Relating Quarks and Leptons without Grand-Unification

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Reference:

- "Relating quarks and leptons without grand-unification," S. Morisi, E. Peinado, Y. S., J. W. F. Valle, [arXiv:1104.1633 [hep-ph]].

Introduction

Neutrinos: Windows to New Physics

Neutrino Oscillations provided important information

- Tiny Neutrino Masses
- Large Neutrino Flavor Mixing

Recent experiments of the neutrino oscillation go into a new phase of the precise determination of mixing angles and mass squared differences, in SK, KamLAND, SNO, MINOS, T2K, Double CHOOZ, and etc.

In order to explain large mixing, many authors consider

Non-Abelian Discrete Flavor Symmetry !

The experimental data indicate Tri-bimaximal mixing

- Global fit of experimental data of the neutrino oscillations:
  (T. Schwetz, M. Tortola, J. W. F. Valle, New J. Phys. **13** (2011) 063004.)

parameter	best fit $\pm 1\sigma$	2σ	3σ
$\Delta m^2_{\rm sol}$ $[10^{-5}{\rm eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24-7.99	7.09-8.19
$\Delta m^2_{\rm atm}$ $[10^{-3}{\rm eV}^2]$	2.45 ± 0.09	2.28-2.64	2.18-2.73
	$-(2.34^{+0.10}_{-0.09})$	$-(2.17\text{-}2.54)$	$-(2.08\text{-}2.64)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28-0.35	0.27-0.36
$\sin^2\theta_{23}$	0.51 ± 0.06	0.41-0.61	0.39-0.64
	0.52 ± 0.06	0.42-0.61	
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$	≤ 0.027	≤ 0.035
	$0.013^{+0.009}_{-0.007}$	≤ 0.031	≤ 0.039

- Lepton mixing angles:
  (P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530** (2002) 167.)

$$\sin^2\theta_{12} \simeq \frac{1}{3}, \quad \sin^2\theta_{23} \simeq \frac{1}{2}, \quad \sin^2\theta_{13} \simeq 0.$$

Tri-bimaximal mixing !

- Lepton flavor mixing: Tri-bimaximal mixing is written as

$$U_{\text{Tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle.$$

- T2K showed the new results: Normal (Inverted) hierarchy

$$0.03\ (0.04) < \sin^2 2\theta_{13} < 0.28\ (0.34),$$

@ 90% C.L. for $\sin^2 2\theta_{23} = 1$, $|\Delta m^2_{\text{atm}}| = 2.4 \times 10^{-3}$ eV^2, $\delta_{CP} = 0$.
(K. Abe *et al.* [T2K Collaboration], arXiv:1106.2822 [hep-ex].)

- Quark flavor mixing: CKM matrix (PDG 2010)

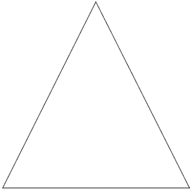
$$\left|V^{\text{exp}}_{\text{CKM}}\right| \simeq \begin{pmatrix} 0.974 & 0.225 & 0.00347 \\ 0.225 & 0.973 & 0.0410 \\ 0.00862 & 0.0403 & 0.999 \end{pmatrix}.$$

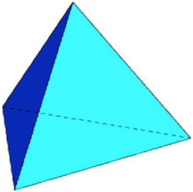
Non-Abelian discrete symmetry

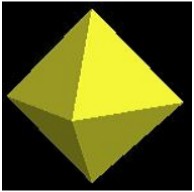
H. Ishimori, T. Kobayashi, H. Ohki, Y. S., H. Okada, M. Tanimoto, Prog. Theor. Phys. Suppl. **183** (2010) 1-163.

Non-Abelian discrete symmetry can lead to tri-bimaximal mixing, since non-Abelian discrete symmetry connects different generations.

- Many authors consider the flavor model: S_3, A_4, S_4, $\cdots$







Our model

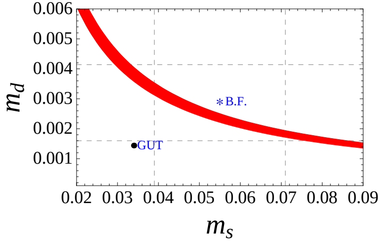
- A_4 flavor model without grand-unification

Since A_4 has triplet, it is natural to explain 3 generations.

Our predictions

- Mass relation:

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}} \, .$$



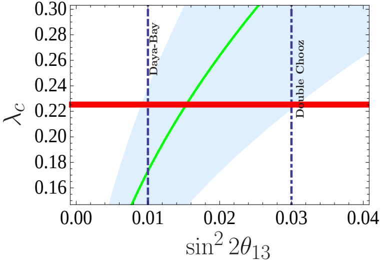
Georgi-Jarlskog mass relation:

$$m_b = m_\tau \, , \; m_s = \frac{1}{3} m_\mu \, , \; m_d = 3 m_e \, .$$

(H. Georgi, C. Jarlskog, Phys. Lett. **B86** (1979) 297-300.)

- Mixing relation:

$$\lambda_C \approx \frac{1}{\sqrt{2}} \frac{m_\mu m_b}{m_\tau m_s} \sqrt{\sin^2 2\theta_{13}} - \sqrt{\frac{m_u}{m_c}} \, .$$



Input data is EW scale (M_Z)

(Z. -z. Xing, H. Zhang, S. Zhou, Phys. Rev. **D77** (2008) 113016.)

A_4 flavor model in $SU(3) \times SU(2) \times U(1)$

- The A_4 charge assignment in our model (SUSY)

fields	$\hat{L}$	$\hat{E}^c$	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}^u$	$\hat{H}^d$
$SU(2)_L$	2	1	2	1	1	2	2
A_4	**3**	**3**	**3**	**3**	**3**	**3**	**3**

- Renormalizable Yukawa Lagrangian for the charged fermions:

$$\mathcal{L}_Y = y^{\ell}_{ijk} \hat{L}_i \hat{H}^d_j \hat{E}^c_k + y^d_{ijk} \hat{Q}_i \hat{H}^d_j \hat{D}^c_k + y^u_{ijk} \hat{Q}_i \hat{H}^u_j \hat{U}^c_k \ .$$

- The Higgs scalar potential invariant under A_4:

$$\begin{aligned} V &= (|\mu|^2 + m^2_{H_u})(|H^u_1|^2 + |H^u_2|^2 + |H^u_3|^2) \\ &+ (|\mu|^2 + m^2_{H_d})(|H^d_1|^2 + |H^d_2|^2 + |H^d_3|^2) \\ &- [b(H^u_1 H^d_1 + H^u_2 H^d_2 + H^u_3 H^d_3) + \text{c.c.}] \\ &+ \frac{1}{8}(g^2 + g'^2)(|H^u_1|^2 + |H^u_2|^2 + |H^u_3|^2 - |H^d_1|^2 - |H^d_2|^2 - |H^d_3|^2) \ . \end{aligned}$$

- Vacuum alignment: possible local minima

$$\langle {H^0}^{u,d} \rangle \sim (1,0,0) \quad \text{and} \quad (1,1,1).$$

- Adding A_4 soft breaking terms:

$$V_{\text{soft}} = \sum_{ij} (\mu^u_{ij} H^{u*}_i H^u_j + \mu^d_{ij} H^{d*}_i H^d_j) + \sum_{ij} b_{ij} H^d_i H^u_j \ .$$

- We get the alignment:

$$\langle H^u \rangle = (v^u, \varepsilon^u_1, \varepsilon^u_2), \quad \langle H^d \rangle = (v^d, \varepsilon^d_1, \varepsilon^d_2) \ ,$$

where $\varepsilon^u_{1,2} \ll v^u$ and $\varepsilon^d_{1,2} \ll v^d$.

Charged fermions

- Charged fermion mass matrix:

$$M_f = \begin{pmatrix} 0 & y_1^f \langle H_3^f \rangle & y_2^f \langle H_2^f \rangle \\ y_2^f \langle H_3^f \rangle & 0 & y_1^f \langle H_1^f \rangle \\ y_1^f \langle H_2^f \rangle & y_2^f \langle H_1^f \rangle & 0 \end{pmatrix},$$

where f denotes any charged lepton, up- or down-type quarks.

- This matrix is rewritten as follow:
$\langle H^u \rangle = (v^u, \varepsilon_1^u, \varepsilon_2^u)$, $\langle H^d \rangle = (v^d, \varepsilon_1^d, \varepsilon_2^d)$

$$M_f = \begin{pmatrix} 0 & a^f \alpha^f & b^f \\ b^f \alpha^f & 0 & a^f r^f \\ a^f & b^f r^f & 0 \end{pmatrix},$$

where $a^f = y_1^f \varepsilon_1^f$, $\quad b^f = y_2^f \varepsilon_1^f$, $\quad r^f = v^f / \varepsilon_1^f$, $\quad \alpha^f = \varepsilon_2^f / \varepsilon_1^f$.

Thanks to the fact that the same Higgs doublet H^d couples to the charged leptons and to the down-type quarks, we obtain

$$r^\ell = r^d \ , \quad \alpha^\ell = \alpha^d \ .$$

We also obtain

$$\frac{r^f}{\sqrt{\alpha^f}} \approx \frac{m_3^f}{\sqrt{m_1^f m_2^f}} \ , \quad a^f \approx \frac{m_2^f}{m_3^f} \frac{\sqrt{m_1^f m_2^f}}{\sqrt{\alpha^f}} \ , \quad b^f \approx \frac{\sqrt{m_1^f m_2^f}}{\sqrt{\alpha^f}} \ .$$

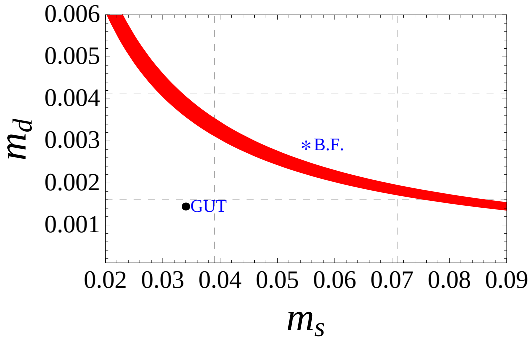
First prediction

- Mass relation

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}} \ .$$

- Down quark mass:

$$m_d \approx m_e \frac{m_\mu}{m_s} \left(\frac{m_b}{m_\tau} \right)^2 .$$



- In the Z boson mass scale (M_Z):

$$1.71 \text{ MeV} < m_d < 4.14 \text{ MeV}, \quad 40 \text{ MeV} < m_s < 71 \text{ MeV}.$$

(Z. -z. Xing, H. Zhang, S. Zhou, Phys. Rev. **D77** (2008) 113016.)

Neutrinos

- The effective dimension-five lepton-number violating operator:

$$\mathcal{L}_{5d} = \frac{f_{ijklm}}{\Lambda} \hat{L}_i \hat{L}_j \hat{H}^u_l \hat{H}^u_m \ .$$

- Neutrino mass matrix: $\langle H^u_1 \rangle \gg \langle H^u_2 \rangle,\ \langle H^u_3 \rangle$

$$M_\nu = \begin{pmatrix} x r^{u2} & \kappa r^u & \kappa r^u \alpha^u \\ \kappa r^u & y r^{u2} & 0 \\ \kappa r^u \alpha^u & 0 & z r^{u2} \end{pmatrix},$$

  where x, y, z, and κ are coupling constants.

- If $y \approx z$ and $\alpha^u \approx 1$, the maximally atmospheric angle is realized in neutrino oscillation data.
  (S. Morisi, E. Peinado, Phys. Rev. **D80** (2009) 113011.)

- We obtain the mixing angle θ_{12}^f from charged fermion matrix:

$$\sin\theta_{12}^f \approx \sqrt{\frac{m_1^f}{m_2^f}\frac{1}{\sqrt{\alpha^f}}} .$$

- If neutrino mass matrix is μ-τ invariant, the reactor angle is:

$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\sin\theta_{12}^\ell = \frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}\frac{1}{\sqrt{\alpha^\ell}}} ,$$

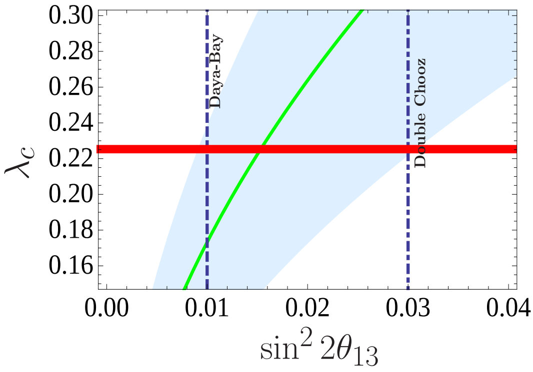
using our mass relation, the Cabibbo angle is written as

$$\lambda_C = \frac{m_b}{m_s}\frac{\sqrt{m_e m_\mu}}{m_\tau}\frac{1}{\sqrt{\alpha^d}} - \sqrt{\frac{m_u}{m_c}} .$$

Second prediction

- Mixing relation

$$\lambda_C \approx \frac{1}{\sqrt{2}} \frac{m_\mu m_b}{m_\tau m_s} \sqrt{\sin^2 2\theta_{13}} - \sqrt{\frac{m_u}{m_c}} \; .$$



Comment on the other two CKM mixing angles

- Mixing parameters of the third family of quarks: $(q = u, d)$

$$U_{13}^q \approx \frac{m_2^q}{m_3^q}\frac{\sqrt{m_1^q m_2^q}}{m_3^q}\frac{1}{\sqrt{\alpha^q}}, \quad U_{23}^q \approx \frac{m_1^q (m_2^q)^2}{(m_3^q)^3}\frac{1}{\alpha^q}$$

are too small.

One of the solution

- We add color triplets vector-like $SU(2)_L$ singlet states. In this case, we obtain V_{ub} and V_{cb} as well as quark $\not{CP}$.

Summary

Conclusion

- We present the A_4 flavor model without grand-unification.
- We predict the mass relation:

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} \approx \frac{m_b}{\sqrt{m_d m_s}} \ .$$

- We also predict the mixing relation:

$$\lambda_C \approx \frac{1}{\sqrt{2}} \frac{m_\mu m_b}{m_\tau m_s} \sqrt{\sin^2 2\theta_{13}} - \sqrt{\frac{m_u}{m_c}} \ .$$

Future work

- We need to generate the other two CKM mixing angles.