Supersymmetric Musings on the Predictivity of Family Symmetries

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1. Introduction

2. Soft Masses from Family Symmetries

3. Comparison with Experimental Constraints
Introduction

Soft Masses from Family Symmetries

Comparison with Experimental Constraints
Curing SUSY Problems by Family Symmetries

- **Non-Abelian** symmetry, matter fields in 3D representation
  \[ \psi, \psi^c \sim 3 \]

- **Gravity**-mediated SUSY breaking
  \[ \Rightarrow \text{SUSY terms generated at } M_{\text{Pl}} \gg \text{flavon vev } \langle \phi \rangle \]
  \[ \Rightarrow \text{Invariant under family symmetry} \]
Curing SUSY Problems by Family Symmetries

- **Non-Abelian** symmetry, matter fields in 3D representation
  \[ \psi, \psi^c \sim 3 \]

- **Gravity-mediated** SUSY breaking
  \[\Rightarrow\] SUSY terms generated at \( M_{Pl} \gg \) flavon vev \( \langle \phi \rangle \)
  \[\Rightarrow\] Invariant under family symmetry

- Allowed scalar mass term:
  \[ \tilde{\psi}_i^* \delta_{ij} m_0^2 \tilde{\psi}_j \Rightarrow \text{Soft mass matrices} \quad \tilde{m}^2 = m_0^2 \mathbb{1} \]

- Trilinear couplings \( a=0 \)
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  \( \Rightarrow \text{SUSY flavor problem solved} \)
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- Extension: **CP** symmetry
- Spontaneously broken by complex flavon vev
  \[ \sim \text{SUSY CP problem solved} \]
Family symmetry breaking

\[ \tilde{m}_{ij}^2 \sim m_0^2 \left( \frac{\langle \phi \rangle}{M} \right)^n \]

\[ a_{ij} \sim \left( \frac{\langle \phi \rangle}{M} \right)^n \]
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New terms

- **Suppressed** \( \leadsto \) flavor and CP violation still under control
- **Predicted** \( \leadsto \) additional experimental test

Abel, Antusch, Calibbi, Feruglio, Hagedorn, Ishimori, Jones-Perez, Khalil, King, Kobayashi, Lebedev, Lin, Malinský, Maurer, Merlo, Nomura, Ohki, Olive, Omura, Ross, Spinrath, Stolarski, Takahashi, Tanimoto, Velasco Sevilla, Vives,
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Example: SU(3)

- **Flavon** $\phi \sim \mathbf{3}$
- **Messengers** $\chi_1 \sim \mathbf{3}, \chi_2 \sim \mathbf{1}$
- $Y \sim \frac{\langle \phi \rangle^2}{M_{\chi_1} M_{\chi_2}} \rightsquigarrow$ only product of messenger masses determined
1. Write down superpotential $W$ of renormalizable theory
2. Write down Kähler potential $K$ (constrained by symmetry)
3. Integrate out heavy messengers
   - Solve $\frac{\partial W}{\partial \chi_1} = 0, \ldots$ for messenger fields
   - Plug results into $W$ and $K$
4. Rough estimate: $(\tilde{m}_\psi^2)_{ij} \sim m_0^2 \frac{\partial^2 K}{\partial \psi_i^* \partial \psi_j}, \ldots$

Illustration:
In Our Example

Forbidden by family symmetry

OK
In Our Example

Forbidden by family symmetry

- No messenger coupling to $\psi \phi \rightarrow \tilde{m}_\psi^2 = m_0^2$
- Tri-bimaximal mixing intact after canonical normalization

Antusch, King, Malinský, 0712.3759
In Our Example

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- No messenger coupling to $\psi \phi \leadsto \tilde{m}_\psi^2 = m_0^2$
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\[ \tilde{m}_{\psi}^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_2}^2} \right) \]

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- $\bar{m}_{\psi c}^2 \sim m_0^2 \left(1 + \frac{\langle \phi \rangle^2}{M_{\chi_2}^2}\right)$ — recall $Y \sim \frac{\langle \phi \rangle^2}{M_{\chi_1} M_{\chi_2}}$
- Only $M_{\chi_2}$ enters soft masses $\rightsquigarrow$ no prediction
In Our Example

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- Only $M_{\chi_2}$ enters soft masses $\sim$ no prediction
- Trilinears: same conclusion
Approaches for Improving Predictivity

- Extend model
  - Explicit theory of messenger sector
    Cf. King, Malinský, hep-ph/0608021 for SO(3)
    (already $M_{\chi} > \langle \phi \rangle$ for all messengers and flavons could help)
  - Fix ratios of flavon vevs
  - Large $Y_{33}$ possibly helpful
- Change messenger sector
More Predictive Messenger Sector

\[ Y \sim \frac{\langle \phi \rangle^2}{M_{\chi_1} M_{\chi_2}} \]

- All messengers SU(3) singlets
Recall $\tilde{m}_\psi^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_1}^2} \right)$, $\tilde{m}_{\psi^c}^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_2}^2} \right)$.
Soft Masses Again

\[ \tilde{m}_\psi^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_1}^2} \right), \quad \tilde{m}_{\psi c}^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_2}^2} \right) \]

- \tilde{\epsilon}_{Q, L}^2
- \tilde{\epsilon}_{u, d, e}^2

- Recall \( Y \sim \frac{\langle \phi \rangle^2}{M_{\chi_1} M_{\chi_2}} =: \epsilon_{u, d}^2 \)
Soft Masses Again

\[ \tilde{m}_\psi^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_1}^2} \right), \quad \tilde{m}_{\psi c}^2 \sim m_0^2 \left( 1 + \frac{\langle \phi \rangle^2}{M_{\chi_2}^2} \right) \]

\[ \tilde{\epsilon}_{Q,L}^2 \]

Recall \( Y \sim \frac{\langle \phi \rangle^2}{M_{\chi_1} M_{\chi_2}} =: \epsilon_{u,d}^2 \)

Off-diagonal elements in all soft mass matrices

All messenger masses appear

Relations between expansion parameters

\[ \tilde{\epsilon}_Q \tilde{\epsilon}_u \sim \epsilon_u^2, \quad \tilde{\epsilon}_Q \tilde{\epsilon}_d \sim \epsilon_d^2, \quad \tilde{\epsilon}_L \tilde{\epsilon}_e \sim \epsilon_d^2 \]

None of them can be arbitrarily small: \( \tilde{\epsilon} \gtrsim 0.01 \)
\[ \tilde{m}_f^2 \sim m_0^2 \left( \begin{array}{ccc}
1 & \tilde{\epsilon}_f^2 & \tilde{\epsilon}_f^2 \\
1 + \tilde{\epsilon}_f^2 & 1 & \tilde{\epsilon}_d^2 \\
\tilde{\epsilon}_f^2 & \tilde{\epsilon}_d^2 & 1
\end{array} \right), \quad f = u, d, Q, e, L \]
Complete Matrices

\[ \tilde{m}_f^2 \sim m_0^2 \begin{pmatrix} 1 & \tilde{\epsilon}_f^2 & \tilde{\epsilon}_f^2 \\ 1 + \tilde{\epsilon}_f^2 & \tilde{\epsilon}_d^2 & \tilde{\epsilon}_d^2 \\ \tilde{\epsilon}_f^2 & \tilde{\epsilon}_d^2 & 1 \end{pmatrix} , \quad f = u, d, Q, e, L \]

- Trilinear couplings: similar, but a bit more complicated
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Running to Low Energy

- Results so far valid at high energy $\sim M_{\text{GUT}}$
- Renormalization group evolution $\rightsquigarrow$ low-energy parameters
- Rough estimate
  Antusch, King, Malinský, 0708.1282,
  Bertolini, Borzumati, Masiero, Ridolfi, Nucl. Phys. B353

- Diagonal entries at low energy:
  \[
  (\tilde{\mathcal{m}}_q)_{ii} \sim 30 \, m_0^2 \\
  (\tilde{\mathcal{m}}_L)_{ii} \sim 4 \, m_0^2 \\
  (\tilde{\mathcal{m}}_e)_{ii} \sim 2 \, m_0^2
  \]

- Off-diagonal elements do not change order of magnitude
- $Y_\nu$ does not contribute because $M_3 > M_{\text{GUT}}$
Super-CKM Basis

- Basis with diagonal Yukawa couplings
- Convenient for low-energy phenomenology

\[
\tilde{m}_{d,RR}^2 \sim m_0^2 \begin{pmatrix}
30 & \tilde{\epsilon}_d^2 \epsilon_d & \tilde{\epsilon}_d^2 \epsilon_d + \epsilon_d^3 \\
. & 30 & \tilde{\epsilon}_d^2 + \epsilon_d^2 \\
. & . & 30
\end{pmatrix}
\]

- 12- and 13-elements enlarged by factor \( \sim 1/\epsilon_d \)
- Cancellations possible in principle
  - \( \tilde{m}_{d,LL}^2 \) analogous with \( \tilde{\epsilon}_d \rightarrow \tilde{\epsilon}_Q \)
  - \( \tilde{m}_{e,LL}^2 \) analogous with \( \tilde{\epsilon}_d \rightarrow \tilde{\epsilon}_L \), 30 \( \rightarrow \) 4
  - \( \tilde{m}_u^2 \) less interesting due to weaker experimental constraints
Flavor-changing neutral currents: $\Delta m_K$, $b \to s\gamma$, $\mu \to e\gamma$ etc.

Mass insertion approximation $\leadsto$ constraints on

$$(\delta^d_{RR})_{ij} := \frac{(\tilde{m}^2_{d,RR})_{ij}}{(\tilde{m}^2_{d,RR})_{ii}}, \quad \ldots$$

Depend on $\tan \beta$, sparticle masses
Experiment vs. Model Predictions

Simple example:

\[ \tilde{\epsilon}_Q = \tilde{\epsilon}_d = \tilde{\epsilon}_L = \tilde{\epsilon}_e = \epsilon_d \approx 0.15 \quad , \quad \tilde{\epsilon}_u = \frac{\epsilon_u^2}{\epsilon_d} \approx 0.02 \]
Experiment vs. Model Predictions

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\[ \tilde{\epsilon}_Q = \tilde{\epsilon}_d = \tilde{\epsilon}_L = \tilde{\epsilon}_e = \epsilon_d \approx 0.15, \quad \tilde{\epsilon}_u = \frac{\epsilon_u^2}{\epsilon_d} \approx 0.02 \]

<table>
<thead>
<tr>
<th>Our example</th>
<th>Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\delta_{RR}^d)_{12})</td>
<td>(\frac{\tilde{\epsilon}_d^2}{30} \epsilon_d \sim 10^{-4})</td>
</tr>
<tr>
<td>((\delta_{LL}^d)_{12})</td>
<td>(\frac{\tilde{\epsilon}_Q^2}{30} \epsilon_d \sim 10^{-4})</td>
</tr>
<tr>
<td>((\delta_{LR}^d)_{12})</td>
<td>(\frac{\epsilon_d^3}{30} \sim 3 \cdot 10^{-5})</td>
</tr>
<tr>
<td>((\delta_{LL}^e)_{23})</td>
<td>(\frac{\tilde{\epsilon}_Q^2}{30} \sim 8 \cdot 10^{-4})</td>
</tr>
<tr>
<td>((\delta_{LL}^e)_{12})</td>
<td>(\frac{\epsilon_L^2}{4} \epsilon_d \sim 8 \cdot 10^{-4})</td>
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</tbody>
</table>

- **Some tension** in 12 sector
  See also Antusch, King, Malinský, 0708.1282

- **Only weak constraints on** \(\delta^u\) and \(\delta_{RR}^e\) \(\sim\) **easily satisfied**
Conclusions

- Non-Abelian family symmetries can solve flavor and CP problems
- Predictions for SUSY-breaking parameters
- Predictivity depends on messenger sector
- Example with SU(3) symmetry: Order-of-magnitude estimates
  - Experimental constraints satisfied
  - Predicted FCNC rates may be within reach