Introduction to K-matrix(Brief!)

Ben Lau

Princeton University

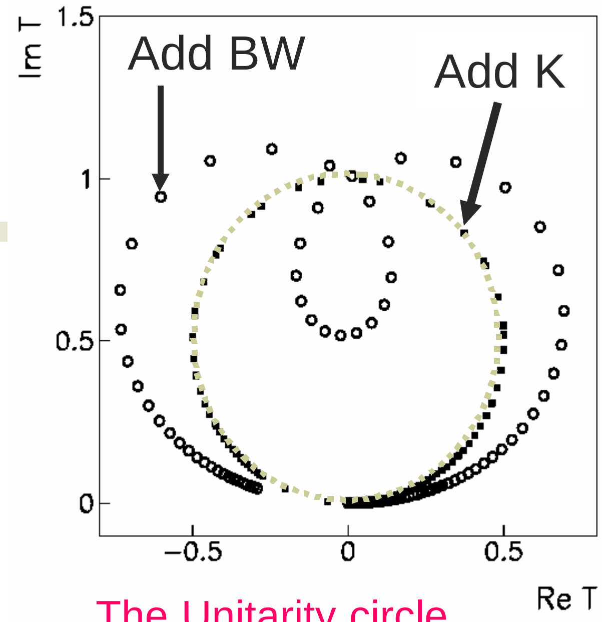
February Collaboration meeting

23 Feb 2005

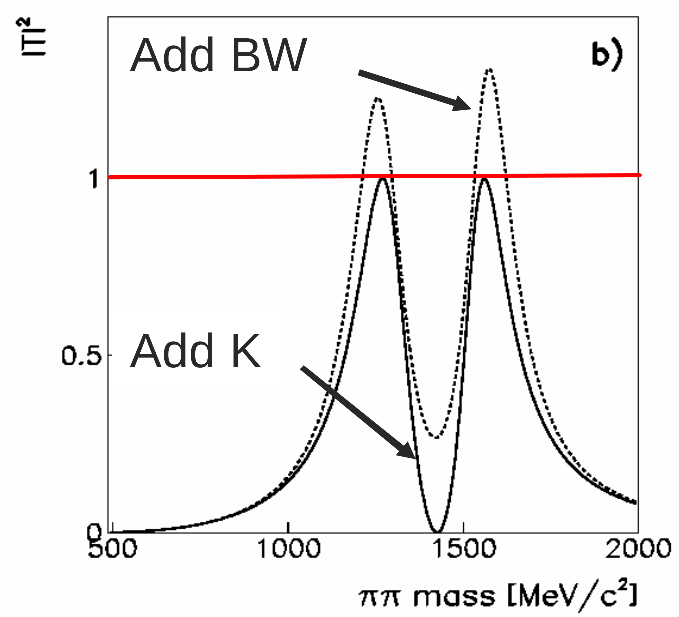
Why K-matrix?

- Adding Breit-Wigners in the “Isobar Model”
  - Breaks the unitarity of the S-Matrix
  - Breit-Wigner is only valid for single, isolated resonance.

For overlapping resonances, (like S-wave), we need more general approach → K-matrix



The Unitarity circle



K-matrix for dummies

We start from first principle: S-matrix respect Unitary:

(unitary means probality conservations)

$$SS^{\dagger} = I$$

(equation 1: unitary of S-matrix)

$$S = I + 2iT$$

(equation2: rewrite S-matrix as T-matrix)

$$(T^{-1} + iI)^{\dagger} = (T^{-1} + iI)$$

(equation3: subsitute 2 in 1)

Now I define K:

Equation4: Most important equation

$$K^{-1} = T^{-1} + iI$$

Notice: (K and T is now related)

→ Unitary condition will be preserved.

From (4): re-write T as a function of K

$$T = K \bullet (I - iK)^{-1}$$

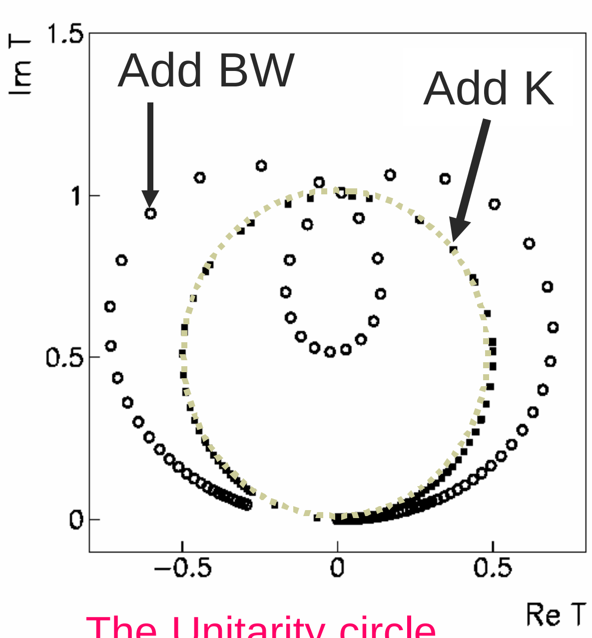
(the T-matrix: The one you observed from the experiment!)

Adding K-matrix!

If we added T-matrix (add BWs)

$$T = BW_1 + BW_2$$

Unitary condition is violated.



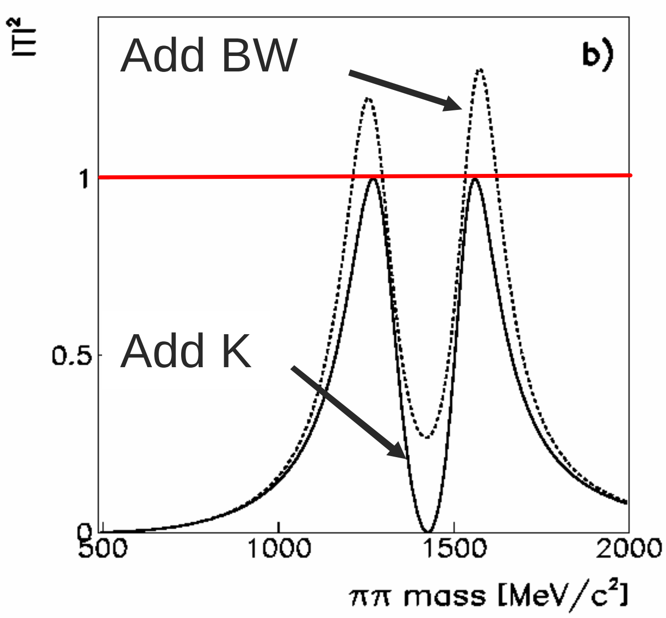
The Unitarity circle

However, if we added K-matrix

$$K = K_1 + K_2$$

Unitary condition is respected.

(Because K is derived from the Unitary of S-matrix from first principle)



Resonance in K-matrix

In Isobar Model, we write the T-matrix as some of Breit-Wigner:

$$T(s) = \sum_a \frac{m_a \Gamma}{(m_a^2 - s) - i m_a \Gamma}$$

But this does NOT perserved unitary!

In K-matrix: The resonances appear as a sum of poles

$$K_{ij}(s) = \sum_a \frac{g_i g_j}{m_a^2 - s}$$

Good! Unitary OK!

Proof of the Breit-Wigner in K-matrix

Now, I am going to prove that in the simplest case the K-matrix formula will derive the usual Breit-Wigner formula

Start from K:

$$K_{ij}(s) = \sum_a \frac{g_i g_j}{m_a^2 - s}$$

If we have one single resonance →1 pole is needed.

$$K = \frac{m\Gamma}{m_a^2 - s}$$

where $g^2 = m\Gamma$

$$1 - iK = \frac{(m_a^2 - s) - im\Gamma}{(m_a^2 - s)}$$

$$\rightarrow K \bullet (I - iK)^{-1} = \frac{m\Gamma}{(m_a^2 - s)} \cdot \frac{(m_a^2 - s)}{(m_a^2 - s) - im\Gamma}$$

$$\rightarrow T = \frac{m\Gamma}{(m_a^2 - s) - im\Gamma}$$

Since:

$$T = K \bullet (I - iK)^{-1}$$

Breit-Wigner formula

Flatte's Formula

K matrix can be generalized to coupled channel decay, eg:

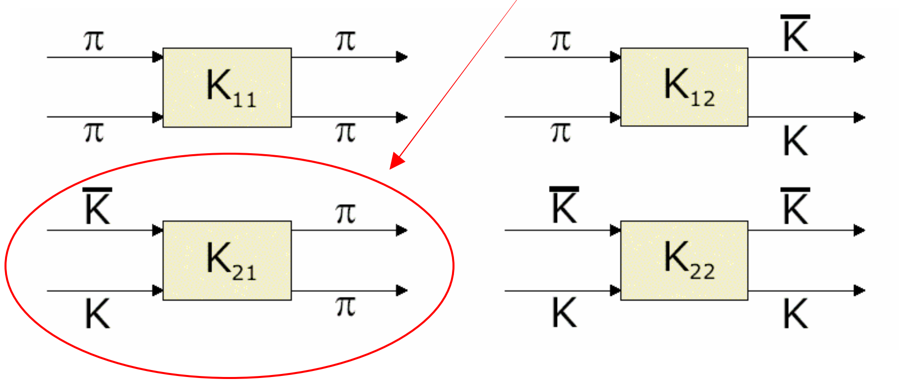
$$f_0(980) \to pp\,, KK$$

Rescattering is NOT included in ISOBAR model

Consider 2 by 2 matrics:

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}$$

In picture:



Follow the pervious exercise, one can derive the famous Flatte's Formula:

$$T = \frac{\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}}{m_a^2 - s - im\Gamma(r_1 g_1^2 + r_2 g_2^2)}$$

How to write down the K-matrix?

In the Breit-Wigner description, there are two parameters, mass M and width Γ, need to be determined from experimental data. And they are all listed in the PDG.

.

$$T(s) = \sum_a \frac{m_a \Gamma}{(m_a^2 - s) - i m_a \Gamma}$$

Similarity, we also need to determine the coupling constant ,g, and pole m, in the K-matrix, from the experimental data!

$$K_{ij}(s) = \sum_a \frac{g_i g_j}{m_a^2 - s}$$

Few words about the K-matrix pole

In the Breit-Wigner description, the mass of the pole term DOES mean the mass of the particle. Eg. rho(770) means the mass of the rho meson has 770MeV.

.

$$T(s) = \sum_a \frac{m_a \Gamma}{(m_a^2 - s) - i m_a \Gamma}$$

However, in the K-matrix, the mass of the pole DOES NOT mean the mass of particle, because T-matrix is the one you can measure in the experiment, not K-matrix.

$$K_{ij}(s) = \sum_a \frac{g_i g_j}{m_a^2 - s}$$

You should:

Perform analytic continuation of the T-matrix, $T = K \bullet (I - iK)^{-1}$ Into complex energy plane to determine the position of the pole

You should NOT:

Perform K-matrix analysis by summing all the K-matrix pole using PDG values!

hep-ph/0204328

Real Example: ππ S-wave

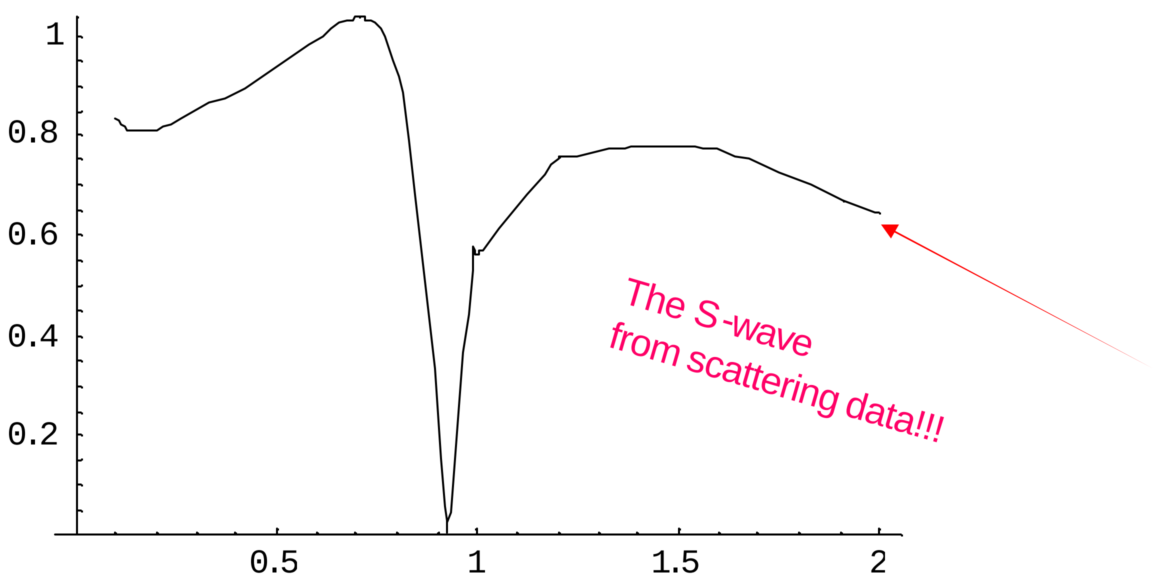
Anisovich and Sarantev perform a global fit of the ππ S-wave, using 5x5 K-matrix with 5 poles.

$$K_{ij}^{00}(s) = \left(\sum_a \frac{g_i^{(a)} g_j^{(a)}}{m_a^2 - s} + f_{ij}^{scatt} \frac{1 - s_0^{scatt}}{s - s_0^{scatt}} \right) \frac{s - s_A m_p^2/2}{(s - s_{A0})(1 - s_{A0})}$$

pole term

slow varying function

Alder zero term.
(suppressed the false kinematics singularity)



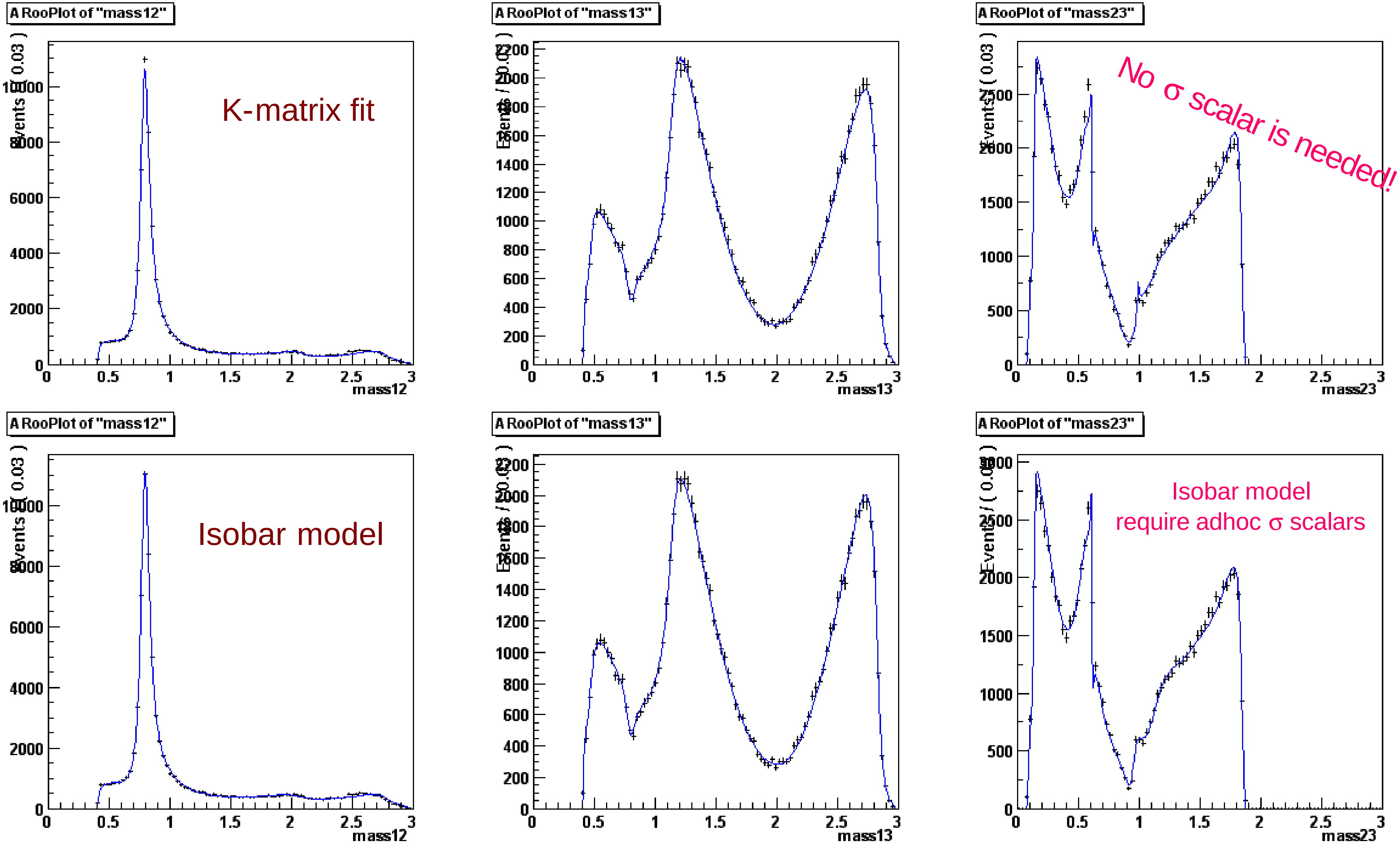
$$K^{-1} = T^{-1} + ir$$

$$T = (I - iK \cdot r)^{-1} K$$

See Luigi's talk for details!

preliminary!

First attempt to $D^0 \to K_s \pi\pi$ dalitz plot via K-matrix approach!



End of the talk!

Comments are welcome.