Predicting Deviations and Alternatives to tri-bimaximal Mixing

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FLASY11, 11/07/11

$m_\nu = m_L - m_D^T M_R^{-1} m_D$

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

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Outline

- Current Status of PMNS
- Tri-bimaximal Mixing (TBM):
  - abundance of models
  - importance of neutrino mass observables
  - deviating from TBM: how to get $\theta_{13} \simeq 0.1$
- Alternatives to TBM
- sterile neutrinos?

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) Matrix

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$

$$\stackrel{?}{=} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \quad \stackrel{?}{=} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \stackrel{?}{=} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$(\sin^2\theta_{23} \stackrel{?}{=} \frac{1}{2})$ $(\sin^2\theta_{13} \stackrel{?}{=} 0)$ $(\sin^2\theta_{12} \stackrel{?}{=} \frac{1}{3})$

$$\stackrel{?}{=} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

talk by Mariam

Mixing close to TBM $\Rightarrow$ expand around it

$$U = R_{23}\left(-\frac{\pi}{4}\right)\, R_{23}(\epsilon_{23})\, \tilde{R}_{13}(\epsilon_{13};\delta)\, R_{12}(\epsilon_{12})\, R_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right)$$

Only one small ϵ_{ij} responsible for deviation of (and only of) θ_{ij} from $\theta_{ij}^{\rm TBM}$

$$\begin{aligned}
\sin^2\theta_{12} &= \frac{1}{3}\left(\cos\epsilon_{12} + \sqrt{2}\,\sin\epsilon_{12}\right)^2 \\
&\simeq \frac{1}{3} + \frac{2\sqrt{2}}{3}\,\epsilon_{12} + \frac{1}{3}\,\epsilon_{12}^2 \\
\sin^2\theta_{23} &= \frac{1}{2} + \sin\epsilon_{23}\,\cos\epsilon_{23} \simeq \frac{1}{2} + \epsilon_{23} \\
U_{e3} &= \sin\epsilon_{13}\, e^{-i\delta}
\end{aligned}$$

Mass Matrix

Special case of μ–τ symmetry

$$(m_\nu)_{\rm TBM} = U_{\rm TBM}^* \, m_\nu^{\rm diag} \, U_{\rm TBM}^\dagger = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

$$A = \frac{1}{3}\left(2\,m_1 + m_2\,e^{-2i\alpha}\right)\,, \quad B = \frac{1}{3}\left(m_2\,e^{-2i\alpha} - m_1\right)\,, \quad D = m_3\,e^{-2i\beta}$$

- $m_{e\mu} = m_{e\tau}$ and $m_{\mu\mu} = m_{\tau\tau}$
- $m_{ee} + m_{e\mu} + m_{e\tau} = m_{\mu e} + m_{\mu\mu} + m_{\mu\tau} = m_{\tau e} + m_{\tau\mu} + m_{\tau\tau}$
- masses independent on mixing (i.e., not $V_{us} = \sqrt{m_d/m_s}$)

Correlations between mass matrix elements $\leftrightarrow$ flavor symmetries

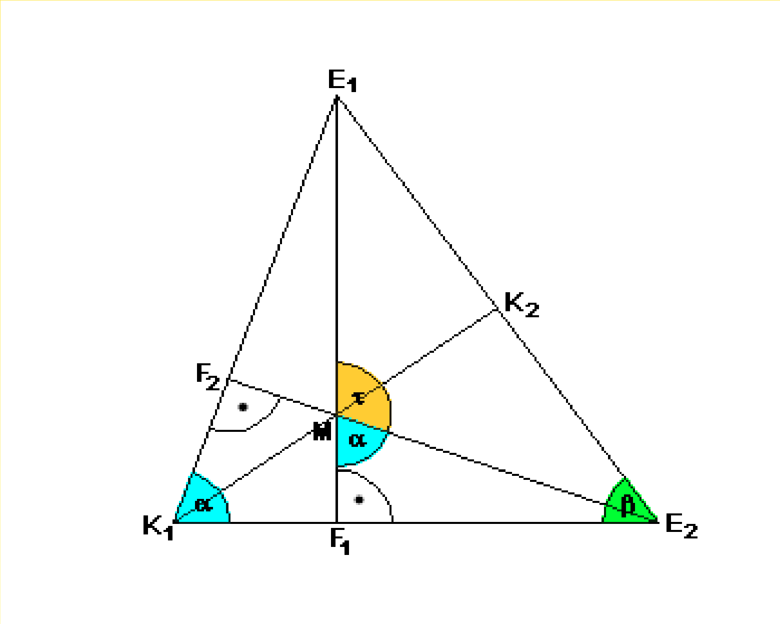
How to choose the group

Group	d	Irr. Repr.'s	Presentation
$D_3 \sim S_3$	6	1, $1'$, 2	$A^3 = B^2 = (AB)^2 = 1$
D_4	8	$1_1, \ldots 1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$
D_5	10	1, $1'$, 2, 2'	$A^5 = B^2 = (AB)^2 = 1$
D_6	12	$1_1, \ldots 1_4$, 2, 2'	$A^6 = B^2 = (AB)^2 = 1$
D_7	14	1, $1'$, 2, $2'$, $2''$	$A^7 = B^2 = (AB)^2 = 1$
A_4	12	1, $1'$, $1''$, 3	$A^3 = B^2 = (AB)^3 = 1$
$A_5 \sim PSL_2(5)$	60	1, 3, $3'$, 4, 5	$A^3 = B^2 = (BA)^5 = 1$
T'	24	1, $1'$, $1''$, 2, $2'$, $2''$, 3	$A^3 = (AB)^3 = R^2 = 1, \; B^2 = R$
S_4	24	1, $1'$, 2, 3, $3'$	$BM: A^4 = B^2 = (AB)^3 = 1$ $TB: A^3 = B^4 = (BA^2)^2 = 1$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, ... 1_9, 3, \overline{3}$	
$PSL_2(7)$	168	$1, 3, \overline{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \overline{1'}, 3, \overline{3}$	$A^7 = B^3 = 1, \; AB = BA^4$

Altarelli, Feruglio, 1002.0211

How to choose the group: A_4

- minimality
  - smallest group with 3 irrep
  - has 3 one-dimensional irreps 1, $1'$, $1''$
- geometry



angle between two faces: $\alpha = 2\,\theta_{\rm TBM}$, where $\sin^2\theta_{\rm TBM} = \frac{1}{3}$

The Zoo

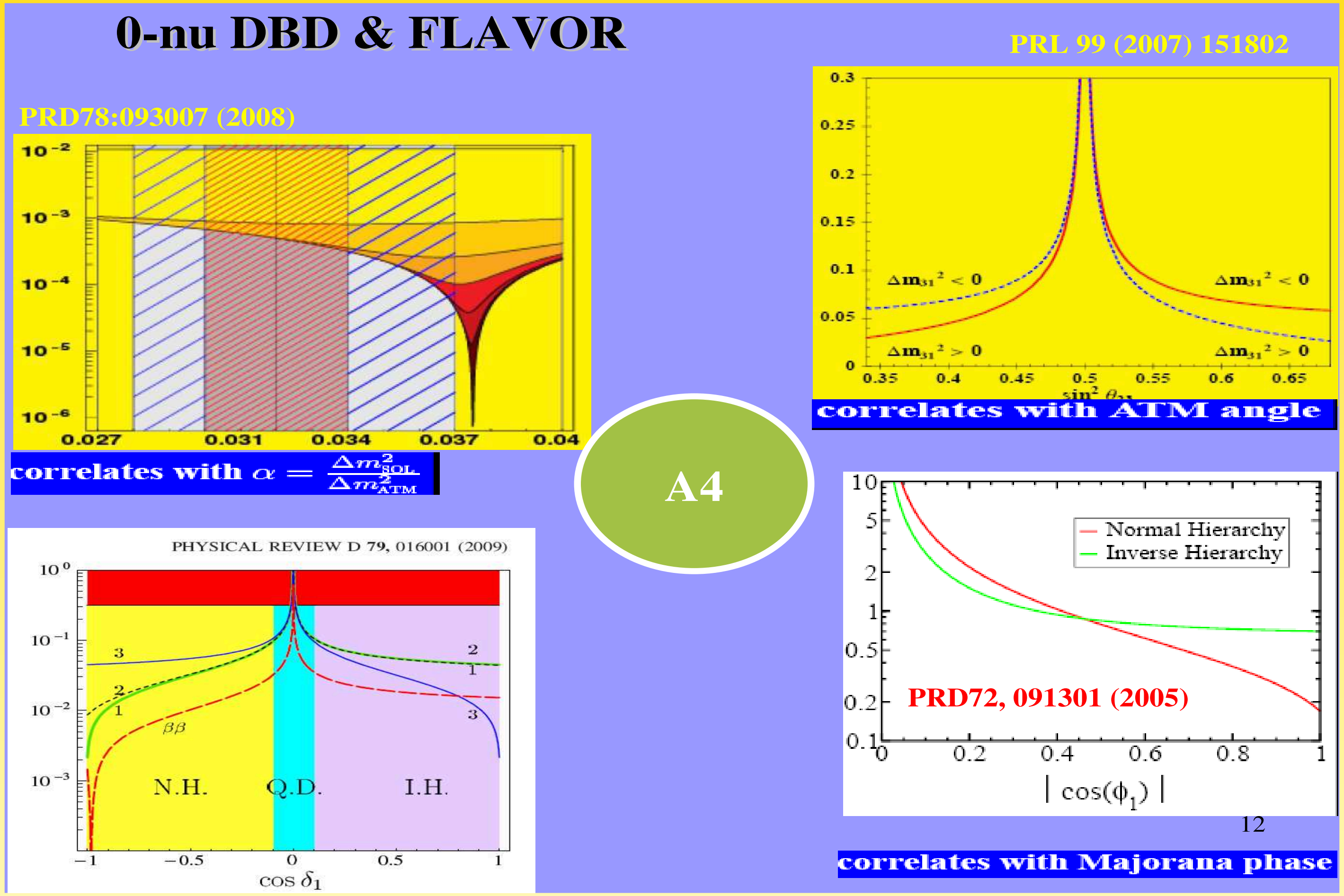
Type	L_i	ℓ_i^c	ν_i^c	Δ	References
A1				-	[1–14] [15]#
A2	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[16–18]
A3				$\underline{1}, \underline{3}$	[19]
B1	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	-	[4, 20–27]# [28–30]* [31–45]
B2				$\underline{1}, \underline{3}$	[46]#
C1				-	[2, 47, 48]
C2	$\underline{3}$	$\underline{3}$	-	$\underline{1}$	[49, 50] [51]#
C3				$\underline{1}, \underline{3}$	[52]
C4				$\underline{1}, \underline{1}', \underline{1}'', \underline{3}$	[53]
D1				-	[54, 55]# [56, 57]* [58]
D2	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	[59] [60]*
D3				$\underline{1}'$	[61]*
D4				$\underline{1}', \underline{3}$	[62]*
E	$\underline{3}$	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	-	[63, 64]
F	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{3}$	$\underline{3}$	$\underline{1}$ or $\underline{1}'$	[65]
G	$\underline{3}$	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}', \underline{1}''$	-	[66]
H	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	-	-	[67]
I	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}, \underline{1}$	-	[68]*
J	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{3}$	-	[12, 39, 69, 70]
K	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}, \underline{1}$	$\underline{1}$	[71]*
L	$\underline{3}$	$\underline{1}, \underline{1}, \underline{1}$	$\underline{1}$	-	[72]*
M	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}'', \underline{1}'$	$\underline{3}, \underline{1}$	-	[73, 74]
N	$\underline{1}, \underline{1}', \underline{1}''$	$\underline{1}, \underline{1}'', \underline{1}'$	$\underline{3}, \underline{1}', \underline{1}''$	-	[75]

Barry, W.R., PRD **81**, 093002 (2010), updated regularly on
http://www.mpi-hd.mpg.de/personalhomes/jamesb/Table_A4.pdf

How to distinguish?

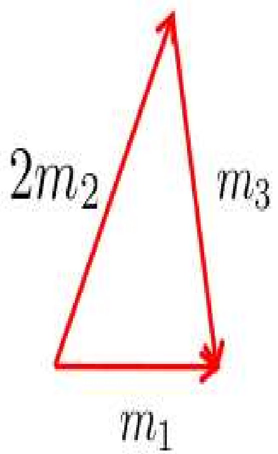
- LFV
- low scale scalars: Higgs, LFV
- compatible with GUTs?
- leptogenesis possible?
- **neutrino mass observables**

correlation with observables



Valle; talk by Merlo

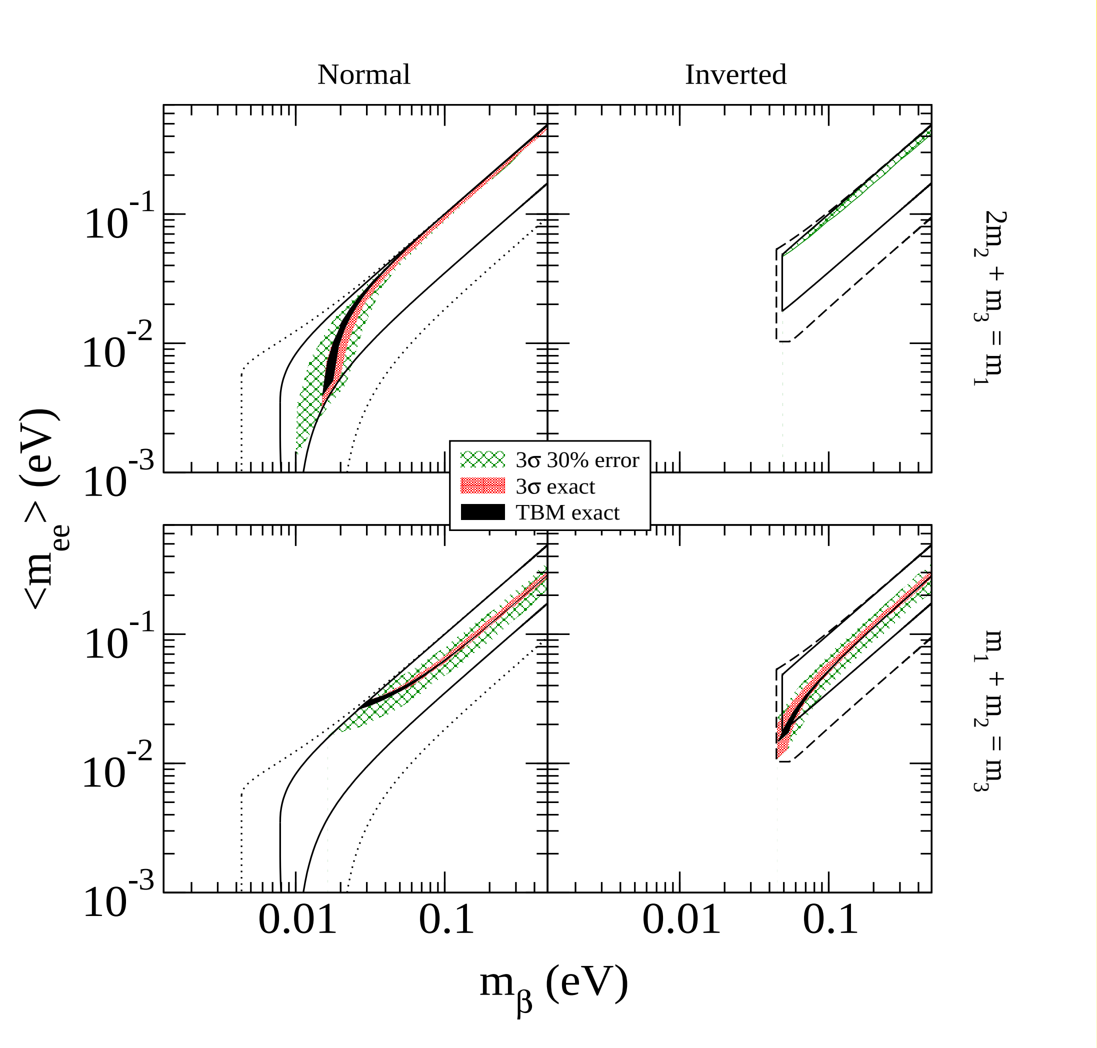
Sum-rules in Models and $0\nu\beta\beta$



Sum-rule	Flavour symmetry
$2m_2 + m_3 = m_1$	$A_4, T', (S_4)$
$m_1 + m_2 = m_3$	$S_4, (A_4)$
$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$	A_4, T'
$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$	S_4

constrains masses and Majorana phases

Barry, W.R., NPB **842**, 33 (2011)



$$m_1 + m_2 - m_3 = \epsilon \, m_{\rm max}$$

stable: new solutions not before $\epsilon \simeq 0.2$

Usual caveat: interpretation of $0\nu\beta\beta$

mechanism	amplitude and particle physics parameter	current limit	test
light neutrino exchange	$\frac{G_F^2}{q^2} \left\| U_{ei}^2\, m_i \right\|$	0.5 eV	oscillations, cosmology, neutrino mass
heavy neutrino exchange	$G_F^2 \left\| \frac{S_{ei}^2}{M_i} \right\|$	2×10^{-8} GeV^{-1}	LFV, collider
heavy neutrino and RHC	$G_F^2\, m_W^4 \left\| \frac{V_{ei}^2}{M_i\, M_{W_R}^4} \right\|$	4×10^{-16} GeV^{-5}	flavor, collider
Higgs triplet and RHC	$G_F^2\, m_W^4 \frac{(M_R)_{ee}}{m_{\Delta_R}^2\, M_{W_R}^4}$	10^{-15} GeV^{-1}	flavor, collider, e^- distribution
λ-mechanism with RHC	$G_F^2 \frac{m_W^2}{q} \left\| \frac{U_{ei}\, \tilde{S}_{ei}}{M_{W_R}^2} \right\|$	1.4×10^{-10} GeV^{-2}	flavor, collider, e^- distribution
η-mechanism with RHC	$G_F^2 \frac{1}{q} \tan\zeta \left\| U_{ei}\, \tilde{S}_{ei} \right\|$	6×10^{-9}	flavor, collider, e^- distribution
short-range $\not{R}$	$\frac{\lambda_{111}'^2}{\Lambda_{\rm SUSY}^5}$ $\Lambda_{\rm SUSY} = f(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\chi_i})$	7×10^{-18} GeV^{-5}	collider, flavor
long-range $\not{R}$	$\frac{G_F}{q} \left\| \sin 2\theta^b\, \lambda_{131}'\, \lambda_{113}' \left(\frac{1}{m_{\tilde{b}_1}^2} - \frac{1}{m_{\tilde{b}_2}^2} \right) \right\|$ $\sim \frac{G_F}{q} m_b \frac{\lambda_{131}'\, \lambda_{113}'}{\Lambda_{\rm SUSY}^3}$	2×10^{-13} GeV^{-2} 1×10^{-14} GeV^{-3}	flavor, collider
Majorons	$\propto \left\| \langle g_\chi \rangle \right\|$ or $\left\| \langle g_\chi \rangle \right\|^2$	$10^{-4} \ldots 1$	spectrum, cosmology

review on $0\nu\beta\beta$ and particle physics: `W.R., 1006.1334`

	Bari*	GM-I	GM-II	STV	TBM
$\sin\theta_{13}$	$0.145^{+0.022}_{-0.031}$	$0.097^{+0.053}_{-0.047}$	$0.089^{+0.051}_{-0.057}$	$0.130^{+0.025}_{-0.041}$	0
$\sin^2\theta_{23}$	$0.42^{+0.08}_{-0.03}$	$0.462^{+0.082}_{-0.050}$	$0.462^{+0.082}_{-0.050}$	$0.51^{+0.06}_{-0.06}$	0.5
$\sin^2\theta_{12}$	$0.306^{+0.018}_{-0.015}$	$0.319^{+0.016}_{-0.016}$	$0.321^{+0.016}_{-0.016}$	$0.316^{+0.016}_{-0.016}$	0.333

all groups find deviations from one or more TBM values

$$\delta\theta_{13} > \delta\theta_{12} \gtrsim \delta\theta_{23}$$

Taking Bari results as example:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.762 & -0.684 \\ 0 & -0.684 & 0.762 \end{pmatrix} \begin{pmatrix} 0.989 & 0 & 0.145\, e^{-i\delta} \\ 0 & 1 & 0 \\ -0.145\, e^{i\delta} & 0 & 0.989 \end{pmatrix} \begin{pmatrix} 0.833 & 0.553 & 0 \\ -0.553 & 0.883 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

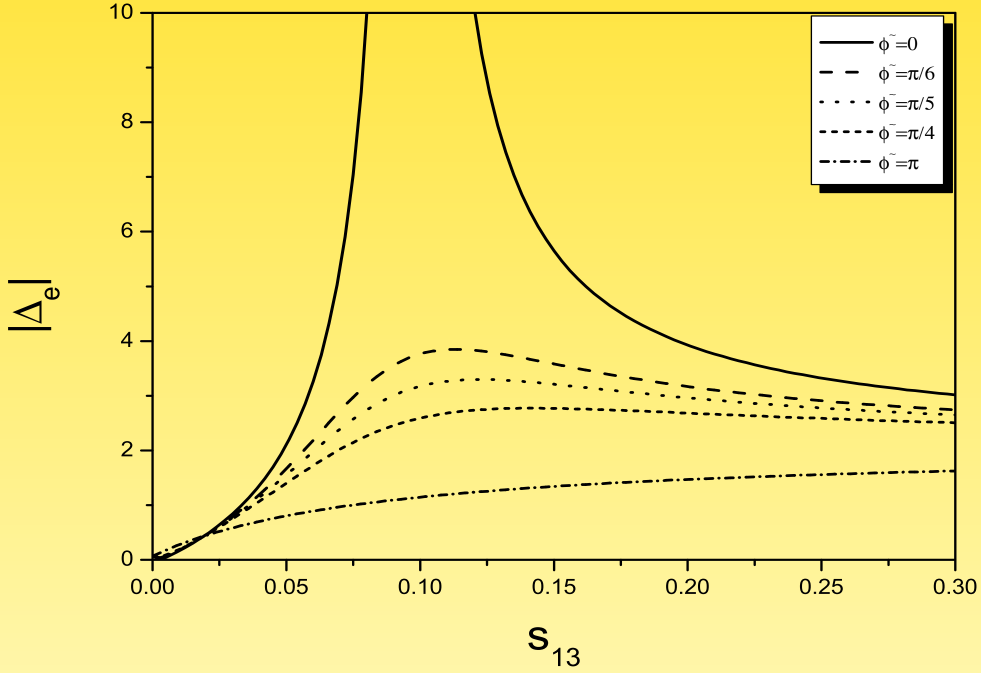
$$= \begin{pmatrix} 0.824 & 0.547 & 0.145\, e^{-i\delta} \\ -0.421 - 0.078\, e^{i\delta} & 0.634 - 0.052\, e^{i\delta} & 0.641 \\ 0.359 - 0.092\, e^{i\delta} & -0.540 - 0.061\, e^{i\delta} & 0.754 \end{pmatrix}$$

`Abbas, Smirnov, PRD` **`82`**`, 013008 (2010)`:

deviations from $m_\nu^{\rm TBM}$ possible: "TBM accidental?"

Define

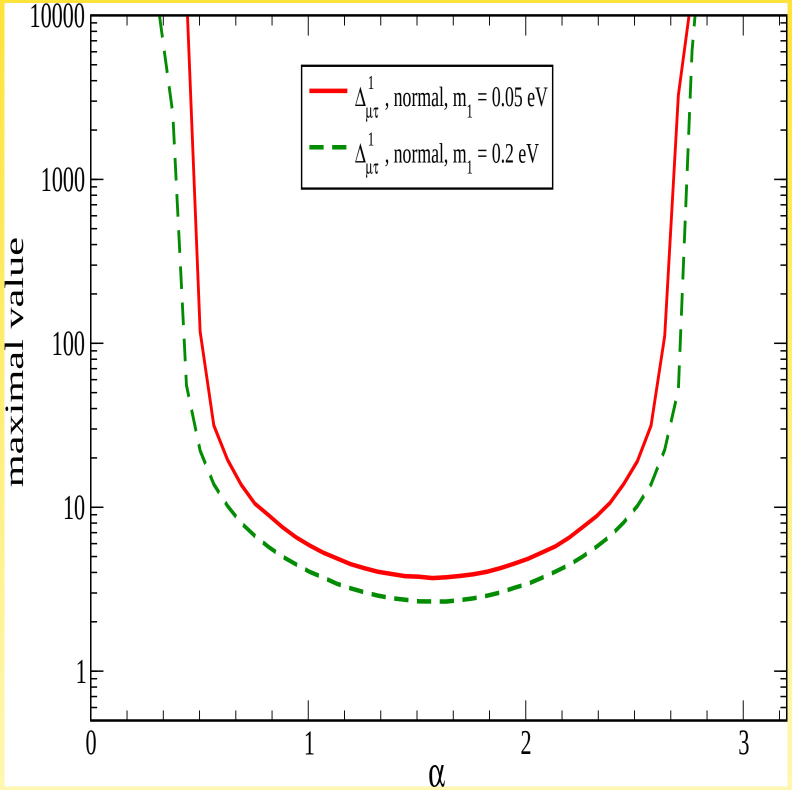
$$\Delta_e = \frac{(m_\nu)_{e\mu} - (m_\nu)_{e\tau}}{(m_\nu)_{e\mu}} \ , \quad \Delta_\mu = \frac{(m_\nu)_{\mu\mu} - (m_\nu)_{\tau\tau}}{(m_\nu)_{\tau\tau}}, \quad \Delta_\Sigma$$

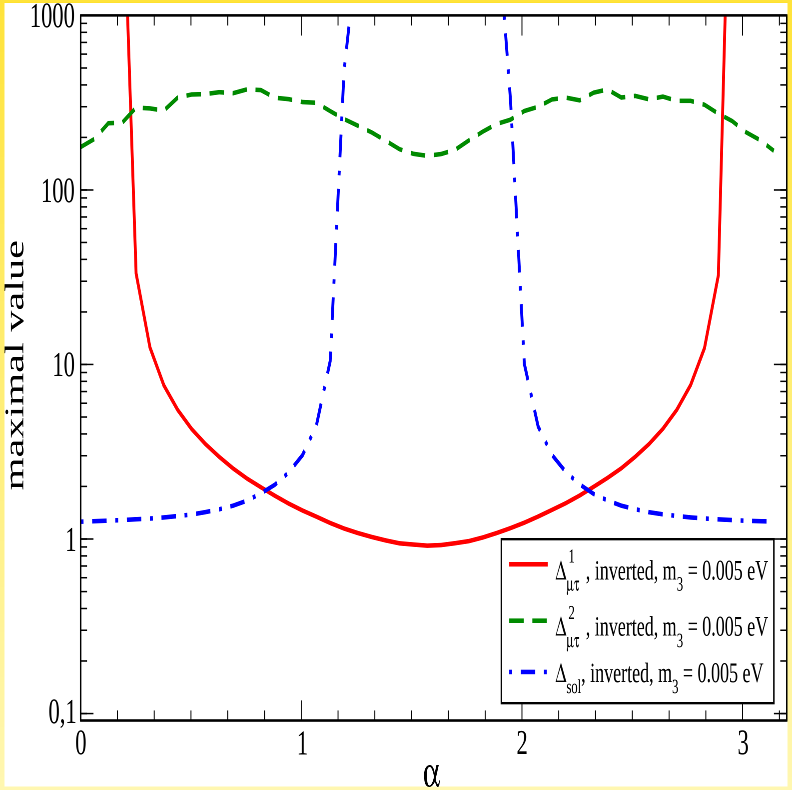


Abbas, Smirnov, PRD **82**, 013008 (2010)

Define

$$\Delta^1_{\mu\tau} = \left|\frac{(m_\nu)_{e\mu} - (m_\nu)_{e\tau}}{(m_\nu)_{e\mu} + (m_\nu)_{e\tau}}\right| \;,\quad \Delta^2_{\mu\tau} = \left|\frac{(m_\nu)_{\mu\mu} - (m_\nu)_{\tau\tau}}{(m_\nu)_{\mu\mu} + (m_\nu)_{\tau\tau}}\right|, \quad \Delta_{\rm sol}$$





Morisi, W.R., in preparation

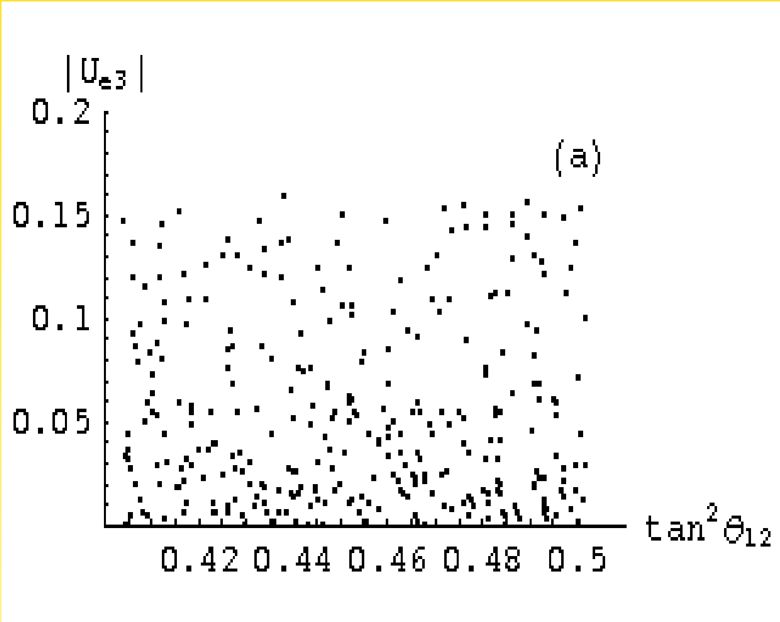
How to perturb a mixing scenario/model

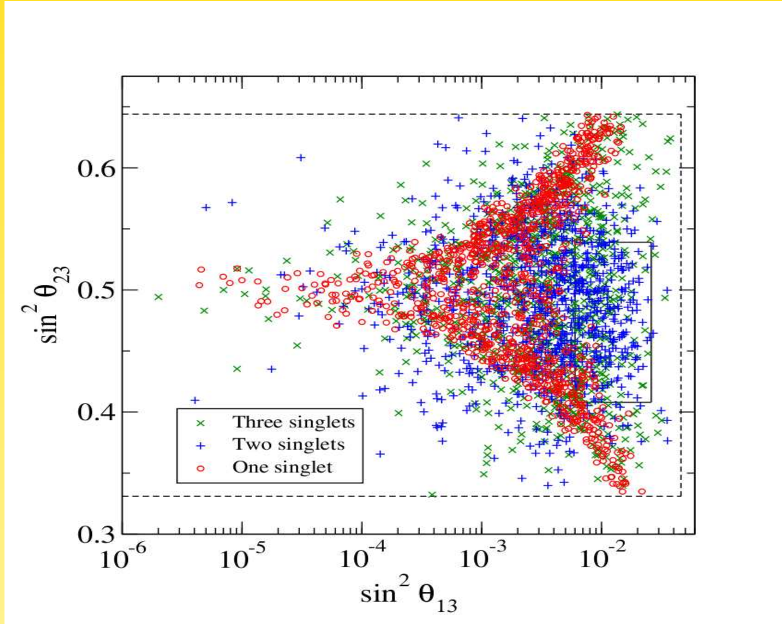
- VEV misalignment, NLO terms
- explicit naive breaking
- renormalization
- charged leptons

VEV misalignment, NLO terms

- **"naive misalignment"**:

  if $\langle \text{flavon} \rangle = (1, 1, 1)^T$, perturb it to $\langle \text{flavon} \rangle = (1, 1 + \epsilon_1, 1 + \epsilon_2)^T$





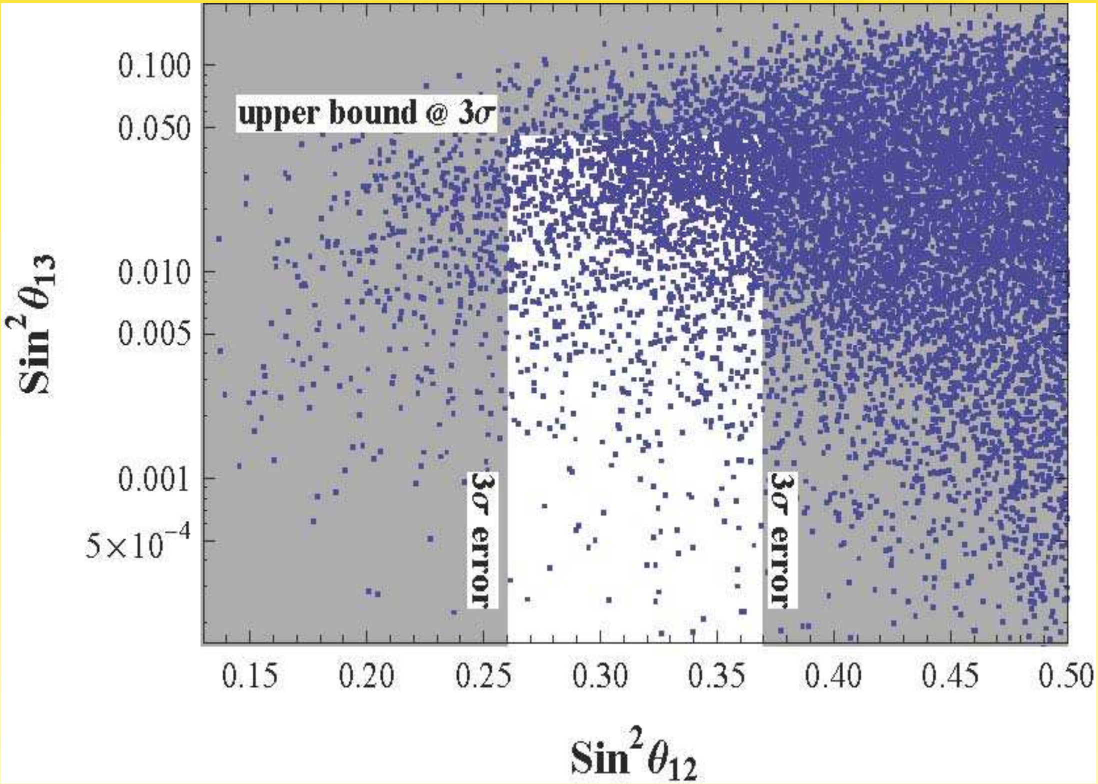
Honda, Tanimoto

Barry, W.R.

- of order $\langle \text{flavon} \rangle / \Lambda$ or $\langle \text{flavon} \rangle / M_R$, typically $\mathcal{O}(0.1)$ or $\mathcal{O}(\lambda_C)$ or $\mathcal{O}(0.01)$
- typically of the same order for θ_{23} and $|U_{e3}|$
- solar neutrino mixing angle receives larger corrections

VEV misalignment, NLO terms

- NLO terms, VEV misalignment due to terms allowed by the symmetry
  $\Rightarrow$ model-dependent!
  - Altarelli, Feruglio, Merlo, JHEP 0905:



$\delta \sin^2 \theta_{12} \simeq \delta |U_{e3}| = \mathcal{O}(\lambda)$ and $\delta \sin^2 \theta_{23} = \mathcal{O}(\lambda^2)$

- Altarelli, Feruglio, Hagedorn, JHEP 0803:

  corrections $\mathcal{O}(\lambda^2)$ to all mixing angles

- Lin, NPB 824:

  $\delta|U_{e3}| = \mathcal{O}(\lambda)$ and $\delta \sin^2\theta_{12} \simeq \delta \sin^2\theta_{23} = \mathcal{O}(\lambda^2)$

- Hagedorn, Ziegler, 1007.1888:

  $\delta|U_{e3}|^2 = \mathcal{O}(\lambda^2)$ and $\delta \sin^2\theta_{12} = \mathcal{O}(\lambda)$

- Ishimori *et al.*, 1004.5004:

  $\delta|U_{e3}|^2 = \mathcal{O}(\lambda^2)$ and $\delta \sin^2\theta_{12} = \mathcal{O}(\lambda)$ and $\delta \sin^2\theta_{23} = \mathcal{O}(\lambda^2)$

- etc.:

  etc.

How to perturb a mixing scenario/model

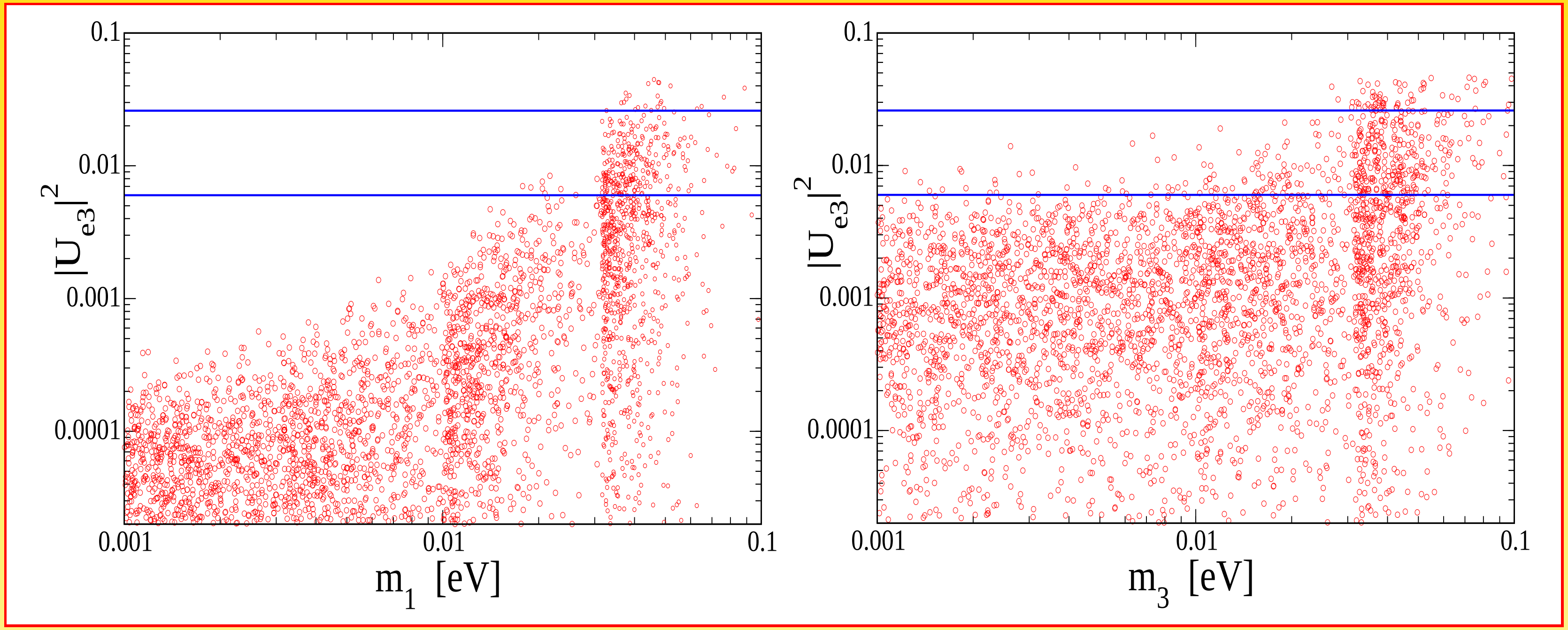
- "explicit" (naive) breaking

$$m_\nu = \begin{pmatrix} A\,(1+\epsilon_1) & B\,(1+\epsilon_2) & B\,(1+\epsilon_3) \\ \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_4) & \frac{1}{2}(A+B-D)\,(1+\epsilon_5) \\ \cdot & \cdot & \frac{1}{2}(A+B+D)\,(1+\epsilon_6) \end{pmatrix}$$

small complex parameters $\epsilon_i = |\epsilon_i|\, e^{i\phi_i}$ with $|\epsilon_i| \leq 0.2$

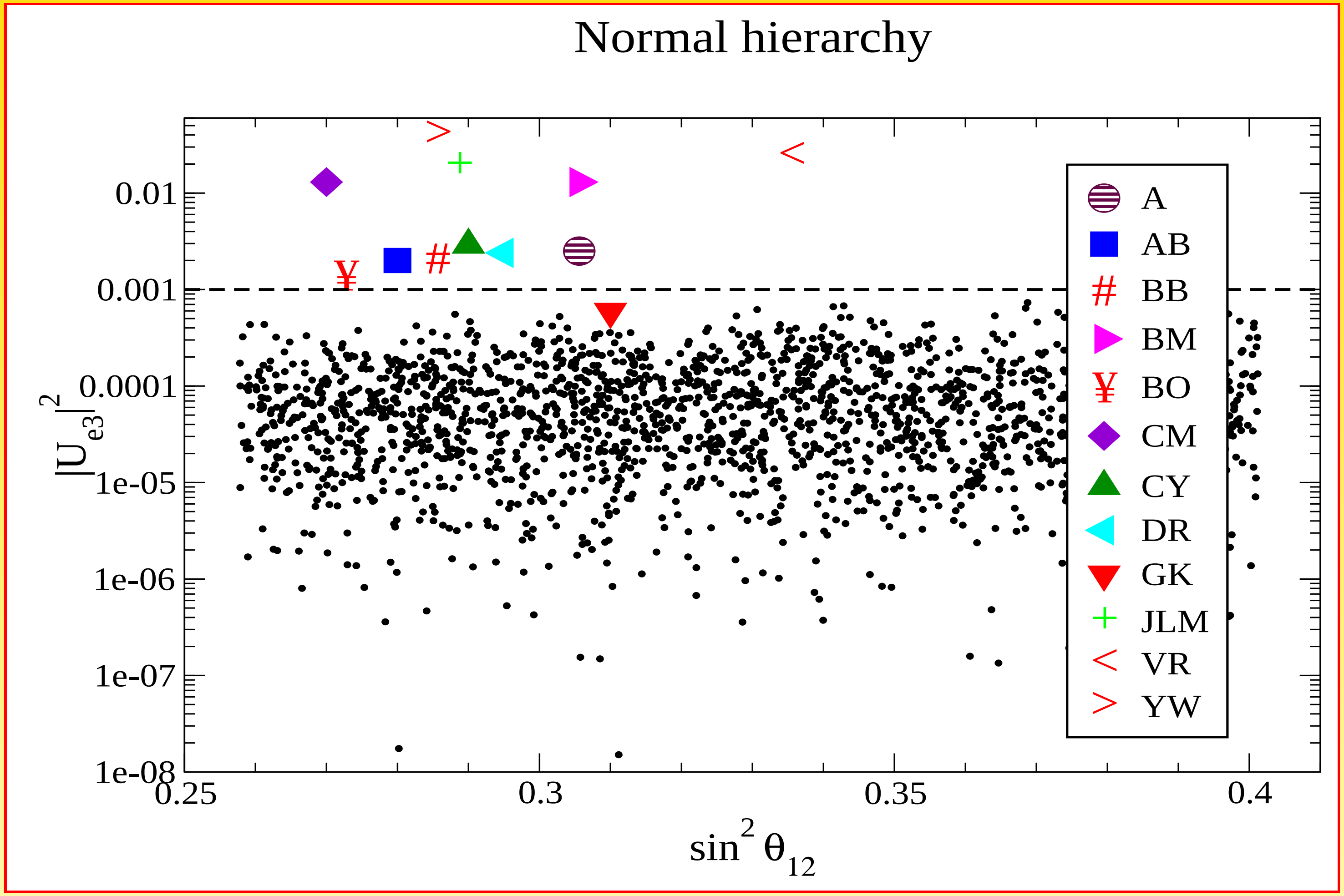
Albright, W.R., Phys. Lett. B **665**, 378 (2008)

- $\delta U_{e3} \sim \delta\theta_{23} < \delta\theta_{12}$ (θ_{12} connected to almost degenerate eigenvalues)
- $|U_{e3}| \simeq 0.1$ requires large $m_1 \gtrsim 0.02$ eV in normal ordering
- $|U_{e3}| \simeq 0.1$ requires nothing in inverted ordering



$|U_{e3}|^2 \simeq 0.01$ requires

- $m_1 \gtrsim 0.02$ eV in normal ordering ($\propto \epsilon^2\,(m_1^2 + \Delta m_\odot^2)/\Delta m_\mathrm{A}^2$)
- nothing in inverted ordering ($\propto \epsilon^2$)



$\sin^2\theta_{12}$ very unstable $\leftrightarrow$ connected to two very close eigenvalues

Sign and size of RG correction

model	mass ordering	θ_{12}	θ_{23}
SM	$\Delta m^2_{31} > 0$	$\searrow$	$\searrow$
	$\Delta m^2_{31} < 0$	$\searrow$	$\nearrow$
MSSM	$\Delta m^2_{31} > 0$	$\nearrow$	$\nearrow$
	$\Delta m^2_{31} < 0$	$\nearrow$	$\searrow$

angle	NH	IH	QD
$\delta\theta_{12}$	1	$\Delta m^2_{\rm A}/\Delta m^2_{\odot}$	$m_0^2/\Delta m^2_{\odot}$
$\delta\theta_{13}$	1	1	$m_0^2/\Delta m^2_{\rm A}$
$\delta\theta_{23}$	1	1	$m_0^2/\Delta m^2_{\rm A}$

Note: potentially huge effect for θ_{12} unless (Majorana) phase suppression

Large $|U_{e3}|$ and RG

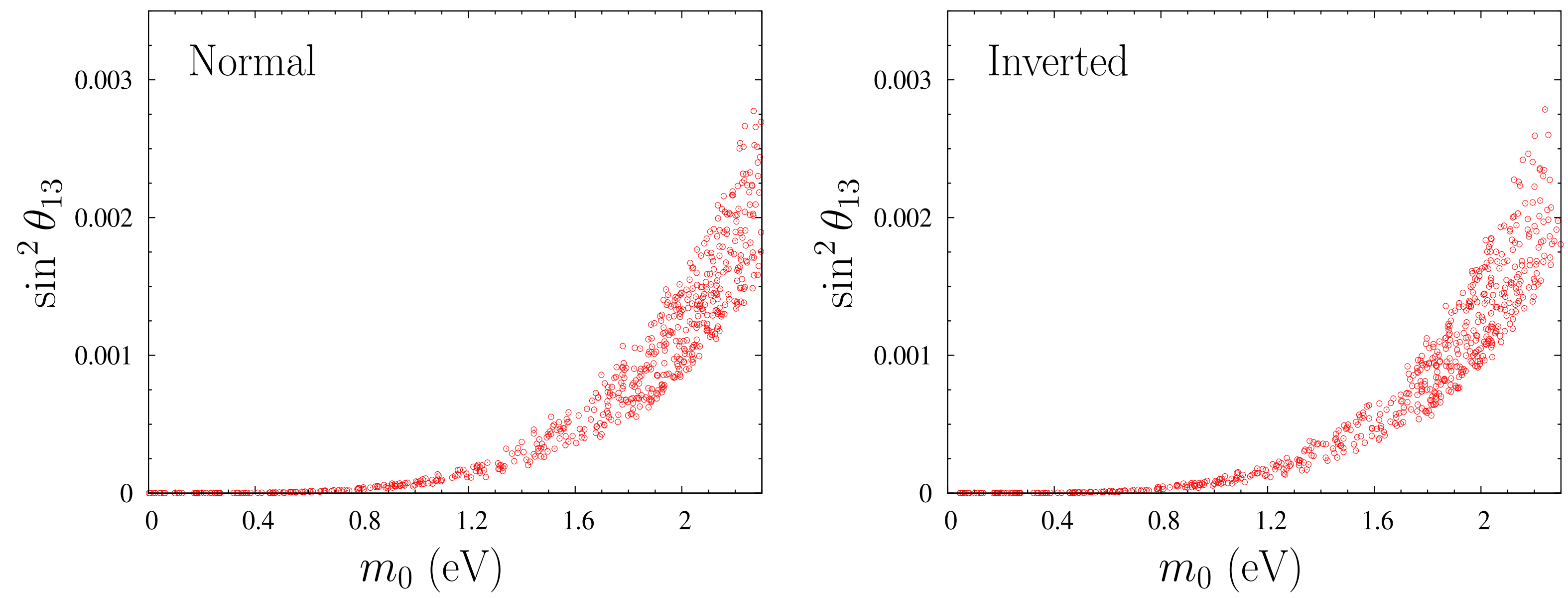
aim: get $|U_{e3}| = 0.1$ from TBM

- constraint: keep $\sin^2\theta_{12}$ close to TBM value
- what is $\sin^2\theta_{23}$?

Goswami, Petcov, Ray, W.R., PRD **80** (2009) 053013

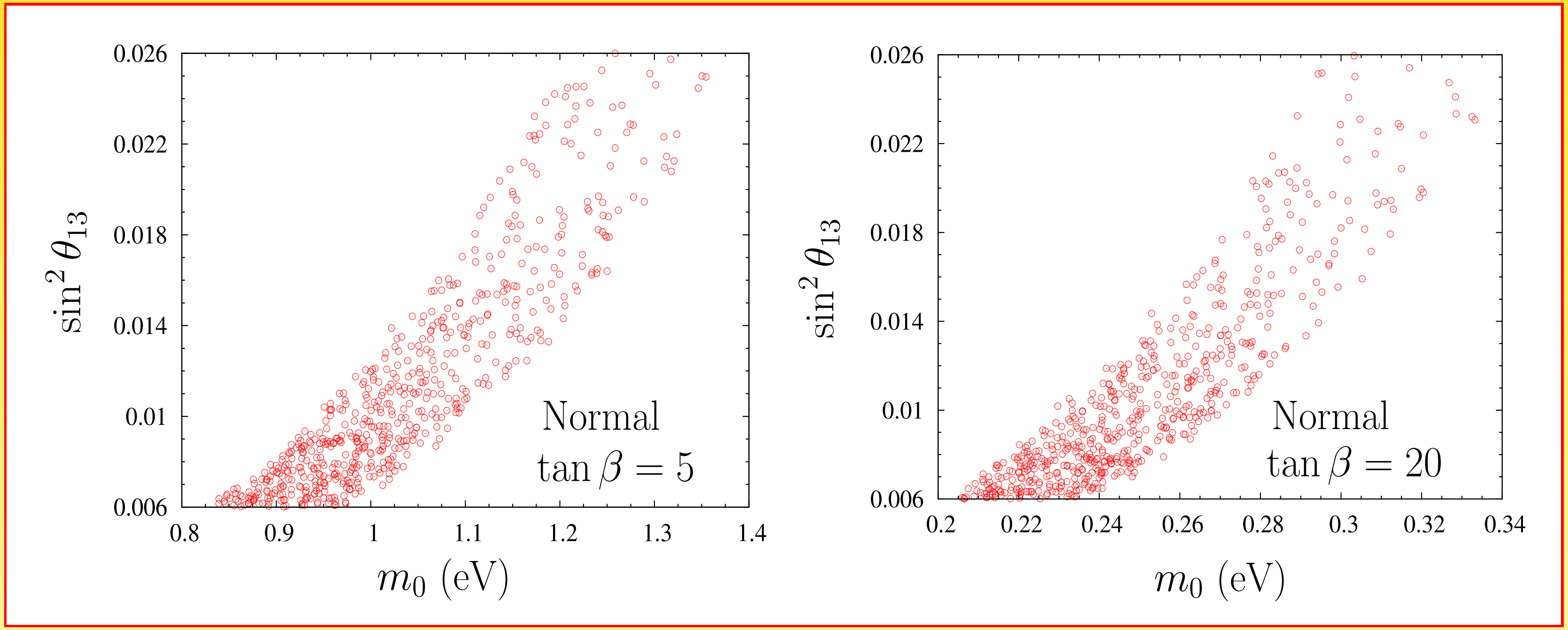
- we took "Bari hint": $0.077 \le |U_{e3}| \le 0.161$
- T2K: $|U_{e3}| \gtrsim 0.08$ (0.09)
- new Fogli *et al.*: $0.11 \le |U_{e3}| \le 0.16$

Renormalization and $|U_{e3}| \simeq 0.1$

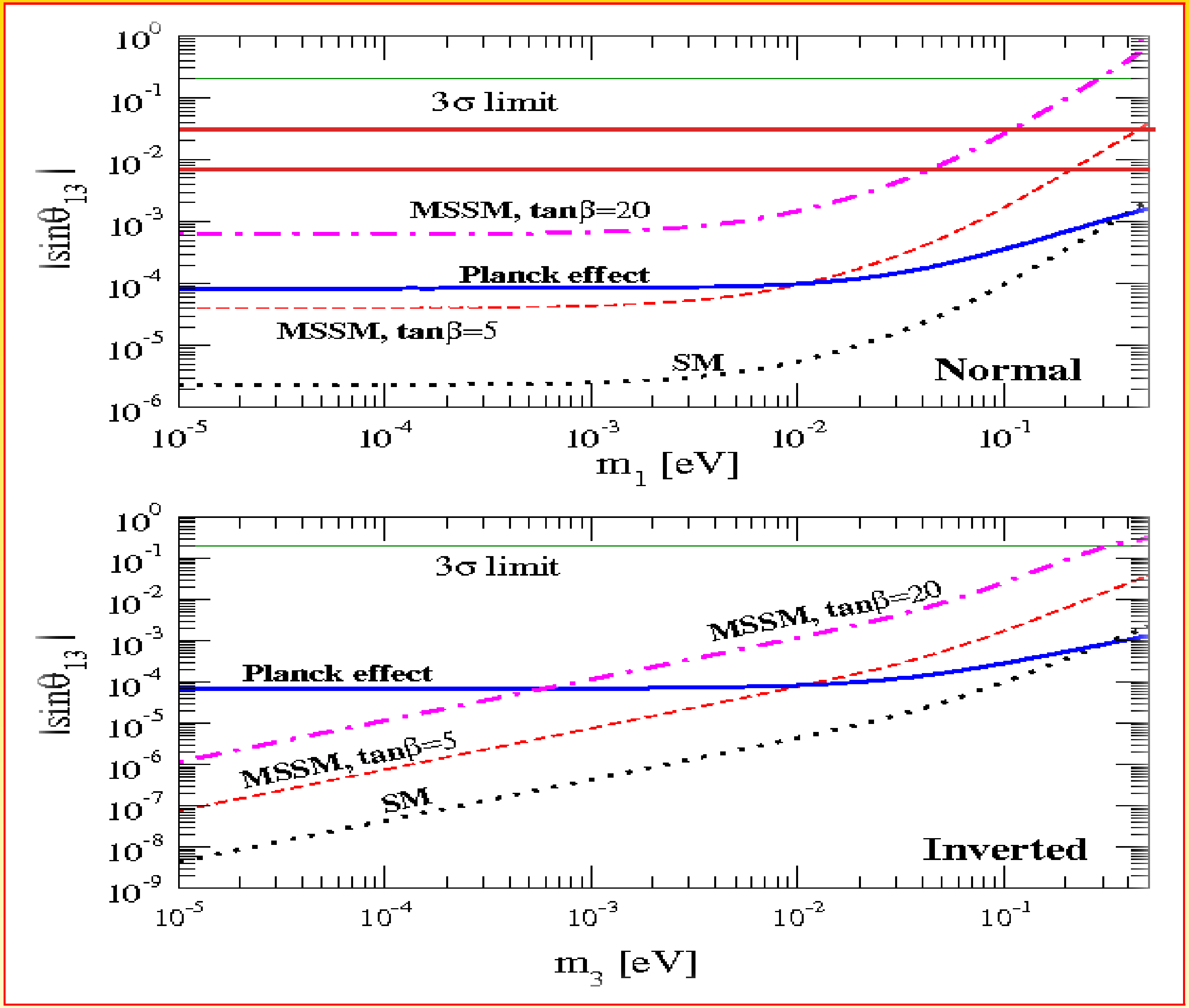


- SM: doesn't work

Renormalization and $|U_{e3}| \simeq 0.1$



- MSSM: quasi-degenerate neutrinos and $4 \lesssim (m_0/\text{eV}) \tan\beta \lesssim 7$



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$|\sin\theta_{13}|$
3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Normal
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
m_1 [eV]
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$|\sin\theta_{13}|$
3σ limit
MSSM, tanβ=20
Planck effect
MSSM, tanβ=5
SM
Inverted
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
m_3 [eV]

Effect on θ_{12}

$$k_{12} = \frac{\sqrt{2}}{3} \, \frac{\left|m_1 + m_2 \, e^{i\alpha_2}\right|^2}{\Delta m^2_\odot} \propto \begin{cases} 1 & \text{NH} \\ \frac{\Delta m^2_{\rm A}}{\Delta m^2_\odot} \, (1 + e^{i\alpha_2}) & \text{IH} \\ \frac{m_0^2}{\Delta m^2_\odot} \, (1 + e^{i\alpha_2}) & \text{QD} \end{cases}$$

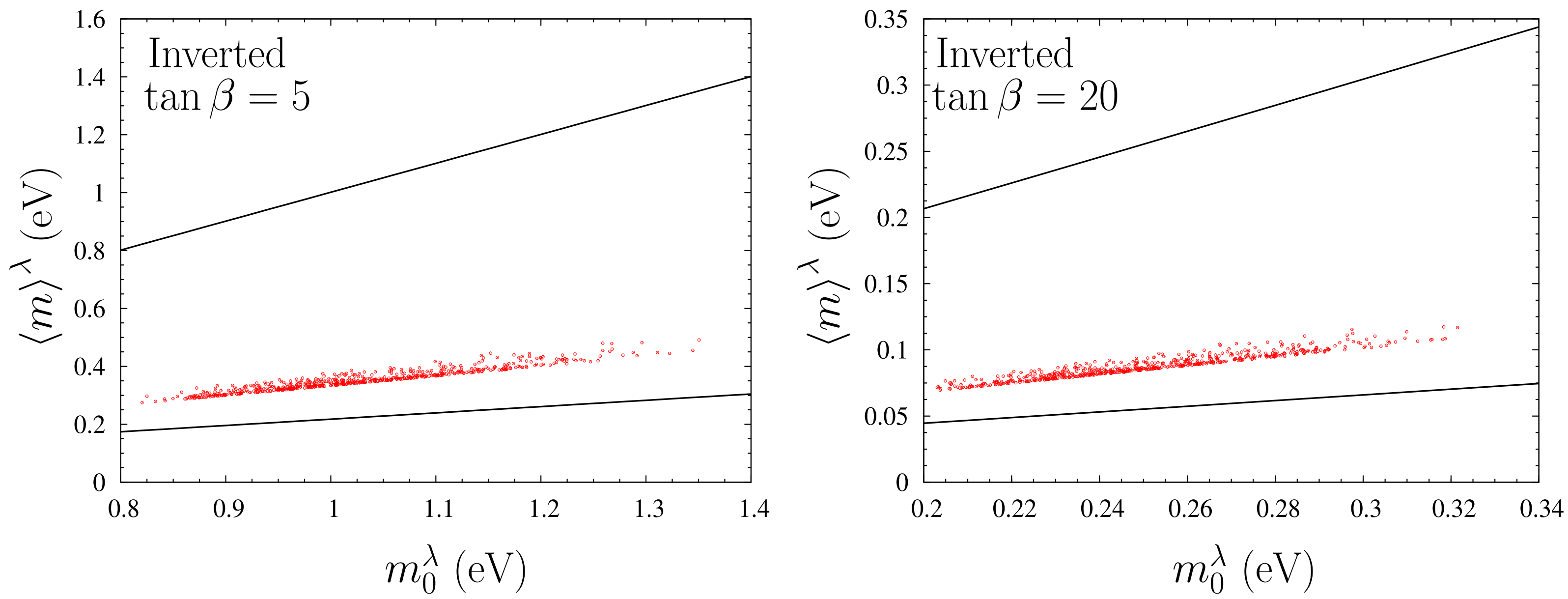
$\Rightarrow$ strong effect for IH and QD

$\Rightarrow$ suppress with $\alpha_2 = \pi$

$$|m_{ee}| \simeq m_0 \, \sqrt{1 - \sin^2 2\theta_{12} \, \sin^2 \alpha_2/2} \stackrel{\alpha_2 = \pi}{\longrightarrow} m_0 \, \cos 2\theta_{12}$$

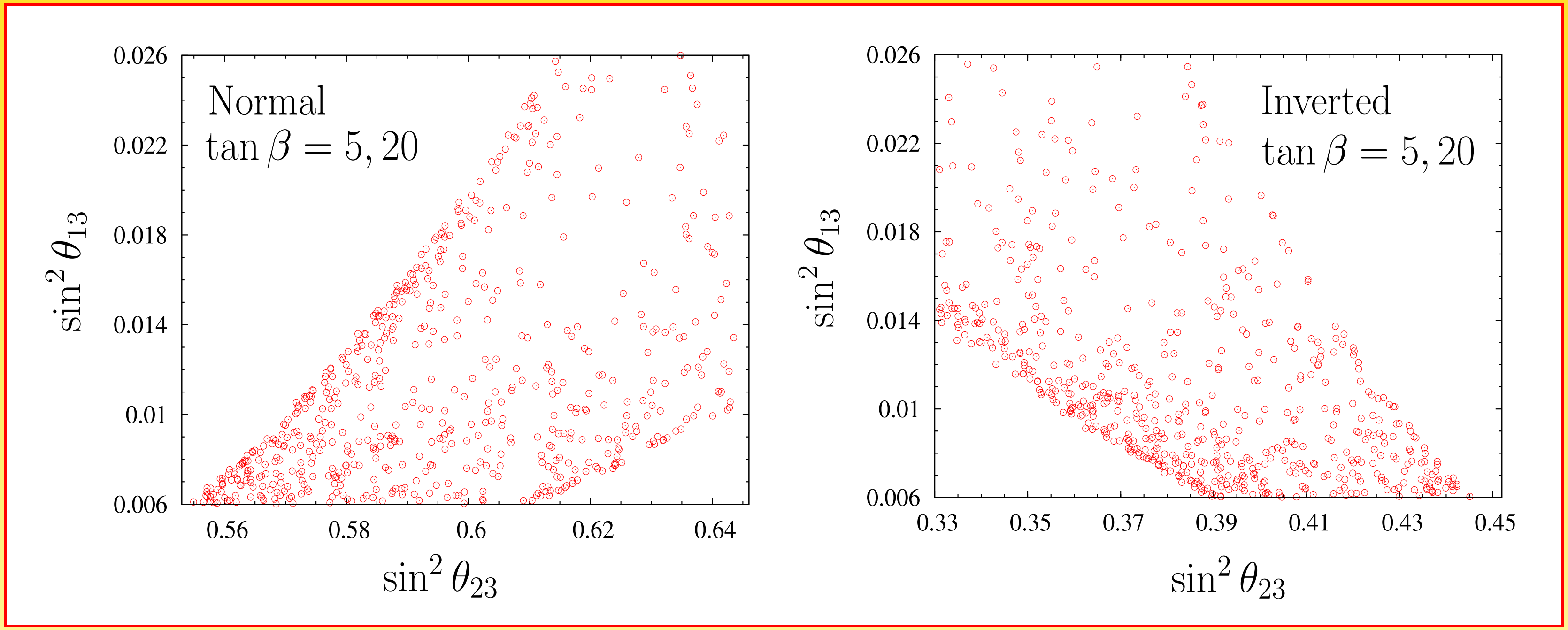
large cancellations in $0\nu\beta\beta$!

Renormalization and $|U_{e3}| \simeq 0.1$



- $|m_{ee}| \simeq c_{13}^2\, m_0\, |c_{12}^2 + s_{12}^2\, e^{i\alpha_2}|$
- $\tan\beta = 5$: $|m_{ee}|$ takes values between 0.26 and 0.50 eV; general upper and lower limits: 0.2 eV and 1.4 eV
- $\tan\beta = 20$: $|m_{ee}|$ takes values between 0.07 and 0.11 eV; general upper and lower limits: 0.05 eV and 0.34 eV

Renormalization and $|U_{e3}| \simeq 0.1$



- $|\theta_{23} - \pi/4| = \mathcal{O}(|U_{e3}|)$
- can NOT be maximal

	charged leptons	renormalization (MSSM)	explicit breaking
$\sin^2\theta_{23}$	$0.44 - 0.53$	$0.55 - 0.64 \quad (\Delta m^2_{\rm A} > 0)$ $0.33 - 0.45 \quad (\Delta m^2_{\rm A} < 0)$	—
$\lvert U_{e3}\rvert$	$\simeq \frac{\lambda}{\sqrt{2}}$	$\propto \frac{m_0^2}{\Delta m^2_{\rm A}}\,(1+\tan^2\beta)$	$\propto \epsilon \quad$ (IH) $\propto \epsilon\, m_1/\sqrt{\Delta m^2_{\rm A}} \quad$ (PD/QD)
mass	—	QD: $m_0\,\tan\beta \simeq (4-7)$ eV	IH, PD, QD
$\lvert m_{ee}\rvert$	—	$m_0\, c_{13}^2\,\cos 2\theta_{12}$	$m_0\, c_{13}^2\,\cos 2\theta_{12} \quad$ (QD) $\sqrt{\Delta m^2_{\rm A}}\, c_{13}^2\,\cos 2\theta_{12} \quad$ (IH)
CP	oscillations: almost maximal CP violation	$\alpha_2 \simeq \pi$	large $\lvert U_{e3}\rvert$ requires suppressed $\lvert m_{ee}\rvert$ only when initially $\alpha_2 \simeq \pi$

$$0.077 \le |U_{e3}| \le 0.161$$

Summary so far

- Corrections to U_{e3} and θ_{23} similar
- do we need precision experiments for θ_{12}?
  - could distinguish very different approaches

$$\sin^2\theta_{12} = \frac{1}{2} - \lambda/\sqrt{2} \simeq 0.339 \quad \text{vs.} \quad \sin^2\theta_{12} = \frac{1}{3}$$

  - BUT: Corrections to θ_{12} tend to be largest...

Alternatives to TBM

- μ–τ symmetry $(Z_2, D_4, \ldots)$:

$$m_\nu = \begin{pmatrix} a & b & b \\ \cdot & d & e \\ \cdot & \cdot & d \end{pmatrix} \Rightarrow U_{e3} = 0,\ \theta_{23} = \pi/4$$

solar neutrino mixing unconstrained $(\theta_{12} = \mathcal{O}(1))$

`countless papers`

Alternatives to TBM

- Golden Ratio φ_1 (A_5)

$$\boxed{\cot\theta_{12} = \varphi} \quad \Rightarrow \sin^2\theta_{12} = \frac{1}{1+\varphi^2} = \frac{2}{5+\sqrt{5}} \quad \simeq 0.276$$

(Datta, Ling, Ramond; Kajiyama, Raidal, Strumia; Everett, Stuart)

- Golden Ratio φ_2 (D_5)

$$\boxed{\cos\theta_{12} = \frac{\varphi}{2}} \quad \Rightarrow \sin^2\theta_{12} = \tfrac{1}{4}\,(3-\varphi) = \frac{5-\sqrt{5}}{8} \quad \simeq 0.345$$

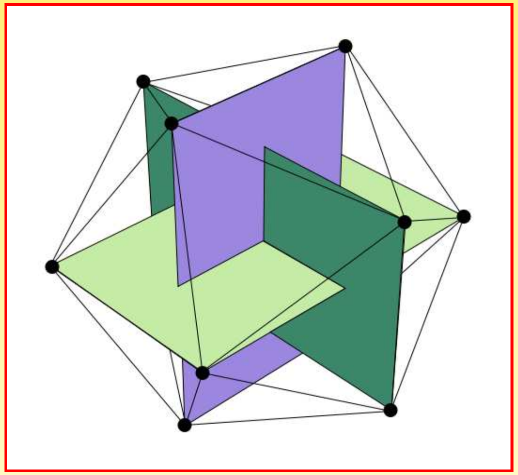
(W.R.; Adulpravitchai, Blum, W.R.)

Golden Ratio Prediction φ_1

$\boxed{\cot\theta_{12} = \varphi}$ or: $\tan 2\theta_{12} = 2$

can be generated by $m_\nu = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow Z_2: \; S = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$

Model based on A_5 (isomorphic to rotational icosahedral symmetry group)?



Cartesian coordinates of its 12 vertices:

$$(0, \pm 1, \pm\varphi)$$

$$(\pm 1, \pm\varphi, 0)$$

$$(\pm\varphi, 0, \pm 1)$$

Golden Ratio Prediction φ_1

A_5 has irreps $\mathbf{1}$, $\mathbf{3}$, $\mathbf{3}'$, $\mathbf{4}$, $\mathbf{5}$

e.g., generators for triplet representation $\mathbf{3}$

$$S_3 = \frac{1}{2}\begin{pmatrix} -1 & \varphi & 1/\varphi \\ \varphi & 1/\varphi & 1 \\ 1/\varphi & 1 & -\varphi \end{pmatrix} \text{ and } T_3 = \frac{1}{2}\begin{pmatrix} 1 & \varphi & 1/\varphi \\ -\varphi & 1/\varphi & 1 \\ 1/\varphi & -1 & \varphi \end{pmatrix}$$

Everett, Stuart, PRD **79**, 085005 (2009)

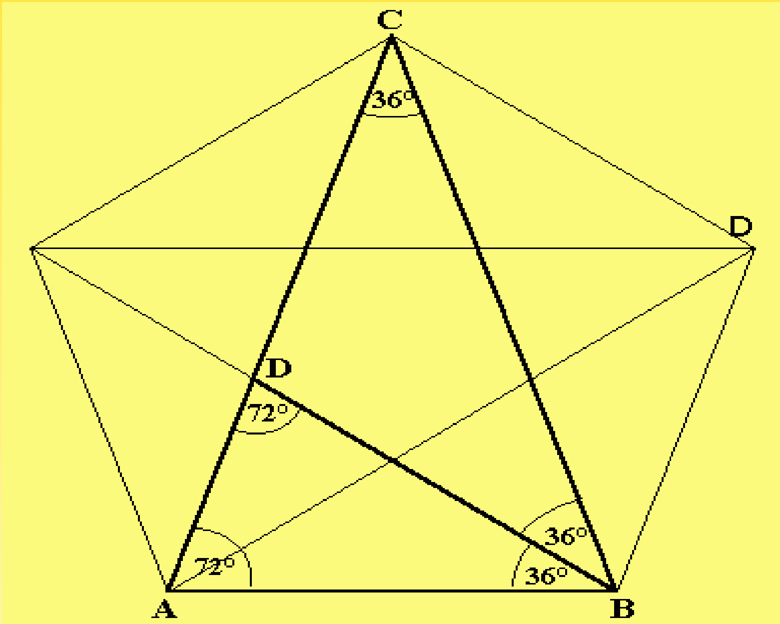
4th generation model Chen, Kephart, Yuan, 1011.3199

Golden Ratio Prediction φ_2

$$\boxed{\cos\theta_{12} = \varphi/2} \quad \text{or: } \theta_{12} = \frac{\pi}{5}$$

$$\sin^2\theta_{12} = \sin^2 \pi/5 = \frac{5-\sqrt{5}}{8} \simeq 0.345$$

(W.R., PLB **671**, 267 (2009))

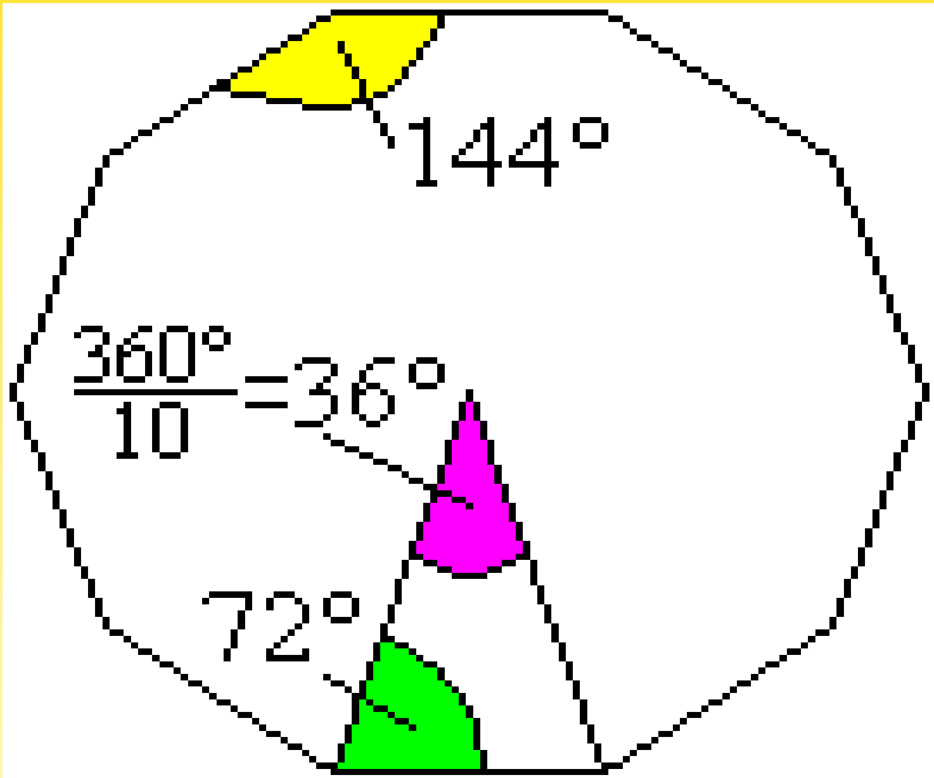


$$\overline{\mathrm{AD}} = \varphi\,\overline{\mathrm{AB}}$$

symmetry group of pentagon: D_5

Golden Ratio Prediction φ_2

symmetry group of decagon: D_{10}



Dihedral Groups

Blum, Hagedorn, Lindner, Hohenegger, PRD **77**, 076004 (2008):

D_n has several $\mathbf{2_j}$, generated by

$$A = \begin{pmatrix} e^{2\pi i \frac{j}{n}} & 0 \\ 0 & e^{-2\pi i \frac{j}{n}} \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and Z_2 is generated by

$$B\,A^k = \begin{pmatrix} 0 & e^{-2\pi i \frac{j}{n}\,k} \\ e^{2\pi i \frac{j}{n}\,k} & 0 \end{pmatrix}$$

Thus, break D_n such that m_ν invariant under $B\,A^{k_\nu}$ and m_ℓ under $B\,A^{k_\ell}$:

$$|U_{e1}|^2 = \left| \cos \pi \frac{j}{n} \left(k_\nu - k_\ell \right) \right|^2$$

Again, D_5 or D_{10} to obtain $\pi/5$

A Model based on D_{10}

Adulpravitchai, Blum, W.R., New J. Phys. **11**, 063026 (2009)

Field	$l_{1,2}$	l_3	$e^c_{1,2}$	e^c_3	$h_{u,d}$	σ^e	$\chi^e_{1,2}$	$\xi^e_{1,2}$	$\rho^e_{1,2}$	σ^ν	$\varphi^\nu_{1,2}$	$\chi^\nu_{1,2}$	$\xi^\nu_{1,2}$
D_{10}	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$
Z_5	ω	ω	ω^2	ω^2	1	ω^2	ω^2	ω^2	ω^2	ω^3	ω^3	ω^3	ω^3

Alternatives to TBM

Bi-maximal

$$U_{\rm BM} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

S_4: Altarelli, Feruglio, Merlo, JHEP **0905**, 020 (2009) (needs large NLO corrections)

CKM(-like) charged lepton corrections may also resurrect it:

- $\mathrm{QLC}_0 : \theta_{12} = \frac{\pi}{4} - \theta_C \Rightarrow \sin^2\theta_{12} \simeq 0.280$
- $\mathrm{QLC}_1 : U = V^\dagger\, U_{\rm BM} \Rightarrow \sin^2\theta_{12} \simeq \frac{1}{2} - \lambda/\sqrt{2}\,\cos\phi \simeq 0.331\ldots0.670$
- $\mathrm{QLC}_2 : U = U_{\rm BM}\, V^\dagger \Rightarrow \sin^2\theta_{12} \simeq \frac{1}{2} - \lambda\,\cos\phi' \simeq 0.276\ldots0.762$

"Quark-Lepton Complementarity"

Alternatives to TBM

Tri-maximal Mixing(s)

- TM$_2$ $(S_{3,4}, \Delta(27))$

$$\begin{pmatrix} |U_{e2}|^2 \\ |U_{\mu 2}|^2 \\ |U_{\tau 2}|^2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

(`Lam; Grimus, Lavoura`)

- TM$_1$, TM$_3$, TM^1, TM^2, TM^3, e.g.,

$$\text{TM}^1: \qquad \left(|U_{e1}|^2\,,\ |U_{e2}|^2\,,\ |U_{e3}|^2\right) = \left(\tfrac{2}{3}\,,\ \tfrac{1}{3}\,,\ 0\right)$$

$$\text{TM}_1: \qquad \begin{pmatrix} |U_{e1}|^2 \\ |U_{\mu 1}|^2 \\ |U_{\tau 1}|^2 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 1/6 \\ 1/6 \end{pmatrix}$$

(`Lam; Albright, W.R.; Friedberg, Lee; He, Zee`)

Minimal Modification of TBM

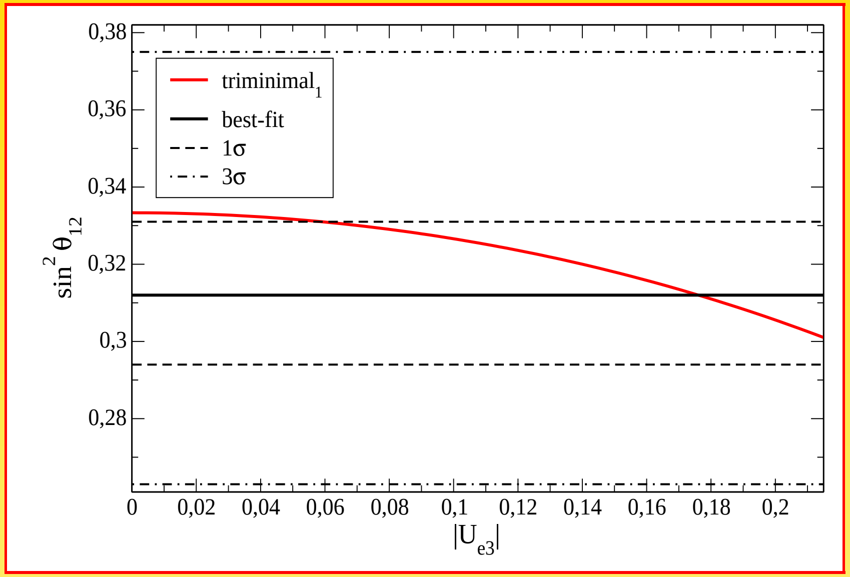
want non-zero θ_{13} and $\sin^2\theta_{12} \leq \frac{1}{3}$:

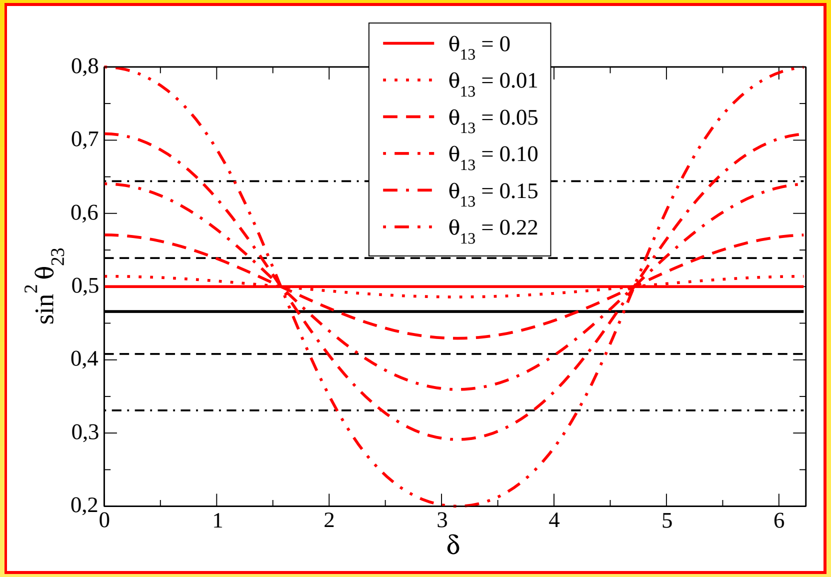
$$\mathrm{TM}_1: \quad \begin{pmatrix} |U_{e1}|^2 \\ |U_{\mu 1}|^2 \\ |U_{\tau 1}|^2 \end{pmatrix} = \begin{pmatrix} 2/3 \\ 1/6 \\ 1/6 \end{pmatrix}$$

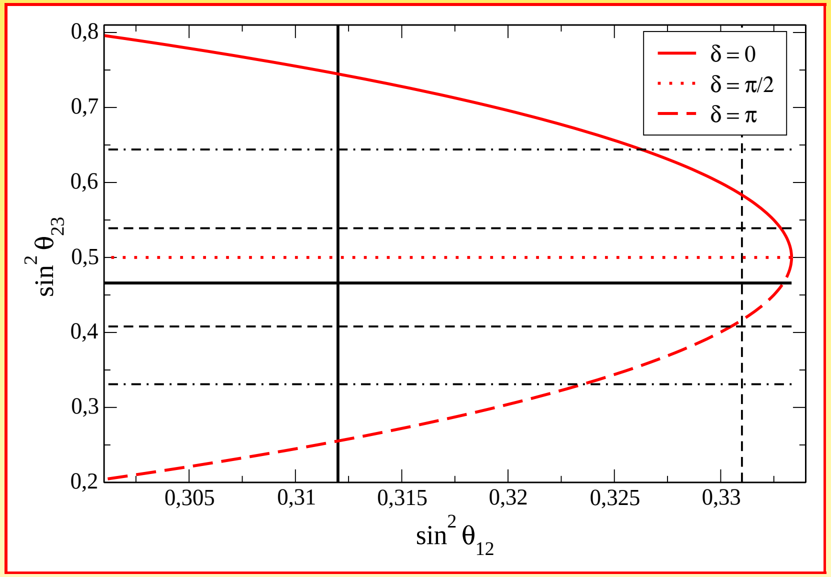
gives observables

$$\sin^2\theta_{12} = \frac{1}{3}\,\frac{1-3\,|U_{e3}|^2}{1-|U_{e3}|^2} \simeq \frac{1}{3}\,\left(1-2\,|U_{e3}|^2\right) \leq \frac{1}{3}$$

$$\cos\delta\,\tan 2\theta_{23} = -\frac{1-5\,|U_{e3}|^2}{2\sqrt{2}\,|U_{e3}|\,\sqrt{1-3\,|U_{e3}|^2}} \simeq \frac{-1}{2\sqrt{2}\,|U_{e3}|} + \frac{7}{4\sqrt{2}}\,|U_{e3}|$$


triminimal$_1$
best-fit
1σ
3σ
$\sin^2\theta_{12}$
$|U_{e3}|$
0,38
0,36
0,34
0,32
0,3
0,28
0
0,02
0,04
0,06
0,08
0,1
0,12
0,14
0,16
0,18
0,2


$\theta_{13} = 0$
$\theta_{13} = 0.01$
$\theta_{13} = 0.05$
$\theta_{13} = 0.10$
$\theta_{13} = 0.15$
$\theta_{13} = 0.22$
$\sin^2\theta_{23}$
δ
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0
1
2
3
4
5
6


δ = 0
δ = π/2
δ = π
$\sin^2\theta_{23}$
$\sin^2\theta_{12}$
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,305
0,31
0,315
0,32
0,325
0,33

Alternatives to TBM

- tetra-maximal (Xing; He, Zhou)

$$U = \mathrm{diag}(1,1,i)\,\tilde{R}_{23}(\pi/4;\pi/2)\,\tilde{R}_{13}(\pi/4;0)\,\tilde{R}_{12}(\pi/4;0)\,\tilde{R}_{13}(\pi/4;\pi)$$

- symmetric mixing $U = U^T$ (Joshipura, Smirnov; Hochmuth, W.R.)

$$|U_{e3}| = \frac{\sin\theta_{12}\,\sin\theta_{23}}{\sqrt{1-\sin^2\delta\,\cos^2\theta_{12}\,\cos^2\theta_{23}}+\cos\delta\,\cos\theta_{12}\,\cos\theta_{23}}$$

- various other proposals after T2K: Xing, 1106.3244; Qui, Ma, 1106.3284; He, Zee, 1106.4359; Zheng, Ma, 1106.4040; Zhou, 1106.4808; Araki, 1106.5211; Haba, Takahashi, 1106.5926; Morisi, Patel, Peinado, 1107.0696, Chao, Zheng, 1107.0738; Zhang, Zhou, 1107.1097...

- hexagonal mixing (D_6)

$$\theta_{12} = \pi/6 \Rightarrow \sin^2\theta_{12} = \frac{1}{4}$$

(`Albright, Dueck, W.R.; Kim, Seo`)

`Kim, Seo, 1005.4684`: D_{12} model

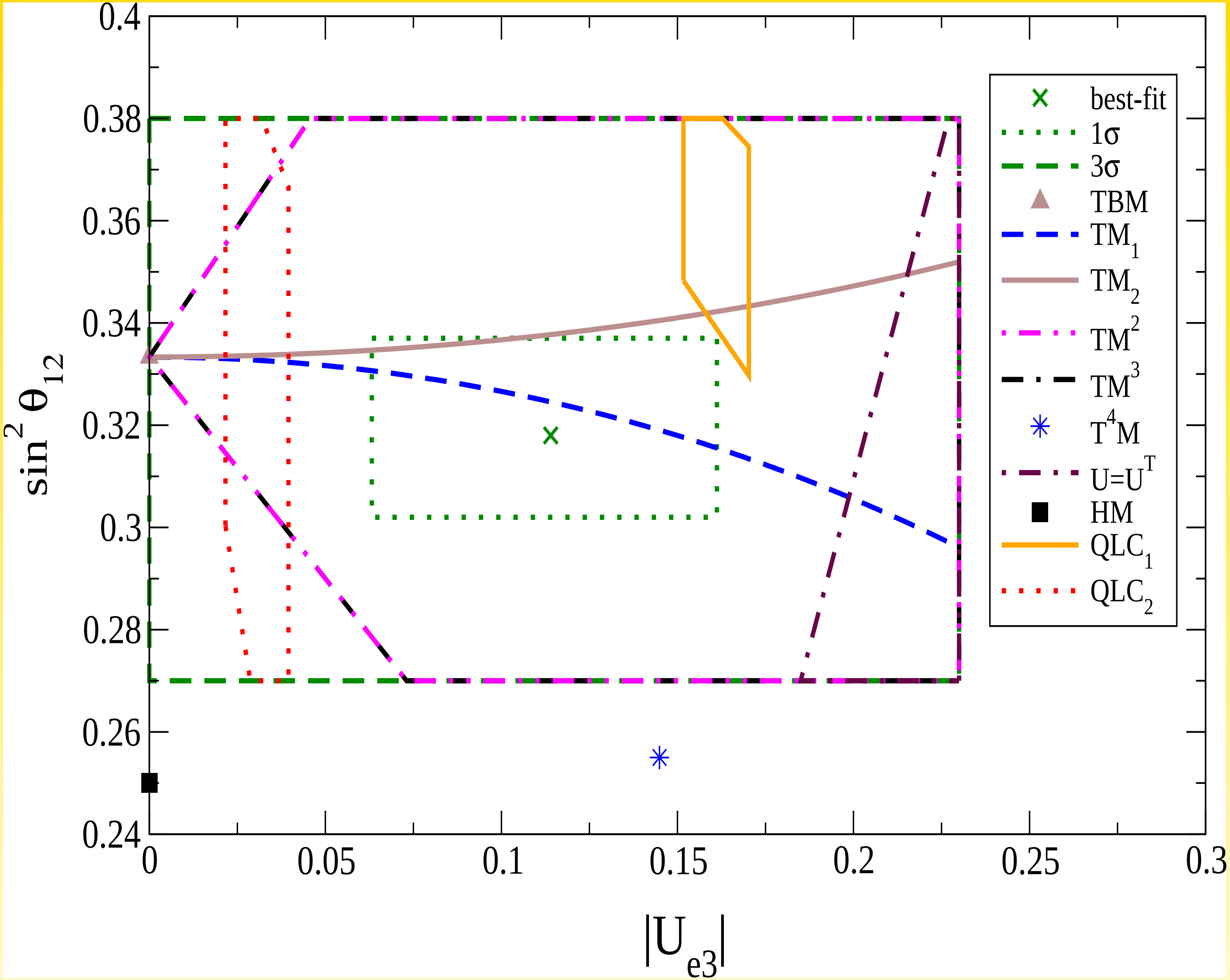
introduces 13 Higgs doublets(...), but achieved (QLC) $\theta_{12} = \pi/6$, $\theta_C = \pi/12 = 15^0 \simeq \theta_C + 1.8^0$ and called it dodecal

Once you start playing with numbers...

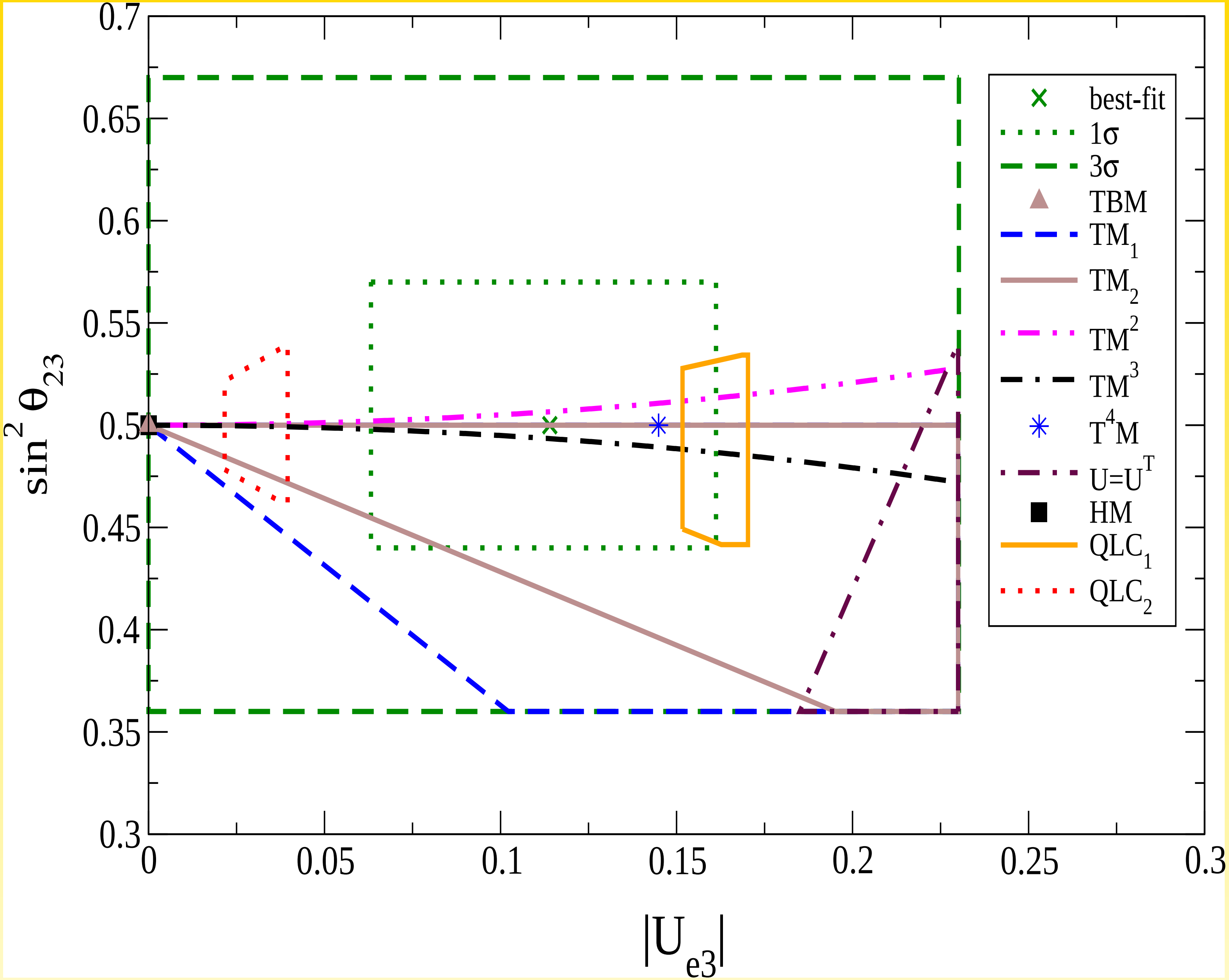
- "transcendental mixing":
  - $\sin\theta_{13} = \theta_{13} \Rightarrow \theta_{13} = 0$
    is the fixed point of $\sin(x)$
  - $\cos\theta_{23} = \theta_{23} = 0.739085133\ldots \Rightarrow \sin^2\theta_{23} = 0.454$
    is the fixed point of $\cos(x)$
    "Dottie's number" $d = \lim_{n\to\infty} \cos_n(x)$ is irrational like π, e, $\sqrt{2}$
- Euler-Mascheroni constant:
  - $\theta_{12} = \gamma = 0.577215664\ldots \Rightarrow \sin^2\theta_{12} = 0.298$
  - $|U_{e2}| = \gamma \Rightarrow \sin^2\theta_{12} = 0.298\ldots0.314$
- Euler's number:
  - $\tan 2\theta_{12} = e = 2.718281828\ldots \Rightarrow \sin^2\theta_{12} = 0.327$
- etc   :-))

Scenario	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	T2K
TBM	0.333	0.500	0.000	–
$\mu-\tau$	—	0.500	0.000	–
TM_1	0.296 0.333	**	—	✓
TM_2	0.333 0.352	**	—	✓
TM_3	—	0.500	0.000	–
TM^1	0.333	—	0.000	–
TM^2	**	0.500 0.528	—	✓
TM^3	**	0.472 0.500	—	✓
T^4M	0.255	0.500	0.021	✓
$U=U^T$	0.000 0.389	0.000 0.504	0.0343 0.053	✓
BM	0.500	0.500	0.000	–
HM	0.250	0.500	0.000	–
φ_1	0.276	0.500	0.000	–
φ_2	0.345	0.500	0.000	–
QLC_0	0.280	0.459	—	–
QLC_1	0.331 0.670	0.442 0.534	0.023 0.029	✓
QLC_2	0.276 0.726	0.462 0.540	0.0005 0.0016	–

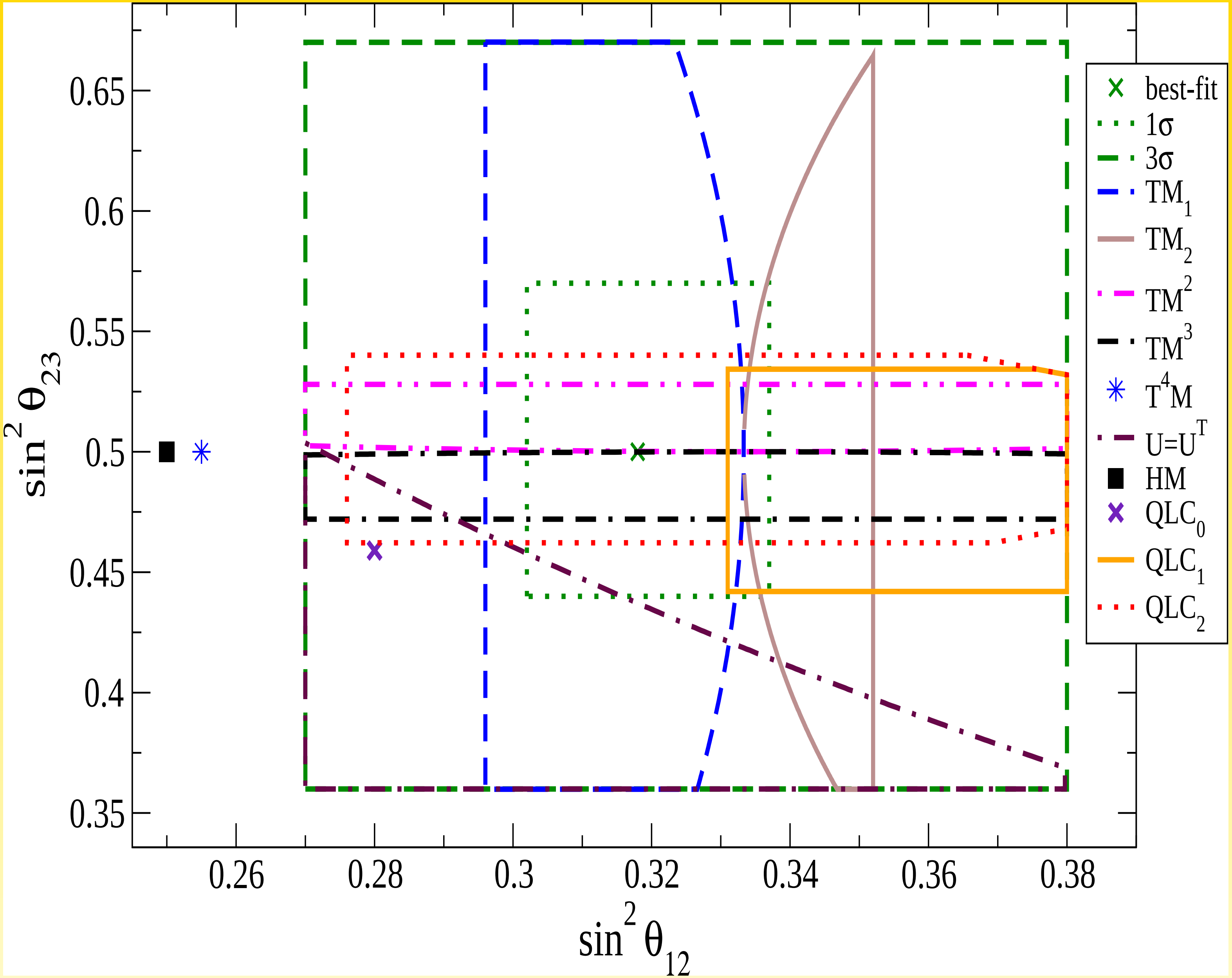
Albright, Dueck, W.R., 1004.2798



0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
$\sin^2\theta_{12}$
0
0.05
0.1
0.15
0.2
0.25
0.3
$|U_{e3}|$
best-fit
1σ
3σ
TBM
TM_1
TM_2
TM^2
TM^3
T^4M
$U=U^T$
HM
QLC_1
QLC_2



$\sin^2\theta_{23}$
$|U_{e3}|$
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0
0.05
0.1
0.15
0.2
0.25
0.3
best-fit
1σ
3σ
TBM
TM_1
TM_2
TM^2
TM^3
T^4M
$U=U^T$
HM
QLC_1
QLC_2



0.65
0.6
0.55
0.5
0.45
0.4
0.35
$\sin^2\theta_{23}$
0.26
0.28
0.3
0.32
0.34
0.36
0.38
$\sin^2\theta_{12}$
best-fit
1σ
3σ
TM_1
TM_2
TM^2
TM^3
T^4M
$U=U^T$
HM
QLC_0
QLC_1
QLC_2

What's special about TBM?

The TBM mass matrix

$$\begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A+B+D) & \frac{1}{2}(A+B-D) \\ \cdot & \cdot & \frac{1}{2}(A+B+D) \end{pmatrix}$$

is the **only one** invariant under

$$R_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and } S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Note: S and $T = \mathrm{diag}(1, \omega^2, \omega)$ generate A_4 via $S^2 = T^3 = (ST)^3 = \mathbb{1}$

in many A_4 models: $R_{\mu\tau}$ accidental, charged leptons preserve Z_3 invariance via T, neutrinos preserve Z_2 invariance via S

Symmetry and Flavor Symmetry

Each Majorana mass matrix is invariant under a $Z_2 \times Z_2$

$$m_\nu = \begin{pmatrix} a & b & b \\ \cdot & d & e \\ \cdot & \cdot & d \end{pmatrix} \text{ invariant under } R_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

look for S such that

$$[S, R_{\mu\tau}] = 0 \; , \quad SS = \mathbb{1}$$

Solvable for general θ_{12} and for special scenarios, e.g.

$d + e = a$	bimaximal
$d + e = a + b$	tri-bimaximal
$d + e = a + \sqrt{2}\, b$	golden ratio$_1$
$d + e = a + 2\sqrt{2}\, b \cot 2\theta_{12}$	general

Symmetry and Flavor Symmetry

$$S = \begin{pmatrix} \cos 2\theta_{12} & -\sqrt{2}\cos\theta_{12}\,\sin\theta_{12} & \sqrt{2}\cos\theta_{12}\,\sin\theta_{12} \\ \cdot & \sin^2\theta_{12} & \cos^2\theta_{12} \\ \cdot & \cdot & \sin^2\theta_{12} \end{pmatrix}$$

charged leptons are diagonal

$$T^\dagger\, m_\ell^\dagger\, m_\ell\, T = m_\ell^\dagger\, m_\ell$$

With $T = \mathrm{diag}(-1, i, -i)$ it follows for $\theta_{12} = \pi/4$ (bimaximal)

$$S^2 = T^4 = (S\,T)^3 = \mathbb{1} \Longrightarrow S_4$$

Interpretation: flavor symmetry G_f generated by S, T broken such that m_ν invariant under S and charged leptons under T ($R_{\mu\tau}$ is accidental) (see also Lam)

invariance only under "hidden" Z_2: Dicus, Ge, Repko, 1012.2571, 1104.0602

Scenario	S	T	relations	group
bimaximal	$\sqrt{\frac{1}{2}}\begin{pmatrix} 0 & -1 & 1 \\ \cdot & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ \cdot & \cdot & \sqrt{\frac{1}{2}} \end{pmatrix}$	$\mathrm{diag}(-1, i, -i)$	$T^4 = (ST)^3 = 1$	S_4
tri-bimaximal	$\frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ \cdot & -1 & -2 \\ \cdot & \cdot & -1 \end{pmatrix}$	$\mathrm{diag}(e^{-2i\pi/3}, e^{2i\pi/3}, 1)$	$T^3 = (ST)^3 = 1$	A_4
golden ratio (A)	$\frac{-1}{\sqrt{5}}\begin{pmatrix} 1 & -\sqrt{2} & \sqrt{2} \\ \cdot & 1/\varphi & \varphi \\ \cdot & \cdot & 1/\varphi \end{pmatrix}$	$\mathrm{diag}(1, e^{-4i\pi/5}, e^{4i\pi/5})$	$T^5 = (ST)^3 = 1$	A_5

from Feruglio, Paris, 1101.0393


And now for
something
completely
different...

Light sterile neutrinos?

Consider the "role model" Altarelli, Feruglio, NPB 720, 64 (2005)

(effective model)

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	ν_s
$SU(2)_L$	2	1	1	1	2	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1
$U(1)_{\rm FN}$	-	4	2	0	-	-	-	-	6

- Z_3 to separate charged leptons and neutrinos
- $U(1)_{\rm FN}$ for charged lepton mass hierarchy
- ν_s added by us, no extra symmetries or fields

Barry, W.R., Zhang, 1105.3911

allowed terms

$$\mathcal{L}_{\mathrm{Y}_s} = \frac{x_e}{\Lambda^2}\xi(\varphi' L h_u)\nu_s + m_s \nu_s^c \nu_s + \mathrm{h.c.}$$

lies at eV scale due to FN

mass matrix

$$M_\nu^{4\times 4} = \begin{pmatrix} a+\frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a-\frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix}$$

with the usual VEV alignment $\langle\xi\rangle = u$, $\langle\varphi\rangle = (v,0,0)$ and $\langle\varphi'\rangle = (v',v',v')$

diagonalized by

$$U \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & 0 & 0 & \frac{e}{m_s} \\ 0 & -\frac{\sqrt{3}e}{m_s} & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & -\frac{\sqrt{3}e^2}{2m_s^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{3e^2}{2m_s^2} \end{pmatrix}$$

giving the eigenvalues

$$m_1 = a + d\,, \quad m_2 = a - \frac{3e^2}{m_s}\,, \quad m_3 = -a + d\,, \quad m_4 = m_s + \frac{3e^2}{m_s}$$

and sum-rules

$$\sin^2\theta_{14} \simeq \sin^2\theta_{24} \simeq \sin^2\theta_{34} \simeq \left(\frac{e}{m_s}\right)^2 \simeq \frac{1}{2}(1 - 3\sin^2\theta_{12}) \simeq 2\sin^2\theta_{23} - 1$$

Still $U_{e3} = 0\ldots$

Can also add second ν_s, giving

$$M_\nu^{5\times 5} = \begin{pmatrix} a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e & f \\ \cdot & \frac{2d}{3} & a - \frac{d}{3} & e & f \\ \cdot & \cdot & \frac{2d}{3} & e & f \\ \cdot & \cdot & \cdot & m_{s_1} & 0 \\ \cdot & \cdot & \cdot & \cdot & m_{s_2} \end{pmatrix}$$

Trivial change of FN charge and scales gives keV sterile neutrinos

Summary

- Corrections θ_{13} and $\theta_{23} - \pi/4$ are typically similar
- to test them on a level of 0.1 is a good idea...
- once θ_{13} will be determined, deviation from maximal θ_{23} will become crucial
- interesting theoretical speculations on θ_{12}, but receives typically large corrections (of phenomenological interest for $0\nu\beta\beta$)
- modify flavor models for other things?
- distinguishing them will be challenging task

Is $\theta_{13} = 0$ or $\theta_{23} = \pi/4$?

$$m_\nu = \begin{pmatrix} x_1 & x_2 & x_3 \\ \cdot & x_4 & x_5 \\ \cdot & \cdot & x_6 \end{pmatrix}$$

Vanishing θ_{13} implies

$$m_\nu \begin{pmatrix} 0 \\ -a \\ b \end{pmatrix} = m_3 \begin{pmatrix} 0 \\ -a \\ b \end{pmatrix} \Rightarrow m_\nu = \begin{pmatrix} x_1 & x_2 & c\,x_2 \\ \cdot & x_4 & c\,(x_4 - m_3) \\ \cdot & \cdot & m_3 + c^2\,(m_3 - x_4) \end{pmatrix}$$

Maximal θ_{23} implies

$$m_\nu \begin{pmatrix} \epsilon \\ -a \\ a \end{pmatrix} = m_3 \begin{pmatrix} \epsilon \\ -a \\ a \end{pmatrix} \Rightarrow m_\nu = \begin{pmatrix} x_1 & x_2 & x_2 + d\,(m_3 - x_1) \\ \cdot & x_4 & x_4 - m_3 - d\,x_2 \\ \cdot & \cdot & x_4 - d^2\,(m_3 - x_1) - 2\,d\,x_2 \end{pmatrix}$$

How to perturb a mixing scenario/model

- Radiative corrections

$$\theta_{ij} \simeq \theta_{ij}^{\rm TBM} + k_{ij}\,\epsilon_{\rm RG}$$

$$k_{12} = \frac{\sqrt{2}}{3}\,\frac{\left|m_1 + m_2\,e^{i\alpha_2}\right|^2}{\Delta m^2_\odot}$$

$$k_{23} = -\left(\frac{2}{3}\,\frac{\left|m_2 + m_3\,e^{i(\alpha_3-\alpha_2)}\right|^2}{m_3^2 - m_2^2} + \frac{1}{3}\,\frac{\left|m_1 + m_3\,e^{i\alpha_3}\right|^2}{m_3^2 - m_1^2}\right)$$

$$k_{13} = -\frac{\sqrt{2}}{3}\left(\frac{\left|m_2 + m_3\,e^{i(\alpha_3-\alpha_2)}\right|^2}{m_3^2 - m_2^2} - \frac{\left|m_1 + m_3\,e^{i\alpha_3}\right|^2}{m_3^2 - m_1^2}\right)$$

$$\epsilon_{\rm RG} = c\,\frac{m_\tau^2}{16\pi^2\,v^2}\ln\frac{M_X}{m_Z}$$ and $c = -3/2$ or $1 + \tan^2\beta$

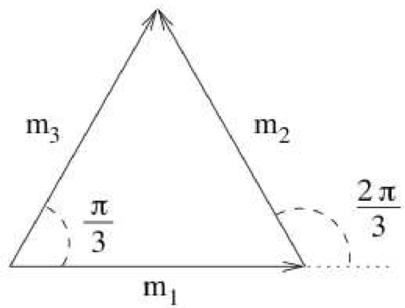
Note: potentially huge effect for θ_{12} unless (Majorana) phase suppression

Sum-rules in Models and $0\nu\beta\beta$

- constrains masses and Majorana phases:

$$m_1 + m_2 = m_3$$

$$1/m_1 + 1/m_2 = 1/m_3$$



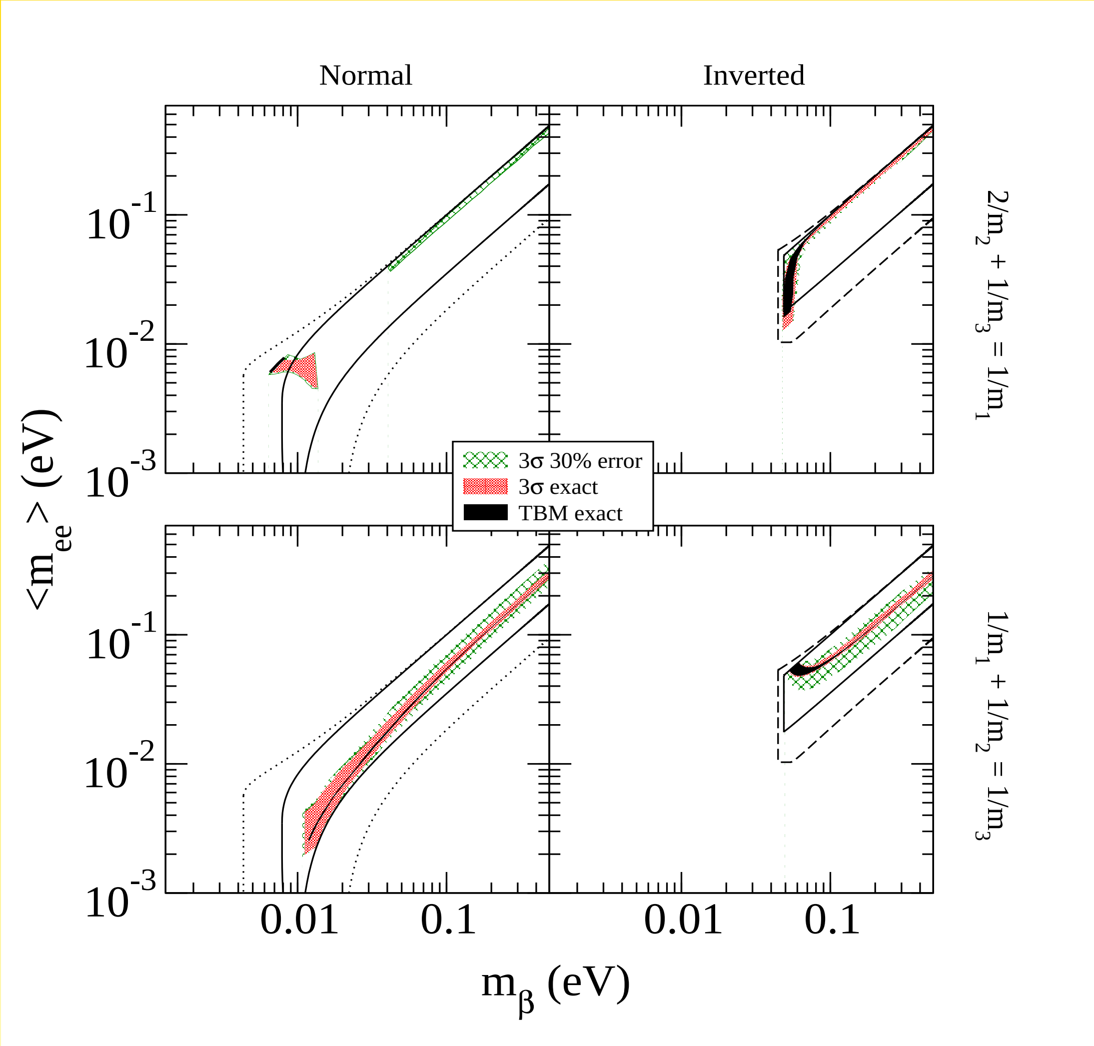
$$\langle m_{ee} \rangle \overset{\text{TBM}}{\approx} \frac{m_0}{\sqrt{3}}$$

- stable:

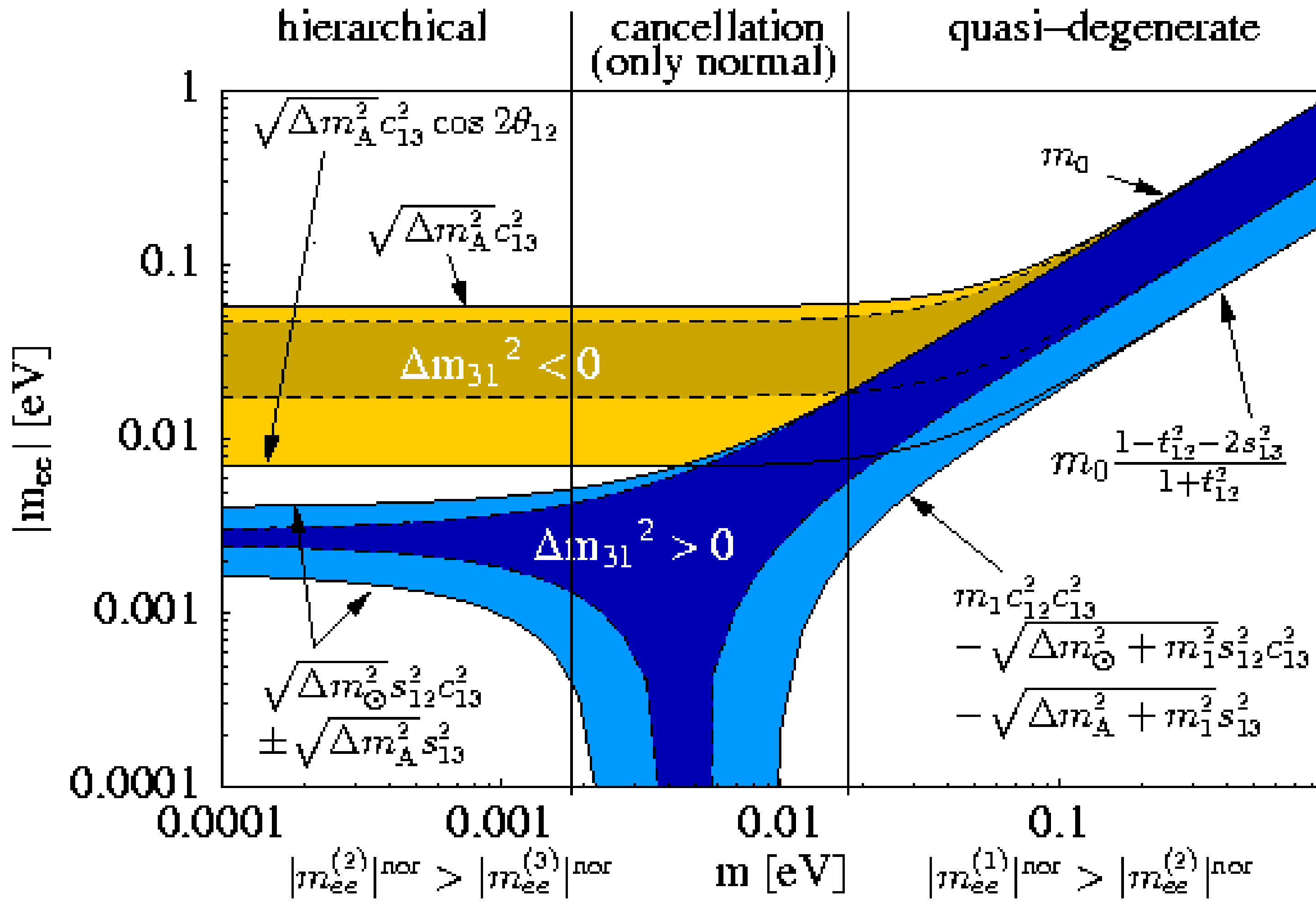
$$m_1 + m_2 - m_3 = \epsilon\, m_{\max}$$

new solutions not before $\epsilon \simeq 0.3$

Sum-rules in Models and $0\nu\beta\beta$



Barry, W.R., NPB **842**, 33 (2011)



hierarchical
cancellation (only normal)
quasi–degenerate
1
0.1
0.01
0.001
0.0001
|m_ee| [eV]
$\sqrt{\Delta m_A^2}c_{13}^2 \cos 2\theta_{12}$
$\sqrt{\Delta m_A^2}c_{13}^2$
m_0
$\Delta m_{31}^2 < 0$
$\Delta m_{31}^2 > 0$
$m_0 \frac{1-t_{12}^2-2s_{13}^2}{1+t_{12}^2}$
$m_1 c_{12}^2 c_{13}^2$
$-\sqrt{\Delta m_\odot^2 + m_1^2} s_{12}^2 c_{13}^2$
$-\sqrt{\Delta m_A^2 + m_1^2} s_{13}^2$
$\sqrt{\Delta m_\odot^2} s_{12}^2 c_{13}^2$
$\pm\sqrt{\Delta m_A^2} s_{13}^2$
0.0001
0.001
0.01
0.1
1
$|m_{ee}^{(2)}|^{nor} > |m_{ee}^{(3)}|^{nor}$
m [eV]
$|m_{ee}^{(1)}|^{nor} > |m_{ee}^{(2)}|^{nor}$

Testing Inverted Ordering

Nature gives us a scale:

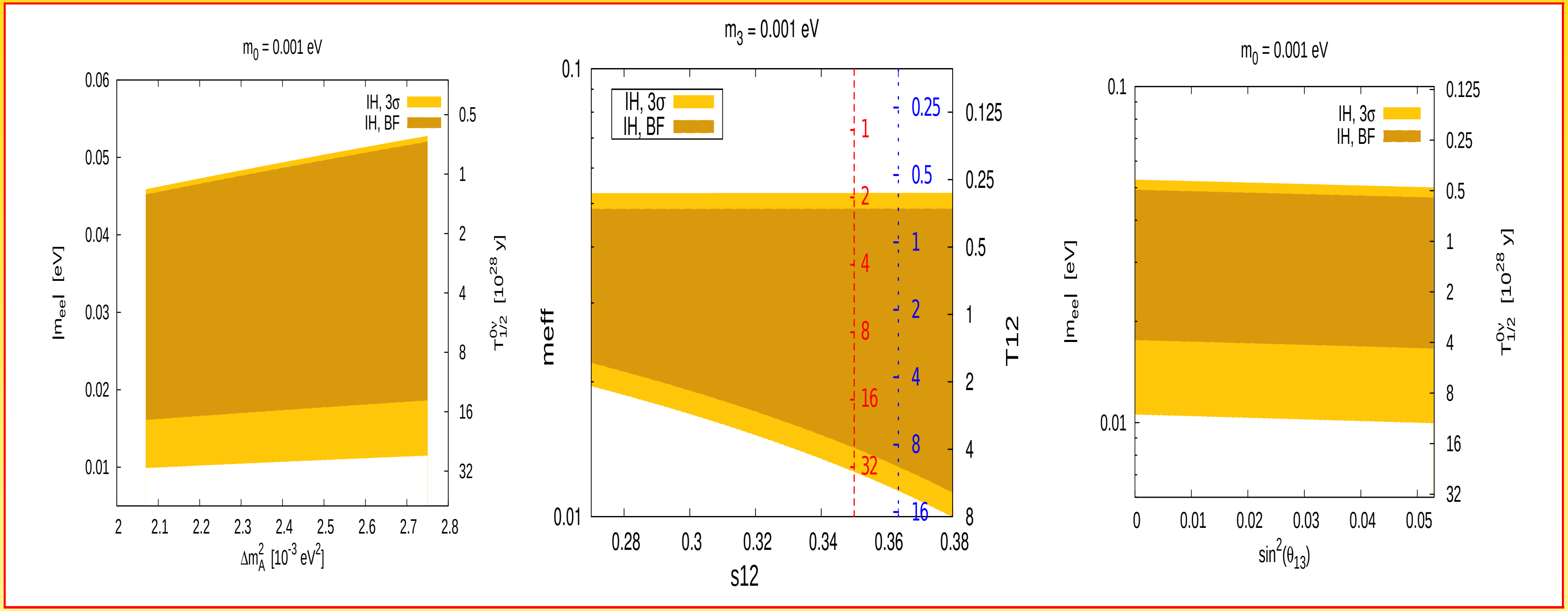
$$|m_{ee}|^{\rm IH}_{\rm min} = \left(1 - |U_{e3}|^2\right) \sqrt{|\Delta m^2_{\rm A}|} \left(1 - 2 \sin^2 \theta_{12}\right) = \begin{cases} (0.015 \ldots 0.020) \text{ eV} & 1\sigma \\ (0.010 \ldots 0.024) \text{ eV} & 3\sigma \end{cases}$$

factor 2 range due to uncertainty in $\sin^2 \theta_{12}$

Recall: a limit $|m_{ee}|_{\rm lim}$ scales with $\left(\frac{\Delta E\, B}{M\, t}\right)^{\frac{1}{4}}$

$\Rightarrow$ factor 2 in $|m_{ee}|^{\rm IH}_{\rm min}$ is a factor of $2^4 = 16$ in $\Delta E\, B/(M\, t)$

Dueck, W.R., Zuber, 1103.4152



$|m_{ee}|$ vs. $\Delta m^2_{\rm A}$ $|m_{ee}|$ vs. $\sin^2\theta_{12}$ $|m_{ee}|$ vs. $|U_{e3}|^2$

$\sin^2\theta_{12}$ gives strongest dependence!

A Model based on D_{10}

$$\begin{pmatrix} \langle\chi_1^e\rangle \\ \langle\chi_2^e\rangle \end{pmatrix} = v_e \begin{pmatrix} 1 \\ e^{\frac{2\pi i k}{5}} \end{pmatrix}, \begin{pmatrix} \langle\xi_1^e\rangle \\ \langle\xi_2^e\rangle \end{pmatrix} = w_e \begin{pmatrix} 1 \\ e^{\frac{3\pi i k}{5}} \end{pmatrix}, \begin{pmatrix} \langle\rho_1^e\rangle \\ \langle\rho_2^e\rangle \end{pmatrix} = z_e \begin{pmatrix} 1 \\ e^{\frac{4\pi i k}{5}} \end{pmatrix}$$

where k is an odd integer between 1 and 9, and

$$\begin{pmatrix} \langle\varphi_1^\nu\rangle \\ \langle\varphi_2^\nu\rangle \end{pmatrix} = v_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle\chi_1^\nu\rangle \\ \langle\chi_2^\nu\rangle \end{pmatrix} = w_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle\xi_1^\nu\rangle \\ \langle\xi_2^\nu\rangle \end{pmatrix} = z_\nu \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\langle\sigma^e\rangle = x_e,\ \langle\sigma^\nu\rangle = x_\nu$$

$$U_\ell = \mathrm{diag}(e^{-2i\Phi}, 1, e^{-i(\Phi+\delta)}) \begin{pmatrix} -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

where $\Phi = \frac{4\pi}{5}$

$$U_\nu = \begin{pmatrix} -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

- $\theta_{12} = \pi/5$
- vanishing U_{e3}
- in general non-maximal θ_{23}

VEV alignment

SUSY and "driving fields"

Field	ψ^{0e}	$\varphi^{0e}_{1,2}$	$\xi^{0e}_{1,2}$	$\psi^{0\nu}$	$\chi^{0\nu}_{1,2}$	$\xi^{0\nu}_{1,2}$
D_{10}	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$
Z_5	ω	ω	ω	ω^4	ω^4	ω^4

flavon superpotential $w_f = w_{f,e} + w_{f,\nu}$

flavor symmetry broken at high scale, thus minimize in supersymmetric limit determine supersymmetric minimum by setting F-terms of driving fields to zero:

$$
\begin{aligned}
\frac{\partial w_{f,e}}{\partial \psi^{0e}} &= a_e \left(\chi_1^e \, \xi_1^e + \chi_2^e \, \xi_2^e\right) = 0 \\
\frac{\partial w_{f,e}}{\partial \varphi_1^{0e}} &= b_e \, \chi_1^e \, \xi_2^e + c_e \, \xi_1^e \, \rho_2^e = 0 \\
\frac{\partial w_{f,e}}{\partial \varphi_2^{0e}} &= b_e \, \chi_2^e \, \xi_1^e + c_e \, \xi_2^e \, \rho_1^e = 0 \\
\frac{\partial w_{f,u}}{\partial \xi_1^{0e}} &= d_e \, \xi_2^e \sigma^e + f_e \, \xi_1^e \, \rho_1^e = 0 \\
\frac{\partial w_{f,u}}{\partial \xi_2^{0e}} &= d_e \, \xi_1^e \, \sigma^e + f_e \, \xi_2^e \, \rho_2^e = 0
\end{aligned}
$$

solved by vev configuration given above. . .