Supersymmetric Musings on the Predictivity of Family Symmetries

Jörn Kersten

University of Hamburg

Based on Kenji Kadota, JK, Liliana Velasco Sevilla, Phys. Rev. **D82** (2010),
arXiv:1007.1532 [hep-ph]

Outline

Curing SUSY Problems by Family Symmetries

- Non-Abelian symmetry, matter fields in 3D representation

$$\psi, \psi^c \sim \mathbf{3}$$

- Gravity-mediated SUSY breaking
  - $\Rightarrow$ ~~SUSY~~ terms generated at $M_{\mathrm{Pl}} \gg$ flavon vev $\langle\phi\rangle$
  - $\Rightarrow$ Invariant under family symmetry

Curing SUSY Problems by Family Symmetries

- Non-Abelian symmetry, matter fields in 3D representation

$$\psi, \psi^c \sim \mathbf{3}$$

- Gravity-mediated SUSY breaking
  $\Rightarrow$ ~~SUSY~~ terms generated at $M_{\mathrm{Pl}} \gg$ flavon vev $\langle\phi\rangle$
  $\Rightarrow$ Invariant under family symmetry
- Allowed scalar mass term:

$$\widetilde{\psi}_i^* \, \delta_{ij} \, m_0^2 \, \widetilde{\psi}_j \quad \Rightarrow \quad \text{Soft mass matrices } \widetilde{m}^2 = m_0^2 \, \mathbb{1}$$

- Trilinear couplings a=0

Curing SUSY Problems by Family Symmetries

- Non-Abelian symmetry, matter fields in 3D representation

$$\psi, \psi^c \sim \mathbf{3}$$

- Gravity-mediated SUSY breaking
  $\Rightarrow$ $\not{\text{SUSY}}$ terms generated at $M_{\text{Pl}} \gg$ flavon vev $\langle\phi\rangle$
  $\Rightarrow$ Invariant under family symmetry
- Allowed scalar mass term:

$$\widetilde{\psi}_i^* \, \delta_{ij} \, m_0^2 \, \widetilde{\psi}_j \quad \Rightarrow \quad \text{Soft mass matrices } \widetilde{m}^2 = m_0^2 \, \mathbb{1}$$

- Trilinear couplings a=0

$\rightsquigarrow$ SUSY flavor problem solved

Curing SUSY Problems by Family Symmetries

- Non-Abelian symmetry, matter fields in 3D representation

$$\psi, \psi^c \sim \mathbf{3}$$

- Gravity-mediated SUSY breaking
  - $\Rightarrow$ ~~SUSY~~ terms generated at $M_{\text{Pl}} \gg$ flavon vev $\langle\phi\rangle$
  - $\Rightarrow$ Invariant under family symmetry
- Allowed scalar mass term:

$$\widetilde{\psi}_i^* \, \delta_{ij} \, m_0^2 \, \widetilde{\psi}_j \quad \Rightarrow \quad \text{Soft mass matrices } \widetilde{m}^2 = m_0^2 \, \mathbb{1}$$

- Trilinear couplings a=0

$\rightsquigarrow$ SUSY flavor problem solved

- Extension: CP symmetry
- Spontaneously broken by complex flavon vev

$\rightsquigarrow$ SUSY CP problem solved

Added Bonus

Family symmetry breaking

$\rightsquigarrow$ Off-diagonal entries

$$\widetilde{m}^2_{ij} \sim m_0^2 \left(\frac{\langle \phi \rangle}{M} \right)^n$$

$\rightsquigarrow$ Non-zero trilinears

$$a_{ij} \sim \left(\frac{\langle \phi \rangle}{M} \right)^n$$

Added Bonus

Family symmetry breaking

$\rightsquigarrow$ Off-diagonal entries

$$\widetilde{m}_{ij}^2 \sim m_0^2 \left(\frac{\langle \phi \rangle}{M} \right)^n$$

$\rightsquigarrow$ Non-zero trilinears

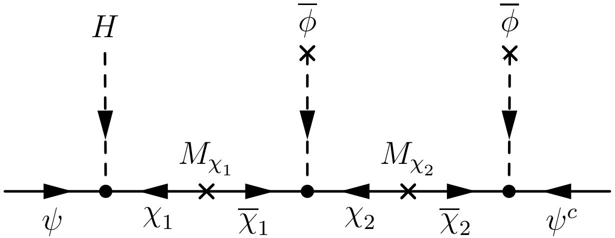
$$a_{ij} \sim \left(\frac{\langle \phi \rangle}{M} \right)^n$$

New terms

- Suppressed $\rightsquigarrow$ flavor and CP violation still under control
- Predicted $\rightsquigarrow$ additional experimental test

Abel, Antusch, Calibbi, Feruglio, Hagedorn, Ishimori, Jones-Perez, Khalil, King, Kobayashi, Lebedev, Lin, Malinský, Maurer, Merlo, Nomura, Ohki, Olive, Omura, Ross, Spinrath, Stolarski, Takahashi, Tanimoto, Velasco Sevilla, Vives,
hep-ph/0112260, hep-ph/0211279, hep-ph/0401064, 0708.1282, 0801.0428, 0803.0796, 0804.4620, 0807.3160, 0807.4625, 0807.5047, 0808.1380, 1104.3040

Example: SU(3)



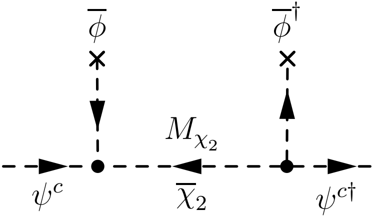
de Medeiros Varzielas, Ross, hep-ph/0507176

- Flavon $\overline{\phi} \sim \mathbf{\overline{3}}$
- Messengers $\chi_1 \sim \mathbf{\overline{3}}$, $\chi_2 \sim \mathbf{1}$
- $Y \sim \dfrac{\langle\overline{\phi}\rangle^2}{M_{\chi_1} M_{\chi_2}}$ $\rightsquigarrow$ only product of messenger masses determined

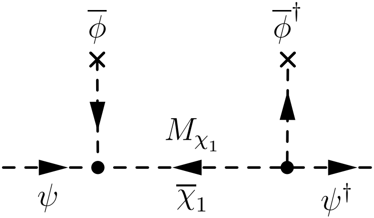
Soft Masses

1. Write down superpotential W of renormalizable theory
2. Write down Kähler potential K (constrained by symmetry)
3. Integrate out heavy messengers
   - Solve $\dfrac{\partial W}{\partial \chi_1} = 0, \ldots$ for messenger fields
   - Plug results into W and K
4. Rough estimate: $\left(\widetilde{m}^2_\psi\right)_{ij} \sim m_0^2 \, \dfrac{\partial^2 K}{\partial \psi_i^* \partial \psi_j} \quad , \quad \ldots$

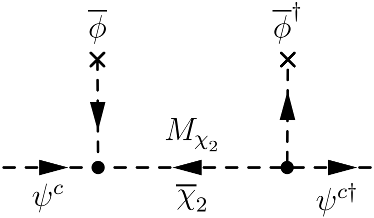
Illustration:



In Our Example

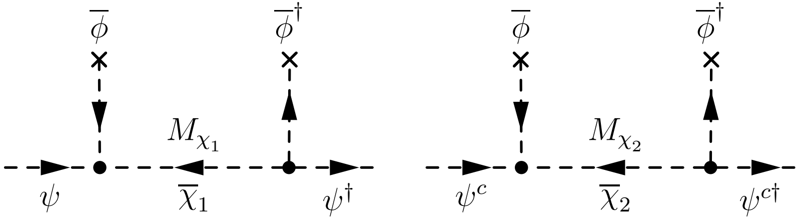


Forbidden by family symmetry



OK

In Our Example

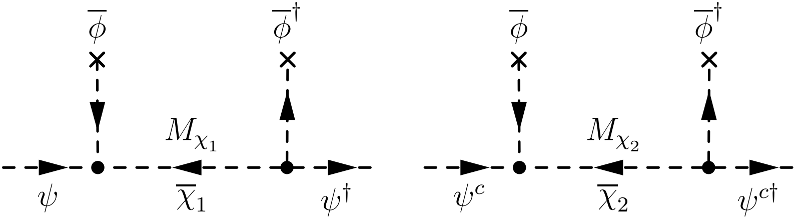


Forbidden by family symmetry — OK

- No messenger coupling to $\psi\overline{\phi} \rightsquigarrow \widetilde{m}^2_\psi = m_0^2$
- Tri-bimaximal mixing intact after canonical normalization

Antusch, King, Malinský, 0712.3759

In Our Example



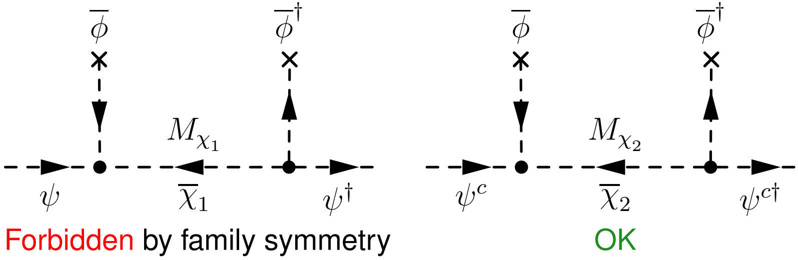
Forbidden by family symmetry OK

- No messenger coupling to $\psi\overline{\phi} \rightsquigarrow \widetilde{m}^2_\psi = m_0^2$
- Tri-bimaximal mixing intact after canonical normalization

  Antusch, King, Malinský, `0712.3759`

- $\widetilde{m}^2_{\psi^c} \sim m_0^2 \left(1 + \frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_2}}\right)$

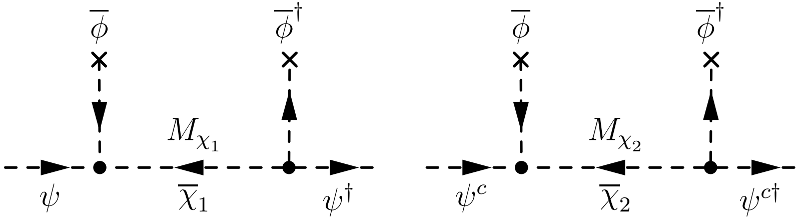
In Our Example



- No messenger coupling to $\psi\overline{\phi} \rightsquigarrow \widetilde{m}_\psi^2 = m_0^2$
- Tri-bimaximal mixing intact after canonical normalization

  Antusch, King, Malinský, 0712.3759
- $\widetilde{m}_{\psi^c}^2 \sim m_0^2 \left(1 + \frac{\langle\overline{\phi}\rangle^2}{M_{\chi_2}^2}\right)$ — recall $Y \sim \frac{\langle\overline{\phi}\rangle^2}{M_{\chi_1} M_{\chi_2}}$
- Only M_{χ_2} enters soft masses $\rightsquigarrow$ no prediction

In Our Example



Forbidden by family symmetry — OK

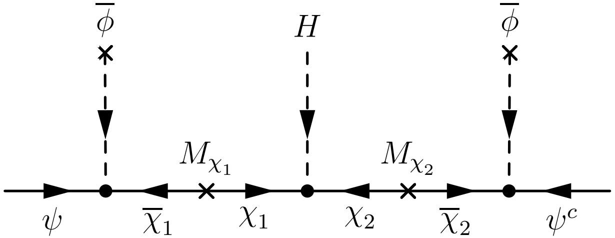
- No messenger coupling to $\psi\overline{\phi} \rightsquigarrow \widetilde{m}^2_\psi = m^2_0$
- Tri-bimaximal mixing intact after canonical normalization

  Antusch, King, Malinský, 0712.3759
- $\widetilde{m}^2_{\psi^c} \sim m^2_0 \left(1 + \frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_2}}\right)$ — recall $Y \sim \frac{\langle\overline{\phi}\rangle^2}{M_{\chi_1} M_{\chi_2}}$
- Only M_{χ_2} enters soft masses $\rightsquigarrow$ no prediction
- Trilinears: same conclusion

Approaches for Improving Predictivity

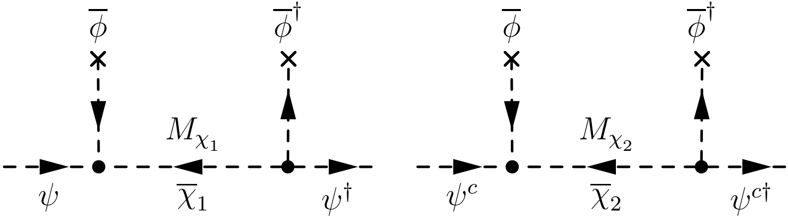
- Extend model
  - Explicit theory of messenger sector
    Cf. King, Malinský, hep-ph/0608021 for SO(3)
    (already $M_\chi > \langle\overline{\phi}\rangle$ for *all* messengers and flavons could help)
  - Fix ratios of flavon vevs
  - Large Y_{33} possibly helpful
- Change messenger sector

More Predictive Messenger Sector



- $Y \sim \dfrac{\langle\overline{\phi}\rangle^2}{M_{\chi_1} M_{\chi_2}}$
- All messengers SU(3) singlets

Soft Masses Again



- $\widetilde{m}^2_\psi \sim m_0^2 \left(1 + \underbrace{\frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_1}}}_{\widetilde{\epsilon}^2_{Q,L}}\right) \quad , \quad \widetilde{m}^2_{\psi^c} \sim m_0^2 \left(1 + \underbrace{\frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_2}}}_{\widetilde{\epsilon}^2_{u,d,e}}\right)$

Soft Masses Again

- $\widetilde{m}^2_{\psi} \sim m_0^2 \left(1 + \underbrace{\frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_1}}}_{\widetilde{\epsilon}^2_{Q,L}}\right) \quad , \quad \widetilde{m}^2_{\psi^c} \sim m_0^2 \left(1 + \underbrace{\frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_2}}}_{\widetilde{\epsilon}^2_{u,d,e}}\right)$
- Recall $Y \sim \frac{\langle\overline{\phi}\rangle^2}{M_{\chi_1} M_{\chi_2}} =: \epsilon^2_{u,d}$

Soft Masses Again

- $$\widetilde{m}^2_{\psi} \sim m_0^2 \left(1 + \underbrace{\frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_1}}}_{\widetilde{\epsilon}^2_{Q,L}}\right) \quad , \quad \widetilde{m}^2_{\psi^c} \sim m_0^2 \left(1 + \underbrace{\frac{\langle\overline{\phi}\rangle^2}{M^2_{\chi_2}}}_{\widetilde{\epsilon}^2_{u,d,e}}\right)$$
- Recall $Y \sim \dfrac{\langle\overline{\phi}\rangle^2}{M_{\chi_1} M_{\chi_2}} =: \epsilon^2_{u,d}$
- Off-diagonal elements in all soft mass matrices
- All messenger masses appear
- Relations between expansion parameters

$$\widetilde{\epsilon}_Q \, \widetilde{\epsilon}_u \sim \epsilon_u^2 \quad , \quad \widetilde{\epsilon}_Q \, \widetilde{\epsilon}_d \sim \epsilon_d^2 \quad , \quad \widetilde{\epsilon}_L \, \widetilde{\epsilon}_e \sim \epsilon_d^2$$

- None of them can be arbitrarily small: $\widetilde{\epsilon} \gtrsim 0.01$

Complete Matrices

$$\widetilde{m}_f^2 \sim m_0^2 \begin{pmatrix} 1 & \widetilde{\epsilon}_f^2\,\epsilon_d^2 & \widetilde{\epsilon}_f^2\,\epsilon_d^2 \\ \cdot & 1+\widetilde{\epsilon}_f^2 & \widetilde{\epsilon}_f^2 \\ \cdot & \cdot & 1 \end{pmatrix} \quad , \quad f = u,\, d,\, Q,\, e,\, L$$

Complete Matrices

$$\widetilde{m}_f^2 \sim m_0^2 \begin{pmatrix} 1 & \widetilde{\epsilon}_f^2\, \epsilon_d^2 & \widetilde{\epsilon}_f^2\, \epsilon_d^2 \\ \cdot & 1 + \widetilde{\epsilon}_f^2 & \widetilde{\epsilon}_f^2 \\ \cdot & \cdot & 1 \end{pmatrix} \quad , \quad f = u, d, Q, e, L$$

- Trilinear couplings: similar, but a bit more complicated

Running to Low Energy

- Results so far valid at high energy $\sim M_{\text{GUT}}$
- Renormalization group evolution $\rightsquigarrow$ low-energy parameters
- Rough estimate
  Antusch, King, Malinský, 0708.1282,
  Bertolini, Borzumati, Masiero, Ridolfi, Nucl. Phys. **B353**
  - Diagonal entries at low energy:

$$(\widetilde{m}^2_q)_{ii} \sim 30\, m_0^2$$
$$(\widetilde{m}^2_L)_{ii} \sim 4\, m_0^2$$
$$(\widetilde{m}^2_e)_{ii} \sim 2\, m_0^2$$

  - Off-diagonal elements do not change order of magnitude
  - Y_ν does not contribute because $M_3 > M_{\text{GUT}}$

Super-CKM Basis

- Basis with diagonal Yukawa couplings
- Convenient for low-energy phenomenology

$$\widetilde{m}^2_{d,\mathrm{RR}} \sim m_0^2 \begin{pmatrix} 30 & \widetilde{\epsilon}_d^2\,\epsilon_d & \widetilde{\epsilon}_d^2\,\epsilon_d + \epsilon_d^3 \\ \cdot & 30 & \widetilde{\epsilon}_d^2 + \epsilon_d^2 \\ \cdot & \cdot & 30 \end{pmatrix}$$

- 12- and 13-elements enlarged by factor $\sim 1/\epsilon_d$

  Ross, Velasco Sevilla, Vives, hep-ph/0401064
- Cancellations possible in principle
- $\widetilde{m}^2_{d,\mathrm{LL}}$ analogous with $\widetilde{\epsilon}_d \to \widetilde{\epsilon}_Q$
- $\widetilde{m}^2_{e,\mathrm{LL}}$ analogous with $\widetilde{\epsilon}_d \to \widetilde{\epsilon}_L$, $30 \to 4$
- $\widetilde{m}^2_u$ less interesting due to weaker experimental constraints

Experimental Constraints

Ciuchini, Masiero, Paradisi, Silvestrini, Vempati, Vives, `hep-ph/0702144`

- Flavor-changing neutral currents: Δm_K, $b \to s\gamma$, $\mu \to e\gamma$ etc.
- Mass insertion approximation $\rightsquigarrow$ constraints on

$$(\delta^d_{\mathrm{RR}})_{ij} := \frac{(\widetilde{m}^2_{d,\mathrm{RR}})_{ij}}{(\widetilde{m}^2_{d,\mathrm{RR}})_{ii}} \quad , \quad \ldots$$

- Depend on $\tan\beta$, sparticle masses

Experiment vs. Model Predictions

Simple example:

$$\widetilde{\epsilon}_Q = \widetilde{\epsilon}_d = \widetilde{\epsilon}_L = \widetilde{\epsilon}_e = \epsilon_d \approx 0.15 \quad , \quad \widetilde{\epsilon}_u = \frac{\epsilon_u^2}{\epsilon_d} \approx 0.02$$

Experiment vs. Model Predictions

Simple example:

$$\widetilde{\epsilon}_Q = \widetilde{\epsilon}_d = \widetilde{\epsilon}_L = \widetilde{\epsilon}_e = \epsilon_d \approx 0.15 \quad , \quad \widetilde{\epsilon}_u = \frac{\epsilon_u^2}{\epsilon_d} \approx 0.02$$

	Our example	Bound
$(\delta^d_{\mathrm{RR}})_{12}$	$\frac{\widetilde{\epsilon}_d^2\,\epsilon_d}{30} \sim 10^{-4}$	$9 \cdot 10^{-3}$
$(\delta^d_{\mathrm{LL}})_{12}$	$\frac{\widetilde{\epsilon}_Q^2\,\epsilon_d}{30} \sim 10^{-4}$	$1 \cdot 10^{-2}$
$(\delta^d_{\mathrm{LR}})_{12}$	$\frac{\propto \epsilon_d^3}{30} \sim 3 \cdot 10^{-5}$	$1 \cdot 10^{-5}$
$(\delta^d_{\mathrm{LL}})_{23}$	$\frac{\widetilde{\epsilon}_Q^2}{30} \sim 8 \cdot 10^{-4}$	$2 \cdot 10^{-1}$
$(\delta^e_{\mathrm{LL}})_{12}$	$\frac{\widetilde{\epsilon}_L^2\,\epsilon_d}{4} \sim 8 \cdot 10^{-4}$	$6 \cdot 10^{-4}$

- Some tension in 12 sector
  See also Antusch, King, Malinský, 0708.1282
- Only weak constraints on δ^u and δ^e_{RR} $\rightsquigarrow$ easily satisfied

Conclusions

- Non-Abelian family symmetries can solve flavor and CP problems
- Predictions for SUSY-breaking parameters
- Predictivity depends on messenger sector
- Example with SU(3) symmetry: Order-of-magnitude estimates
  - $\rightsquigarrow$ Experimental constraints satisfied
  - $\rightsquigarrow$ Predicted FCNC rates may be within reach