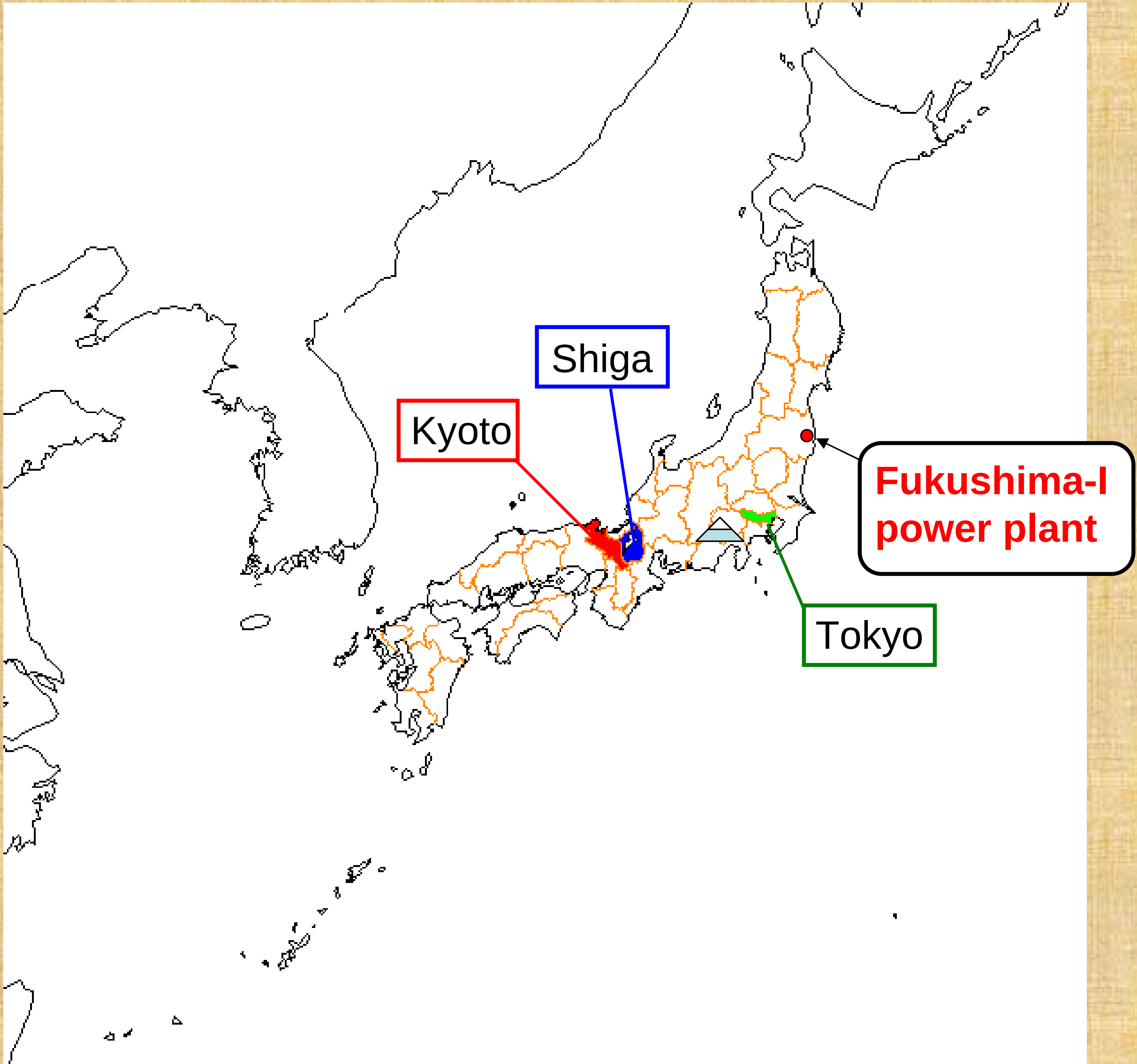
I use words “triplet” for $SU(2)_L$, “3-rep.” for A_4

Phenomenology in the Higgs Triplet Model with A_4 Symmetry

Hiroaki SUGIYAMA
(Ritsumeikan Univ., Shiga, Japan)

Contents

based on ‘T. Fukuyama, H.S., K. Tsumura, PRD82, 036004’
(arXiv:1005.5338)

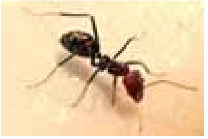

Shiga
Kyoto
Fukushima-I power plant
Tokyo

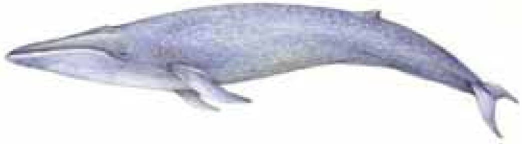
Introduction

Curious features in the lepton sector

Neutrino masses are extremely smaller than other fermion masses.

$\text{neutrino} \lesssim 1\,\text{eV}$ $\quad \text{electron} = 0.5\,\text{MeV}$ $\quad \text{tau} = 1.8\,\text{GeV}$ $\quad \text{top} = 172\,\text{GeV}$



$\longrightarrow$ neutrino-specific mechanism to generate their masses ?

Lepton mixing is nontrivial and very different from quark mixing

$$U_{\mathrm{MNS}} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{0.5} & \sqrt{0.5} \\ 0 & -\sqrt{0.5} & \sqrt{0.5} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{0.68} & \sqrt{0.32} & 0 \\ -\sqrt{0.32} & \sqrt{0.68} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{\mathrm{CKM}} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \sin\theta_{23} \simeq \frac{1}{\sqrt{2}} \qquad \sin\theta_{13} \simeq 0 \qquad \sin\theta_{12} \simeq \frac{1}{\sqrt{3}}$$

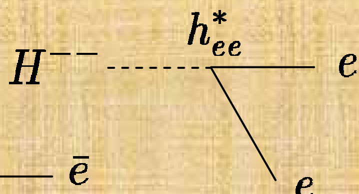
$\longrightarrow$ some underlying physics for the lepton flavor ?

Motivation

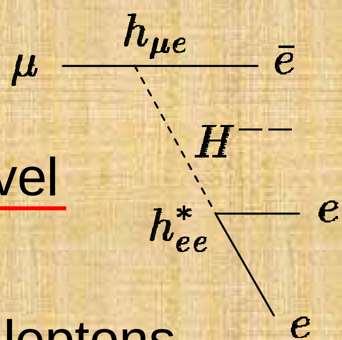
Higgs Triplet Model : a simple model to generate neutrino masses

→ restricted phenomenology on Higgs bosons by the neutrino oscillation measurements

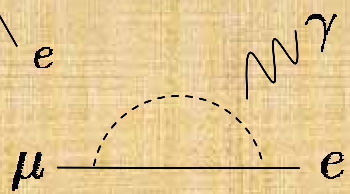
- leptonic decays of the doubly charged Higgs boson
  $H^{--} \to \ell\ell'$



- flavor violating decays of charged leptons at the tree level
  $\mu \to \bar{e}ee \qquad \tau \to \bar{\ell}\ell'\ell''$



- flavor violating radiative decays of charged leptons
  $\ell \to \ell'\gamma$



A_4 symmetry : a possibility to reproduce the MNS matrix in a simple way

→ sharp prediction about flavor structure in the lepton sector

our goal

sharp predictions about $H^{--} \to \ell\ell'$, $\mu \to \bar{e}ee$, $\tau \to \bar{\ell}\ell'\ell''$, $\ell \to \ell'\gamma$ etc. in the Higgs Triplet Model with A_4 symmetry

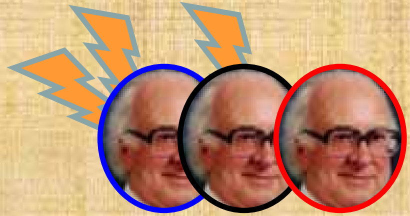
Phenomenology in the Higgs Triplet Model with A_4 Symmetry

Hiroaki SUGIYAMA
(Ritsumeikan Univ., Shiga, Japan)

Contents

based on 'T. Fukuyama, H.S., K. Tsumura, PRD82, 036004'
(arXiv:1005.5338)

Higgs Triplet Model



the SM + a complex Higgs triplet of $SU(2)_L$

no ν_R (no seesaw mechanism)

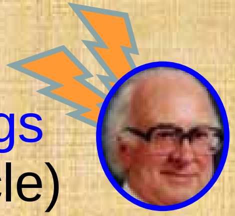
Yukawa interaction with a complex Higgs triplet

$$h_{\ell\ell'}\left(-\overline{(\ell_L)^c}, \overline{(\nu_{\ell L})^c}\right)\overbrace{\begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}}^{\text{Higgs triplet } (Y=2,\, \mathrm{L}\#=-2)}\begin{pmatrix} \nu_{\ell' L} \\ \ell'_L \end{pmatrix} + \mathrm{h.c.}$$

$$\Downarrow \; v_\Delta \equiv \sqrt{2}\langle \Delta^0 \rangle$$

$$\frac{1}{2}\underline{\sqrt{2}\, v_\Delta h_{\ell\ell'}}\, \overline{(\nu_{\ell L})^c}\nu_{\ell' L} + \mathrm{h.c.} + \cdots$$

H^{++} : doubly charged Higgs
(characteristic particle)



Majorana ν_L mass matrix

$$(M_\nu)_{\ell\ell'} = \sqrt{2}\, v_\Delta\, h_{\ell\ell'}$$

W. Konetschny and W. Kummer, PLB**70**, 433
J. Schechter and J.W.F. Valle, PRD**22**, 2227
T.P. Cheng and L.F. Li, PRD**22**, 2860

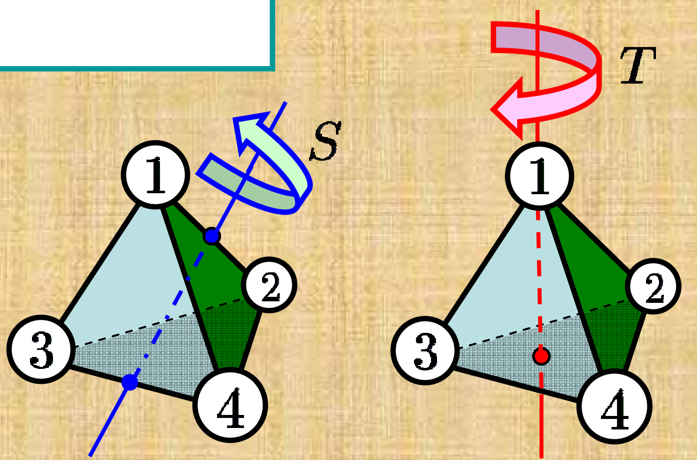
A_4 Group

alternating group on 4 letters

12 elements of A_4 group: even-permutations of 4 letters

$e, a_1, \cdots, a_{11}$

$$
(1,2,3,4)\left\{
\begin{array}{l}
\xrightarrow{\quad e \quad} (1,2,3,4) \\
\xrightarrow{a_1 \equiv S} (\underline{2,1},\underline{4,3}) \\
\xrightarrow{a_2 \equiv T} (1,\underline{3,4,2}) \\
\quad \vdots \\
\xrightarrow{a_{11} \equiv T^2 S} (2,3,1,4)
\end{array}
\right.
$$



S - invariant & T - invariant $\Longrightarrow$ A_4 - symmetric

fits for three flavors

irreducible representations

1-dim. rep. : $\underline{\mathbf{1}}, \ \underline{\mathbf{1}}', \ \underline{\mathbf{1}}''$ 3-dim. rep. : $\underline{\mathbf{3}} = \begin{pmatrix} \mathbf{3}_x \\ \mathbf{3}_y \\ \mathbf{3}_z \end{pmatrix}$

A_4 is minimal one which includes 3-dim. irreducible rep.

Phenomenology in the Higgs Triplet Model with A_4 Symmetry

Hiroaki SUGIYAMA
(Ritsumeikan Univ., Shiga, Japan)

Contents

based on ‘T. Fukuyama, H.S., K. Tsumura, PRD82, 036004’
(arXiv:1005.5338)

A_4 - Symmetric Higgs Triplet Model (A4HTM)

Particle contents no $SU(2)_L \times U(1)_Y$ singlet fields (ν_R ,"flavon")

	ψ^-_{1R}	ψ^-_{2R}	ψ^-_{3R}	$\Psi_{AL} = \begin{pmatrix} \psi^0_{AL} \\ \psi^-_{AL} \end{pmatrix}$
A_4	$\underline{\mathbf{1}}$	$\underline{\mathbf{1}'}$	$\underline{\mathbf{1}''}$	$\underline{\mathbf{3}}$
$SU(2)_L$	singlet	singlet	singlet	doublet
$U(1)_Y$	-2	-2	-2	-1

$A = x, y, z$

$\Phi_A = \begin{pmatrix} \phi^+_A \\ \phi^0_A \end{pmatrix}$	$\delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$	$\Delta_A = \begin{pmatrix} \frac{\Delta^+_A}{\sqrt{2}} & \Delta^{++}_A \\ \Delta^0_A & -\frac{\Delta^+_A}{\sqrt{2}} \end{pmatrix}$
$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{3}}$
doublet	triplet	triplet
1	2	2

three Higgs doublets and four Higgs triplets

Vacuum alignment

$$\begin{pmatrix}\langle\phi_x^0\rangle\\ \langle\phi_y^0\rangle\\ \langle\phi_z^0\rangle\end{pmatrix} = \begin{pmatrix}v/\sqrt{6}\\ v/\sqrt{6}\\ v/\sqrt{6}\end{pmatrix}, \qquad \langle\delta^0\rangle = \frac{v_\delta}{\sqrt{2}}, \qquad \begin{pmatrix}\langle\Delta_x^0\rangle\\ \langle\Delta_y^0\rangle\\ \langle\Delta_z^0\rangle\end{pmatrix} = \begin{pmatrix}v_\Delta/\sqrt{2}\\ 0\\ 0\end{pmatrix}$$

$A_4 \to Z_3$

$A_4 \to Z_2$

flavor eigenstates

$e,\ \mu,\ \tau,\ \nu_{eL},\ \nu_{\mu L},\ \nu_{\tau L}$

neutrino **mass** eigenstates

$\nu_{1L},\ \nu_{2L},\ \nu_{3L}$

$U_{\rm MNS} =$ "tri-bimaximal mixing" form

attractive feature of A_4 symmetry

Realization of tri-bimaximal mixing can be expressed **in terms of symmetry breaking pattern**.

G. Altarelli and F. Feruglio, NPB**720**, 64 (with singlet scalars "flavon")

Phenomenology in the Higgs Triplet Model with A_4 Symmetry

Hiroaki SUGIYAMA
(Ritsumeikan Univ., Shiga, Japan)

Contents

based on ‘T. Fukuyama, H.S., K. Tsumura, PRD82, 036004’
(arXiv:1005.5338)

Vacuum alignment : revisited

$$\begin{pmatrix} \langle \phi_x^0 \rangle \\ \langle \phi_y^0 \rangle \\ \langle \phi_z^0 \rangle \end{pmatrix} = \begin{pmatrix} v/\sqrt{6} \\ v/\sqrt{6} \\ v/\sqrt{6} \end{pmatrix}, \quad \langle \delta^0 \rangle = \frac{v_\delta}{\sqrt{2}}, \quad \begin{pmatrix} \langle \Delta_x^0 \rangle \\ \langle \Delta_y^0 \rangle \\ \langle \Delta_z^0 \rangle \end{pmatrix} = \begin{pmatrix} v_\Delta/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$$

$$v = 246\,\mathrm{GeV} \qquad \sqrt{v_\delta^2 + v_\Delta^2} \lesssim 1\mathrm{GeV}$$

$$\left.\begin{array}{r} v : A_4 \to Z_3 \\ v_\Delta : A_4 \to Z_2 \\ v_\Delta \ll v \end{array}\right\} \longrightarrow$$ An approximate Z_3 symmetry remains in the A4HTM

Mass eigenstates of particles (excpt. for ν_i) should have definite Z_3- charges

= eigenstates of T

	$H_3^{++},\ H_4^{++}$ $L_e,\ e_R$	H_2^{++} $L_\mu,\ \mu_R$	H_1^{++} $L_\tau,\ \tau_R$
Z_3–charge	1	ω	ω^2

A to Z (from mathematics to physics)

$$\begin{array}{c} 1\\ \omega \\ \omega^2\end{array}\begin{pmatrix} e_R\\ \mu_R\\ \tau_R\end{pmatrix} \equiv \begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}\psi^-_{1R}\\ \psi^-_{2R}\\ \psi^-_{3R}\end{pmatrix}\begin{array}{c}\underline{1}\\ \underline{1'}\\ \underline{1''}\end{array}$$

$$\begin{array}{c} 1\\ \omega \\ \omega^2\end{array}\begin{pmatrix} L_e\\ L_\mu\\ L_\tau\end{pmatrix} \equiv \frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}1&1&1\\1&\omega^2&\omega\\1&\omega&\omega^2\end{pmatrix}\left.\begin{pmatrix}\Psi_{xL}\\ \Psi_{yL}\\ \Psi_{zL}\end{pmatrix}\right\}\underline{3}$$

$$\begin{array}{c} \omega^2\\ \omega \\ 1 \\ 1\end{array}\begin{pmatrix} H_1^{++}\\ H_2^{++}\\ H_3^{++}\\ H_4^{++}\end{pmatrix} \equiv \frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&c_{\pm\pm}&s_{\pm\pm}\\0&0&-s_{\pm\pm}&c_{\pm\pm}\end{pmatrix}\begin{pmatrix}1&\omega&\omega^2&0\\1&\omega^2&\omega&0\\1&1&1&0\\0&0&0&\sqrt{3}\,e^{-i\alpha_{\pm\pm}}\end{pmatrix}\begin{pmatrix}\Delta_x^{++}\\ \Delta_y^{++}\\ \Delta_z^{++}\\ \delta^{++}\end{pmatrix}\begin{array}{l}\left.\begin{matrix}\\ \\ \\ \end{matrix}\right\}\underline{3}\\ \underline{1}\end{array}$$

Triplet Yukawa in physics basis (renormalizable)

$$(h_{i\pm\pm})_{\ell\ell'}\, \overline{(\ell_L)^c}\, \ell'_L\, H_i^{++} + \text{h.c.}$$

$$\frac{2}{\sqrt{3}} h_\Delta \left\{ -\overset{\color{red}1}{\overline{(e_L)^c}}\, \overset{\color{red}\omega}{\mu_L} + \overset{\color{red}\omega^2}{\overline{(\tau_L)^c}}\, \overset{\color{red}\omega^2}{\tau_L} \right\} \overset{\color{red}\omega^2}{H_1^{++}}$$

$$h_{1\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

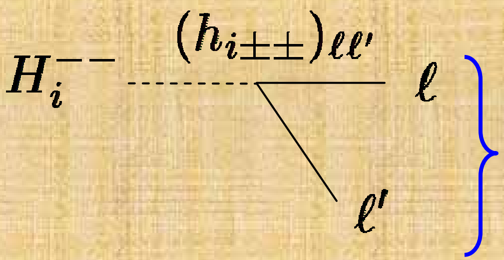
$$h_{2\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$h_{3\pm\pm} = \frac{1}{\sqrt{3}} h_\Delta c_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$h_{4\pm\pm} = -\frac{1}{\sqrt{3}} h_\Delta s_{\pm\pm} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Phenomenology

$\underline{H_i^{--} \to \ell\ell'}$



same-signed charged leptons

$\downarrow$

clear signal

testable at LHC

if some of them are light

prediction in A4HTM

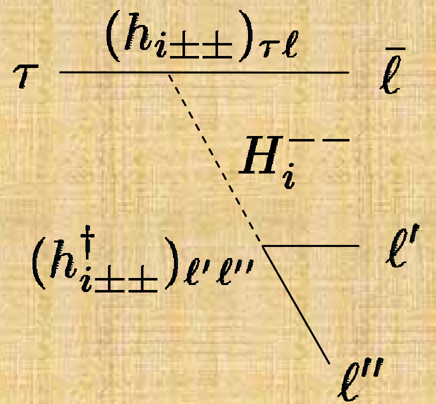
		$\mathrm{BR}(H_i^{--} \to \ell\ell')$
		$\overset{11}{ee} : \overset{\omega\,\omega}{\mu\mu} : \overset{\omega^2\omega^2}{\tau\tau} : \overset{1\omega}{e\mu} : \overset{1\omega^2}{e\tau} : \overset{\omega\omega^2}{\mu\tau}$
ω	H_1^{--}	$0 : 0 : 2 : 1 : 0 : 0$
ω^2	H_2^{--}	$0 : 2 : 0 : 0 : 1 : 0$
1	H_3^{--}	$R_3^{\pm\pm} : 0 : 0 : 0 : 0 : 1$
1	H_4^{--}	$R_4^{\pm\pm} : 0 : 0 : 0 : 0 : 1$

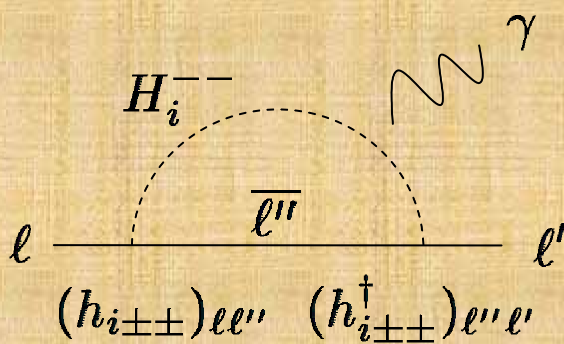
$$R_3^{\pm\pm} \equiv \frac{|2h_\Delta c_{\pm\pm} + \sqrt{3}\, h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}{2|h_\Delta c_{\pm\pm} - \sqrt{3}\, h_\delta s_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}$$

$$R_4^{\pm\pm} \equiv \frac{|2h_\Delta s_{\pm\pm} - \sqrt{3}\, h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}{2|h_\Delta s_{\pm\pm} + \sqrt{3}\, h_\delta c_{\pm\pm} e^{i\alpha_{\pm\pm}}|^2}$$

parts of zero : Z_3 nonzero values : A_4

$\tau \to \bar{\ell}\ell'\ell''$, $\mu \to \bar{e}ee$, $\ell \to \ell'\gamma$





prediction in A4HTM

		$\tau \to \bar{\ell}\ell'\ell''$
ω	H_1^{--}	none
ω^2	H_2^{--}	$\underset{\omega^2}{\tau_L} \to \underset{1}{\overline{e_L}}\,\underset{\omega}{\mu_L}\,\underset{\omega}{\mu_L}$
1	H_3^{--}	$\underset{\omega^2}{\tau_L} \to \underset{\omega^2}{\overline{\mu_L}}\,\underset{1}{e_L}\,\underset{1}{e_L}$
1	H_4^{--}	$\tau_L \to \overline{\mu_L}e_Le_L$

no $\underset{\omega}{\mu} \to \underset{1}{e}\,\underset{1}{\gamma}$

no $\underset{\omega}{\mu} \to \underset{1}{\bar{e}}\,\underset{1}{e}\,\underset{1}{e}$

Stringent constraint is avoided

$$\mathrm{BR}(\mu \to \bar{e}ee) < 1.0 \times 10^{-12}$$

SINDRUM collab., NPB**299**,1 (1988)

testable at B-factories and MEG

Summary

A4HTM is a **renormalizable model** for neutrino masses and mixings (no “flavons”)

A4HTM is **testable at LHC**

		$\mathrm{BR}(H_i^{--} \to \ell\ell')$					
		11	$\omega\,\omega$	$\omega^2\omega^2$	1ω	$1\omega^2$	$\omega\omega^2$
		ee :	$\mu\mu$:	$\tau\tau$:	$e\mu$:	$e\tau$:	$\mu\tau$
ω	H_1^{--}	0 :	0 :	2 :	1 :	0 :	0
ω^2	H_2^{--}	0 :	2 :	0 :	0 :	1 :	0
1	H_3^{--}	$R_3^{\pm\pm}$:	0 :	0 :	0 :	0 :	1
1	H_4^{--}	$R_4^{\pm\pm}$:	0 :	0 :	0 :	0 :	1

A4HTM is **testable at B-factories and MEG**

possible : $\tau \to \overset{\omega^2}{\bar{e}}\overset{1\omega\omega}{\mu\mu},\ \tau \to \overset{\omega^2 11}{\bar{\mu}ee}$

almost forbidden : $\tau \to \bar{e}ee,\ \tau \to \bar{e}e\mu,\ \tau \to \bar{\mu}\mu\mu,\ \tau \to \bar{\mu}e\mu$

$\mu \to \bar{e}ee,\ \ \mu \to e\gamma$

backup

Possible Realization of the Vacuum

Higgs potential in HTM

$$V = -m^2(\Phi^\dagger\Phi) + M^2\mathrm{Tr}(\Delta^\dagger\Delta) + \left(\frac{1}{\sqrt{2}}\mu(\Phi^T i\sigma^2\Delta^\dagger\Phi) + \mathrm{h.c}\right) + (\text{quartic terms})$$

$v \equiv \sqrt{2}\langle\phi^0\rangle = 246\,\mathrm{GeV}$: spontaneous breaking of $SU(2)_L$

$v_\Delta \equiv \sqrt{2}\langle\Delta^0\rangle \simeq \dfrac{\mu v^2}{2M^2}$: explicit breaking of $\mathrm{L}\#$ $\longrightarrow$ no NG-boson for $\cancel{L}\#$ ("Majoron")

Soft breaking terms of $L\#$ and A_4 in A4HTM (a possibility)

$$\tilde{V}_\mu = \frac{1}{\sqrt{2}}\mu_\delta(\overset{\mathbf{3}}{\Phi_\alpha}\overset{\mathbf{3}}{\Phi_\beta})_{\mathbf{1}}(i\sigma^2\overset{\mathbf{1}}{\delta^\dagger})_{\alpha\beta} + \frac{1}{\sqrt{2}}\mu_{\Delta_x}(2\overset{\mathbf{3}_y}{\Phi_{y\alpha}}\overset{\mathbf{3}_z}{\Phi_{z\beta}})(i\sigma^2\overset{\mathbf{3}_x}{\Delta_x^\dagger})_{\alpha\beta} + \mathrm{h.c.}$$

$v_\delta \equiv \sqrt{2}\,\langle\delta^0\rangle$

$v_\Delta \equiv \sqrt{2}\,\langle\Delta_x^0\rangle \qquad \langle\Delta_y^0\rangle = \langle\Delta_z^0\rangle = 0$

one necessary condition : $v_\delta\mathrm{Re}(\lambda'_{4s} + \lambda'_{5s}) + v_\Delta(\lambda_{4\Delta ss} + \lambda_{5\Delta ss}) = 0$