Maximal atmospheric neutrino mixing from texture zeros and quasi-degenerate neutrino masses

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Lepton mixing

Lepton mass terms:

$$\mathcal{L} = -\bar{\ell}'_R \mathcal{M}_\ell \ell'_L + \frac{1}{2} {\nu'_L}^T C^{-1} \mathcal{M}_\nu \nu'_L + \mathrm{H.c.} =$$
$$= -\bar{\ell}_R \hat{\mathcal{M}}_\ell \ell_L + \frac{1}{2} {\nu_L}^T C^{-1} \hat{\mathcal{M}}_\nu \nu_L + \mathrm{H.c.}$$

Diagonalization by a biunitary transformation:

$$\mathcal{M}_\ell = U_R^\ell \hat{\mathcal{M}}_\ell U_L^{\ell\dagger}, \quad \mathcal{M}_\nu = U_L^{\nu *} \hat{\mathcal{M}}_\nu U_L^{\nu\dagger}$$

$$\Rightarrow U_L^{\nu\dagger} \nu'_L = \nu_L, \quad U_L^{\ell\dagger} \ell'_L = \ell_L$$

Charged current interaction:

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}'_L \gamma^\mu \nu'_L + \mathrm{H.c.} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu \underbrace{U_L^{\ell\dagger} U_L^\nu}_{U_{\mathrm{PMNS}}} \nu_L + \mathrm{H.c.}$$

Parameters of the lepton mixing matrix

U_{PMNS} can be parameterized by three mixing angles, the CP-violating phase δ and two physical Majorana phases.

parameter	best fit	2σ	HPS
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	$0.28 - 0.35$	$\frac{1}{3}$
$\sin^2\theta_{23}$	0.51 ± 0.06	$0.41 - 0.61$	$\frac{1}{2}$
	0.52 ± 0.06	$0.42 - 0.61$	$\frac{1}{2}$
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$	≤ 0.027	0
	$0.013^{+0.009}_{-0.007}$	≤ 0.031	0

T. Schwetz, M. Tórtola and J.W.F. Valle (2011)

Extremal mixing angles

U_{PMNS} is compatible with the Harrison Perkins Scott (HPS) mixing matrix within 2σ.

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\rightarrow$ Idea: **symmetries in the lepton sector**

HPS mixing matrix: two extremal mixing angles:

$$\theta_{13} = 0^\circ, \quad \theta_{23} = 45^\circ$$

C.I. Low (2005):

The only extremal mixing angle which can be obtained by an Abelian symmetry is $\theta_{13} = 0^\circ$.

Near extremal atmospheric mixing

$\theta_{23} = 45^\circ \Rightarrow$ **non-Abelian symmetry needed**

$\longrightarrow$ Can we achieve $\theta_{23} \approx 45^\circ$ using Abelian symmetries?

$\longrightarrow$ **Yes, we can!**

Basic idea: U_{PMNS} close to U_{HPS}

U_{PMNS} looks like a matrix consisting of "pure numbers"

Famous relation in the quark sector:

$$\sin\theta_c \approx \sqrt{\frac{m_d}{m_s}}$$

$\longrightarrow$ perhaps similar situation in the lepton sector!

Near extremal atmospheric mixing

Ansatz: If $\mathcal{M}_\ell$ (approximately) diagonal

$\Rightarrow$ lepton mixing from bidiagonalization of $\mathcal{M}_\nu$ only.

Suppose we would have a relation between the mixing angles and the neutrino mass ratios of the form

$$\sin^2\theta_{ij} = f_{ij}(m_a/m_b).$$

$\Rightarrow$ In the limit of a quasi-degenerate neutrino mass spectrum

$$\frac{m_a}{m_b} \to 1 \quad \Rightarrow \quad \sin^2\theta_{ij} \to f_{ij}(1) \quad \text{independent of the masses.}$$

$\Rightarrow$ U_{PMNS} could look like a matrix consisting of "pure numbers".

Cosmological bounds: PDG (2010):

$$\sum_i m_i < 1\,\mathrm{eV} \Rightarrow \text{quasi-degenerate spectrum not excluded yet.}$$

The framework of our analysis

Analysis of W. Grimus and PL (2011):

Assumptions:

- The charged lepton mass matrix $\mathcal{M}_\ell$ is (approximately) diagonal.
- We have an Abelian family symmetry in the lepton sector.
- The neutrino mass spectrum is quasi-degenerate.
- We assume Majorana nature for the neutrinos.

Only restrictions of Abelian symmetries on $\mathcal{M}_\nu$: **texture zeros**.

$\Rightarrow$ We search for **symmetric** neutrino mass matrices with **texture zeros** compatible with the experimental data.

Zeros of the neutrino mass matrix

P.H. Frampton, S.L. Glashow and D. Marfatia (2002):

In the basis where $\mathcal{M}_\ell$ is diagonal, there are seven cases of two-texture zeros that are compatible with the experimental data.

$$A_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad B_1 = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},$$

$$B_2 = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}, \quad B_3 = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad B_4 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.$$

The cases B_3 and B_4

$$B_3 : \mathcal{M}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad B_4 : \mathcal{M}_\nu \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$$

$\mathcal{M}_\ell$ diagonal $\Rightarrow U_{\mathrm{PMNS}} = U_L^\nu =: U$.

$$\mathcal{M}_\nu = U^* \,\mathrm{diag}(m_1, m_2, m_3)\, U^\dagger$$

Two texture zeros in $\mathcal{M}_\nu \Rightarrow$ two equations of the form

$$(\mathcal{M}_\nu)_{ij} = \sum_k m_k U_{ik}^* U_{jk}^* = 0.$$

Using the standard parameterization:

$$U = e^{i\hat{\alpha}} V e^{i\hat{\sigma}}, \quad \hat{\alpha}, \hat{\sigma} \text{ diagonal}$$

$$\sum_k \mu_k V_{ik}^* V_{jk}^* = 0, \quad \mu_k = m_k e^{2i\sigma_k}.$$

The cases B_3 and B_4

Two equations of the form:

$$\sum_k \mu_k V_{ik}^* V_{jk}^* = 0, \quad \mu_k = m_k e^{2i\sigma_k}.$$

Input: m_1, m_2, m_3, θ_{12}, θ_{13} $\Rightarrow$ predictions: θ_{23}, δ, $\sigma_{1,2}$

Note: $\sin^2\theta_{13} > 0$, because else $m_1 = m_2$.

One obtains a cubic equation for $\lambda = \tan^2\theta_{23}$

$$\lambda^3 + \lambda^2 \left[s_{13}^2 + c_{13}^2 (c_{12}^2 \rho_1 + s_{12}^2 \rho_2) \right] - \lambda \left[s_{13}^2 \rho_1 \rho_2 + c_{13}^2 (s_{12}^2 \rho_1 + c_{12}^2 \rho_2) \right] - \rho_1 \rho_2 = 0.$$

with

$$\rho_i = \left(\frac{m_i}{m_3} \right)^2 \quad \text{(for } B_3\text{)} \quad \text{and} \quad \rho_i \to \frac{1}{\rho_i} \quad \text{(for } B_4\text{)}.$$

The limit of a quasi-degenerate spectrum

In the limit of a quasi-degenerate neutrino mass spectrum $\rho_i \to 1$ the cubic equation becomes

$$\lambda^3 + \lambda^2 - \lambda - 1 = 0$$

$$\Rightarrow \lambda = \tan^2\theta_{23} = 1 \Rightarrow \theta_{23} = 45^\circ.$$

Limit independent of the values of θ_{12} and θ_{13}!

Approximate solution of the exact equation:

$$\sin^2\theta_{23} \simeq \frac{1}{2} \mp \frac{1}{8}\frac{\Delta m^2_{31}}{m_1^2}(1 + \sin^2\theta_{13})$$

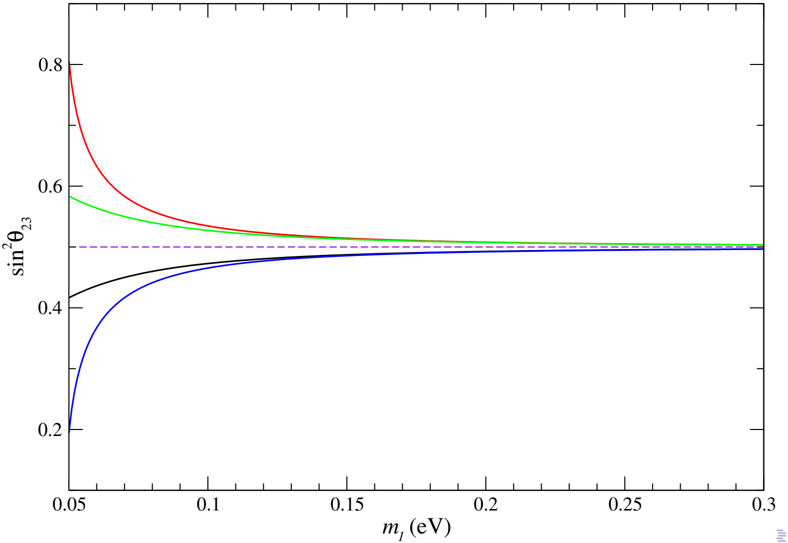
This indeed has the desired form

$$\sin^2\theta_{23} = f(m_a/m_b).$$

The limit of a quasi-degenerate spectrum

(Δm^2_{21}, Δm^2_{31}, $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ fixed to best fit values.)

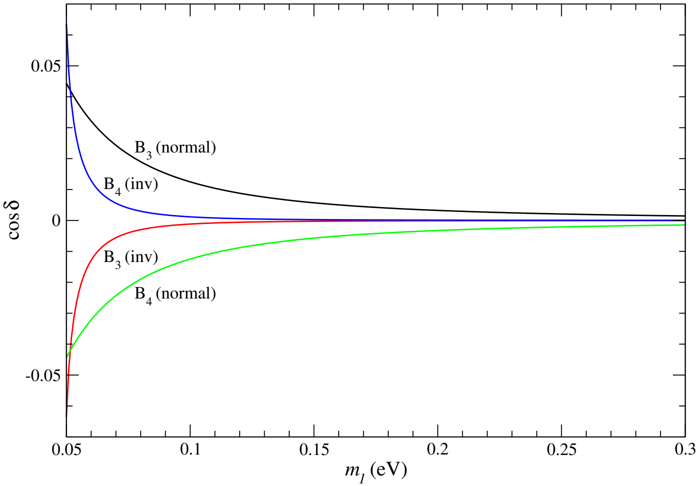
B_3 (inverted) B_4 (normal) B_3 (normal) B_4 (inverted)



The limit of a quasi-degenerate spectrum

In the limit of a quasi-degenerate mass spectrum:

$$\tan\theta_{12} \sin\theta_{13} \cos\delta \rightarrow 0.$$



(Δm^2_{21}, Δm^2_{31}, $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ fixed to best fit values.)

Lower bounds on m_1

In the limit of small m_1: $|\cos\delta| \leq 1 \Rightarrow$ lower bound on m_1

Lower bound dependent on the value of $\sin^2\theta_{13}$:

- inverted spectrum: bound $m_1 \gtrsim 0.05\,\mathrm{eV}$ stable for $\sin^2\theta_{13} \gtrsim 0.0001$.
- normal spectrum:

$$m_1 \gtrsim 0.03\,\mathrm{eV} \text{ at } \sin^2\theta_{13} \simeq 0.0001.$$
$$m_1 \gtrsim 0.01\,\mathrm{eV} \text{ at } \sin^2\theta_{13} \simeq 0.01.$$

$\Rightarrow$ Quasi-degenerate spectrum not implied by texture zeros B_3 and B_4. $\Rightarrow$ has to be postulated.

Conclusions

- We have found two instances in the framework of two texture zeros in $\mathcal{M}_\nu$ ($\mathcal{M}_\ell$ diagonal) which lead to maximal atmospheric neutrino mixing in the limit of a quasi-degenerate neutrino mass spectrum.
- In this scenario
$$\lim_{m_1 \to \infty} \sin^2\theta_{23} = \frac{1}{2}$$
is independent of the values of θ_{13} and θ_{12}!
- $\theta_{23} \approx 45^\circ$ and $\cos\delta \approx 0$ achievable by the use of an Abelian symmetry. This could be an alternative road to explain near maximal atmospheric neutrino mixing.