Right unitarity triangles and tri-bimaximal mixing from discrete symmetries and unification

Martin Spinrath

SISSA

FLASY 2011 - 12th July

Based on collaborations with S. Antusch, S.F. King, C. Luhn and M. Malinsky:
PRD 81 (2010) 033008, PRD 83 (2011) 013005, NPB 850 (2011) 477

Outline

- Motivation
- Quark Mixing Sum Rules
- Discrete Vacuum Alignment
- Examples
- Summary and Conclusions

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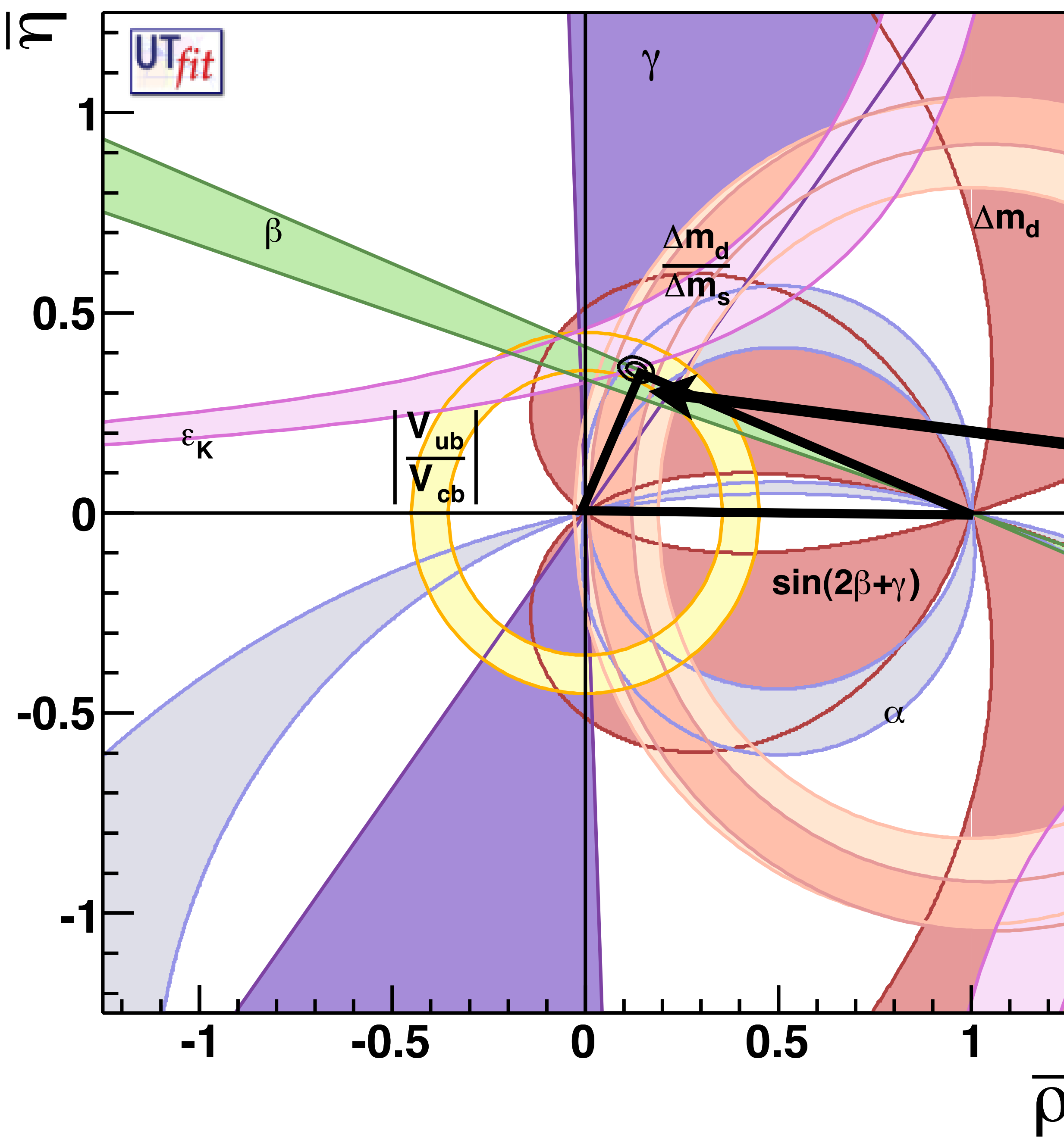
The CKM Unitarity Triangle



$$V_{bi}^\dagger V_{id} = 0$$

Fit result [UTfit]:

$$\alpha = (87.8 \pm 3.0)^\circ$$

Plot and number taken from the UTfit collaboration summer 2010 results (pre-ICHEP)

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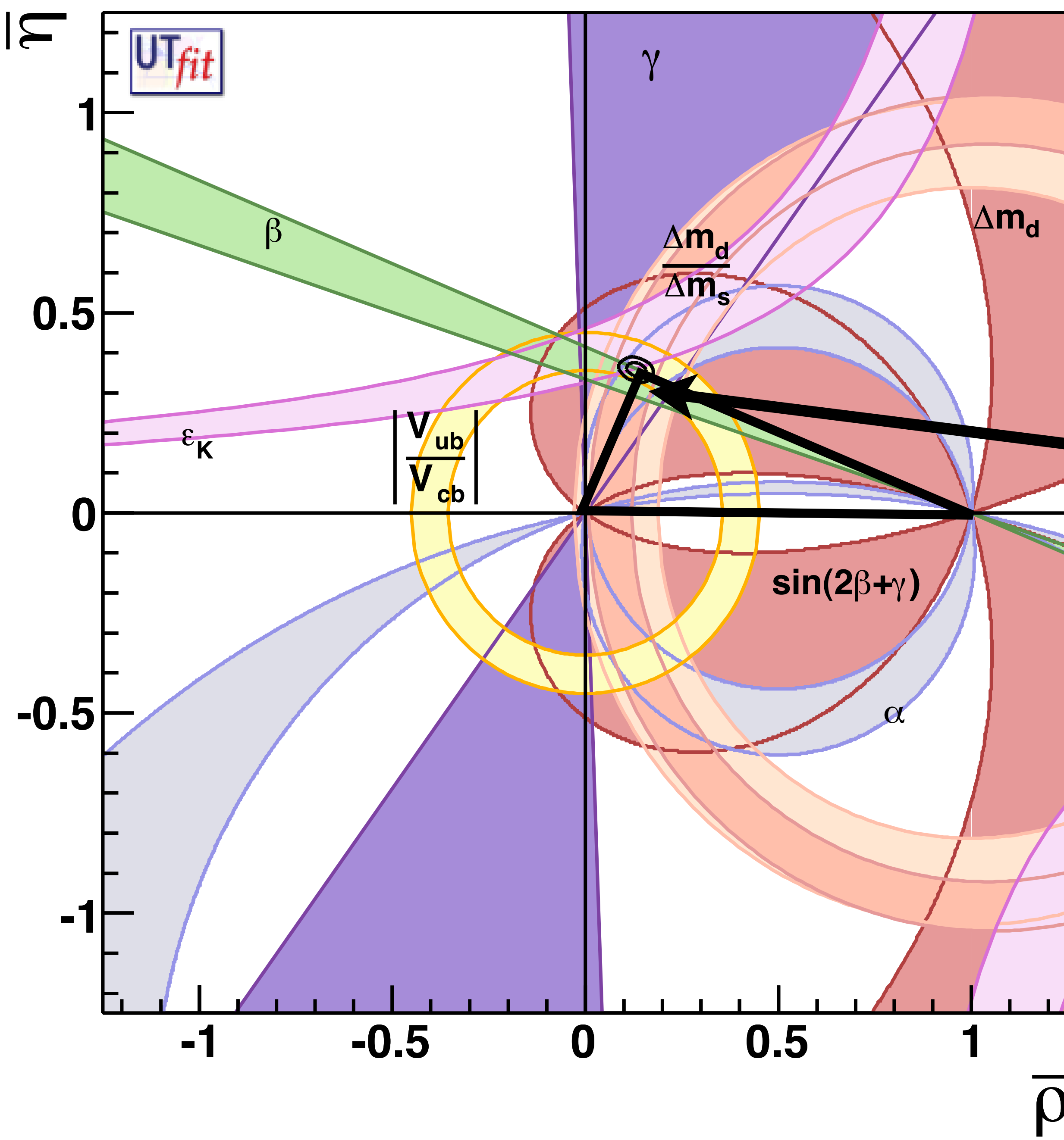

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Accident???

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Assumptions

[Antusch, King, Malinsky, MS 2010]

- Hierarchical Quark Mass Matrices
- Texture Zeros in the 1-3 Elements

The Sum Rules

[Antusch, King, Malinsky, MS 2010]

$$\theta_{12}^u = \frac{\theta_{13}}{\theta_{23}} = (4.96 \pm 0.30)^\circ$$

$$\theta_{12}^d = \left| \theta_{12} - \frac{\theta_{13}}{\theta_{23}} e^{-i\delta_{\mathrm{CKM}}} \right| = \left(12.0^{+0.39}_{-0.22}\right)^\circ$$

$$\alpha = \delta_{12}^d - \delta_{12}^u = \left(89.0^{+4.4}_{-4.2}\right)^\circ$$

[Numbers based on PDG]

Idea: Mass matrices with purely real/imaginary elements!

[Antusch, King, Malinsky, MS 2010]
see also [Fritzsch and Xing; Masina and Savoy 2006; Harrison, Dallison, Roythorne, Scott 2009]

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Ingredients

[Antusch, King, Luhn, MS 2011]

- Discrete Family Symmetry (A_4, S_4, ...)
- Discrete Shaping Symmetry (Z_n`s)
- Spontaneous CP violation
- SUSY GUT (SU(5), ...)

The Method I

[Antusch, King, Luhn, MS 2011]

- Use family symmetry to align flavon, e.g.:

$$\langle\phi\rangle \propto (0,0,x)^T \quad \text{or} \quad \langle\phi\rangle \propto (x,x,x)^T$$

- Add term to W compatible with shaping symmetry:

$$P\left(\frac{\phi^n}{\Lambda^{n-2}} \mp M^2\right)$$

- Solve F-term condtions ($|F_P|$=0)

$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n}q\ , \quad q = 1,\ldots,n & \text{for ``}-\text{''} \\ \frac{2\pi}{n}q + \frac{\pi}{n}\ , \quad q = 1,\ldots,n & \text{for ``}+\text{''} \end{cases}$$

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First Example: Ingredients

[Antusch, King, Luhn, MS 2011]

- Symmetries: SU(5) x A_4 x $G_{shaping}$
- 5 Flavons
- Matter sector: F, $T_{1,2,3}$, $N_{1,2}$
- Higgs Fields in 5-, 24-, 45-dim. SU(5) Reps.

New GUT relations!!!
[Antusch, MS ´09]

First Example: Alignment

[Antusch, King, Luhn, MS 2011]

We use the following flavon alignment:

$$\langle\phi_1\rangle \propto \begin{pmatrix}1\\0\\0\end{pmatrix}, \ \langle\phi_2\rangle \propto \begin{pmatrix}0\\-\mathrm{i}\\0\end{pmatrix}, \ \langle\phi_3\rangle \propto \begin{pmatrix}0\\0\\1\end{pmatrix},$$

$$\langle\phi_{23}\rangle \propto \begin{pmatrix}0\\1\\-1\end{pmatrix}, \ \langle\phi_{123}\rangle \propto \begin{pmatrix}1\\1\\1\end{pmatrix}$$

For details of the alignment, see paper...

The Matter Sector

[Antusch, King, Luhn, MS 2011]

We work in an SU(5) GUT:

$$F_i = \begin{pmatrix} d_R^c \\ d_B^c \\ d_G^c \\ e \\ -\nu \end{pmatrix}_i , \quad T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_G^c & u_B^c & -u_R & -d_R \\ u_G^c & 0 & -u_R^c & -u_B & -d_B \\ -u_B^c & u_R^c & 0 & -u_G & -d_G \\ u_R & u_B & u_G & 0 & -e^c \\ d_R & d_B & d_G & e^c & 0 \end{pmatrix}_i , \quad N_{1,2}$$

We know the family and the shaping symmetries...

The Quark Sector

[Antusch, King, Luhn, MS 2011]

For the Yukawa matrices in the quark sector we find:

$$Y_d = \begin{pmatrix} 0 & \mathrm{i}\,\epsilon_2 & 0 \\ \epsilon_{123} & \epsilon_{23} + \epsilon_{123} & -\epsilon_{23} + \epsilon_{123} \\ 0 & 0 & \epsilon_3 \end{pmatrix} \,,$$

$$Y_u = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & a_{23} \\ 0 & a_{23} & a_{33} \end{pmatrix} \,.$$

The sum rule is applicable and we have a right-angled CKM unitarity triangle!!!

The Lepton Sector

[Antusch, King, Luhn, MS 2011]

In the lepton sector we find:

$$M_R = \begin{pmatrix} M_{R_1} & 0 \\ 0 & M_{R_2} \end{pmatrix} \; , \; Y_\nu = \begin{pmatrix} 0 & a_{\nu_2} \\ a_{\nu_1} & a_{\nu_2} \\ -a_{\nu_1} & a_{\nu_2} \end{pmatrix} \; ,$$

$$Y_e^T = -\frac{3}{2} \begin{pmatrix} 0 & \mathrm{i}\, \epsilon_2 & 0 \\ \epsilon_{123} & -3\, \epsilon_{23} + \epsilon_{123} & 3\, \epsilon_{23} + \epsilon_{123} \\ 0 & 0 & \epsilon_3 \end{pmatrix} \; .$$

- New GUT relations
- No unitarity triangle
- Small deviations from tri-bimaximal mixing

Another Model

(same same, but different)

[Antusch, King, MS 2010]

- Symmetries: SU(5) x A_4 x $G_{shaping}$
- Matter sector as before: F, T, N
- Flavons: ϕ_{23}, ϕ_{123} and ϕ_3 as before
  New Flavon: $\langle \tilde{\phi}_{23} \rangle \propto \begin{pmatrix} 0 & -i & w \end{pmatrix}^T$
- H_{15} for type-II-seesaw

The SM Yukawas

[Antusch, King, MS 2010]

$$Y_u = \begin{pmatrix} 2a_{11}\epsilon_{23}^2 & 0 & a_{13}\epsilon_{23}\epsilon_3 \\ 0 & 3a_{22}\epsilon_{123}^2 + (w^2 - 1)\tilde{a}_{22}^2\tilde{\epsilon}_{23}^2 & a_{23}\epsilon_{123}\epsilon_3 \\ a_{13}\epsilon_{23}\epsilon_3 & a_{23}\epsilon_{123}\epsilon_3 & a_{33} \end{pmatrix}$$

$$Y_d = \begin{pmatrix} 0 & \epsilon_{23} & -\epsilon_{23} \\ \epsilon_{123} & \epsilon_{123} + i\,\tilde{\epsilon}_{23} & \epsilon_{123} + w\tilde{\epsilon}_{23} \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

$$Y_e^T = \begin{pmatrix} 0 & \epsilon_{23} & -\epsilon_{23} \\ \epsilon_{123} & \epsilon_{123} + \frac{9}{2}\,i\,\tilde{\epsilon}_{23} & \epsilon_{123} + \frac{9}{2}w\tilde{\epsilon}_{23} \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

Phase Sum Rule
<u>not</u> valid!!

What about the CKM CP phase?

[Antusch, King, MS 2010]

From y_e, y_μ, and θ_{12} we can determine it to:

$$\delta_{CKM} \approx 69.9^\circ$$

This is in perfect agreement with experiment:

$$\delta_{CKM} \approx (68.8^{+4.0}_{-2.3})^\circ$$

This is only due to the 9/2!!

The Neutrino Sector

[Antusch, King, MS 2010]

The effective neutrino mass matrix:

$$m_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2'}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3'}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

We predict (at the GUT scale):

$$\theta_{12}^{MNS} \approx 35.1^\circ, \quad \theta_{23}^{MNS} = 45^\circ, \quad \theta_{13}^{MNS} \approx 3.3^\circ,$$

$$\delta_{MNS} \approx 90^\circ, \quad \alpha_1 \approx 9.1^\circ, \quad \alpha_2 = 0^\circ$$

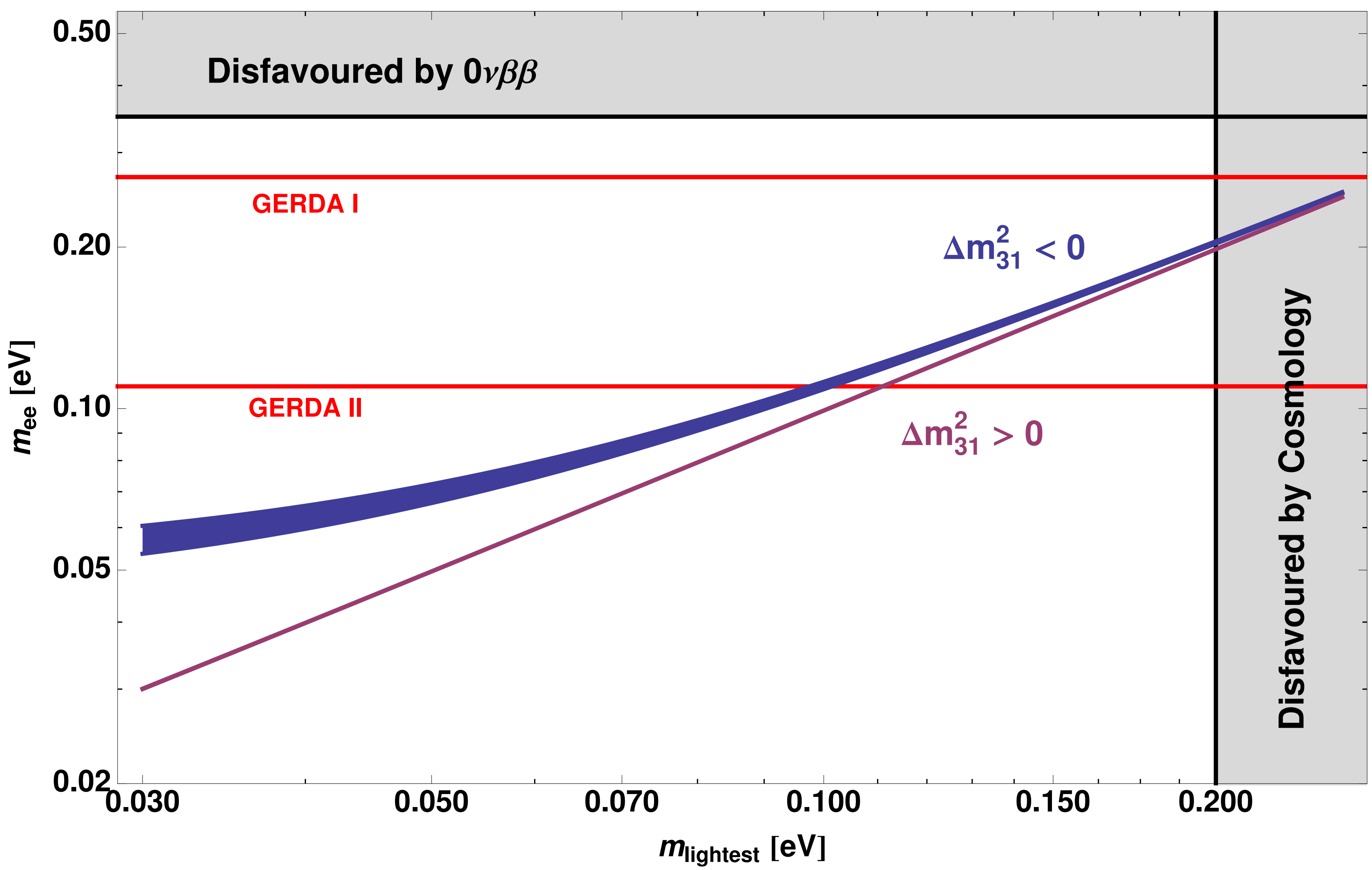
At low energies (RGE effects below 1°):

[Gonzalez-Garcia, Maltoni, Salvado 2010]

$$\theta_{12}^{MNS} = (34.5 \pm 1.0)^\circ, \quad \theta_{23}^{MNS} = (42.3^{+5.3}_{-2.8})^\circ, \quad \theta_{13}^{MNS} = (5.7^{+3.0}_{-3.9})^\circ,$$

$$\delta_{MNS} \approx ?, \quad \alpha_1 \approx ?, \quad \alpha_2 = ?$$

Neutrinoless Double Beta Decay



[Antusch, King, MS 2010]

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Summary & Conclusions

- Quark Phase Sum Rule
- Simple method to fulfill this Phase Sum Rule
  - Spontaneous CP violation
  - Family Symmetry: A_4, S_4, ...
  - Shaping Symmetries: Z_2's and/or Z_4's
- In combination with GUTs:
  - (Small) Deviations from Tri-Bimaximal Mixing
  - Predictions for Leptonic CP Phases

Backup

Symmetries (Flavon Sector)

	$SU(5)$	A_4	$\mathbb{Z}_4^{(1)}$	$\mathbb{Z}_4^{(2)}$	$\mathbb{Z}_4^{(3)}$	$\mathbb{Z}_4^{(4)}$	$\mathbb{Z}_2^{(1)}$	$\mathbb{Z}_2^{(2)}$	$U(1)_R$
Flavons									
ϕ_1	**1**	**3**	3	0	0	0	0	0	0
ϕ_2	**1**	**3**	3	3	0	0	0	0	0
ϕ_3	**1**	**3**	0	0	0	0	1	0	0
ϕ_{123}	**1**	**3**	0	0	3	0	0	0	0
ϕ_{23}	**1**	**3**	0	0	3	3	0	0	0
ξ	**1**	**1**	0	0	1	0	0	0	0
Driving Fields									
P_i	**1**	**1**	0	0	0	0	0	0	2
A_1	**1**	**3**	2	0	0	0	0	0	2
A_2	**1**	**3**	2	2	0	0	0	0	2
A_3	**1**	**3**	0	0	0	0	0	0	2
A_{123}	**1**	**3**	0	0	2	0	0	0	2
$O_{1;2}$	**1**	**1**	2	1	0	0	0	0	2
$O_{1;3}$	**1**	**1**	1	0	0	0	1	0	2
$O_{2;3}$	**1**	**1**	1	1	0	0	1	0	2
$O_{1;23}$	**1**	**1**	1	0	1	1	0	0	2
$O_{123;23}$	**1**	**1**	0	0	2	1	0	0	2

Proof of Principle!

[Antusch, King, Luhn, MS 2011]

Symmetries (Matter and Higgs Sector)

[Antusch, King, Luhn, MS 2011]

	$SU(5)$	A_4	$\mathbb{Z}_4^{(1)}$	$\mathbb{Z}_4^{(2)}$	$\mathbb{Z}_4^{(3)}$	$\mathbb{Z}_4^{(4)}$	$\mathbb{Z}_2^{(1)}$	$\mathbb{Z}_2^{(2)}$	$U(1)_R$
Matter Fields									
F	$\bar{\mathbf{5}}$	$\mathbf{3}$	0	0	0	0	0	1	1
T_1	$\mathbf{10}$	$\mathbf{1}$	1	0	0	0	0	0	1
T_2	$\mathbf{10}$	$\mathbf{1}$	0	0	1	0	0	0	1
T_3	$\mathbf{10}$	$\mathbf{1}$	0	0	0	0	1	0	1
N_1	$\mathbf{1}$	$\mathbf{1}$	0	0	3	3	0	0	1
N_2	$\mathbf{1}$	$\mathbf{1}$	0	0	3	0	0	0	1
Higgs Fields									
$\bar{H}_1$	$\bar{\mathbf{5}}$	$\mathbf{1}$	0	1	0	0	0	0	0
$\bar{H}_2$	$\overline{\mathbf{45}}$	$\mathbf{1}$	0	0	0	1	0	0	0
$\bar{H}_3$	$\bar{\mathbf{5}}$	$\mathbf{1}$	0	0	0	0	0	0	0
H	$\mathbf{5}$	$\mathbf{1}$	0	0	0	0	0	0	0
H_{24}	$\mathbf{24}$	$\mathbf{1}$	0	0	0	0	0	1	0

New GUT relations!!! [Antusch, MS ´09]