An A_4 Flavor Model for Quarks and Leptons in Warped Geometry

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Based on: 1)A.Kadosh and E.Pallante, JHEP08(2010)115

2)A.Kadosh and E.Pallante, JHEP06(2011)121

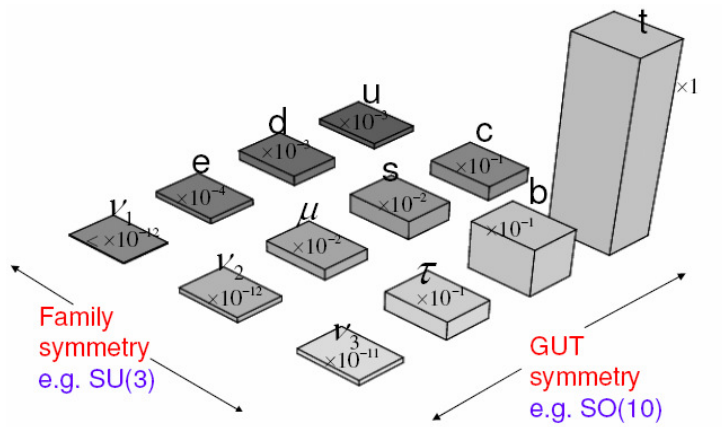
Outline

- Motivation
- The RS-A_4 setup
- Main Features
- Phenomenology and comparison with flavor anarchy
- Conclusions

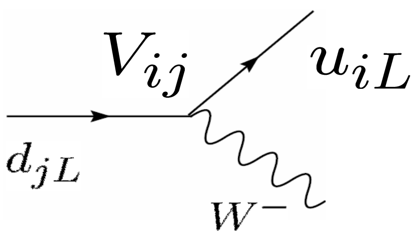
Motivation

Find a unified framework to account for the masses and mixing patterns of quarks and leptons.

Fermion mass hierarchy



Smallness and hierarchy of quark mixing angles

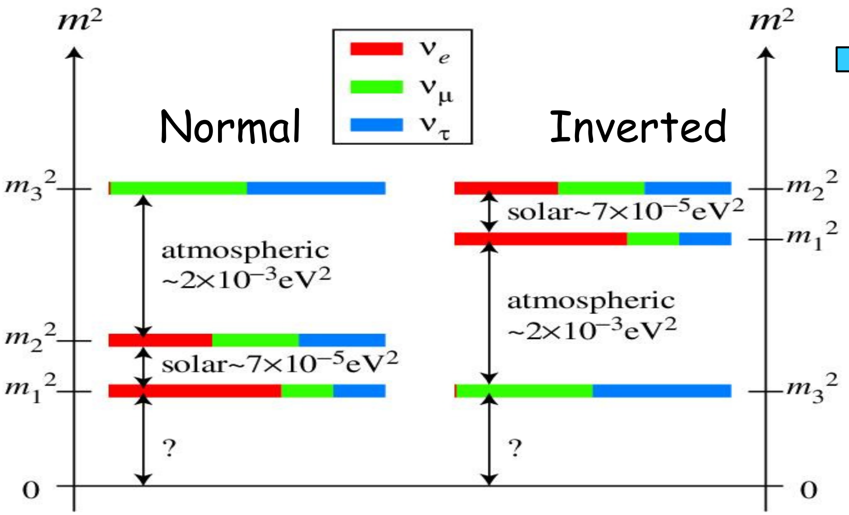


$$V_{ij} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$$\lambda_{CKM} \simeq 0.2257$$

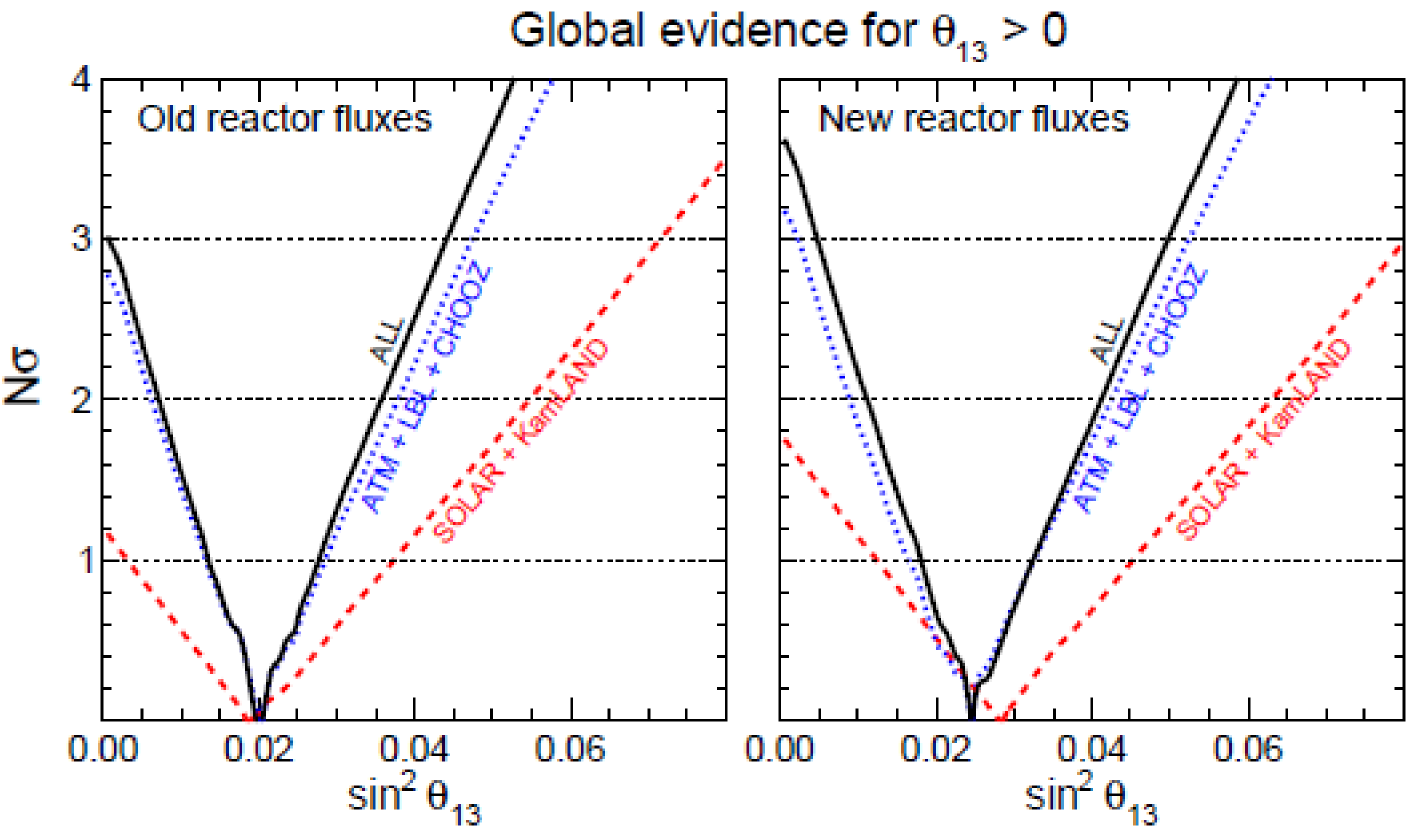
Largeness of neutrino mixing angles and smallness of neutrino masses

TBM $$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$ +H.O ?



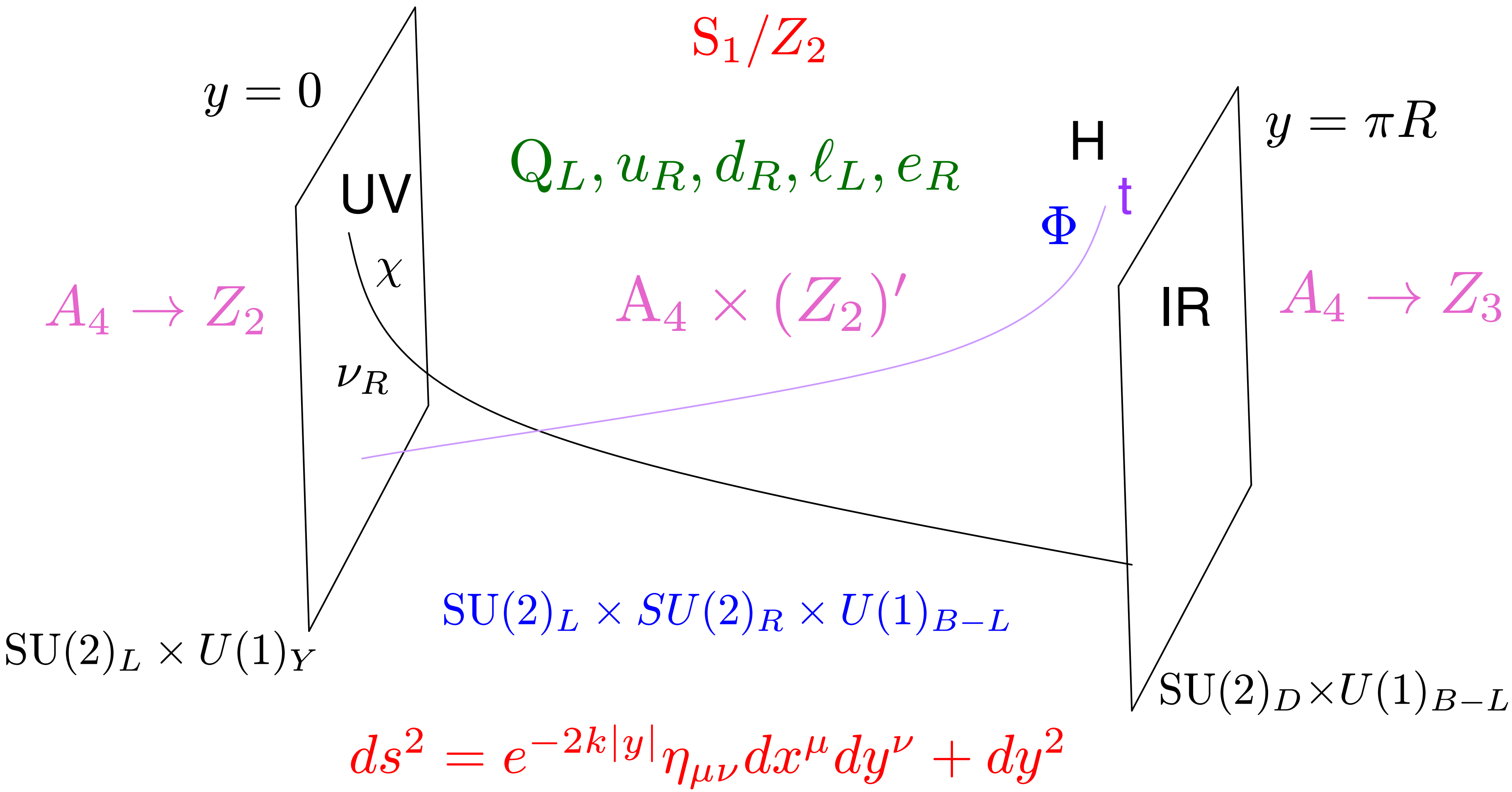
See-saw mechanism is the most elegant solution

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	< 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	< 0.046	0.331 – 0.644	2.06 – 2.81



$$\sin^2(\theta_{13}) = 0.025^{+0.025}_{-0.02}(3\sigma)$$

Fogli et al.- arXiv:1106.6028 [hep-ph]


S_1/Z_2
$y = 0$
$y = \pi R$
H
UV
$Q_L, u_R, d_R, \ell_L, e_R$
t
Φ
χ
$A_4 \to Z_2$
$A_4 \times (Z_2)'$
IR
$A_4 \to Z_3$
ν_R
$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(2)_L \times U(1)_Y$
$SU(2)_D \times U(1)_{B-L}$
$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dy^\nu + dy^2$

The Yukawa Lagrangian

$$\mathcal{L}^{Yuk}_{5D} = \underbrace{\mathcal{L}_{LO}}_{\text{UV/IR}} + \underbrace{\mathcal{L}_{NLO}}_{\text{Cross brane}}$$

$$\Lambda_{5D}^{-1/2}\bar{\ell}_L H \nu_R$$

$$(\Lambda_{5D}^{-1/2}\chi, M)\nu_R \nu_R^c$$

$$\Lambda_{5D}^{-2}\bar{Q}_L(\bar{\ell}_L)\Phi H(u_R, d_R, e_R)$$

$$\Lambda_{5D}^{-3/2}\bar{\ell}_L H \chi \nu_R$$

$$\Lambda_{5D}^{-7/2}\bar{Q}_L(\ell_L)\Phi\chi H(u_R, d_R, (e_R))$$

A_4 Assignments

$\mathrm{SU}(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times A_4$

$$\Phi \sim (1,1,1,0)\,(\underline{\mathbf{3}}), \quad \chi \sim (1,1,1,0)\,(\underline{\mathbf{3}}), \quad H\,(1,2,2,0)\,(\underline{\mathbf{1}})$$

$$Q_L \sim \left(3,2,1,\frac{1}{3}\right)(\underline{\mathbf{3}}) \qquad \ell_L \sim (1,2,1,-1)\,(\underline{\mathbf{3}})$$

$$u_R \oplus u'_R \oplus u''_R \sim \left(3,1,2,\frac{1}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') \qquad \nu_R \sim (1,1,2,0)\,(\underline{\mathbf{3}})$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3,1,2,\frac{1}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') \qquad e_R \oplus e'_R \oplus e''_R \sim (1,1,2,-1)\,(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')$$

LO Results

SSB of A_4

$$\langle \Phi \rangle = (v_\Phi, v_\Phi, v_\Phi)$$

$$A_4 \to Z_3$$

$$\langle \chi \rangle = (0, v_\chi, 0)$$

$$A_4 \to Z_2$$

$$V_L^{u,d,e} = U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

$$V_R^{u,d,e} = 1_{3\times 3}$$

$$\omega = e^{2\pi i/3}$$

$$V_L^{\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

No quark mixing at LO!

TBM neutrino mixing

NLO Corrections to the CKM Matrix

- Cross brane flavon interactions induce deviations of the CKM matrix from unity

$$M_q + \Delta M_q = U(\omega)\sqrt{3}\begin{pmatrix} \tilde{y}_{q_1} v + (x_1^q + y_1^q)/3 & (x_2^q + y_2^q)/3 & (x_3^q + y_3^q)/3 \\ (x_1^q + \omega y_1^q)/3 & \tilde{y}'_{q_2} v + (x_2^q + \omega y_2^q)/3 & (x_3^q + \omega y_3^q)/3 \\ (x_1^q + \omega^2 y_1^q)/3 & (x_2^q + \omega^2 y_2^q)/3 & \tilde{y}''_{q_3} v + (x_3^q + \omega^2 y_3^q)/3 \end{pmatrix}$$

q=u,d

12 complex parameters $(x_i^q, y_i^q) = f_\chi^q C_\chi(\tilde{x}_i^q, \tilde{y}_i^q) \simeq 0.05(\tilde{x}_i^q, \tilde{y}_i^q)$

•Parameterizing V_{CKM} in terms of λ :

$$\mathcal{O}(x_i^{u,d}, y_i^{u,d}) \Longrightarrow V_{CKM} = \begin{pmatrix} 1 & a\lambda & b\lambda^3 \\ -a^*\lambda & 1 & c\lambda^2 \\ -b^*\lambda^3 & -c^*\lambda^2 & 1 \end{pmatrix}$$

$V_{ii} \neq 1$, $|V_{ub}| \neq |V_{td}|$ and phase structure $\Longrightarrow$ $\mathcal{O}\left((x_i^{u,d})^2, (y_i^{u,d})^2\right)$

NLO Corrections to the PMNS Matrix

- Cross brane flavon interactions induce deviations of the PMNS matrix from TBM

$$M_\ell + \Delta M_\ell = U(\omega)\,\sqrt{3}\begin{pmatrix} \tilde{y}_e v + (x_1^\ell + y_1^\ell)/3 & (x_2^\ell + y_2^\ell)/3 & (x_3^\ell + y_3^\ell)/3 \\ (x_1^\ell + \omega y_1^\ell)/3 & \tilde{y}_\mu v + (x_2^\ell + \omega y_2^\ell)/3 & (x_3^\ell + \omega y_3^\ell)/3 \\ (x_1^\ell + \omega^2 y_1^\ell)/3 & (x_2^\ell + \omega^2 y_2^\ell)/3 & \tilde{y}_\tau v + (x_3^\ell + \omega^2 y_3^\ell)/3 \end{pmatrix}$$

12 complex parameters

- Parameterizing V_{PMNS} in terms of $\lambda_\ell = f_\chi^\ell C_\chi \simeq 0.05$, $\epsilon_\nu \simeq 0.08$

$$V_L^\ell = \begin{pmatrix} 1 & \lambda_\ell(\tilde{x}_2^\ell + \tilde{y}_2^\ell) & \lambda_\ell(\tilde{x}_3^\ell + \tilde{y}_3^\ell) \\ -\lambda_\ell(\tilde{x}_2^{\ell *} + \tilde{y}_2^{\ell *}) & 1 & \lambda_\ell(\tilde{x}_3^\ell + \omega\tilde{y}_3^\ell) \\ -\lambda_\ell(\tilde{x}_3^{\ell *} + \tilde{y}_3^{\ell *}) & -\lambda_\ell(\tilde{x}_3^{\ell *} + \omega^2\tilde{y}_3^{\ell *}) & 1 \end{pmatrix}$$

$$(\mathrm{V}_L^{\nu(NLO)})_{13,31} = \epsilon_\nu \Longrightarrow V_{PMNS} = (U(\omega)V_L^\ell)^\dagger V_L^{\nu(NLO)}$$

NLO Corrections to the PMNS matrix-(cont.)

$$\theta_{13}^{NLO} = \frac{e^{i\delta}-1}{\sqrt{6}} - \frac{a\epsilon_\nu}{\sqrt{3}} + \frac{1-\omega e^{i\delta}}{\sqrt{6}}(\tilde{x}_2^\ell + \tilde{y}_2^\ell + \omega\tilde{x}_3^\ell + \omega\tilde{y}_3^\ell)\lambda_\ell$$

δ Is the CP violating phase in neutrino oscillations generated only at the NLO and satisfies: $|\delta| \lesssim 0.16$

$$\theta_{23} = \frac{\omega e^{i\delta}-1}{\sqrt{6}} - \frac{\epsilon_\nu}{\sqrt{3}} + \frac{e^{i\delta}-1}{\sqrt{6}}(\tilde{x}_2^{\ell*} + \tilde{y}_2^{\ell*})\lambda_\ell + \frac{1-\omega^2 e^{i\delta}}{\sqrt{6}}(\tilde{x}_3^{\ell*} + \omega^2\tilde{y}_3^{\ell*})\lambda_\ell$$

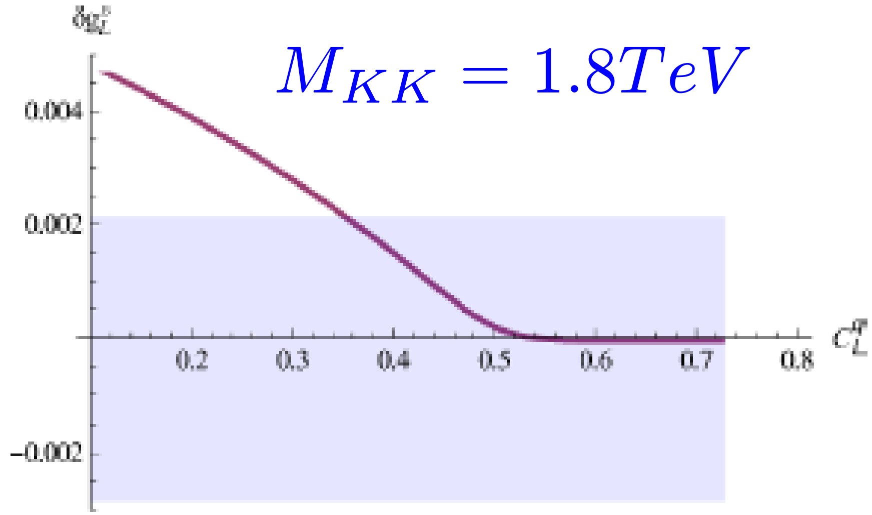
$$\theta_{12} = \frac{1}{\sqrt{3}} - \frac{\omega^2}{\sqrt{3}}(\tilde{x}_2^\ell + \tilde{y}_2^\ell)\lambda_\ell - \frac{\omega}{\sqrt{3}}(\tilde{x}_3^\ell + \tilde{y}_3^\ell)\lambda_\ell$$

Current Neutrino oscillation data (Including T2K) can still be explained with natural O(1) parameters.

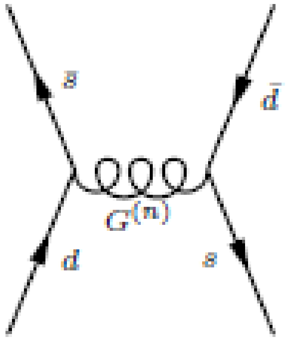
Main Features of RS-A_4 setup

Degenerate C_L

EWPM $\Rightarrow$ $Zb\bar{b}$



$\epsilon_K^{NP} = 0$



$\Rightarrow$ $M_{KK} \gtrsim 10\,TeV$

(In flavor anarchy)

Neutron EDM at 1-loop and HMFCNC $\left(C_{2,4}^{K,D}\right)$ at tree level, strongly suppressed.

Phenomenology-Dipole Operators

As a first step we work in the mass insertion approximation

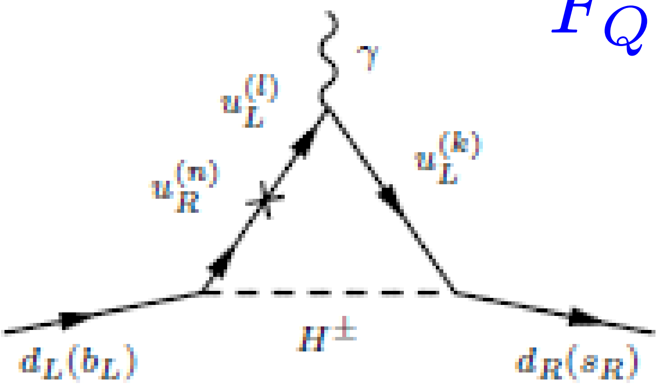
-Flavor part of amplitude in terms of spurions $F_Q, F_{u,d}, \hat{Y}_{u,d}$

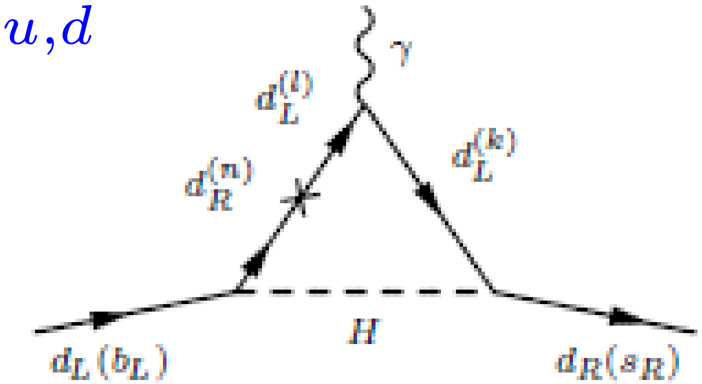
(Agashe, Soni, Perez 2004)

-IR Higgs vs. Bulk Higgs couplings $r_{nm}^{H\Phi(\chi)}(c_{Q_i}, c_{u_j,d_j}, \beta)$

$$O_7^\gamma = \bar{q}_L^i \sigma^{\mu\nu} F_{\mu\nu} q_R^j \qquad i = j = d \ (EDM)$$

$$F_Q Y_{u,d} Y_{u,d}^\dagger Y_d F_{u,d}$$





Dipole Operators (cont.)

$$(C^{d-type}_{7\gamma(8g)})_{ij} = \frac{m_{d_i} A^{1L} f_Q^2}{v^2 M_{KK}} \Big[V_R^{d\dagger} \mathrm{d}iag(f^2_{d,s,b}) (\hat{r}^d_{00})^{-1} \tilde{r}^d_{01}\, \tilde{r}^d_{11} (\hat{r}^d_{00})^{-1} V_R^d$$
$$\times \mathrm{d}iag(m^2_{d,s,b}) V_R^{d\dagger}\, (\hat{r}^d_{00})^{-1} \hat{r}^d_{10}\, V_R^d \Big]_{ij}$$

$\tilde{r}^{u,d}_{01} \tilde{r}^{u,d}_{11} = \hat{r}^{u,d}_{01} \hat{r}^{u,d}_{11} + \hat{r}^{u,d}_{01-+} \hat{r}^{u,d}_{1-+1}$ Overlap corrections

Various levels of Approximation

$(V_R^d)_{LO} = 1_{3\times 3}$ EDM=0

$(V_R^d)_{NLO}$ + Degenerate Overlaps EDM=0

$(V_R^d)_{NLO}$ + Non degenerate Overlaps

$$EDM \sim \mathcal{O}\left((m_d/m_s) f_\chi^{u,d} \Delta r\right) \approx 10^{-29} e \cdot cm$$

Dipole Operators (cont.)

Main drawback of spurion analysis $\Rightarrow$ Failure to account for the explicit coupling to the various types (BC) of KK modes.

Second step- diag. of the 1 gen. KK mass matrix

$$(\mathcal{A}_{ij})_D^{overlap} = \frac{\left(\sum_n (\hat{Y}^{d_i}_{KK})^{mass}_{1n} (\hat{Y}^{d_j}_{KK})^{mass}_{n1}\right)\Big|_{overlap}}{M^{(n)d_j}_{KK}}$$

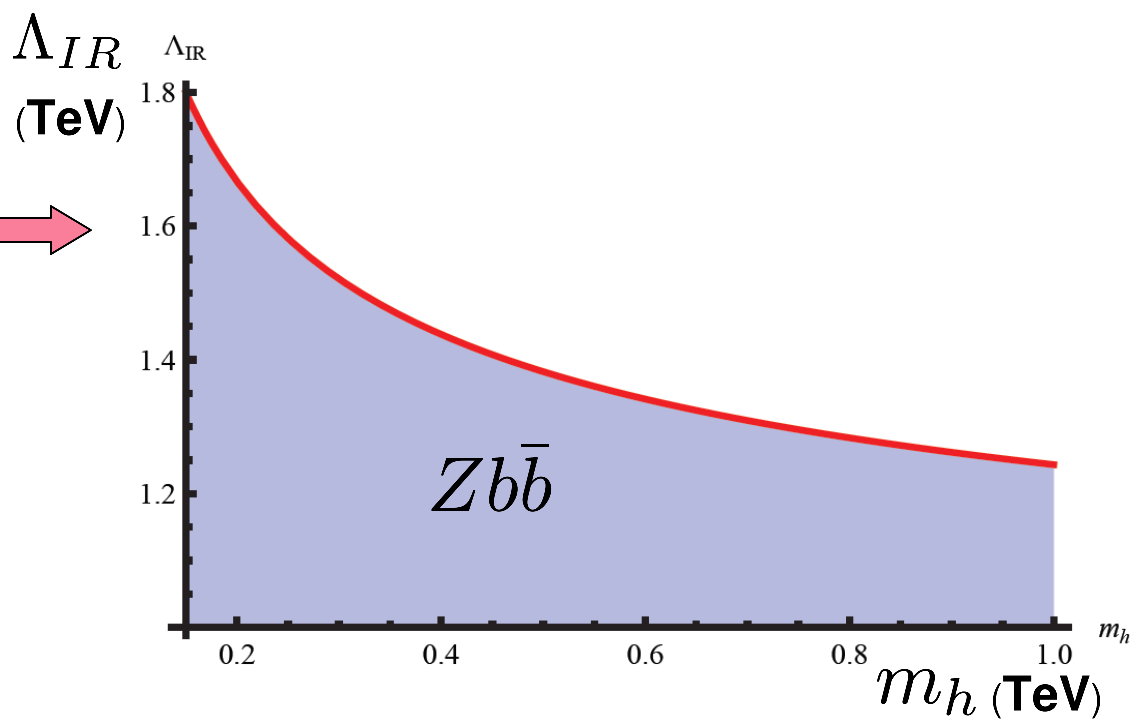
Generational mixing effects $\Rightarrow$ Mass insertion approx.

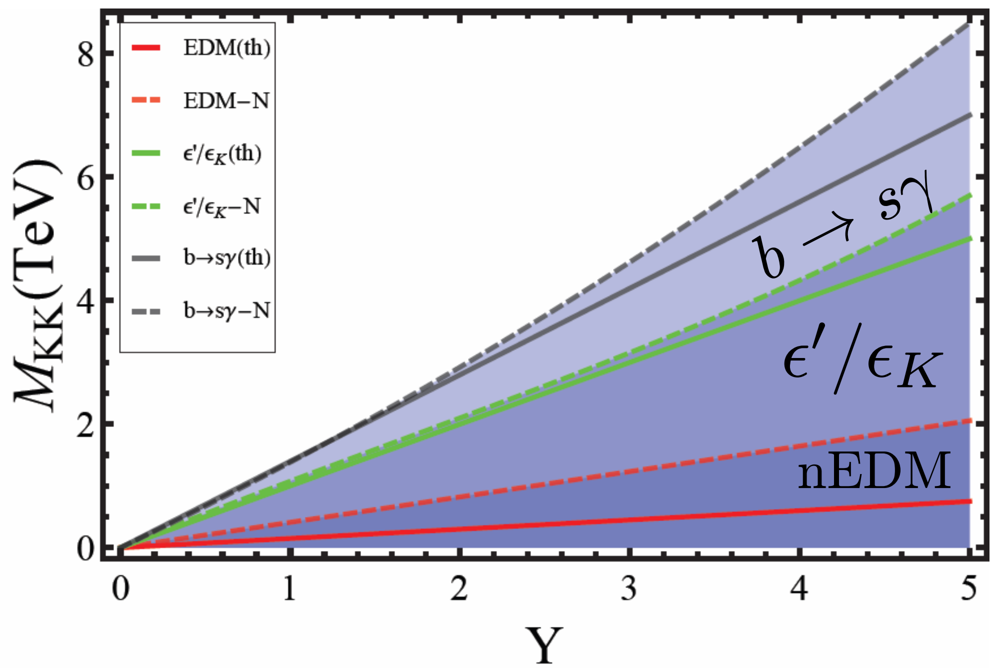
(Agashe-Azatov-Zhu 2009, Gedalia-Isidori-Perez 2009)

Third step- Approximate analytical and numerical diag. of 3 gen. zero $+1^{st}$ KK mass matrices. (12x12)

The constraint of Zbb with running of Higgs mass

(Casagrande, Neubert et al. , 2010)





Most significant constraints from dipole operators

Conclusions

RS-A_4 → Vacuum Alignment, Flavor Hierarchy
EW-Planck Hierarchy, CKM, TBM+…,
Neutrino masses. Naturalness!
EWPM constrain bulk masses!

Significant Relaxation of Pheno. constraints compared to flavor anarchy, due to degeneracy of C_L!!!

Possible extensions-

P_{LR} extended Custodial Symm. → ZMA remains the same!

Larger (other…) flavor symmetries

“Soft Wall”

Questions

(. . .)

$$
\begin{aligned}
\mathcal{L}^{4D} \supset \; & \hat{Y}^{u,d}_{ij} h^{(\,,*)}_{0(4D)} \Big[\psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{0}_{u_j,d_j} f^{-1}_{u_j,d_j} r^{H}_{00}(c_{Q_i}, c_{u_j,d_j}, \beta) + \sum\nolimits_n \psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{n}_{u_j,d_j} r^{H}_{0n}(c_{Q_i}, c_{u_j,d_j}, \beta) \\
& + \sum\nolimits_n \psi^{0\dagger}_{Q_i} f^{-1}_{Q_i} \psi^{n-+}_{u_j,d_j} r^{H}_{0n-+}(c_{Q_i}, c_{d_j,u_j}, \beta) + \sum\nolimits_n \psi^{n\dagger}_{Q_i} \psi^{0}_{u_j,d_j} f^{-1}_{u_j,d_j} r^{H}_{n0}(c_{Q_i}, c_{u_j,d_j}, \beta) \\
& + \sum\nolimits_{n,m} \psi^{n\dagger}_{Q_i} \psi^{m}_{u_j,d_j} r^{H}_{nm}(c_{Q_i}, c_{u_j,d_j}, \beta) + \sum\nolimits_{n,m} \psi^{n--}_{Q_i} (\psi^{m--}_{u_j,d_j})^{\dagger} r^{H}_{n^-m^-}(c_{Q_i}, c_{u_j,d_j}, \beta) \\
& + \sum\nolimits_{n,m} \psi^{n\dagger}_{Q_i} \psi^{m-+}_{u_j,d_j} r^{H}_{nm-+}(c_{Q_i}, c_{d_j,u_j}, \beta) + \sum\nolimits_{n,m} \psi^{n--}_{Q_i} (\psi^{m+-}_{u_j,d_j})^{\dagger} r^{H}_{n^-m^{+-}}(c_{Q_i}, c_{d_j,u_j}, \beta) \Big]
\end{aligned}
$$

$$
\hat{Y}^{u(d)}_{ij} = \frac{1}{v} r^{-1}_{00}(c_{Q_i}, c_{u_j(d_j)}, \beta) \left(F^{-1}_{Q} V^{u(d)}_{L} \mathrm{diag}(m_{u,c,t(d,s,b)}) V^{u(d)\dagger}_{R} F^{-1}_{u(d)} \right)_{ij}
$$

$$
\begin{aligned}
(C^{d-type}_{7\gamma(8g)})_{ij} \; = \; & \frac{A_{1L}}{v^2 M_{KK}} \Big[(V^{d}_{L})^{\dagger}_{i\ell} (\hat{r}^{d}_{0n})_{\ell\ell_1} (\hat{r}^{d}_{00})^{-1}_{\ell\ell_1} \left(V^{d}_{L} \mathrm{diag}(m_{d,s,b}) (V^{d}_{R})^{\dagger} \mathrm{diag}(f^{2}_{d,s,b}) \right)_{\ell\ell_1} \\
& \times (\hat{r}^{d}_{nm})_{\ell_2\ell_1} (\hat{r}^{d}_{00})^{-1}_{\ell_2\ell_1} \left(V^{d}_{R} \mathrm{diag}(m_{d,s,b}) V^{d\dagger}_{L} \mathrm{diag}(f^{2}_{Q1,Q2,Q3}) \right)_{\ell_1\ell_2} (\hat{r}^{d}_{m0})_{\ell_2\ell_3} \\
& \times (\hat{r}^{d}_{00})^{-1}_{\ell_2\ell_3} \left(V^{d}_{L} \mathrm{diag}(m_{d,s,b}) (V^{d}_{R})^{\dagger} \right)_{\ell_2\ell_3} (V^{d}_{R})_{\ell_3 j} \Big] ,
\end{aligned}
$$

A_4 Simplifications

$$\hat{r}^{u,d}_{00,10,01} = diag(r_{00,10,01}(c^L_q, c_{u_i,d_i}, \beta)) \qquad \hat{r}^{u,d}_{11} = diag(r_{11}(c_{u_i,d_i}, c^L_q, \beta))$$

$$\hat{r}^{u,d}_{01-+} = diag(r_{01-+}(c^L_q, c_{d_j,u_j}, \beta)) \quad \hat{r}^{u,d}_{1-+1} = diag(r_{1-+1}(c_{d_i,u_i}, c^L_q, \beta))$$

$$\begin{aligned}(C^{d-type}_{7\gamma(8g)})_{ij} &= \frac{m_{d_i} A^{1L} f_Q^2}{v^2 M_{KK}} \Big[V_R^{d\dagger} \mathrm{d}iag(f^2_{d,s,b})(\hat{r}^d_{00})^{-1} \tilde{r}^d_{01}\, \tilde{r}^d_{11} (\hat{r}^d_{00})^{-1} V_R^d\, \mathrm{d}iag(m_{d,s,b}) \\ &\quad \times \mathrm{d}iag(m_{d,s,b}) V_R^{d\dagger} \, (\hat{r}^d_{00})^{-1} \hat{r}^d_{10}\, V_R^d \Big]_{ij},\end{aligned}$$

$$B_P^{u,d} = \max \left((\hat{r}^{u,d}_{00})^{-3} (\hat{r}^{u,d}_{01} \hat{r}^{u,d}_{11} + \hat{r}^{u,d}_{01-+} \hat{r}^{u,d}_{1-+1})\, \hat{r}^{u,d}_{10} \right)$$

Near degeneracy
Of overlap corrections

$$\begin{aligned}(C^{d-type}_7)_{ij} &= \frac{A^{1L} m_{d_i} m_{d_j} B_P^d}{v^2 M_{KK}} \left[V_R^{d\dagger} \mathrm{d}iag(f^2_{d_1,d_2,d_3}) V_R^d \mathrm{d}iag(m_{d,s,b}) V_L^{d\dagger} \mathrm{d}iag(f^2_{Q_1,Q_2,Q_3}) V_L^d \right]_{ij} \\ &= \frac{A^{1L} f_Q^2 m_{d_i} m_{d_j}^2 B_P^d}{v^2 M_{KK}} \sum_{n=1}^{3} (V_R^d)^*_{ni} (V_R^d)_{nj} f^2_{d_n}\end{aligned}$$

1 generation KK Yukawa matrices

$$\mathrm{Y}^{d}_{KK} = \begin{pmatrix} \bar{Q}^{d(0)}_{L} \\ \bar{d}^{(1^{--})}_{L} \\ \bar{Q}^{d(1)}_{L} \\ \bar{\tilde{d}}^{(1^{+-})}_{L} \end{pmatrix}^{T} \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} & 0 & \breve{y}_d f_q^{-1} r_{01} & \breve{y}_u f_q^{-1} r_{101} \\ 0 & \breve{y}_d^{*} r_{22} & 0 & 0 \\ \breve{y}_d f_d^{-1} r_{10} & 0 & \breve{y}_d r_{11} & \breve{y}_u r_{111} \\ 0 & \breve{y}_u^{*} r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} d^{(0)}_{R} \\ Q^{d(1^{--})}_{R} \\ d^{(1)}_{R} \\ \tilde{d}^{(1^{-+})}_{R} \end{pmatrix}$$

$$\hat{Y}^{d(h_-)}_{KK} = \begin{pmatrix} \bar{Q}^{d(0)}_{L} \\ \bar{d}^{(1^{--})}_{L} \\ \bar{Q}^{d(1)}_{L} \\ \bar{\tilde{d}}^{(1^{+-})}_{L} \end{pmatrix}^{T} \begin{pmatrix} -\breve{y}_u f_q^{-1} f_u^{-1} r_{00} & 0 & -\breve{y}_u f_q^{-1} r_{01} & -\breve{y}_d f_q^{-1} r_{101} \\ 0 & \breve{y}_d^{*} r_{22} & 0 & 0 \\ -\breve{y}_u f_u^{-1} r_{10} & 0 & -\breve{y}_u r_{11} & -\breve{y}_d r_{111} \\ 0 & \breve{y}_u^{*} r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} u^{(0)}_{R} \\ Q^{u(1^{--})}_{R} \\ u^{(1)}_{R} \\ \tilde{u}^{(1^{-+})}_{R} \end{pmatrix}$$

$$\hat{Y}^{d(h_+)}_{KK} = \begin{pmatrix} \bar{Q}^{u(0)}_{L} \\ \bar{u}^{(1^{--})}_{L} \\ \bar{Q}^{u(1)}_{L} \\ \bar{\tilde{u}}^{(1^{+-})}_{L} \end{pmatrix}^{T} \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} & 0 & \breve{y}_d f_q^{-1} r_{01} & \breve{y}_u f_q^{-1} r_{101} \\ 0 & -\breve{y}_u^{*} r_{22} & 0 & 0 \\ \breve{y}_d f_d^{-1} r_{10} & 0 & \breve{y}_d r_{11} & \breve{y}_u r_{111} \\ 0 & -\breve{y}_d^{*} r_{222} & 0 & 0 \end{pmatrix} \begin{pmatrix} d^{(0)}_{R} \\ Q^{d(1^{--})}_{R} \\ d^{(1)}_{R} \\ \tilde{d}^{(1^{-+})}_{R} \end{pmatrix}$$

4X4 one gen. KK mass matrix

$$\frac{\hat{\mathrm{M}}_d^{KK}}{(M_{KK})} = \begin{pmatrix} \bar{Q}_L^{d(0)} \\ \tilde{d}_L^{(1--)} \\ \bar{Q}_L^{d(1)} \\ \tilde{d}_L^{(1+-)} \end{pmatrix}^T \begin{pmatrix} \breve{y}_d f_q^{-1} f_d^{-1} r_{00} x & 0 & \breve{y}_d f_q^{-1} r_{01} x & \breve{y}_u f_q^{-1} r_{101} x \\ 0 & \breve{y}_d^* r_{22} x & 1 & 0 \\ \breve{y}_d f_d^{-1} r_{10} x & 1 & \breve{y}_d r_{11} x & \breve{y}_u r_{111} x \\ 0 & \breve{y}_u^* r_{222} x & 0 & 1 \end{pmatrix} \begin{pmatrix} d_R^{(0)} \\ Q_R^{d(1--)} \\ d_R^{(1)} \\ \tilde{d}_R^{(1-+)} \end{pmatrix}$$

$$\breve{y}_{u,d} \equiv (\hat{y}_{u,d}^{LO})_{11} v_\Phi^{4D} e^{k\pi R}/k$$

12X12 three gen. KK mass matrix

$$\hat{\mathrm{M}}_{Full}^D = M_{KK} \begin{pmatrix} \hat{\mathrm{M}}_d^{KK}/M_{KK} & x\hat{Y}_{KK}^s(\hat{y}_{12}^{LO}, f_s) & x\hat{Y}_{KK}^b(\hat{y}_{13}^{LO}, f_b) \\ x\hat{Y}_{KK}^d(\hat{y}_{21}^{LO}, f_d) & \hat{\mathrm{M}}_s^{KK}/M_{KK} & x\hat{Y}_{KK}^b(\hat{y}_{23}^{LO}, f_b) \\ x\hat{Y}_{KK}^d(\hat{y}_{31}^{LO}, f_d) & x\hat{Y}_{KK}^s(\hat{y}_{32}^{LO}, f_s) & \hat{\mathrm{M}}_b^{KK}/M_{KK} \end{pmatrix}$$

$O_L^{s\,K\,K}$

One example of KK diag. matrix

$$\begin{pmatrix} 1 & 0.94 f_Q^{-1} r_{01} \breve{y}_s x & \frac{f_Q^{-1}}{\sqrt{2}} r_{101} \breve{y}_c x & \frac{f_Q^{-1}}{\sqrt{2}} r_{101} \breve{y}_c x \\ -0.94 f_Q^{-1} r_{01} \breve{y}_s x & 1 & (6.06 r_{11} + 4.72 r_{22}) e^{i\theta_c} \breve{y}_s^* x & -(6.06 r_{11} + 4.72 r_{22}) e^{i\theta_c} \breve{y}_s^* x \\ \mathcal{O}(x^2) & (8.6(r_{11} + 8.1 r_{22}) \breve{y}_s & -\frac{e^{i\theta_c}}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/\breve{y}_c^*)}{4\sqrt{2}(r_{111}+r_{222})} x & \frac{e^{i\theta_c}}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/\breve{y}_c^*)}{4\sqrt{2}(r_{111}+r_{222})} x \\ -f_Q^{-1} r_{101} \breve{y}_c^* x & \mathcal{O}(x^2) & \frac{1}{\sqrt{2}} + \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/|\breve{y}_c|)}{4\sqrt{2}(r_{111}+r_{222})} x & \frac{1}{\sqrt{2}} - \frac{(r_{111}^2 - r_{222}^2)\breve{y}_c + r_{11}^2 (|\breve{y}_s|^2/|\breve{y}_c|)}{4\sqrt{2}(r_{111}+r_{222})} x \end{pmatrix}$$

12X12 additional A_4 rotation

$$\hat{\mathrm{O}}_{L,R}^{(U,D)A_4} = \begin{pmatrix} (V_{L,R}^{u,d})_{11} \times \tilde{\mathbb{1}}_{4\times 4} & (V_{L,R}^{u,d})_{12} \times \tilde{\mathbb{1}}_{4\times 4} & (V_{L,R}^{u,d})_{13} \times \tilde{\mathbb{1}}_{4\times 4} \\ (V_{L,R}^{u,d})_{21} \times \tilde{\mathbb{1}}_{4\times 4} & (V_{L,R}^{u,d})_{22} \times \tilde{\mathbb{1}}_{4\times 4} & (V_{L,R}^{u,d})_{23} \times \tilde{\mathbb{1}}_{4\times 4} \\ (V_{L,R}^{u,d})_{31} \times \tilde{\mathbb{1}}_{4\times 4} & (V_{L,R}^{u,d})_{32} \times \tilde{\mathbb{1}}_{4\times 4} & (V_{L,R}^{u,d})_{33} \times \tilde{\mathbb{1}}_{4\times 4} \end{pmatrix}$$

Dynamical Completion Issues

- Vacuum Alignment – We will have to make sure that the scalar potential doesn't ruin the specific VEV structure we are interested in.
- Most importantly in any flavor model one should explain the origin of quark and lepton masses and their hierarchy (FN, GUT's, WED, UED etc.…).
- Ultimately, a dynamical origin for the A_4 symmetry should be supplemented.
- One of the possibilities is obtaining A_4 via compactification of a 6 dimensional flat space on an orbifold T_2/Z_2. The various fields reside on the four orbifold fixed points (Branes).

(Feruglio and Altarelli)

Off diagonal CKM elements

$$V_{us} = -V_{cd}^{*} \simeq \left((\tilde{x}_2^d + \tilde{y}_2^d) f_\chi^s - (\tilde{x}_2^u + \tilde{y}_2^u) f_\chi^c \right)$$

$$V_{cb} = -V_{ts}^{*} \simeq \left((\tilde{x}_3^d + \omega \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \omega \tilde{y}_3^u) f_\chi^t \right)$$

$$V_{ub} = -V_{td}^{*} \simeq \left((\tilde{x}_3^d + \tilde{y}_3^d) f_\chi^b - (\tilde{x}_3^u + \tilde{y}_3^u) f_\chi^t \right)$$

$$\mathrm{V}_{CKM} = \left(\begin{array}{ccc} 1 & V_{us} & V_{ub} \\ -V_{us}^{*} & 1 & V_{cb} \\ -V_{ub}^{*} & -V_{cb}^{*} & 1 \end{array} \right)$$

$$\mathcal{H} = \left(\begin{array}{cc} H & \tilde{H} \end{array} \right) = \left(\begin{array}{cc} h_0^* & h_+ \\ -h_+^* & h_0 \end{array} \right) \qquad h_0(x, y) = v_H(\beta_H, y) + \sum_n h_0^{(n)}(x) \phi_n(y)$$

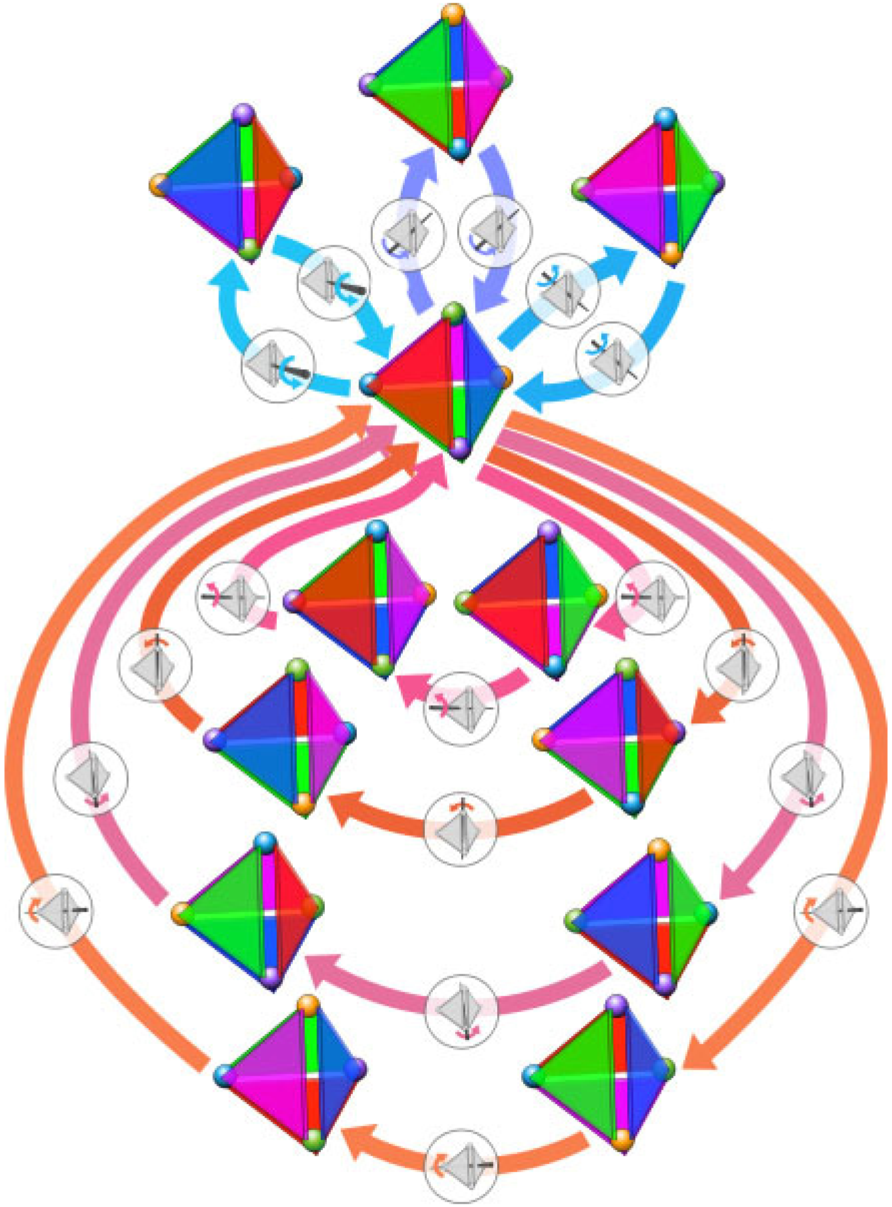
ZMA RH diag. Matrices

$$V_R^q = \begin{pmatrix} 1 & \Delta_1^q & \Delta_2^q \\ -(\Delta_1^q)^* & 1 & \Delta_3^q \\ -(\Delta_2^q)^* & -(\Delta_3^q)^* & 1 \end{pmatrix}$$

$$\Delta_1^q = \frac{m_{q1}}{m_{q2}} \left[f_\chi^{q_1} \left((\tilde{x}_1^q)^* + \omega^2 (\tilde{y}_1^q)^* \right) + f_\chi^{q_2} \left(\tilde{x}_2^q + \tilde{y}_2^q \right) \right]$$

$$\Delta_2^q = \frac{m_{q1}}{m_{q3}} \left[f_\chi^{q_1} \left((\tilde{x}_1^q)^* + \omega (\tilde{y}_1^q)^* \right) + f_\chi^{q_3} \left(\tilde{x}_3^q + \tilde{y}_3^q \right) \right]$$

$$\Delta_3^q = \frac{m_{q2}}{m_{q3}} \left[f_\chi^{q_2} \left((\tilde{x}_2^q)^* + \omega (\tilde{y}_2^q)^* \right) + f_\chi^{q_3} \left(\tilde{x}_3^q + \omega \tilde{y}_3^q \right) \right]$$



The Tetrahedral Group A_4

- A(4) is the group of even permutations of 4 objects
- It is also isomorphic to the symmetry group of a regular tetrahedron, and is a subgroup of SO(3)
- Other extensions include:

  T' $\quad \Delta(27) \quad \Sigma(81)$
- Will be used to explain proximity of mixing in the lepton sector to TBM, and proximity of mixing in the quark sector to unity.
- Differs from other types of flavor models: “Anarchic”, continuous flavor groups, GUT’s, (SUSY),…

Some A(4) Basic properties:

•A(4) has one real triplet, $\underline{3}$ and three “singlets”: $\underline{1}$, $\underline{1}'$ and $\underline{1}''$

$$\underline{3}\otimes\underline{3}=\underline{3}_s\oplus\underline{3}_a\oplus\underline{1}\oplus\underline{1}'\oplus\underline{1}'', \quad \text{and} \quad \underline{1}'\otimes\underline{1}'=\underline{1}''$$

$$(\underline{3}\otimes\underline{3})_{\underline{3}_s} = (x_2y_3+x_3y_2\,,\,x_3y_1+x_1y_3\,,\,x_1y_2+x_2y_1),$$

$$(\underline{3}\otimes\underline{3})_{\underline{3}_a} = (x_2y_3-x_3y_2\,,\,x_3y_1-x_1y_3\,,\,x_1y_2-x_2y_1),$$

$$(\underline{3}\otimes\underline{3})_{\underline{1}} = x_1y_1+x_2y_2+x_3y_3,$$

$$(\underline{3}\otimes\underline{3})_{\underline{1}'} = x_1y_1+\omega\,x_2y_2+\omega^2\,x_3y_3, \qquad \omega = e^{i2\pi/3}$$

$$(\underline{3}\otimes\underline{3})_{\underline{1}''} = x_1y_1+\omega^2\,x_2y_2+\omega\,x_3y_3,$$

A simple A(4) Model

(Ma, Feruglio, Altarelli, Babu, Volkas…)

•We assign the SM fermions to the following representations:

$$Q_L \sim \left(3,2,\tfrac{1}{3}\right)(\underline{\mathbf{3}}) \qquad \ell_L \sim (1,2,-1)(\underline{\mathbf{3}})$$

Under A(4)

SU(3)xSU(2)xU(1)

$$u_R \oplus u'_R \oplus u''_R \sim \left(3,1,\tfrac{4}{3}\right)(\underline{1} \oplus \underline{1}' \oplus \underline{1}'') \qquad \nu_R \sim (1,1,0)(\underline{\mathbf{3}})$$

$$d_R \oplus d'_R \oplus d''_R \sim \left(3,1,-\tfrac{2}{3}\right)(\underline{1} \oplus \underline{1}' \oplus \underline{1}'') \qquad e_R \oplus e'_R \oplus e''_R \sim (1,1,-2)(\underline{1} \oplus \underline{1}' \oplus \underline{1}'')$$

•The scalar sector of this model will be given by:

$$\Phi \sim (1,2,-1)(\underline{\mathbf{3}}), \quad \phi \sim (1,2,-1)(\underline{\mathbf{1}}), \quad \chi \sim (1,1,0)(\underline{\mathbf{3}}).$$

•We will also need an additional U(1) symmetry which will be explicitly broken to Z_2 under which ϕ, χ and Q_L are odd and the rest of the fields are even.

•The Yukawa Lagrangian is:

$$\mathcal{L}_{\rm Yuk} = \lambda_u\ (\overline{Q}_L\Phi)_{\underline{1}}\, u_R + \lambda'_u(\overline{Q}_L\Phi)_{\underline{1}'}\, u''_R + \lambda''_u(\overline{Q}_L\Phi)_{\underline{1}''}\, u'_R +$$

$$+\ \lambda_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}}\, d_R + \lambda'_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}'}\, d''_R + \lambda''_d(\overline{Q}_L\tilde{\Phi})_{\underline{1}''}\, d'_R +$$

$$+\ \lambda_\nu(\bar{\ell}_L\nu_R)_{\underline{1}}\,\phi + M\,[\overline{\nu}_R(\nu_R)^c]_{\underline{1}} + \lambda_\chi[\overline{\nu}_R(\nu_R)^c]_{\underline{3}_s}\cdot\chi +$$

$$+\ \lambda_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}}\, e_R + \lambda'_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}'}\, e''_R + \lambda''_e(\bar{\ell}_L\tilde{\Phi})_{\underline{1}''}\, e'_R + h.c.$$

•And the resulting mass matrix in each sector, f=(u,d,e):

$$\left(\overline{f}_{1L},\overline{f}_{2L},\overline{f}_{3L}\right)\begin{pmatrix}\lambda v_1 & \lambda' v_1 & \lambda'' v_1\\ \lambda v_2 & \omega\lambda' v_2 & \omega^2\lambda'' v_2\\ \lambda v_3 & \omega^2\lambda' v_3 & \omega\lambda'' v_3\end{pmatrix}\begin{pmatrix}f_R\\ f''_R\\ f'_R\end{pmatrix} + h.c.$$

The Neutrino Sector

- From the Yukawa Lagrangian we get that the Dirac and the bare Majorana mass matrices are proportional to the identity:

$$M_\nu^D = \lambda_\nu \, v_\phi \, 1 \equiv m_\nu^D \, 1 \quad \text{and} \quad M_{\nu Bare}^{Maj.} = M\mathbf{1}$$

The required non-trivial structure is supplied by the Yukawa coupling to the field, χ , which turns out to be:

$$\lambda_\chi \ \left(\overline{\nu}_{1R}, \overline{\nu}_{2R}, \overline{\nu}_{3R}\right) \begin{pmatrix} 0 & \chi_3 & \chi_2 \\ \chi_3 & 0 & \chi_1 \\ \chi_2 & \chi_1 & 0 \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix}$$

•Inserting the VEV of χ the resulting 6x6 mass matrix is:

$$\begin{pmatrix} 0 & 0 & 0 & m_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_\nu^D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_\nu^D \\ m_\nu^D & 0 & 0 & M & 0 & M_\chi \\ 0 & m_\nu^D & 0 & 0 & M & 0 \\ 0 & 0 & m_\nu^D & M_\chi & 0 & M \end{pmatrix} \qquad M_\chi \equiv \lambda_\chi v_\chi$$

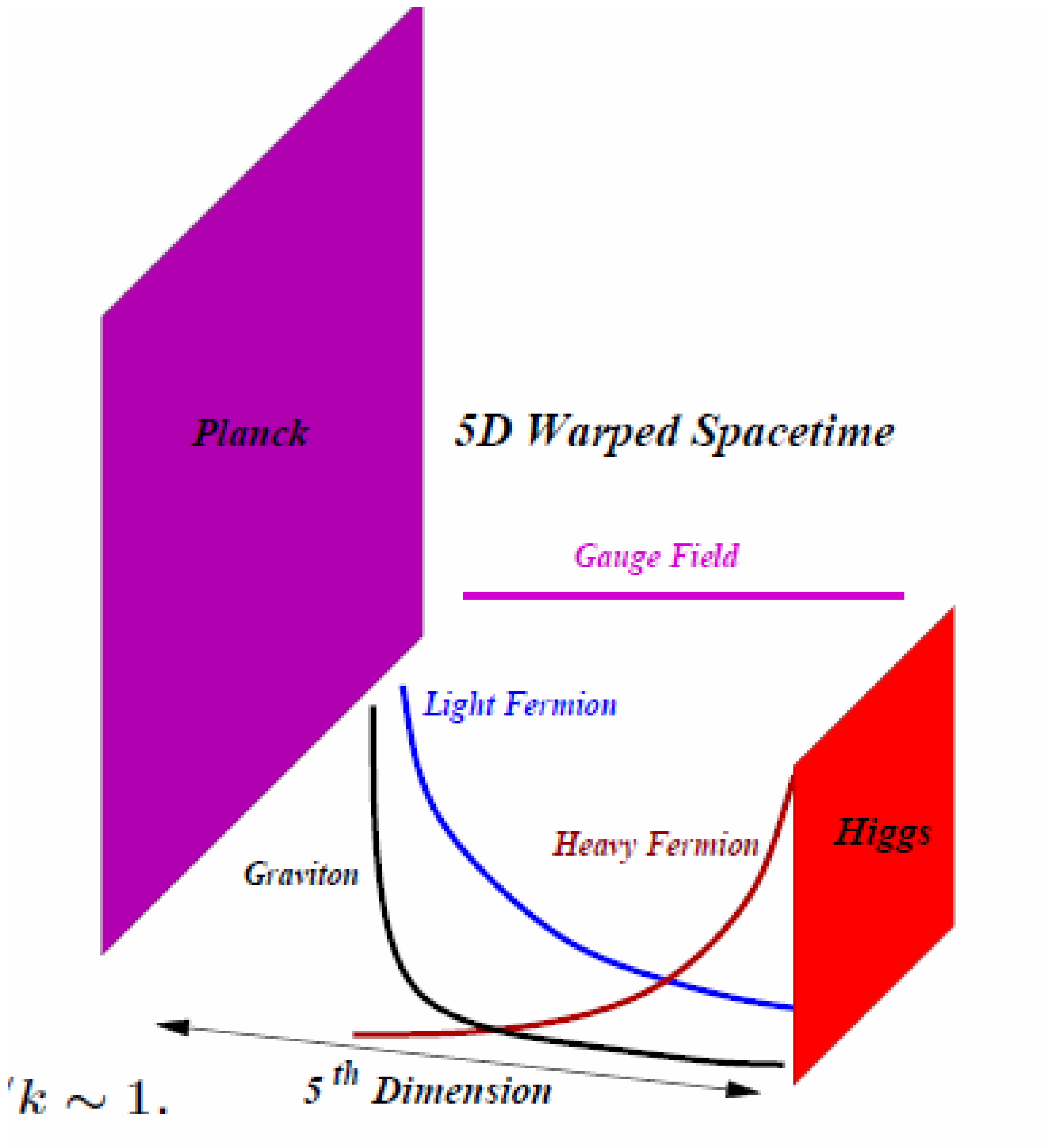
- In the see-saw limit, $|M|, |M_\chi| \gg m_\nu^D$ the effective 3x3 mass matrix for the light neutrinos is given by:

$$M_L = -M_\nu^D M_R^{-1} (M_\nu^D)^T = -\frac{(m_\nu^D)^2}{M} \begin{pmatrix} \frac{M^2}{M^2-M_\chi^2} & 0 & -\frac{M M_\chi}{M^2-M_\chi^2} \\ 0 & 1 & 0 \\ -\frac{M M_\chi}{M^2-M_\chi^2} & 0 & \frac{M^2}{M^2-M_\chi^2} \end{pmatrix}$$

•The diagonalization matrix turns out to be : $V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$

•So the MNSP matrix at this order is Tri-bi-maximal:

$$V_{MNSP} = V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{-i\pi/6}}{\sqrt{2}} \end{pmatrix}$$


Planck
5D Warped Spacetime
Gauge Field
Light Fermion
Higgs
Heavy Fermion
Graviton
5 th Dimension
'k ∼ 1.