	0.545
fp1_Im	-9.61	0.276	-9.67	0.28	0.146
fp1_Re	-7.13	0.282	-7.28	0.28	0.389

use of cartesian coordinates to better evaluate the low amplitudes component

the value are differs less than 1σ changing bkg parametrization

Fit fractions

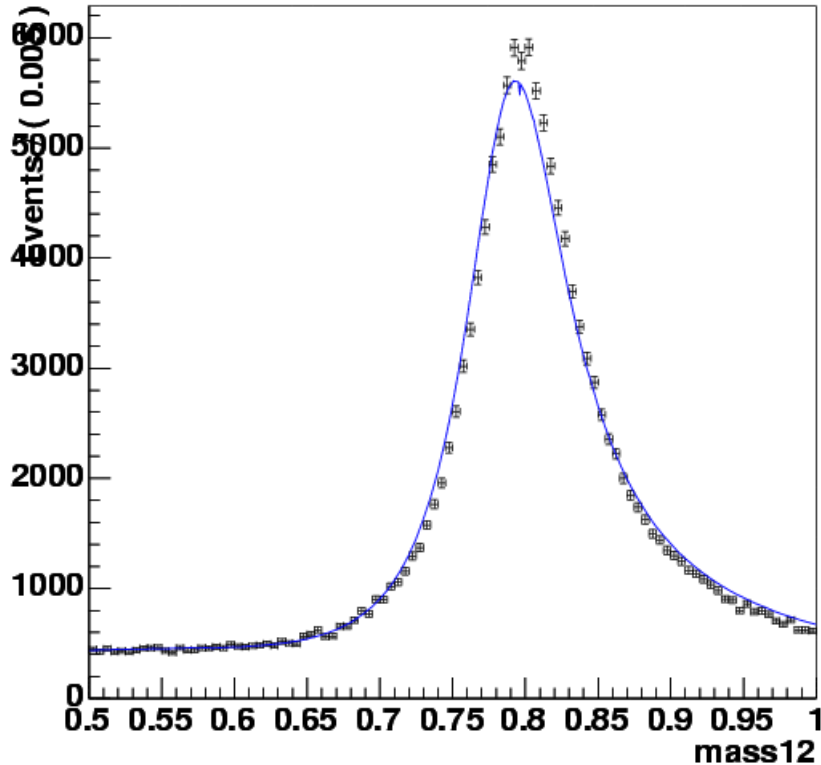
	K-M fit [%]	B-W fit [%]
Rho770	23.28	22.33
Kst1410	0.01	0.39
K2*1430_DCS	0.04	0.01
K2*1430	2.30	2.70
Kst1430_DCS	0.28	0.60
Kst1430	11.46	8.37
Kst1680	6.46	0.35
Kstminus	60.25	58.51
Kstplus	0.54	0.59
F2_1270	2.99	2.95
Omega782	0.52	0.56
Rho1450	1.05	0.28
S-wave	14.20	19.29
NonReson	-----	6.82
SUM	123.39	123.75

the sum of the fit fraction
is quite similar
in the two fits

The S-wave fit fraction is
larger in the BW fit as
expected

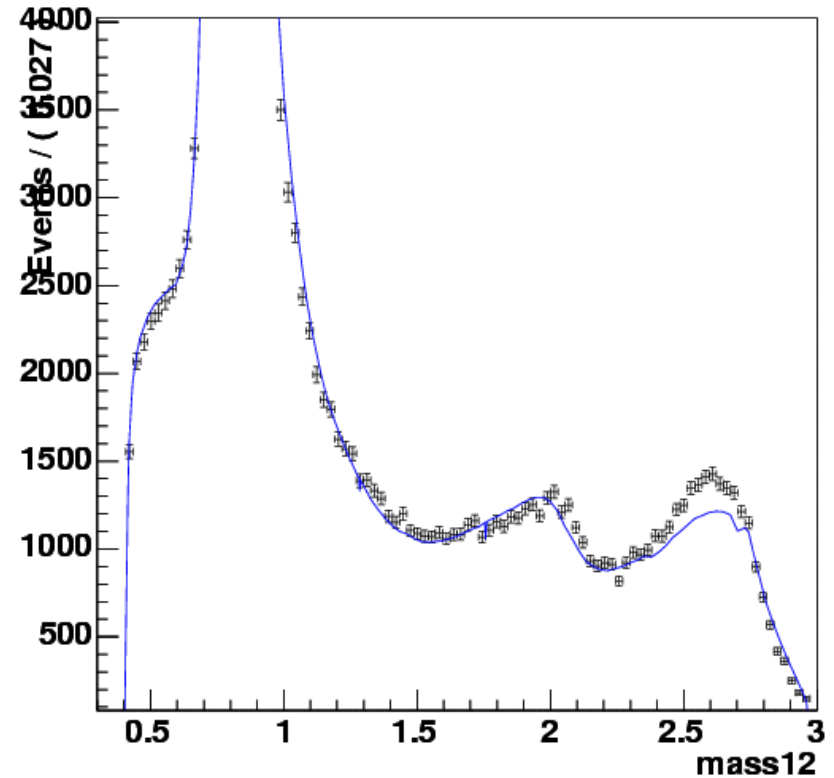
Fit problems

A RooPlot of "mass12"



$K^*(890)$ shape maybe due to resolution effect

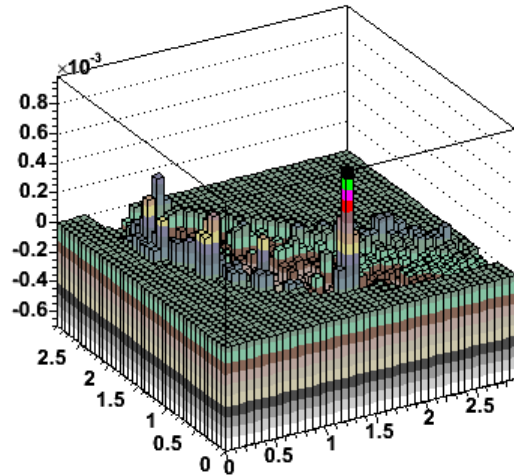
A RooPlot of "mass12"



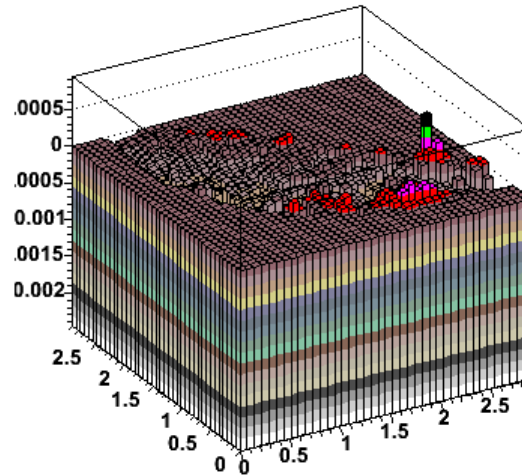
high $m(K_s \pi^-)$ value maybe due to $K-\pi$ S-wave (only $K_0^*(1430)$ in the fit)

Resolution function

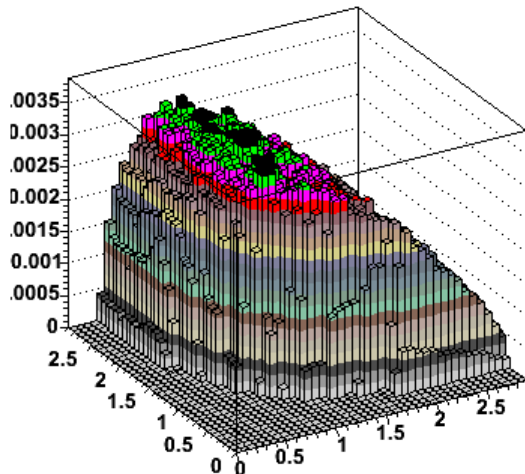
mean m12



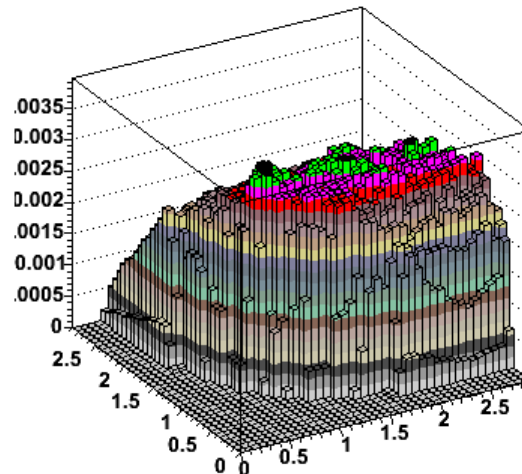
mean m13



sigma m12



sigma m13



Resolution function evaluated as a function of the Dalitz plot.

$m_{ij}(\text{gen.}) - m_{ij}(\text{rec.})$ fitted with 2 gaussians in bins of the Dalitz plot

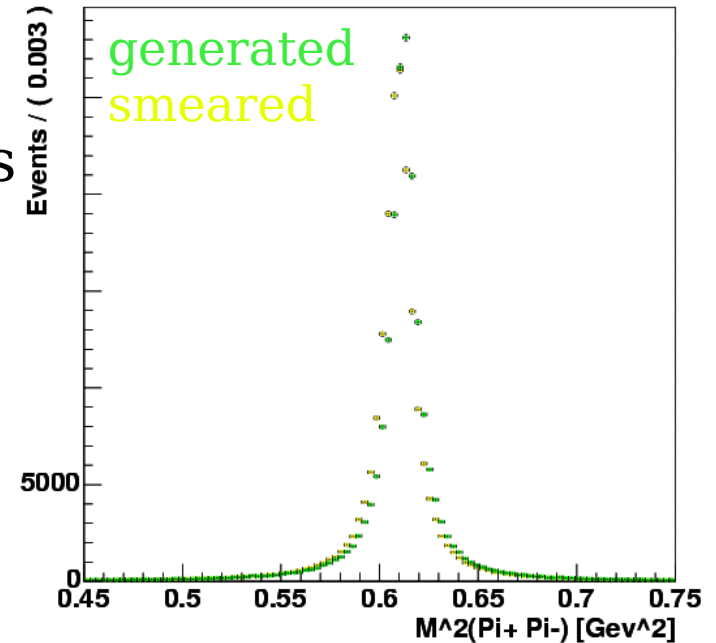
a **maximum** of the smearing effect is present **in the region** of $K^*(890)$ CA and DCS

Smearing effect

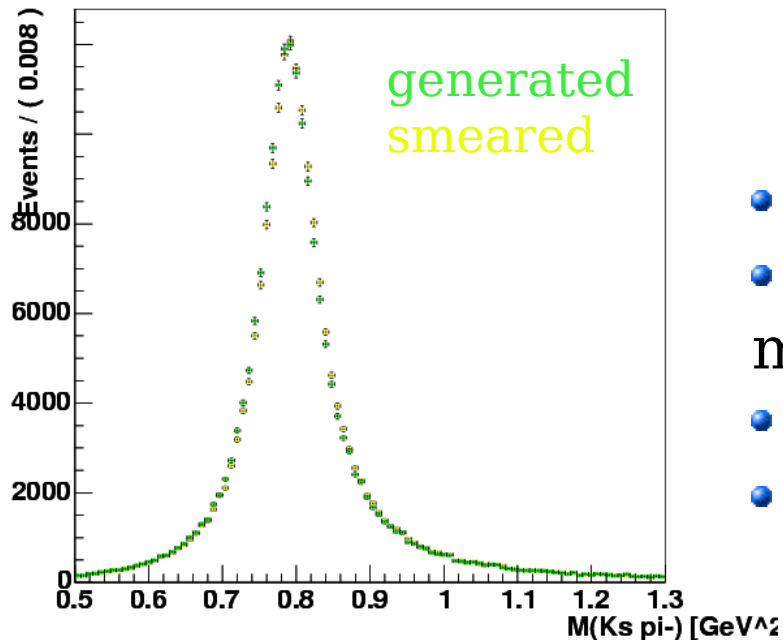
- Generate a sample of $K^*(892)$ or $\omega(782)$
- Smear the sample according to the resolution function
- Fit the **generated** and the **smear** samples

parameter	generated	smear
$K^*(892)$ mass	$8.9222e-01 \pm 8.00e-05$	$8.9348e-01 \pm 8.02e-05$
$K^*(892)$ width	$5.1208e-02 \pm 1.86e-04$	$5.1232e-02 \pm 1.86e-04$
$\omega(782)$ mass	$7.8246e-01 \pm 1.48e-05$	$7.8053e-01 \pm 1.36e-05$
$\omega(782)$ width	$8.8380e-03 \pm 2.67e-05$	$8.8995e-03 \pm 2.93e-05$

omega: green=generated yellow=smear



Kst distribution



- Negligible effect on the **width**
- **1.3 (2.1) MeV/c²** shift for the $K^*(892)$ ($\omega(782)$) mass
- Low $\omega(782)$ amplitude, **neglect** the affect
- **Shift** the $K^*(892)$ mass

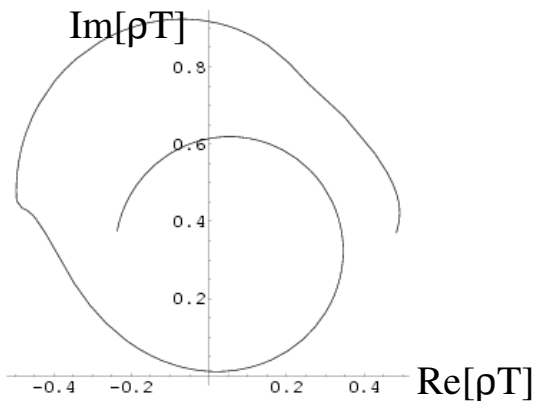
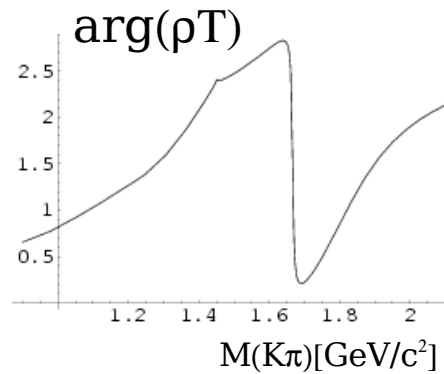
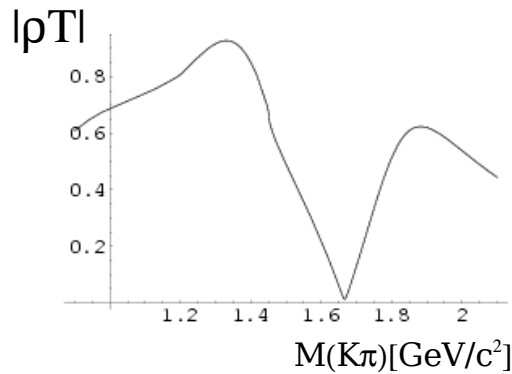
K- π S-wave: K-matrix

V.V. Anisovich, A.V. Sarantsev (Phys. Lett. B 413 (1997) 137-146)

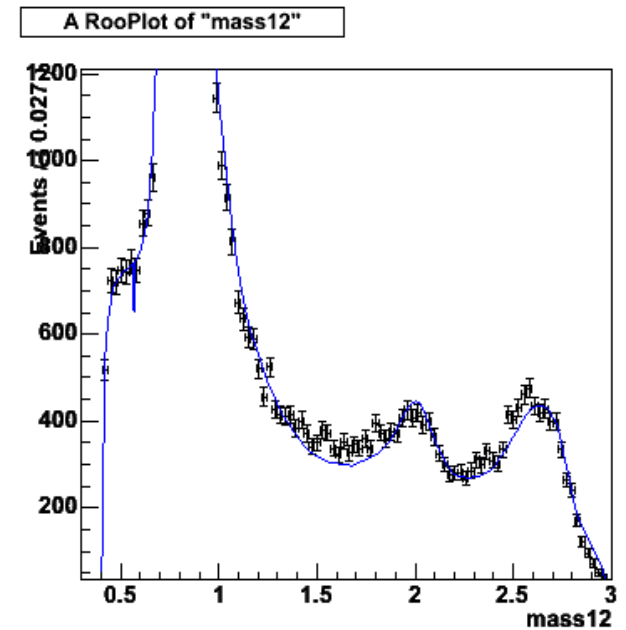
$$K_{ij}(s) = \sum_{\alpha} \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_{\alpha}^2 - s} + f_{ij}^{scatt.} \frac{1.5 \text{ GeV}^2 - s_0^{scatt.}}{s - s_0^{scatt.}}$$

modes(i,j): K π , K η' , K 3π
 2 poles: 1.16, 1.89 GeV/c²

problem: defined in [0.9–2.1] GeV/c²



we cannot extend
 the parametrization
 below 0.9 GeV/c²

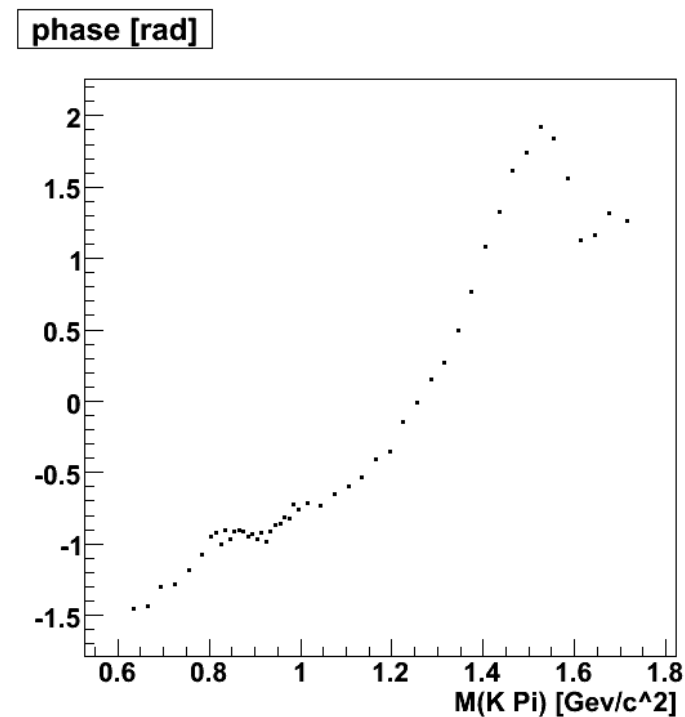
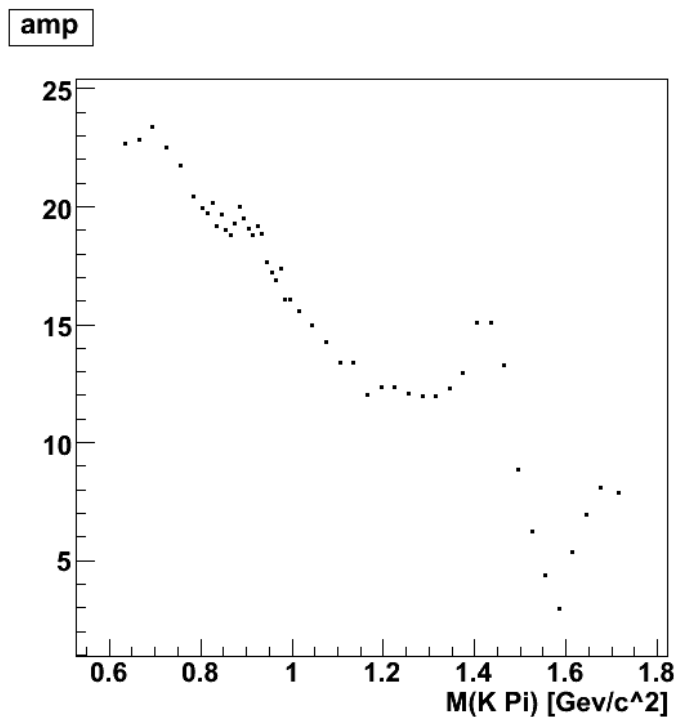


Better high $m(K_S \pi^-)$
 value description

K- π S-wave from D decays

K- π S-wave extracted from A.Palano in $D^+ \rightarrow K^- \pi^+ \pi^+$ decay

problems: errors not available, impossible to separate the different production component.



χ^2 evaluation

- The χ^2 value depend from the binning then is impossible to give to the χ^2 a meaning of goodness of fit
- Anyhow the χ^2 can be used to **select from different models**.

Starting with a **300x300** bins in the Dalitz plot and then **merging the bins** until having more than **10 events** per bin:

for the fit with **flat background**:

$$\chi^2 = 6640 / (3970 - 32) = 1.69$$

for the fit with **sidebands background**:

$$\chi^2 = 6593 / (3970 - 32) = 1.67$$

Conclusion

- Run 1-4 selection give more than 25000 events
- Purity of the sample 97.7 %
- Absence of $M(D^0)$ peaking background, it is possible to use the $M(D^0)$ sidebands to parametrize the background
- The indetermination of the background shape can be taken into account with a systematic (2.3 % of background).
- The V.V. Anisovich, A.V. Sarantsev $\pi\pi$ S-wave parametrization works fine for the $D^0 \rightarrow K_S \pi^- \pi^+$ decay.
- Problem in the $K^*(890)$ shape, maybe due to resolution effects. Try to shift $K^*(890)$ mass.
- Problem in the high $m(K_S \pi^-)$ value maybe due to $K-\pi$ S-wave
- A satisfactory $K-\pi$ S-wave parametrization not present, work ongoing.
- Final model still not decided, a χ^2 can help to select a model.
- Documentation in BAD #1237