Revisiting Quark-Lepton Complementarity and Triminimal Parametrization of Neutrino Mixing Matrix

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Outline

- Introduction
  - motivation
  - current status of neutrino mixing
- Parametrization of neutrino mixing
- Quark mixing angles and Quark-Lepton Complenementarity (QLC)
- QLC parametrization and predictions of physical observables
- Comparing with Triminimal parametrization

Motivation

- To investigate whether QLC reflecting bi-maximal mixing deviated by CKM mixing matrix (QLC parametrization) is consistent with current neutrino data or not.

- To compare this QLC parametrization with Triminimal parametrization reflecting tri-bimaximal mixing deviated by some perturbation.

Neutrino Mixing Matrix

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Standard Model states

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrino mass states

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric angle

Reactor angle and Dirac CP phase

Solar angle

Majorana phases

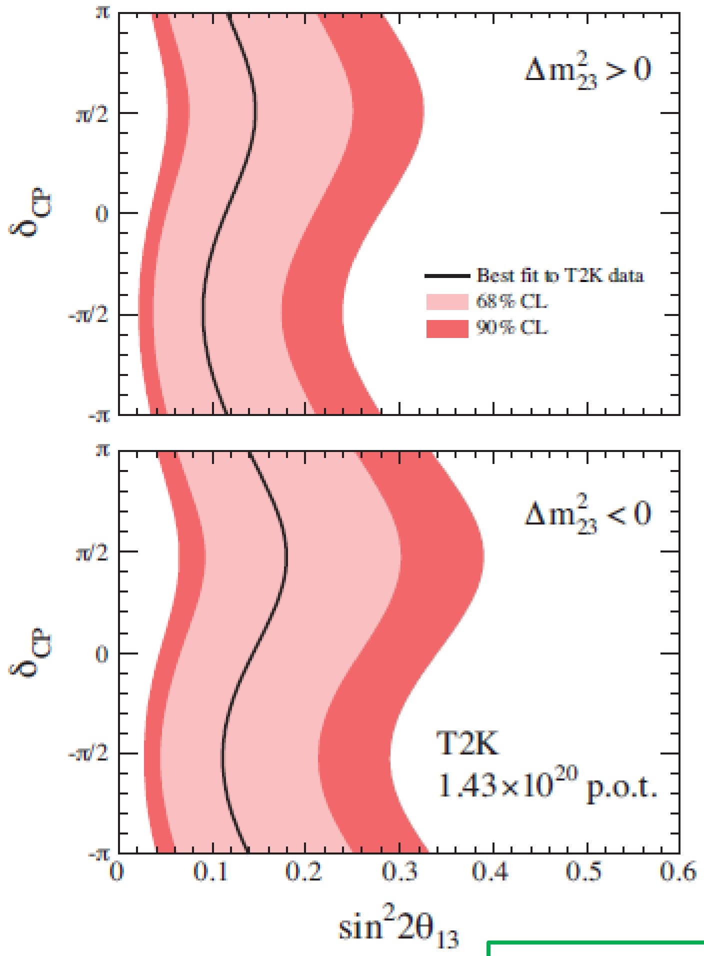
➢ Current data of neutrino mixing angles

(Schwetz, Tortola, Valle, **New J.Phys.13:063004,2011**.)

Parameter	Best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} (10^{-5} eV^2)	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} (10^{-3} eV^2)	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	2.28–2.64 $-$(2.17–2.54)	2.18–2.73 $-$(2.08–2.64)
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.010^{+0.009}_{-0.006}$ $0.013^{+0.009}_{-0.007}$	$\leqslant 0.027$ $\leqslant 0.031$	$\leqslant 0.035$ $\leqslant 0.039$

$$\boxed{\begin{array}{c} \theta_{12} = 33.9^\circ\,,\ \theta_{23} = 45.6^\circ\,,\ \theta_{13} = 5.75^\circ \\ 31.9^\circ < \theta_{12} < 36.4^\circ\,,\ 39.9^\circ < \theta_{23} < 51.4^\circ\,,\ \ \theta_{13} < 10.2^\circ\ (2\sigma) \end{array}}$$

Recent T2K experiment



The results show

$$\boxed{0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)}$$

at 90% C.L.

$$\text{for} \begin{pmatrix} \sin^2 2\theta_{23} = 1.0 \\ |\Delta m^2_{23}| = 2.4\times10^{-3} \ \ \text{eV}^2 \end{pmatrix} \quad \text{(CPC)}$$

For non-maximal 23 mixing

$$\sin^2 2\theta_{13} \Rightarrow 2\sin^2\theta_{23}\cos^2 2\theta_{13}$$

More data needed to firmly establish ν_e appearance and to better determine θ_{13}

- Current data for neutrino mixing angles is consistent with Tri-bimaximal mixing pattern at 2σ

$$U=\begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\boldsymbol{\theta_{12} = 35.3^\circ\ ,\ \theta_{23} = 45^\circ\ ,\ \theta_{13} = 0^\circ}$$

- Although TB mixing can be achieved by imposing some flavor symmetries, it is widely accepted that TB is a good zeroth order approximation to reality and there may be deviations from TB in general.

- To accommodate deviations from TB, neutrino mixing angles can be parameterized (S.F.King 2008)

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s), \sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a), \sin\theta_{13} = \frac{r}{\sqrt{2}}$$

- The global fit of the neutrino mixing angles can be translated into the 3σ

$$0 < r < 0.31,\ -0.10 < \mathrm{s} < 0.03,\ -0.18 < \mathrm{a} < 0.14$$

- Alternatively, triminimal parametrization has been proposed

(Pakvasa, Rodejohann, Weiler, Phys. Rev. Lett.100 ,2008)

$$U_{TMin} = R_{23}\left(\frac{\pi}{4}\right)U_{\varepsilon}(\varepsilon_{23},\varepsilon_{13},\delta,\varepsilon_{21})R_{12}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right)$$

$$= \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \frac{U_{\varepsilon}}{\sqrt{6}} \begin{pmatrix} \sqrt{2} & 1 & 0 \\ -1 & \sqrt{2} & 1 \\ 0 & 0 & \sqrt{3} \end{pmatrix}$$

$$U_{\varepsilon} = R_{23}(\varepsilon_{23})U_{\delta}^{+}R_{13}(\varepsilon_{13})U_{\delta}R_{21}(\varepsilon_{21})$$

- The neutrino mixing observables up to 2nd order in ε_{ij}

$$\sin^2\theta_{21} \approx \frac{1}{3} + \frac{2\sqrt{2}}{3}\varepsilon_{21} + \frac{1}{3}\varepsilon_{21}^2 \quad , \qquad \sin^2\theta_{32} \approx \frac{1}{2} + \varepsilon_{32}$$

$$U_{e3} = \sin\varepsilon_{13}e^{-i\delta}$$

Quark Mixing Angles

- The standard parameterization of V_{CKM}

$$V_{CKM,S} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Rotation of s- and b-quark}} \cdot \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\substack{\text{Rotation of d- and b-quark} \\ \text{plus phase factor } \delta}} \cdot \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{Rotation of d- and s-quark} \\ \text{(remember case } N=2)}}$$

- Current data for quark mixing angles

$$|V_{CKM,S}| = \begin{pmatrix} 0.9739-0.9751 & 0.221-0.227 & 0.0029-0.0045 \\ 0.221-0.227 & 0.9730-0.9744 & 0.039-0.044 \\ 0.0048-0.014 & 0.037-0.043 & 0.9990-0.9992 \end{pmatrix}$$

Wolfenstein Parameterization of quark mixing matrix

L. Wolfenstein, Phys. Rev. Lett. 51 (1983)

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\boxed{\lambda = 0.2243 \pm 0.0016,}$$

$$A = 0.82, \qquad \rho = 0.20, \qquad \eta = 0.33.$$

Quark Lepton Complimentarity

Lepton mixings

Parameter	Best−fit	1 sigma range
θ_{12}	33.9°	$33.0^\circ - 35.0^\circ$
θ_{23}	45.6°	$42.1^\circ - 49.0^\circ$
θ_{13}	5.75°	$< 11.4^\circ$

Quark mixings

Parameter	Best−fit	1 sigma range
$\theta_{12}=\theta_C$	12.95°	$12.88^\circ - 13.02^\circ$
θ_{23}	2.36°	$2.29^\circ - 2.43^\circ$
θ_{13}	0.21°	$0.17^\circ - 0.25^\circ$

Present experimental data allows for relations like:

$$\boxed{\begin{aligned}\theta_{12}+\theta_C &= 45^\circ\\ \theta_{23}+\theta_{23}^q &= 45^\circ\end{aligned}}$$

Raidal (2004)

Smirnov, Minakata (2004)

'Quark-Lepton Complementarity'

QLC is well in consistent with the current data within 2σ

- QLC may serve as a clue to quark-lepton symmetry or unification.
- So, QLC motivates measurements of neutrino mixing angles to at least the accuracy of the measured quark mixing angles.
- In the light of QLC, neutrino mixing matrix can be composed of CKM mixing matrix and maximal mixing matrix.
- Three possible combinations of maximal mixing and CKM as parametrization of neutrino mixing matrix

(Cheung, Kang, Kim, Lee (2005), Datta, Everett, Ramond(2005), Everett(2006))

$$(1)\quad U^{+}_{CKM} R_{23}(\pi/4) R_{12}(\pi/4)$$

$$(2)\quad R_{23}(\pi/4) R_{12}(\pi/4) U^{+}_{CKM}$$

$$(3)\quad R_{23}(\pi/4) U^{+}_{CKM} R_{12}(\pi/4)$$

- The flavor mixings stem from the mismatch between the LH rotations of the up-type and down-type quarks, and the charged leptons and neutrinos.

- In general, the quark Yukawa matrices

$$Y_U = U_U Y_U^{diag} V_U^+$$

$$Y_D = U_D Y_D^{diag} V_D^+$$

$$\Rightarrow \quad V_{CKM} = U_U^+ U_D$$

- **For charged lepton sector** : $Y_L = U_L Y_L^{diag} V_L^+$

For neutrino sector :

- Introducing one RH singlet neutrino per family which leads to the seesaw mechanism

$$M_\nu = M_{\rm Dirac}\frac{1}{M_R}M_{\rm Dirac}^T = \left(U_0 M_{\rm Dirac}^{\rm diag} V_0^\dagger\right)\frac{1}{M_R}\left(V_0^* M_{\rm Dirac}^{\rm diag} U_0^T\right)$$

- Rewriting neutrino mass, $M_\nu = U_0 V_M M_\nu^{\rm diag} V_M^T U_0^T$

$$\Rightarrow \quad \boxed{U_{PMNS} = U_L^+ U_0 V_M}$$

- In SO(10), $Y_\nu = Y_U$, $Y_L = Y_D^T$ $\Rightarrow$ $U_L = V_D^*$, $U_0 = U_U$

Requiring symmetric Y_D , $\boxed{U_{PMNS} = V_{CKM}^+ V_M}$

Requiring $V_D^T U_D = R_{23}(\pi/4)$ $\boxed{U_{PMNS} = R_{23}(\pi/4) V_{CKM}^+ V_M}$

- Among three possible combinations, we consider

$$\boxed{U_{PMNS} = R_{23}(\pi/4)\, V_{CKM}^{+} R(\pi/4)}$$

(QLC parametrization)

- Motivation to consider this :

it is well compared and has similar merit to the triminimal parametrization so that we can simply examine if the effects of deviations from the TB mixing can be compatible with the QLC relation or not by investigating a few observables presented by simple formulas.

- Mixing angles in powers of λ

$$\theta_{sol} \approx \frac{\pi}{4} - \lambda, \quad \theta_{atm} \approx \frac{\pi}{4} - A\lambda^2, \quad \theta_{13} \approx A\lambda^3$$

Predictions for Physical Observables

- Neutrino mixing probabilities for phased averaged propagation

($\Delta m^2 L/4E >> 1$)

$$P_{\nu_\alpha \to \nu_\beta} = \sum |U_{\alpha i}|^2 |U_{\beta i}|^2 \equiv \sum \underline{U_{\alpha i}} \cdot \underline{U_{\beta i}}$$

(Pakvasa, Rodejohann, Weiler, Phys. Rev. Lett.100 ,2008)

$$\underline{U} = \frac{1}{4}\left\{\begin{pmatrix} 2 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 & -4 & 0 \\ -2 & 2 & 0 \\ -2 & 2 & 0 \end{pmatrix} \right.$$

$$\left. + A\lambda^2 \begin{pmatrix} 0 & 0 & 0 \\ 2 & 2 & -1 \\ -2 & -2 & 1 \end{pmatrix} + \lambda^3 \begin{pmatrix} -2 & 2 & 0 \\ 1-S & -1+S & 0 \\ 1+S & -1-S & 0 \end{pmatrix} \right\}$$

$$S = 2A + 2A\rho$$

- Using the previous formulae, we obtain

$$P_{\nu_e \to \nu_e} \approx \frac{1}{2} + 2\lambda^2, \qquad P_{\nu_\mu \to \nu_\mu} \approx \frac{3}{8} + \frac{1}{2}(1 - A)\lambda^2 ,$$

$$P_{\nu_e \to \nu_\mu} \approx \frac{1}{4} - (1 - \frac{1}{2}A)\lambda^2, \qquad P_{\nu_\mu \to \nu_\tau} \approx \frac{3}{8} + \frac{1}{2}\lambda^2 ,$$

$$P_{\nu_e \to \nu_\tau} \approx \frac{1}{4} - (1 + \frac{1}{2}A)\lambda^2, \qquad P_{\nu_\tau \to \nu_\tau} \approx \frac{3}{8} + \frac{1}{2}(1 + A)\lambda^2 ,$$

- Using the best fit values for the parameters

$$P_{\nu_e \to \nu_e} \approx 0.602, \qquad P_{\nu_\mu \to \nu_\mu} \approx 0.220 ,$$

$$P_{\nu_e \to \nu_\mu} \approx 0.178, \qquad P_{\nu_\mu \to \nu_\tau} \approx 0.380 ,$$

$$P_{\nu_e \to \nu_\tau} \approx 0.401, \qquad P_{\nu_\tau \to \nu_\tau} \approx 0.421,$$

- The phase-averaged mixing matrix $\underline{U_{\alpha i}}$ modifies the flavor composition of the neutrino fluxes.

- The most common source for atmospheric and astrophysical neutrinos is thought to be pion production and decay.
- The pion decay chain generates an initial neutrino flux with a flavor composition given approximately for the neutrino fluxes : (P. Lipari, M. Lusignoli, and D. Meloni (2007))

$$\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0 = 1 : 2 : 0$$

- At the Earth :

$$\Phi_e : \Phi_\mu : \Phi_\tau = (1 + 4A\lambda^2) : (1 - 0.5A\lambda^2) : (1 - 0.5A\lambda^2)$$
$$\approx 1.2 : 1 : 1$$

▪ Lepton Flavor Violation in the SUSY

⇨ caused by the misalignment of lepton and slepton mass matrices

RG induces $$m^2_{\tilde{l}_{ij}} \approx -\frac{1}{8\pi}(3m_0^2 + A_0^2)(Y'_\nu Y'_\nu)_{ij} \log\frac{M}{M_X}$$

$$Y'_\nu Y'^{+}_\nu = R_{23}(\pi/4)U^{+}_{CKM}(Y^D_\nu)^2 U_{CKM} R^{+}_{23}(\pi/4)$$

Taking $Y^D_\nu = y_t Diag[\lambda^8, \lambda^4, 1]$

$$\boxed{BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) \approx \lambda^6 : \lambda^6 : 1}$$

(cf)

$$U^{+}_{CKM} R_{23}(\pi/4) R_{12}(\pi/4) \Rightarrow BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) \approx \lambda^8 : \lambda^4 : 1$$

$$R_{23}(\pi/4) R_{12}(\pi/4) U^{+}_{CKM} \Rightarrow BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) \approx \lambda^4 : \lambda^4 : 1$$

- Matching with triminimal parametrization

$$
\begin{aligned}
U_{PMNS} &= R_{23}(\pi/4)\, V_{CKM}^{+} R_{12}(\pi/4) \\
&= R_{23}(\pi/4) U_{\varepsilon}(\varepsilon_{23}, \varepsilon_{13}, \delta, \varepsilon_{12}) R_{12}\left(\sin^{-1} 1/\sqrt{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\sin \varepsilon_{13} e^{-i\delta} \approx \varepsilon_{13} e^{-i\delta} \approx A\lambda^{3}(1-\rho+i\eta) \\
&\sin \varepsilon_{32} \approx \varepsilon_{32} \approx -A\lambda^{2} \\
&\sin \varepsilon_{12} \approx \frac{s}{c}\left(1-\frac{\lambda}{cs}+\frac{s^{2}}{c^{2}}\lambda^{2}\right) \quad \left(\begin{matrix} s=(\sqrt{2}-1)/\sqrt{6} \\ c=(\sqrt{2}+1)/\sqrt{6} \end{matrix}\right)
\end{aligned}
$$

Final Remark

- In the light of recent data from T2K,

$$U_{CKM}^{+} R_{23}(\pi/4) R_{12}(\pi/4)$$

more favorable because it predicts

$$\boxed{\sin\theta_{13} \approx \frac{\lambda}{\sqrt{2}}}$$

Conclusion

- We have investigated QLC parametrization refelcting bi-maixmal mixing deviated by CKM matrix is consistent with the current neutrino data at 2σ.

- The result for θ_{13} from T2K disfavors QLC parametrization at 90% CL.

- We have compared QLC parametrization in which physical observables can be presented in terms of λ with Triminimal parametrization reflecting tri-bimaximal mixing deviated by some perturbation.