Maximal atmospheric neutrino mixing from texture zeros and quasi-degenerate neutrino masses

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Lepton mass terms:

\[ \mathcal{L} = -\bar{\ell}_R \mathcal{M}_\ell \ell_L' + \frac{1}{2} \nu_L' T C^{-1} \mathcal{M}_\nu \nu_L' + \text{H.c.} = \]

\[ = -\bar{\ell}_R \hat{\mathcal{M}}_\ell \ell_L + \frac{1}{2} \nu_L T C^{-1} \hat{\mathcal{M}}_\nu \nu_L + \text{H.c.} \]

Diagonalization by a biunitary transformation:

\[ \mathcal{M}_\ell = U_R^\ell \hat{\mathcal{M}}_\ell U_L^{\ell \dagger}, \quad \mathcal{M}_\nu = U_L^{\nu \ast} \hat{\mathcal{M}}_\nu U_L^{\nu \dagger} \]

\[ \Rightarrow U_L^{\nu \dagger} \nu_L' = \nu_L, \quad U_L^{\ell \dagger} \ell_L' = \ell_L \]

Charged current interaction:

\[ \mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}} \bar{W}_\mu^L \ell_L' \gamma^\mu \nu_L' + \text{H.c.} = -\frac{g}{\sqrt{2}} \bar{W}_\mu^L \ell_L \gamma^\mu \underbrace{U_L^{\ell \dagger} U_L^\nu}_{U_{\text{PMNS}}} \nu_L + \text{H.c.} \]
$U_{PMNS}$ can be parameterized by three mixing angles, the CP-violating phase $\delta$ and two physical Majorana phases.

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit</th>
<th>$2\sigma$</th>
<th>HPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.312^{+0.017}_{-0.015}$</td>
<td>$0.28 - 0.35$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.51 \pm 0.06$</td>
<td>$0.41 - 0.61$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$0.52 \pm 0.06$</td>
<td>$0.42 - 0.61$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.010^{+0.009}_{-0.006}$</td>
<td>$\leq 0.027$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$0.013^{+0.009}_{-0.007}$</td>
<td>$\leq 0.031$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Extremal mixing angles

$U_{\text{PMNS}}$ is compatible with the Harrison Perkins Scott (HPS) mixing matrix within $2\sigma$.

$$U_{\text{HPS}} = \begin{pmatrix}
    \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
    -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
    \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.$$

→ Idea: symmetries in the lepton sector

HPS mixing matrix: two extremal mixing angles:

$$\theta_{13} = 0^\circ, \quad \theta_{23} = 45^\circ$$

C.I. Low (2005):

The only extremal mixing angle which can be obtained by an Abelian symmetry is $\theta_{13} = 0^\circ$. 
Near extremal atmospheric mixing

\[ \theta_{23} = 45^\circ \Rightarrow \text{non-Abelian symmetry needed} \]

\[ \Rightarrow \text{Can we achieve } \theta_{23} \approx 45^\circ \text{ using Abelian symmetries?} \]

\[ \Rightarrow \text{Yes, we can!} \]

Basic idea: \( U_{\text{PMNS}} \) close to \( U_{\text{HPS}} \)

\( U_{\text{PMNS}} \) looks like a matrix consisting of “pure numbers”

Famous relation in the quark sector:

\[ \sin \theta_c \approx \sqrt{\frac{m_d}{m_s}} \]

\[ \Rightarrow \text{perhaps similar situation in the lepton sector!} \]
Ansatz: If $\mathcal{M}_\ell$ (approximately) diagonal

$\Rightarrow$ lepton mixing from bidiagonalization of $\mathcal{M}_\nu$ only.

Suppose we would have a relation between the mixing angles and the neutrino mass ratios of the form

$$\sin^2 \theta_{ij} = f_{ij}(m_a/m_b).$$

$\Rightarrow$ In the limit of a quasi-degenerate neutrino mass spectrum

$$\frac{m_a}{m_b} \to 1 \Rightarrow \sin^2 \theta_{ij} \to f_{ij}(1)$$

independent of the masses.

$\Rightarrow U_{\text{PMNS}}$ could look like a matrix consisting of “pure numbers”.

Cosmological bounds: PDG (2010):

$$\sum_i m_i < 1 \text{ eV} \Rightarrow \text{quasi-degenerate spectrum not excluded yet.}$$
The framework of our analysis

Analysis of W. Grimus and PL (2011):

Assumptions:

- The charged lepton mass matrix $M_\ell$ is (approximately) diagonal.
- We have an Abelian family symmetry in the lepton sector.
- The neutrino mass spectrum is quasi-degenerate.
- We assume Majorana nature for the neutrinos.

Only restrictions of Abelian symmetries on $M_\nu$: *texture zeros*.

⇒ We search for symmetric neutrino mass matrices with *texture zeros* compatible with the experimental data.
In the basis where $\mathcal{M}_\ell$ is diagonal, there are seven cases of two-texture zeros that are compatible with the experimental data.

\[
A_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad B_1 = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix},
\]

\[
B_2 = \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}, \quad B_3 = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad B_4 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix},
\]

\[
C = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.
\]
The cases $B_3$ and $B_4$

\[ B_3 : \mathcal{M}_\nu \sim \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad B_4 : \mathcal{M}_\nu \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix} \]

$\mathcal{M}_\ell$ diagonal $\Rightarrow$ $U_{\text{PMNS}} = U_\nu^L : = U$.

\[ \mathcal{M}_\nu = U^* \text{diag}(m_1, m_2, m_3) U^\dagger \]

Two texture zeros in $\mathcal{M}_\nu$ $\Rightarrow$ two equations of the form

\[ (\mathcal{M}_\nu)_{ij} = \sum_k m_k U^*_{ik} U^*_{jk} = 0. \]

Using the standard parameterization:

\[ U = e^{i\hat{\alpha}} V e^{i\hat{\sigma}}, \quad \hat{\alpha}, \hat{\sigma} \text{ diagonal} \]

\[ \sum_k \mu_k V^*_{ik} V^*_{jk} = 0, \quad \mu_k = m_k e^{2i\sigma_k}. \]
The cases $B_3$ and $B_4$

Two equations of the form:

$$\sum_k \mu_k V^*_k V^*_k = 0, \quad \mu_k = m_k e^{2i\sigma_k}.$$  

Input: $m_1, m_2, m_3, \theta_{12}, \theta_{13} \Rightarrow$ predictions: $\theta_{23}, \delta, \sigma_{1,2}$

Note: $\sin^2\theta_{13} > 0$, because else $m_1 = m_2$.

One obtains a cubic equation for $\lambda = \tan^2\theta_{23}$

$$\lambda^3 + \lambda^2 \left[ s_{13}^2 + c_{13}^2(c_{12}^2\rho_1 + s_{12}^2\rho_2) \right] - \lambda \left[ s_{13}^2\rho_1\rho_2 + c_{13}^2(s_{12}^2\rho_1 + c_{12}^2\rho_2) \right] - \rho_1\rho_2 = 0.$$  

with

$$\rho_i = \left( \frac{m_i}{m_3} \right)^2 \quad \text{(for } B_3 \text{)} \quad \text{and} \quad \rho_i \rightarrow \frac{1}{\rho_i} \quad \text{(for } B_4).$$
The limit of a quasi-degenerate spectrum

In the limit of a quasi-degenerate neutrino mass spectrum \( \rho_i \to 1 \) the cubic equation becomes

\[
\lambda^3 + \lambda^2 - \lambda - 1 = 0
\]

\[
\Rightarrow \lambda = \tan^2 \theta_{23} = 1 \Rightarrow \theta_{23} = 45^\circ.
\]

**Limit independent of the values of \( \theta_{12} \) and \( \theta_{13} \)!**

Approximate solution of the exact equation:

\[
\sin^2 \theta_{23} \simeq \frac{1}{2} + \frac{1}{8} \frac{\Delta m^2_{31}}{m^2_1} (1 + \sin^2 \theta_{13})
\]

This indeed has the desired form

\[
\sin^2 \theta_{23} = f(m_a/m_b).
\]
The limit of a quasi-degenerate spectrum

\[(\Delta m_{21}^2, \Delta m_{31}^2, \sin^2 \theta_{12} \text{ and } \sin^2 \theta_{13} \text{ fixed to best fit values.})\]

\[B_3 \text{ (inverted)} \quad B_4 \text{ (normal)} \quad B_3 \text{ (normal)} \quad B_4 \text{ (inverted)}\]
The limit of a quasi-degenerate spectrum

In the limit of a quasi-degenerate mass spectrum:

$$\tan \theta_{12} \sin \theta_{13} \cos \delta \rightarrow 0.$$
Lower bounds on $m_1$

In the limit of small $m_1$: $|\cos \delta| \leq 1 \Rightarrow$ lower bound on $m_1$

Lower bound dependent on the value of $\sin^2 \theta_{13}$:

- **inverted spectrum**: bound $m_1 \gtrsim 0.05$ eV stable for $\sin^2 \theta_{13} \gtrsim 0.0001$.
- **normal spectrum**:

  $m_1 \gtrsim 0.03$ eV at $\sin^2 \theta_{13} \simeq 0.0001$.

  $m_1 \gtrsim 0.01$ eV at $\sin^2 \theta_{13} \simeq 0.01$.

$\Rightarrow$ Quasi-degenerate spectrum not implied by texture zeros $B_3$ and $B_4$. $\Rightarrow$ has to be postulated.
Conclusions

- We have found two instances in the framework of two texture zeros in $\mathcal{M}_\nu$ ($\mathcal{M}_\ell$ diagonal) which lead to maximal atmospheric neutrino mixing in the limit of a quasi-degenerate neutrino mass spectrum.
- In this scenario
  \[
  \lim_{m_1 \to \infty} \sin^2 \theta_{23} = \frac{1}{2}
  \]
  is independent of the values of $\theta_{13}$ and $\theta_{12}$!
- $\theta_{23} \approx 45^\circ$ and $\cos \delta \approx 0$ achievable by the use of an Abelian symmetry. This could be an alternative road to explain near maximal atmospheric neutrino mixing.