Viability of the exact Tri-Bimaximal mixing at the GUT scale in SO(10)

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Based on

- **A. S. Joshipura and K. M. Patel, [arXiv:1105.5943]**

TBM and Quark-Lepton Unification

- The observed mixing pattern among leptons indicates some underlying flavor symmetries *if not accidental*.
- Incorporating such symmetries or the tri-bimaximal mixing (TBM) into grand unified theories (GUTs), particularly based on the $SO(10)$ gauge group is quite challenging.

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- Since all fermions in a given generation are unified into a single 16_F, imposition of the TBM structure on the leptonic mass matrices also constrains the quark mass matrices.
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INTRODUCTION

Leptonic Mixing Matrices and TBM

- The TBM structure for $\mathcal{M}_{\nu f}$ (neutrino mass matrix in flavor basis)

$$\mathcal{M}_{\nu f} = \frac{1}{3}\begin{pmatrix} 2f_1 + f_2 & f_2 - f_1 & f_2 - f_1 \\ f_2 - f_1 & \frac{1}{2}(f_1 + 2f_2 + 3f_3) & \frac{1}{2}(f_1 + 2f_2 - 3f_3) \\ f_2 - f_1 & \frac{1}{2}(f_1 + 2f_2 - 3f_3) & \frac{1}{2}(f_1 + 2f_2 + 3f_3) \end{pmatrix}$$

- This matrix is diagonalized by

$$U_{PMNS} = O_{TBM}\,Q$$

where Q is a diagonal phase matrix.

- It is known that $\mathcal{M}_{\nu f}$ is invariant under $Z_2 \times Z_2$ symmetry

$$S_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad \text{and} \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

and satisfy

$$S_{2,3}^T \mathcal{M}_{\nu f} S_{2,3} = \mathcal{M}_{\nu f}$$

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$$S_{2,3}^T M_\nu S_{2,3} = M_\nu$$

- In this basis, if U_l itself is $Z_2 \times Z_2$ symmetric, *i.e.* satisfies

$$S_{2,3}^T U_l S_{2,3} = U_l$$

then $\mathcal{M}_{\nu f}$ will also be invariant under $Z_2 \times Z_2$ symmetry and exhibit the TBM structure.

- Such a U_l can be parameterized as

$$U_l = e^{i\alpha} P_l \tilde{U}_l P_l \ ,$$

$$\tilde{U}_l = \begin{pmatrix} c_\theta & \frac{s_\theta}{\sqrt{2}} & \frac{s_\theta}{\sqrt{2}} \\ \frac{s_\theta}{\sqrt{2}} & -\frac{1}{2}(c_\theta + e^{i\delta}) & -\frac{1}{2}(c_\theta - e^{i\delta}) \\ \frac{s_\theta}{\sqrt{2}} & -\frac{1}{2}(c_\theta - e^{i\delta}) & -\frac{1}{2}(c_\theta + e^{i\delta}) \end{pmatrix},$$

where $P_l = \mathrm{diag.}(1, e^{i\beta}, e^{i\beta})$ is a diagonal phase matrix and $\tan\theta = -2\sqrt{2}\cos\beta$.

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Leptonic Mixing Matrices and TBM

- The $Z_2 \times Z_2$ invariance of U_l is also **necessary** if $\mathcal{M}_{\nu f}$ is to exhibit the TBM structure.
- If $\mathcal{M}_{\nu f}$ has the TBM structure then $U_{PMNS} = O_{TBM} Q$.
- At the same time M_ν also has TBM structure in specified basis which implies $U_\nu = O_{TBM} P$.
- Since $U_l = U_\nu U^\dagger_{PMNS}$,

$$\begin{aligned} U_l &= O_{TBM} P Q^* O^T_{TBM} \ , \\ &= \frac{1}{3}\begin{pmatrix} 2p_1 + p_2 & p_2 - p_1 & p_2 - p_1 \\ p_2 - p_1 & \frac{1}{2}(p_1 + 2p_2 + 3p_3) & \frac{1}{2}(p_1 + 2p_2 - 3p_3) \\ p_2 - p_1 & \frac{1}{2}(p_1 + 2p_2 - 3p_3) & \frac{1}{2}(p_1 + 2p_2 + 3p_3) \end{pmatrix} \ , \end{aligned}$$

  p_i are the elements of the diagonal phase matrix PQ^*.
- The above U_l is obtained from the general TBM $\mathcal{M}_{\nu f}$, by replacing the neutrino masses with the phases p_i and like $\mathcal{M}_{\nu f}$ such a U_l is automatically $Z_2 \times Z_2$ symmetric.

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$SO(10)$ Model and TBM

Embedding leptonic structure in $SO(10)$

- We study the derived structure of M_ν and U_l in context of a well predictive susy GUT $SO(10)$ model.
- A particular class of $SO(10)$ model is studied in which we
  1. consider a supersymmetric $SO(10)$ model with the Higgs transforming as $10, \overline{126}, 120$ representations of $SO(10)$.
  2. impose the generalized parity leading to Hermitian mass matrices.

     [B.Dutta *et al.* (2004), W.Grimus, H.Kuhbock (2007)]
  3. assume that the dominant contribution to M_ν is a type-II seesaw, *i.e.* linear in the $\overline{126}$ Yukawa coupling.

$SO(10)$ Model and TBM

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- The fermion mass relations after electroweak symmetry breaking can be written in their most general forms as

$$M_d = H + F + iG \quad , \quad M_u = r(H + sF + it_u G),$$
$$M_l = H - 3F + it_l G \quad , \quad M_D = r(H - 3sF + it_D G)$$

where H, F are real symmetric and G is real anti-symmetric matrices. r, s, t_f are real dimensionless parameters. The light neutrino mass matrix is given by,

$$\mathcal{M}_\nu = r_L F - r_R M_D F^{-1} M_D^T \equiv \mathcal{M}_\nu^{II} + \mathcal{M}_\nu^{I}$$

- The 16-plet fermions can be rotated in generation space in such a way that

$$M_\nu \propto F \to R^T F R = F_{TBM} \equiv O_{TBM}\ \mathrm{Diag.}(f_1, f_2, f_3)\ O_{TBM}^T$$

is diagonalized by the TBM matrix.

- The model has altogether 17 independent real parameters (3 in F_{TBM}, 6 in H, 3 in G, r, s, t_u, t_l and r_L) which determine the entire 22 low energy observables of the fermion mass spectrum.

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Embedding leptonic structure in $SO(10)$

- Some of these parameters can be fixed by the known values of observables directly.
- The $SO(10)$ relation for the charged lepton mass matrix can be rewritten as

$$H + it_l G = V_l D_l V_l^\dagger + 3F_{TBM}$$

- V_l ($\sim \tilde{V}_l P$) is a unitary matrix that diagonalizes M_l and contains six free parameters in the most general case.
- If we fix $V_l = U_l$, where U_l has the same $Z_2 \times Z_2$ symmetry as $\mathcal{M}_\nu$ and parameterized by only three parameters, then we get an **exact TBM** lepton mixing.
- In such case, most of the free parameters are fixed in terms of the observables of the lepton sector and remaining 6 parameters has to reproduce 10 observables of the quark sector.

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Numerical Study of Model: χ^2 Analysis

- We do the χ^2 fitting to check the viability of the free parameters with the quark sector observables.
- We construct

$$\chi^2(\alpha_j) = \sum_i \left(\frac{X_i(\alpha_j) - O_i}{\sigma_i} \right)^2$$

  Where,
  X_i are the quark masses and mixing as complex nonlinear functions of parameters α_j calculated from the given model at GUT scale.
  O_i (σ_i) are the input mean values(1σ errors) of respective masses and mixing angles evaluated at M_{GUT}=2×10^{16} GeV.
- We assume normal hierarchy in neutrino masses and consider two separate cases corresponding to the individual dominance of type-I and type-II seesaw mechanism.
- Then the data are fitted by minimizing the χ^2 function with respect to parameters α_j using an algorithm based on the downhill simplex method.

$SO(10)$ Model and TBM: Numerical Analysis

Results of χ^2 minimization

	Case A		Case B1		Case B2	
Observables	Fitted value	Pull	Fitted value	Pull	Fitted value	Pull
$m_d[\mathrm{MeV}]$	1.19851	−0.101205	1.22098	−0.0463899	1.02686	−0.519852
$m_s[\mathrm{MeV}]$	22.1374	0.0841145	21.9922	0.0561874	22.0058	0.058806
$m_b[\mathrm{GeV}]$	1.05103	−0.0996223	1.16345	0.738942	1.2842	1.60145
$m_u[\mathrm{MeV}]$	0.550206	0.000824013	0.550234	0.000936368	0.550787	0.00314771
$m_c[\mathrm{GeV}]$	0.209956	−0.00208935	0.209952	−0.00230315	0.210481	0.0229054
$m_t[\mathrm{GeV}]$	82.6175	0.00717855	82.5855	0.00612198	81.7487	−0.0440052
$m_e[\mathrm{MeV}]$	0.3585	–	0.3585	–	0.3585	–
$m_\mu[\mathrm{MeV}]$	75.672	–	75.672	–	75.672	–
$m_\tau[\mathrm{GeV}]$	1.2922	–	1.2922	–	1.2922	–
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.031875	–	0.031875	–	0.031875	–
$\sin\theta^q_{12}$	0.224299	−0.0007232	0.2243	0.0002182	0.224303	0.0019076
$\sin\theta^q_{23}$	0.0350871	−0.0099165	0.0350951	−0.0038047	0.0351294	0.022597
$\sin\theta^q_{13}$	0.00317877	−0.0424606	0.00319436	−0.0112796	0.0031749	−0.0502087
$\sin^2\theta^l_{12}$	0.303622	−0.0171641	**0.3333**	–	**0.3333**	–
$\sin^2\theta^l_{23}$	0.501109	0.015842	**0.5**	–	**0.5**	–
$\sin^2\theta^l_{13}$	**0.0394**	–	**0**	–	**0**	–
J_{CP}	2.24×10^{-5}	0.0732629	2.21×10^{-5}	0.0194165	2.25×10^{-5}	0.0845729
δ_{MNS}	**273.934**	–	–	–	–	–
α_1	**186.801**	–	**160.829**	–	**180**	–
α_2	**70.8178**	–	**318.593**	–	**0**	–
χ^2_{min}		**0.0351**		**0.5519**		**2.8510**

Perturbed TBM

Possible Perturbations to exact TBM

- The TBM is an ideal situation and various perturbations to this can arise in the model.
- We need to analyze these perturbations in order to distinguish this case from the generic case without the built in TBM.
- A deviation from tri-bimaximality can arise due to
  1. renormalization group evolution (RGE) from M_{GUT} to M_Z.
  2. small contribution from the sub dominant type-I seesaw.
  3. the breaking of the $Z_2 \times Z_2$ symmetry in U_l which ensured TBM.
- The effect of (1) is known to be negligible in case of the hierarchical neutrino mass spectrum which we obtain here.

[A. Dighe *et al.* (2006), T. Araki *et al.* (2010)]

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Perturbed TBM

Perturbation from type-I seesaw

$$M_\nu = r_L(F - \xi M_D F^{-1} M_D^T)$$

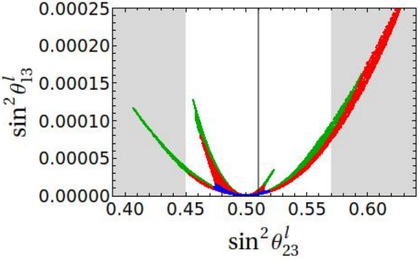
- A sub dominant contribution from type-I seesaw term can perturb the TBM.
- It brings two new parameters ξ and t_D present in M_D which however affect only the neutrino sector.
- ξ and t_D remain unconstrained at this minimum and their values do not change the χ^2 obtained earlier since the latter contains only the observables in the quark sector.
- We randomly vary the parameters ξ and t_D and evaluate the neutrino masses and mixing angles.
- While doing this, we take care that all these observables remain within their 3σ limits.

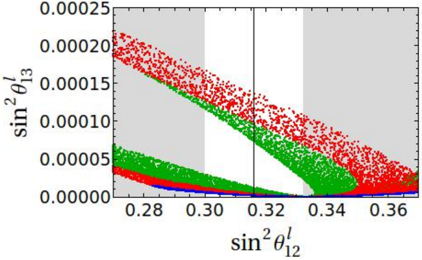
[T. Schwetz *et al.* (2011)]

- Such constrains allow very small values of $|\xi| \leq 10^{-7}$.

Perturbed TBM

Perturbation from type-I seesaw





- Correlations among the lepton mixing angles when two real parameters ξ and t_D are varied randomly.
- The perturbation induced by type-I term cannot generate considerable deviation in the reactor angle.
- Requiring that $\sin^2\theta^l_{12}$ remains within the 3σ range puts an upper bound $\sin^2\theta^l_{13} \leq 0.0002$.

Perturbed TBM

Perturbation from charged lepton mixing

- A different class of perturbation to TBM arise when U_l deviates from its $Z_2 \times Z_2$ symmetric form.
- We simultaneously perturb all three mixing angles by allowing the most general U_l.
- The specific value p_0 of an observable P is pinned down by adding a term

$$\chi_P^2 = \left(\frac{P - p_0}{0.01\ p_0}\right)^2$$

to χ^2 and then minimizing

$$\hat{\chi}^2 \equiv \chi^2 + \chi_P^2 \ .$$

- Artificially introduced small error fixes the value p_0 for the observable P at the minimum of $\hat{\chi}^2$.
- We then look at the variation of

$$\bar{\chi}^2_{min} \equiv (\hat{\chi}^2 - \chi_P^2)|_{min}$$

with p_0.

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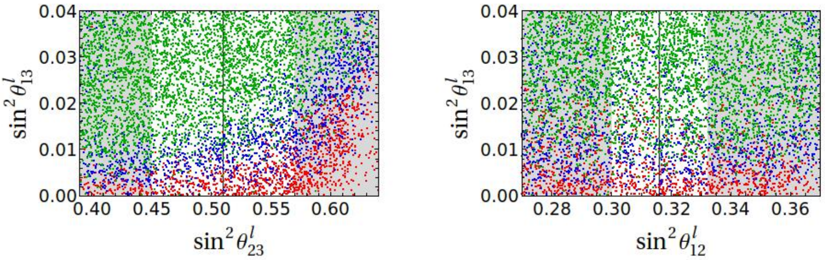

- Correlations among the lepton mixing angles in case of the general charged lepton mixing matrix U_l.
- $\bar{\chi}^2_{min} < 1$ (green); $1 \leq \bar{\chi}^2_{min} < 4$ (blue); $\bar{\chi}^2_{min} \geq 4$ (red)
- The region $\bar{\chi}^2_{min} < 4$ falls largely above $\sin^2\theta^l_{13} > 0.005$ for $\sin^2\theta^l_{23} = 0.5$ which may be regarded as an approximate lower bound on θ^l_{13}.

SUMMARY

- General structures of the charged lepton and the neutrino mixing matrices leading to tri-bimaximal leptonic mixing are determined.
- We have shown that it is possible to construct a class of non-trivial U_l quite different from identity which preserve the TBM structure of M_ν when transformed to the flavour basis.
- Identification of such non-trivial U_l becomes crucial in the context of $SO(10)$ and allows us to obtain a viable fit to fermion spectrum keeping TBM intact.
- The quality of fit obtained in this case is excellent and differs only marginally from a general situation without imposing the TBM structure at the outset.
- The existence of TBM at the GUT scale may be inferred by considering its breaking which can arise in the model and the reactor mixing angle is a good pointer to this.

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- We have shown that it is possible to construct a class of non-trivial U_l quite different from identity which preserve the TBM structure of M_ν when transformed to the flavour basis.
- Identification of such non-trivial U_l becomes crucial in the context of $SO(10)$ and allows us to obtain a viable fit to fermion spectrum keeping TBM intact.
- The quality of fit obtained in this case is excellent and differs only marginally from a general situation without imposing the TBM structure at the outset.
- The existence of TBM at the GUT scale may be inferred by considering its breaking which can arise in the model and the reactor mixing angle is a good pointer to this.

SUMMARY

- The quantum corrections lead to very small θ_{13}^{l} for the hierarchical neutrinos.
- Similarly, corrections coming from type-I seesaw term imply an upper bound, $\sin^2\theta_{13}^{l} \leq 0.0002$
- These two cases are in sharp contrast to a situation in which one does not impose the TBM at M_{GUT} by breaking $Z_2 \times Z_2$ symmetry of U_l. In this case, one is lead to an approximate lower bound $\sin^2\theta_{13}^{l} \geq 0.005$.
- θ_{13}^{l} can thus provide a good way of determining the existence or otherwise of the exact TBM at M_{GUT} in the specific model considered here.

THANKS

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