From Flavour to SUSY Flavour Models

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Based on Antusch, Calibbi, V.M. & Spinrath arXiv:1104.3040v1

Outline

1. Motivation

2. Defining a SUSY Flavour Model

3. Testing a SUSY Flavour Model

4. Summary

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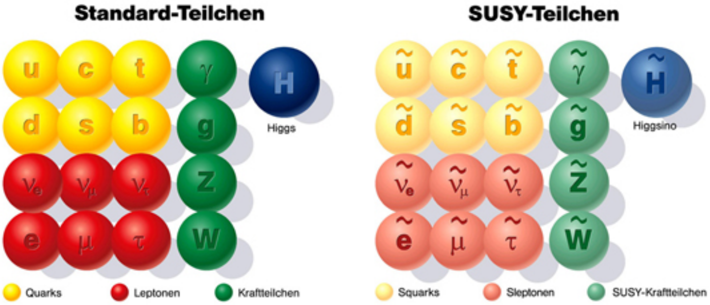
What we want to describe



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Class of Models: Matter Fields

- Symmetries:

$$SU(5) \times G_{\text{family}}$$

- Matter fields:

$$F \sim (\bar{5}, 3) \quad T_{1,2,3} \sim (10, 1) \quad N_{1,2} \sim (1, 1)$$

- $SU(5) \to \text{SM}$

$$F = (d^c, L) \quad T = (Q, u^c, e^c)$$

$$Y_d \sim Y_e^T$$

Class of Models: GUT Symmetry Breaking

- Adjoint of $SU(5)$:

$$H_{24} \sim (24, 1)$$

- Broken into direction

$$H_{24} \propto Y = \begin{pmatrix} \frac{1}{3} & & & & \\ & \frac{1}{3} & & & \\ & & \frac{1}{3} & & \\ & & & -\frac{1}{2} & \\ & & & & -\frac{1}{2} \end{pmatrix}$$

$\Rightarrow$ Different coupling to F-submultiplets

Class of Models: Family Symmetry Breaking

- Flavon fields:

$$\phi_i \sim (1,3)$$

- G_{family} broken by VEVs in the directions [Antusch, King, Spinrath '10]

$$\phi_1 \sim \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \phi_2 \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \tilde{\phi}_2 \sim \begin{pmatrix} 0 \\ \mathrm{i} \\ w \end{pmatrix}, \phi_3 \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- Yukawa matrices of the form

$$Y \sim \frac{1}{M} \begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \langle \phi_1 \rangle & \langle \phi_2 \rangle + \langle \tilde{\phi}_2 \rangle & \langle \phi_3 \rangle \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \frac{H_{24}}{M} \qquad (1)$$

Class of Models: Matrix Textures

M_N diagonal

$$Y_\nu^T = \begin{pmatrix} 0 & y_1 & -y_1 \\ y_2 & y_2 & y_2 \end{pmatrix} \qquad \Rightarrow m_1 = 0, \sim\text{TBM}$$

Y_u diagonal

$$Y_d = \begin{pmatrix} 0 & \epsilon_1 & -\epsilon_1 \\ \epsilon_2 & \epsilon_2 + \mathrm{i}\,\tilde{\epsilon}_2 & \epsilon_2 + w\,\tilde{\epsilon}_2 \\ 0 & 0 & \epsilon_3 \end{pmatrix}$$

$$Y_e^T = \begin{pmatrix} 0 & c_1\,\epsilon_1 & -c_1\,\epsilon_1 \\ c_2\,\epsilon_2 & c_2\,\epsilon_2 + \mathrm{i}\,\tilde{c}_2\,\tilde{\epsilon}_2 & c_2\,\epsilon_2 + w\,\tilde{c}_2\,\tilde{\epsilon}_2 \\ 0 & 0 & c_3\,\epsilon_3 \end{pmatrix}$$

$$\Rightarrow \frac{y_\tau}{y_b} = \frac{3}{2},\ \theta_{13}^{\text{CKM}}$$

with $c_1 = c_2 = c_3 = -\frac{3}{2}, \quad \tilde{c}_2 = 6$ [Antusch, Spinrath '09]

- Kähler potential

$$K = F^\dagger F + T_i^\dagger T_i + \frac{\phi_i^\dagger \phi_i}{M^2} F^\dagger F + \frac{\phi_i^\dagger \phi_i}{M^2} T_i^\dagger T_i$$

- Using hierarchy $\epsilon_3 \sim y_b \gg y_{d,s} \sim \epsilon_{1,2,\tilde{2}}$:

$$\tilde{K}_{FF^\dagger} \approx \mathrm{diag}(1, 1, 1 + \zeta^2)$$

with $\zeta^2 \sim \frac{\phi_3^\dagger \phi_3}{M^2}$

- Non-canonical kinetic terms $\Rightarrow F \to \mathrm{diag}(1, 1, 1 - \frac{1}{2}\zeta^2) F$

[Antusch, King, Malinsky '07] [Antusch, Calibbi, V.M., Spinrath '11]

Class of Models: Matrix Textures in Canonical Basis

M_N diagonal

$$Y_\nu^T = \begin{pmatrix} 0 & y_1 & -y_1 k \\ y_2 & y_2 & y_2 k \end{pmatrix}$$

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$\Rightarrow \frac{y_\tau}{y_b} = \frac{3}{2}$, $\theta_{13}^{\mathrm{CKM}} - \zeta^2$

$$Y_e^T = \ldots$$

with $k = 1 - \frac{1}{2}\zeta^2$

[Antusch, Calibbi, V.M., Spinrath '11]

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S~~USY~~ Mediation and Soft Terms

SUSY breaking mediated by supergravity

- Typically SUSY breaking by all fields with VEVs

$$F_\phi = \mathcal{O}(1) m_{3/2} \langle \phi \rangle$$

- SUGRA & sequestering results in

$$\tilde{m}^2 = m_{3/2}^2 \tilde{K} - F_{\bar{n}} F_m \partial_{\bar{n}} \partial_m \tilde{K}$$
$$A = A_0 Y + F_m \partial_m Y$$

with n, m running over flavons

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Class of Models: Soft Terms

Small Deviations from CMSSM

- Soft masses

$$m^2_{\tilde{F}} = m_0^2 \,\mathrm{diag}(1, 1, 1 - \hat{x}_3^2\, \zeta^2)$$

- Trilinear couplings

$$A_d = A_0 \begin{pmatrix} 0 & x_1\, \epsilon_1 & -x_1\, \epsilon_1\, (1 - \frac{1}{2}\zeta^2) \\ x_2\, \epsilon_2 & x_2\, \epsilon_2 + \mathrm{i}\, \tilde{x}_2\, \tilde{\epsilon}_2 & (x_2\, \epsilon_2 + \tilde{x}_2\, w\, \tilde{\epsilon}_2)\,(1 - \frac{1}{2}\zeta^2) \\ 0 & 0 & x_3\, \epsilon_3\, (1 - \frac{1}{2}\zeta^2) \end{pmatrix}$$

$\not\propto Y_d \Rightarrow$ not diagonal in SCKM basis

- Almost CMSSM spectrum
- Flavour and CP violation effects dominated by A terms

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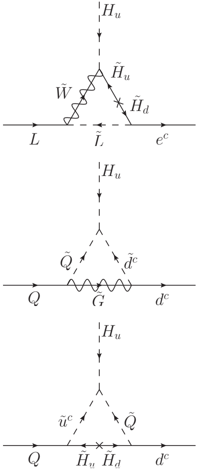
SUSY Threshold Corrections and Parameterisation

Simple formulae for $\tan\beta$ enhanced corrections to Y_d and Y_e



$$y^{\text{SM}}_{e,\mu,\tau} \approx (1 + \epsilon_l \tan\beta)\, y^{\text{MSSM}}_{e,\mu,\tau} \cos\beta$$
$$y^{\text{SM}}_{d,s} \approx (1 + \epsilon_q \tan\beta)\, y^{\text{MSSM}}_{d,s} \cos\beta$$
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$$\theta^{\text{SM}}_{i3} \approx \frac{1 + \epsilon_q \tan\beta}{1 + (\epsilon_q + \epsilon_A)\tan\beta}\, \theta^{\text{MSSM}}_{i3}$$

$$\theta^{\text{SM}}_{12} \approx \theta^{\text{MSSM}}_{12}$$
$$\delta^{\text{SM}}_{\text{CKM}} \approx \delta^{\text{MSSM}}_{\text{CKM}}$$

[Antusch, Calibbi, V.M., Spinrath '11]

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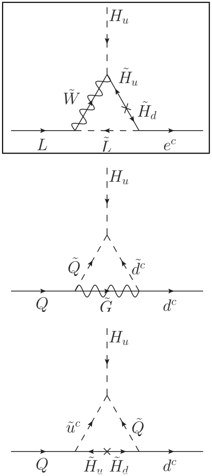

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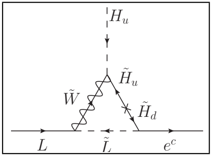

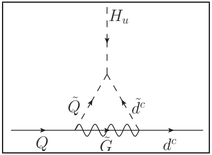

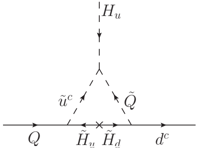

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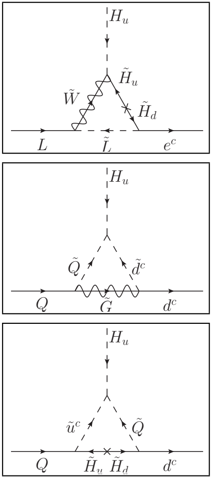

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[Antusch, Calibbi, V.M., Spinrath '11]

Application to Our Matrix Textures

GUT ratios and θ_{13}^{CKM} require (for $\tan\beta = 30$)

- θ_{13}^{CKM}: $3.62 \times (1 - \frac{1}{2}\zeta^2) \overset{!}{=} (1 + \epsilon_A \tan\beta) \times 3.24$

$$\Rightarrow \boxed{\epsilon_A \tan\beta + \frac{1}{2}\zeta^2 = 0.11}$$

- $\frac{y_\tau}{y_b}$: $\frac{3}{2} \overset{!}{=} (1 + (\epsilon_A + \epsilon_q - \epsilon_l)\tan\beta) \times 1.26$

$$\Rightarrow \boxed{(\epsilon_A + \epsilon_q - \epsilon_l)\tan\beta = 0.19}$$

[Antusch, Calibbi, V.M., Spinrath '11]

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$$\Rightarrow \boxed{(\epsilon_A + \epsilon_q - \epsilon_l)\tan\beta = 0.19}$$

- $\frac{y_\mu}{y_s}$: $\Rightarrow \boxed{(\epsilon_q - \epsilon_l)\tan\beta = 0.32}$

- $\frac{y_e}{y_d}$: $\Rightarrow \boxed{(\epsilon_q - \epsilon_l)\tan\beta = 0}$

[Antusch, Calibbi, V.M., Spinrath '11]

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GUT ratios and θ_{13}^{CKM} require (for $\tan\beta = 30$)

- θ_{13}^{CKM}: $3.62 \times (1 - \frac{1}{2}\zeta^2) \overset{!}{=} (1 + \epsilon_A \tan\beta) \times 3.24 \pm 0.15$

$$\Rightarrow \boxed{\epsilon_A \tan\beta + \frac{1}{2}\zeta^2 = 0.11 \pm 0.04}$$

- $\frac{y_\tau}{y_b}$: $\frac{3}{2} \overset{!}{=} (1 + (\epsilon_A + \epsilon_q - \epsilon_l)\tan\beta) \times 1.26 \pm 0.05$

$$\Rightarrow \boxed{(\epsilon_A + \epsilon_q - \epsilon_l)\tan\beta = 0.19 \pm 0.05}$$

- $\frac{y_\mu}{y_s}$: $\Rightarrow \boxed{(\epsilon_q - \epsilon_l)\tan\beta = 0.32 \pm 0.38}$

- $\frac{y_e}{y_d}$: $\Rightarrow \boxed{(\epsilon_q - \epsilon_l)\tan\beta = 0 \pm 0.44}$

[Antusch, Calibbi, V.M., Spinrath '11]

Application to Our Matrix Textures

GUT ratios and $\theta_{13}^{\mathrm{CKM}}$ require (for $\tan\beta = 30$)

- $\theta_{13}^{\mathrm{CKM}}$: $3.62 \times (1 - \frac{1}{2}\zeta^2) \stackrel{!}{=} (1 + \epsilon_A \tan\beta) \times 3.24 \pm 0.15$

$$\Rightarrow \boxed{\epsilon_A \tan\beta + \frac{1}{2}\zeta^2 = 0.11 \pm 0.04}$$

- $\frac{y_\tau}{y_b}$: $\frac{3}{2} \stackrel{!}{=} (1 + (\epsilon_A + \epsilon_q - \epsilon_l)\tan\beta) \times 1.26 \pm 0.05$

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[Antusch, Calibbi, V.M., Spinrath '11]

Matching Spectrum with Matrix Textures

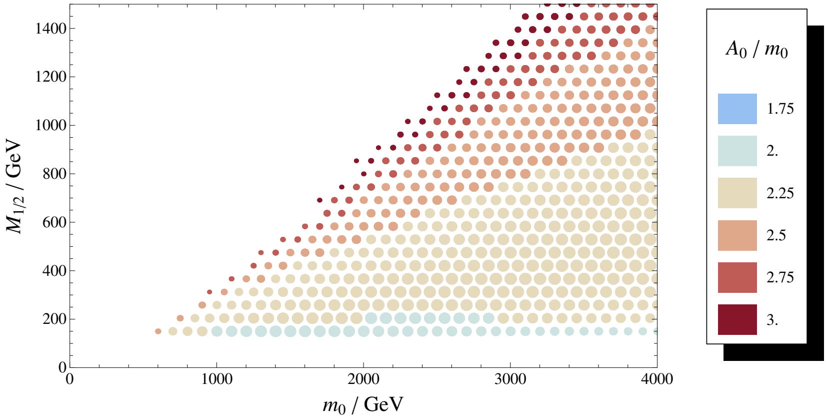

Figure: Points satisfying the ratio and angle constraints ($\tan\beta = 30$ & $\mu > 0$)

[Antusch, Calibbi, V.M., Spinrath ’11]

Matching Spectrum with Matrix Textures: Numerics

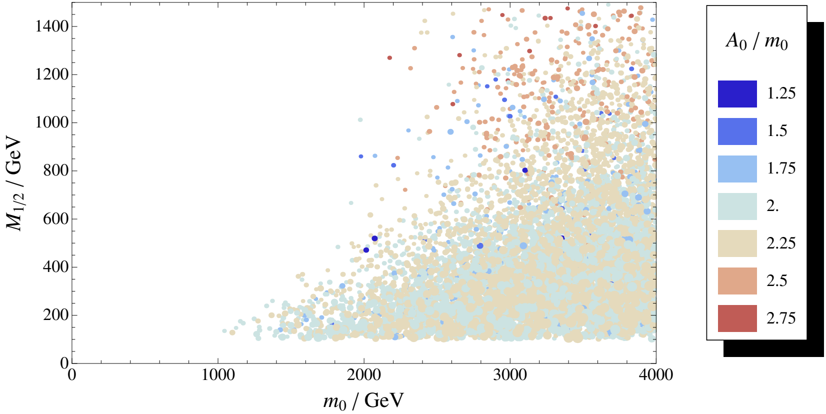

Figure: Points found by MCMC with $\mathcal{P} \geq 5\%$ ($\tan\beta = 30$ & $\mu > 0$)

[Antusch, Calibbi, V.M., Spinrath '11]

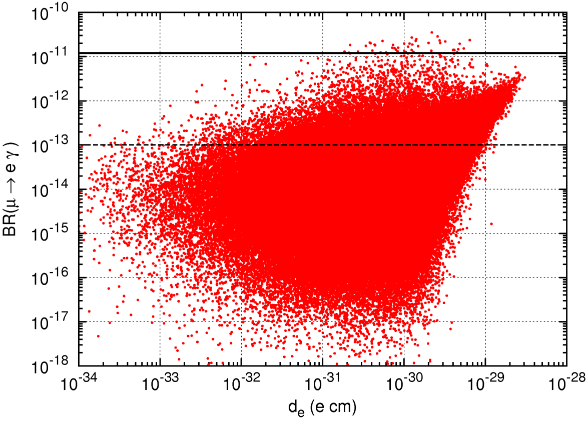

Figure: Points found by varying x_1, x_2, $x_{\tilde{2}}$ around previous points.

[Antusch, Calibbi, V.M., Spinrath '11]

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Summary

- Extend flavour models to SUSY/SUGRA flavour models
- Keep track of all effects!
  - Canonical Normalisation
  - SUSY Threshold Corrections
  - Deviations from CMSSM
- Tests for SUSY/SUGRA flavour models
  - SUSY Spectrum
  - Flavour Violation
  - CP Violation

End of the Talk

Thank you for your attention!

Backup: Concrete Neutrino Values

- Almost Tribimaximal Mixing [cf. Antusch, King, Malinsky 2008]

$$\sin\theta_{12}^{\text{MNS}} \approx \frac{1}{\sqrt{3}}\left(1+\frac{1}{6}\zeta^2\right)$$

$$\sin\theta_{23}^{\text{MNS}} \approx \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\zeta^2\right)$$

$$\theta_{13}^{\text{MNS}} \approx \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\zeta^2\right)\frac{1}{3}\theta_{12}^{\text{CKM}}$$

- Normal Hierarchy with

$$0 = m_1 < m_2 < m_3$$

- Maximal CP Violation

$$\delta_{\text{MNS}} = -90^\circ$$

[Antusch, King '08]

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- Almost Tribimaximal Mixing [cf. Antusch, King, Malinsky 2008]

$$\sin\theta_{12}^{\text{MNS}} \approx \frac{1}{\sqrt{3}}\left(1+\frac{1}{6}\zeta^2\right) \gtrsim 0.577 \quad (\text{Exp: } 0.565 \pm 0.014)$$

$$\sin\theta_{23}^{\text{MNS}} \approx \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\zeta^2\right) \gtrsim 0.707 \quad (\text{Exp: } 0.679^{+0.57}_{-0.038})$$

$$\theta_{13}^{\text{MNS}} \approx \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\zeta^2\right)\frac{1}{3}\theta_{12}^{\text{CKM}} \gtrsim 0.06 \quad (\text{T2K: } 0.17^{+0.05}_{-0.07})$$

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- Maximal CP Violation

$$\delta_{\text{MNS}} = -90^\circ$$

[Antusch, King '08] , [Gonzalez-Garcia, Maltoni, Salvado '10] , [T2K '11]

Backup: Concrete Quarks and Leptons GUT Scale Values

- Yukawa Ratios

$$\frac{y_\tau}{y_b} = \frac{3}{2}$$

$$\frac{y_\mu}{y_s} = (1 - \mathcal{O}(\tan^{-2}\delta_{\mathrm{CKM}}))\ 6 \approx 5.6$$

$$\frac{y_e}{y_d} = \frac{9}{4}\frac{y_s}{y_\mu} \approx 0.4$$

- Mixing Angles

$$\theta_{13}^{\mathrm{CKM}} = \theta_{13}^{\mathrm{CKM}}(y_e, y_\mu, y_\tau, \theta_{12}^{\mathrm{CKM}}, \zeta^2) \approx 3.62 \times 10^{-3}\left(1 - \frac{1}{2}\zeta^2\right)$$

- CP Violation

$$\delta_{\mathrm{CKM}} = 1.28$$