Continuous and Discrete (Flavor) Symmetries

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Feruglio et al.('07), H/Serone ('11)
Blum/H/Lindner ('07), H/Ziegler ('10), Blum/H ('09), H/Meloni (to appear)

Outline

- Observations
- General comments on the flavor symmetry G_F
  properties, possible symmetries
- Examples
  - A_4 and S_4: tri-bimaximal mixing
  - Dihedral groups in general, θ_C and $\mu\tau$ symmetry
- Conclusions

Observations: Charged Fermion Masses

	Mass at M_Z	in units of $m_t(M_Z)$
u	(1.7 ± 0.4) MeV	λ^8
c	(0.62 ± 0.03) GeV	λ^4
t	(171 ± 3) GeV	1

	Mass at M_Z	in units of $m_b(M_Z)$
d	(3.0 ± 0.6) MeV	λ^4
s	(54 ± 8) MeV	λ^2
b	(2.87 ± 0.03) GeV	1

	Mass at M_Z	in units of $m_\tau(M_Z)$
e	$(0.486570161 \pm 0.000000042)$ MeV	$\lambda^{4\div5}$
μ	$(102.7181359 \pm 0.0000092)$ MeV	λ^2
τ	$1.74624^{+0.00020}_{-0.00019}$ GeV	1

Observations: Neutrino Masses

- Mass hierarchy in ν sector is mild compared to charged fermions
- We know two mass squared differences (at 2σ)

  (Fogli et al. ('11))

$$\delta m^2 \equiv \Delta m^2_{\rm sol} \equiv m_2^2 - m_1^2 = (7.58^{+0.41}_{-0.42}) \times 10^{-5}\ {\rm eV}^2$$

$$\Delta m^2 \equiv m_3^2 - \frac{m_2^2 + m_1^2}{2} = (2.35^{+0.22}_{-0.18}) \times 10^{-3}\ {\rm eV}^2$$

- However $m_1 < m_3$ and $m_3 < m_1$ is possible due to $\Delta m^2 \lessgtr 0$

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- However $m_1 < m_3$ and $m_3 < m_1$ is possible due to $\Delta m^2 \lessgtr 0$
- Also unknown: the absolute mass scale m_0

$\sum m_\nu \leq 0.7\,{\rm eV}$	(WMAP+LSS)
$m_\beta < 2.2\,{\rm eV}$	(Mainz)
$\lvert m_{ee}\rvert < (0.2...1)\,{\rm eV}$	(Heidelberg-Moscow, IGEX, NEMO)

Observations: Fermion Masses and Mixings

- The mixing pattern is very peculiar ***(Fogli et al. ('11))***

$$\sin^2(\theta^l_{12}) = 0.306^{+0.036}_{-0.031}\ , \quad \sin^2(\theta^l_{23}) = 0.42^{+0.18}_{-0.06} \quad \text{and} \quad \sin^2(\theta^l_{13}) = 0.021^{+0.015}_{-0.013}$$

$$\theta^l_{12} = (33.6^{+2.2}_{-2.0})^\circ\ , \quad \theta^l_{23} = (40.4^{+10.4}_{-3.5})^\circ \quad \text{and} \quad \theta^l_{13} = (8.3^{+2.6}_{-3.2})^\circ \qquad (2\,\sigma)$$

compare to quark sector

$$\theta^q_{12} \approx 13^\circ\ , \quad \theta^q_{23} \approx 2.4^\circ \quad \text{and} \quad \theta^q_{13} \approx 0.21^\circ$$

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(Still) Hint at a Flavor Symmetry !?!

Special Patterns for Lepton Mixings - Before T2K

- $\mu\tau$ Symmetry

$$\sin^2(\theta^l_{23}) = \frac{1}{2}\ , \quad \sin^2(\theta^l_{13}) = 0$$

$$|U_{PMNS}| = \begin{pmatrix} \cos(\theta^l_{12}) & \sin(\theta^l_{12}) & 0 \\ \frac{\sin(\theta^l_{12})}{\sqrt{2}} & \frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin(\theta^l_{12})}{\sqrt{2}} & \frac{\cos(\theta^l_{12})}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Tri-Bimaximal mixing (TB mixing)

$$\sin^2(\theta^l_{12}) = \frac{1}{3}\ , \quad \sin^2(\theta^l_{23}) = \frac{1}{2}\ , \quad \sin^2(\theta^l_{13}) = 0$$

$$|U_{PMNS}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Special Patterns for Lepton Mixings -Before T2K

Further patterns which might be realized

- Golden ratio $(\phi = \frac{1}{2}\left(1+\sqrt{5}\right))$

$$\sin^2(\theta^l_{12}) = \frac{1}{\sqrt{5}\phi} \approx 0.276\ ,\ \ \sin^2(\theta^l_{23}) = \frac{1}{2}\ ,\ \ \sin^2(\theta^l_{13}) = 0$$

$$|U_{PMNS}| = \left(\begin{array}{ccc} \sqrt{\frac{1}{10}(5+\sqrt{5})} & \sqrt{\frac{2}{5+\sqrt{5}}} & 0 \\ \frac{1}{\sqrt{5+\sqrt{5}}} & \sqrt{\frac{1}{20}(5+\sqrt{5})} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{5+\sqrt{5}}} & \sqrt{\frac{1}{20}(5+\sqrt{5})} & \frac{1}{\sqrt{2}} \end{array} \right)$$

Are these still reasonable Leading Order Patterns ?!

Summary of Observations

- Three families of elementary particles observed
- Strong hierarchy among charged fermions
- Mass hierarchy in the ν sector is much milder, ordering and m_0 unknown
- Only θ_C is sizable in the quark sector
- Special lepton mixing pattern could be realized? Which one?
- No excessive flavor violation observed, all in accordance with Standard Model

$$\Downarrow$$

Necessity of Constraints on Couplings y_{ij}^u, y_{ij}^l, etc.

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Necessity of Flavor Symmetry G_F ?!

Effect of Flavor Symmetry G_F

- Yukawa couplings in the SM

$$y^u_{ij}\, Q_i\, H^c\, u^c_j \quad \text{or} \quad y^d_{ij}\, Q_i\, H\, d^c_j$$
$$\text{with} \quad y^{u,d}_{ij} \in \mathbb{C}$$

- Enforce invariance under G_F
  $\hookrightarrow$ Constraints on $y^{u,d}_{ij}$
  $\hookrightarrow$ Extension of scalar sector needed

$H \longrightarrow H_k$	or	$H \longrightarrow H\, \phi_k/(M, \Lambda)$
multi-Higgs doublets	or	flavon fields
$y^d_{ij,k}\, Q_i\, H_k\, d^c_j$	or	$y^d_{ij,k}\, Q_i\, H\, d^c_j\, \left(\frac{\phi_k}{(M,\Lambda)}\right)$
renormalizable couplings	or	in general non-renormalizable

but in both cases alignment is needed

Choice of Flavor Symmetry G_F

The symmetry G_F could ...

- ... be abelian or non-abelian

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- ... be broken at low or high energies (low: electroweak scale)
  (high: seesaw/GUT scale)

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  (high: seesaw/GUT scale)

Its maximal possible size depends on the gauge group.

Possible Symmetries

- **Continuous Groups**

$$SU(2),\ U(2),\ SO(3),\ SU(3),\ U(3)$$

- **Discrete Groups**
  - Permutation symmetries, S_N and A_N with $N \in \mathbb{N}$
  - Dihedral symmetries, D_n and D'_n with $n \in \mathbb{N}$
  - Further double-valued groups: T', O', I', ...
  - Subgroups of $SU(3)$, series of $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ groups with $n \in \mathbb{N}$, as well as finite number of Σ groups
  - Additional groups such as subgroups of the mentioned groups, e.g. $T_7 \cong Z_7 \rtimes Z_3 \subset \Delta(147)$, subgroups of $U(3)$, e.g. $\Sigma(81)$

Models with Continuous G_F

The group $U(2)$ ***(Barbieri/Hall ('96), Barbieri et al. ('96, '97, '99))***

- $\underline{\mathbf{2}} + \underline{\mathbf{1}}$ structure explains heaviness of the third generation
- Framework of the models: supersymmetric $SU(5)$ ($SO(10)$) GUT
- Symmetry is broken in steps: $U(2) \to U(1) \to$ nothing
- $\underline{\mathbf{2}} + \underline{\mathbf{1}}$ also appropriate for solving the SUSY flavor problem
- Nine relations among fermion masses and mixings
- However, only θ_{23} large in lepton sector and θ_{12}, θ_{13} small

Models with Continuous G_F

The groups $SO(3)$ and $SU(3)$ are promising, ...

(King ('05), King/Ross ('01, '03), de Medeiros Varzielas/Ross ('05))

- ... since they allow to unify all three generations (in $SO(3)$ only left-handed ones)
- ... since the largeness of two mixing angles in the lepton sector indicates that not only two generations are closely related

Models with Continuous G_F

General characteristics of $SO(3)$ & $SU(3)$ flavor models

- Hierarchical breaking
  $SO(3) \to SO(2) \to$ nothing and $SU(3) \to SU(2) \to$ nothing
- SUSY models with SM, PS or (in EDs) $SO(10)$ gauge group
- Masses of SM fermions arise from non-renorm. operators only
- Additionally symmetries (Z_n factors and $U(1)$) to forbid operators
- Different messenger scales allow different expansion parameters in up quark, down quark and charged lepton sector
- Specific vacuum structure necessary to predict TB mixing
- Vacuum alignment achieved by F- or D-terms

Models with Continuous G_F

Lepton mixing in the $SO(3)$ & $SU(3)$ flavor models

- TB mixing is achieved through CSD ***(King ('05))***
- Corrections from charged lepton sector are small

Discrete "spin-offs" of the $SO(3)$ & $SU(3)$ models

(de Medeiros Varzielas et al. ('05, '06), King/Malinsky ('06))

- Use A_4, $\Delta(27)$, $\Delta(108)$ as flavor group
- Very similar results compared to the models with continuous symmetries
- In general the vacuum alignment is simpler
- No D-terms arise from discrete flavor symmetry

Possible Discrete Symmetries

- Permutation symmetries: symmetric groups S_N and alternating groups A_N with $N \in \mathbb{N}$
- Dihedral symmetries: single-valued groups D_n and double-valued groups D'_n with $n \in \mathbb{N}$
- Further double-valued groups: T', O', I', ...
- Subgroups of $SU(3)$: series of $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ groups with $n \in \mathbb{N}$, as well as finite number of Σ groups
- Subgroups of $U(3)$ such as $\Sigma(81)$ and subgroups of the listed groups such as $T_7 \cong Z_7 \rtimes Z_3 \subset \Delta(147)$

TB Mixing from Non-Trivial $\boldsymbol{A_4}$ Breaking

(Altarelli/Feruglio ('05), de Medeiros Varzielas et al. ('05), He et al. ('06))

- $G_F = A_4$ spontaneously broken at high energies
- Low energy effective theory: MSSM
- Breaking of G_F is induced by VEVs of flavon fields which are singlets under the SM gauge group
- (MS)SM fermions transform non-trivially under G_F
- MSSM Higgs doublets $h_{u,d}$ are singlets under G_F
- Elaborate construction of scalar sector needed to ensure vacuum alignment

Particle Content

		$LEPTONS$				$FLAVONS$			
Field	L	e^c_L	μ^c_L	τ^c_L	φ_T	φ_S	$\xi, \tilde{\xi}$	θ	
A_4	3	1	$1''$	$1'$	3	3	1	1	family dependent
Z_3	ω	ω^2	ω^2	ω^2	1	ω	ω	1	family independent
$U(1)_{FN}$	0	2	1	0	0	0	0	-1	family dependent

Additionally needed

- Z_3 symmetry to separate charged lepton and neutrino sector
- $U(1)_{FN}$ for hierarchy $m_e \ll m_\mu \ll m_\tau$

Group Theory of A_4

- The group A_4 is the symmetry group of a regular tetrahedron, group of even permutations of four objects
- Order of the group is 12
- Irred reps are 1, $1'$, $1''$ and 3
- Kronecker products

$$1 \times \mu = \mu \quad \forall \quad \mu \, ,$$
$$1' \times 1' = 1'' \, , \quad 1'' \times 1'' = 1' \, , \quad 1' \times 1'' = 1$$
$$1 \times 3 = 1' \times 3 = 1'' \times 3 = 3$$
$$3 \times 3 = 1 + 1' + 1'' + 3 + 3$$

Group Theory of A_4

- The group A_4 is the symmetry group of a regular tetrahedron, group of even permutations of four objects
- Order of the group is 12
- Irred reps are 1, $1'$, $1''$ and 3
- Generator relations $S^2 = \mathbb{1}\,, \quad T^3 = \mathbb{1}\,, \quad (ST)^3 = \mathbb{1}$

rep.	S	T
1	1	1
$1'$	1	ω^2
$1''$	1	ω
3	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$

$(\omega = e^{\frac{2\pi i}{3}})$

(Altarelli/Feruglio ('05), 2nd paper)

Fermion Masses at LO

- Assume vacuum

$$G_S \quad : \qquad \langle \varphi_S \rangle = (v_S, v_S, v_S) \; , \quad \langle \xi \rangle = u \; , \quad \langle \tilde{\xi} \rangle = 0 \; ,$$
$$G_T \quad : \qquad \langle \varphi_T \rangle = (v_T, 0, 0) \; .$$

- Charged lepton sector

$$\mathcal{M}_l = \frac{v_T}{\Lambda} v_d \, \text{diag} \left(y_e \frac{\langle \theta \rangle^2}{\Lambda^2}, y_\mu \frac{\langle \theta \rangle}{\Lambda}, y_\tau \right)$$

- Neutrino sector

$$M_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix} \Rightarrow \boxed{\text{TB mixing!}}$$

with $\frac{v_u^2}{\Lambda}$ diag$(a+b, a, -a+b)$ and $a = x_a \frac{u}{\Lambda}$, $b = x_b \frac{v_S}{\Lambda}$

Non-Trivial A_4 Breaking

- Observation

$\langle \varphi_S \rangle = (v_S, v_S, v_S)$, $\langle \xi \rangle = u$ breaks $A_4 \to G_S \cong Z_2$ in ν sector

$\langle \varphi_T \rangle = (v_T, 0, 0)$ breaks $A_4 \to G_T \cong Z_3$ in l sector

A_4 *completely* broken in the whole theory

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A_4 *completely* broken in the whole theory

In detail: Z_2 symmetry is generated by the element S of A_4

Since for 1, $1'$ and $1''$ $S = 1$, VEV $\langle \xi \rangle$ preserves S.

For $\langle \varphi_S \rangle$ the vector has to fulfill

$$\langle \varphi_S \rangle \propto v_{+1} \quad \text{with} \quad S\, v_{+1} = +1\, v_{+1}$$

i.e. $$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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- Observation

$\langle \varphi_S \rangle = (v_S, v_S, v_S)$, $\langle \xi \rangle = u$ breaks $A_4 \to G_S \cong Z_2$ in ν sector

$\langle \varphi_T \rangle = (v_T, 0, 0)$ breaks $A_4 \to G_T \cong Z_3$ in l sector

A_4 *completely* broken in the whole theory

In detail: Z_3 symmetry is generated by the element T of A_4

For $\langle \varphi_T \rangle$ the vector has to fulfill

$$\langle \varphi_T \rangle \propto v_{+1} \quad \text{with} \quad T\, v_{+1} = +1\, v_{+1}$$

i.e. $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Non-Trivial A_4 Breaking

- Observation

$\langle\varphi_S\rangle = (v_S, v_S, v_S)$, $\langle\xi\rangle = u$ breaks $A_4 \to G_S \cong Z_2$ in ν sector

$\langle\varphi_T\rangle = (v_T, 0, 0)$ breaks $A_4 \to G_T \cong Z_3$ in l sector

A_4 *completely* broken in the whole theory

A_4	D_2	Z_3	Z_2
1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$1'$	$\mathbf{1}$		$\mathbf{1}$
$1''$	$\mathbf{1}$		$\mathbf{1}$
3		$\mathbf{1}$ $+ \ldots$	$\mathbf{1}$ $+ \ldots$

with $\mathbf{1}$ being a total singlet of the subgroup

Non-Trivial A_4 Breaking

- Observation

$\langle \varphi_S \rangle = (v_S, v_S, v_S)$, $\langle \xi \rangle = u$ breaks $A_4 \to G_S \cong Z_2$ in ν sector
$\langle \varphi_T \rangle = (v_T, 0, 0)$ breaks $A_4 \to G_T \cong Z_3$ in l sector
A_4 *completely* broken in the whole theory

Mismatch of different subgroups generates non-trivial (large) mixing and even more predicts exact values of mixing angles (TB mixing) (independent of choice of parameters, apart from order of eigenvalues)

$\Downarrow$

Interpretation of Mixing

Note:
In the T' extension of this model small quark mixings are explained through the breaking to the same subgroup in up and down quark sector. **(Feruglio et al. ('07))**

Is S_4 better for TB Mixing?

(Lam ('06,'07,'08))

- TB mixing only results from A_4, if two flavons ξ', ξ'' in $1'$ and $1''$ are not present in the neutrino sector
- However: $\langle \xi' \rangle \neq 0$, $\langle \xi'' \rangle \neq 0$ leave $Z_2 \subset A_4$ invariant
- Matrix M_ν

$$M_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix}$$

is not only invariant under S

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \;, \quad \text{i.e.} \quad S^T M_\nu S = M_\nu$$

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but also under P_{23}

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \, , \quad \text{i.e.} \quad P_{23}^T \, M_\nu \, P_{23} = M_\nu$$

Is S_4 better for TB Mixing?

- TB mixing only results from A_4, if two flavons ξ', ξ'' in $1'$ and $1''$ are not present in the neutrino sector
- However: $\langle \xi' \rangle \neq 0$, $\langle \xi'' \rangle \neq 0$ leave $Z_2 \subset A_4$ invariant
- Matrix M_ν is invariant under S and P_{23}

$$\Downarrow$$

M_ν is invariant under $Z_2 \times Z_2$

But only S is an element of A_4 and not P_{23}

$$\Downarrow$$

The actual symmetry must be larger: S_4
$S_4 \quad \longrightarrow \quad Z_2 \times Z_2$ in ν sector
$S_4 \quad \longrightarrow \quad Z_3$ in l sector
S_4 *completely* broken in the whole theory

Probably the best is $\boldsymbol{S_4 \times Z_3}$

(H/Serone ('11))

Problem: Whether the charged lepton masses can still be *naturally* generated by, e.g. a $U(1)_{FN}$, is not obvious.

Solution: $S_4 \times Z_3$

The additional Z_3 factor solves the problem of the charged lepton mass hierarchy, because now the S_4 breaking is

$$S_4 \times Z_3 \quad \rightarrow \quad Z_2 \times Z_2 \times Z_3 \qquad \text{in } \nu \text{ sector}$$
$$S_4 \times Z_3 \quad \rightarrow \quad Z_3^{(D)} \qquad \text{in } l \text{ sector}$$

$S_4 \times Z_3$ *completely* broken in the whole theory

Details:

Field	L	e_R	μ_R	τ_R	ν_R
S_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
Z_3	1	1	ω^2	ω	1

$S_4 \times Z_3 \;\longrightarrow\; Z_3^{(D)}$

- Take the generator T of S_4 for $\mathbf{3}$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

  and break it together with Z_3 to their diagonal subgroup $Z_3^{(D)}$

- Leptons transform under the remnant $Z_3^{(D)}$ as

$$L_1 \sim 1 \quad L_2 \sim \omega^2 \quad L_3 \sim \omega$$
$$e_R \sim 1 \quad \mu_R \sim \omega^2 \quad \tau_R \sim \omega$$

- As consequence, the charged lepton mass matrix is diagonal.

Should we throw this away in light of T2K?

- In all models TB mixing holds only at LO and gets corrected
- For protecting θ_{12} corrections should be $\delta \lesssim 0.05$
- In most models (but not all!) then also $\sin\theta_{13} \sim \delta \approx 0.05$.
- However, usually

$$\sin\theta_{13} \approx c \times \delta$$

with c a complex number. c largish still saves most models.
Early exception: ***Lin ('09)***

- If you do not like this, maybe you like models with bimaximal mixing at LO, because then $\delta \sim 0.2$ also for θ_{12}, but tricks needed for keeping $\delta_{\theta_{23}}$ smaller. ***(Altarelli/Feruglio/Merlo ('09), Meloni ('11))***
- T2K signal has to be confirmed by others/more statistics!

Dihedral Symmetries: Idea of the Prediction

It has been shown in a study on the general properties of dihedral groups used as flavor symmetries that in case of a non-trivial breaking one element of the mixing matrix is given only in terms of group theoretical quantities. ***(Blum et al. ('07))***

The element of the mixing matrix $|V_{\alpha\beta}|$ is

$$|V_{\alpha\beta}| = \left|\cos\left(\frac{\pi\,(k_1 - k_2)\,\mathrm{j}}{n}\right)\right|$$

for

n being the index of the dihedral group D_n

j being the index of the representation $\underline{\mathbf{2}}_{\mathbf{j}}$ under which two of the three generations of left-handed fermion fields transform

k_1 and k_2 being two integers which specify the different subgroups $Z_2 = \langle \mathrm{B}\,\mathrm{A}^{k_{1,2}} \rangle$ preserved in the two different sectors

Dihedral Symmetries: Idea of the Prediction

Step 1:

Take a dihedral group D_n generated by A and B and break it to $Z_2 = \langle \mathrm{BA}^{k_1} \rangle$ in fermion sector 1 and to $Z_2 = \langle \mathrm{BA}^{k_2} \rangle$ in fermion sector 2

Step 2:

Assign left-handed fields $L \sim \underline{\mathbf{1}}_{\mathbf{s}} + \underline{\mathbf{2}}_{\mathbf{j}}$ and right-handed ones are either in singlet representations $L^c \sim \underline{\mathbf{1}}_{\mathbf{i}_p}$ or also $L^c \sim \underline{\mathbf{1}}_{\mathbf{l}} + \underline{\mathbf{2}}_{\mathbf{m}}$

Step 3:

Consider a generic model in which you have two sets of scalars Φ_1 and Φ_2 (for fermion sector 1 and sector 2) in all representations μ of D_n and allow non-zero VEVs only for those which leave invariant $Z_2 = \langle \mathrm{BA}^{k_1} \rangle$ and $Z_2 = \langle \mathrm{BA}^{k_2} \rangle$, respectively, i.e.

$$\text{sector 1} \quad : \quad D_n \xrightarrow{\langle \Phi_1 \rangle} Z_2 = \langle \mathrm{BA}^{k_1} \rangle$$

$$\text{sector 2} \quad : \quad D_n \xrightarrow{\langle \Phi_2 \rangle} Z_2 = \langle \mathrm{BA}^{k_2} \rangle$$

Dihedral Symmetries: Idea of the Prediction

Step 4:

Mass matrices $\mathcal{M}_i$ have the form ($i = 1, 2$)

$$\mathcal{M}_i = \left(\begin{array}{ccc} 0 & A_i & B_i \\ C_i & D_i & E_i \\ -C_i \mathrm{e}^{-i\varphi_i \mathrm{j}} & D_i \mathrm{e}^{-i\varphi_i \mathrm{j}} & E_i \mathrm{e}^{-i\varphi_i \mathrm{j}} \end{array} \right) \quad \text{with} \quad \varphi_i = \frac{2\pi k_i}{n}$$

Dihedral Symmetries: Idea of the Prediction

Step 4:

Mass matrices $\mathcal{M}_i$ have the form ($i = 1, 2$)

$$\text{or} \quad \mathcal{M}_i = \begin{pmatrix} A_i & C_i & C_i \mathrm{e}^{-i\varphi_i \mathrm{m}} \\ B_i & D_i & E_i \\ B_i \mathrm{e}^{-i\varphi_i \mathrm{j}} & E_i \mathrm{e}^{-i\varphi_i (\mathrm{j}-\mathrm{m})} & D_i \mathrm{e}^{-i\varphi_i (\mathrm{j}+\mathrm{m})} \end{pmatrix} \quad \text{with} \quad \varphi_i = \frac{2\pi k_i}{n}$$

Dihedral Symmetries: Idea of the Prediction

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Step 5:

Such matrices have $\mathcal{M}_i \mathcal{M}_i^\dagger$ of the form

$$\mathcal{M}_i \mathcal{M}_i^\dagger = \begin{pmatrix} a_i & b_i \mathrm{e}^{i\zeta_i} & b_i \mathrm{e}^{i(\zeta_i + \varphi_i \mathrm{j})} \\ b_i \mathrm{e}^{-i\zeta_i} & c_i & d_i \mathrm{e}^{i\varphi_i \mathrm{j}} \\ b_i \mathrm{e}^{-i(\zeta_i + \varphi_i \mathrm{j})} & d_i \mathrm{e}^{-i\varphi_i \mathrm{j}} & c_i \end{pmatrix}$$

Dihedral Symmetries: Idea of the Prediction

Step 6:

Mixing matrices U_i are

$$U_i = \begin{pmatrix} 0 & \cos\theta_i\, \mathrm{e}^{i\zeta_i} & \sin\theta_i\, \mathrm{e}^{i\zeta_i} \\ -\frac{1}{\sqrt{2}}\mathrm{e}^{i\varphi_i \mathrm{j}} & -\frac{\sin\theta_i}{\sqrt{2}} & \frac{\cos\theta_i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{\sin\theta_i}{\sqrt{2}}\mathrm{e}^{-i\varphi_i \mathrm{j}} & \frac{\cos\theta_i}{\sqrt{2}}\mathrm{e}^{-i\varphi_i \mathrm{j}} \end{pmatrix} \quad \text{with} \quad \varphi_i = \frac{2\pi k_i}{n}$$

Step 7:

Physical mixings arise from $V = U_1^t U_2^\star$. You can check that one element is of the form

$$|V_{\alpha\beta}| = \frac{1}{2}\left|1 + \mathrm{e}^{i(\varphi_1 - \varphi_2)\mathrm{j}}\right| = \left|\cos\left((\varphi_1 - \varphi_2)\frac{\mathrm{j}}{2}\right)\right| = \left|\cos\left(\frac{\pi\,(k_1 - k_2)\,\mathrm{j}}{n}\right)\right|$$

Rest of elements depends also on $\theta_{1,2}$ and $\zeta_{1,2}$.

D_4 Model for $\mu\tau$ Symmetry

(H/Ziegler ('10))

It has been shown in a study on the general properties of dihedral groups used as flavor symmetries that in case of a non-trivial breaking one element of the mixing matrix is given only in terms of group theoretical quantities.

In this model this is true for the PMNS matrix element $|U_{\mu 3}|$ which is

$$|U_{\mu 3}| = \left| \cos\left(\frac{\pi\,((0,2)-(1,3))\,1}{4} \right) \right| = \left| \cos\left(\frac{\pi}{4}\right) \right| = 1/\sqrt{2}$$

$n = 4$, since G_F contains D_4

$\mathrm{j} = 1$, since $L_D \sim \underline{\mathbf{2}}$

$k_1 = 0, 2$: $Z_2 = \langle \mathrm{B} \rangle$ or $Z_2 = \langle \mathrm{B\,A}^2 \rangle$ in neutrino sector

$k_2 = 1, 3$: $Z_2 = \langle \mathrm{B\,A} \rangle$ or $Z_2 = \langle \mathrm{B\,A}^3 \rangle$ in charged lepton sector

D_{14} *Model: Idea of the Prediction* *(Blum/H ('09))*

It has been shown in a study on the general properties of dihedral groups used as flavor symmetries that in case of a non-trivial breaking one element of the mixing matrix is given only in terms of group theoretical quantities.
In this model this is true for the CKM matrix element $|V_{ud}|$ which is

$$|V_{ud}| = \left|\cos\left(\frac{\pi\,(0-1)\,1}{14}\right)\right| = \left|\cos\left(\frac{\pi}{14}\right)\right| \approx 0.97493$$

for

$n = 14$, since G_F contains D_{14}

$\mathrm{j} = 1$, since $Q_D \sim \underline{\mathbf{2}}_{\mathbf{1}}$

$k_1 = 0$ and $k_2 = 1$ or $k_2 = 13$ so that $Z_2 = \langle \mathrm{B} \rangle$ is conserved in the up and $Z_2 = \langle \mathrm{B\,A} \rangle$ or $Z_2 = \langle \mathrm{B\,A}^{13} \rangle$ in the down quark sector

Group Theory of D_{14}

- D_{14} belongs to the dihedral groups and is the symmetry group of a regular planar 14-gon
- Its order is 28, i.e. it has 28 distinct elements
- It has 4 + 6 real irred. reps., $\underline{\mathbf{1}}_{\mathbf{i}}$, $\mathrm{i} = 1, ..., 4$ and $\underline{\mathbf{2}}_{\mathbf{j}}$, $\mathrm{j} = 1, ..., 6$
- Generator relations of D_{14}

$$\mathrm{A}^{14} = \mathbb{1}\ , \quad \mathrm{B}^2 = \mathbb{1}\ , \quad \mathrm{A\,B\,A} = \mathrm{B}$$

- Generators

$$\begin{array}{lcl} \underline{\mathbf{1}}_{\mathbf{1}} & : & \mathrm{A} = 1\ , \quad \mathrm{B} = 1 \\ \underline{\mathbf{1}}_{\mathbf{2}} & : & \mathrm{A} = 1\ , \quad \mathrm{B} = -1 \\ \underline{\mathbf{1}}_{\mathbf{3}} & : & \mathrm{A} = -1\ , \quad \mathrm{B} = 1 \\ \underline{\mathbf{1}}_{\mathbf{4}} & : & \mathrm{A} = -1\ , \quad \mathrm{B} = -1 \end{array}$$

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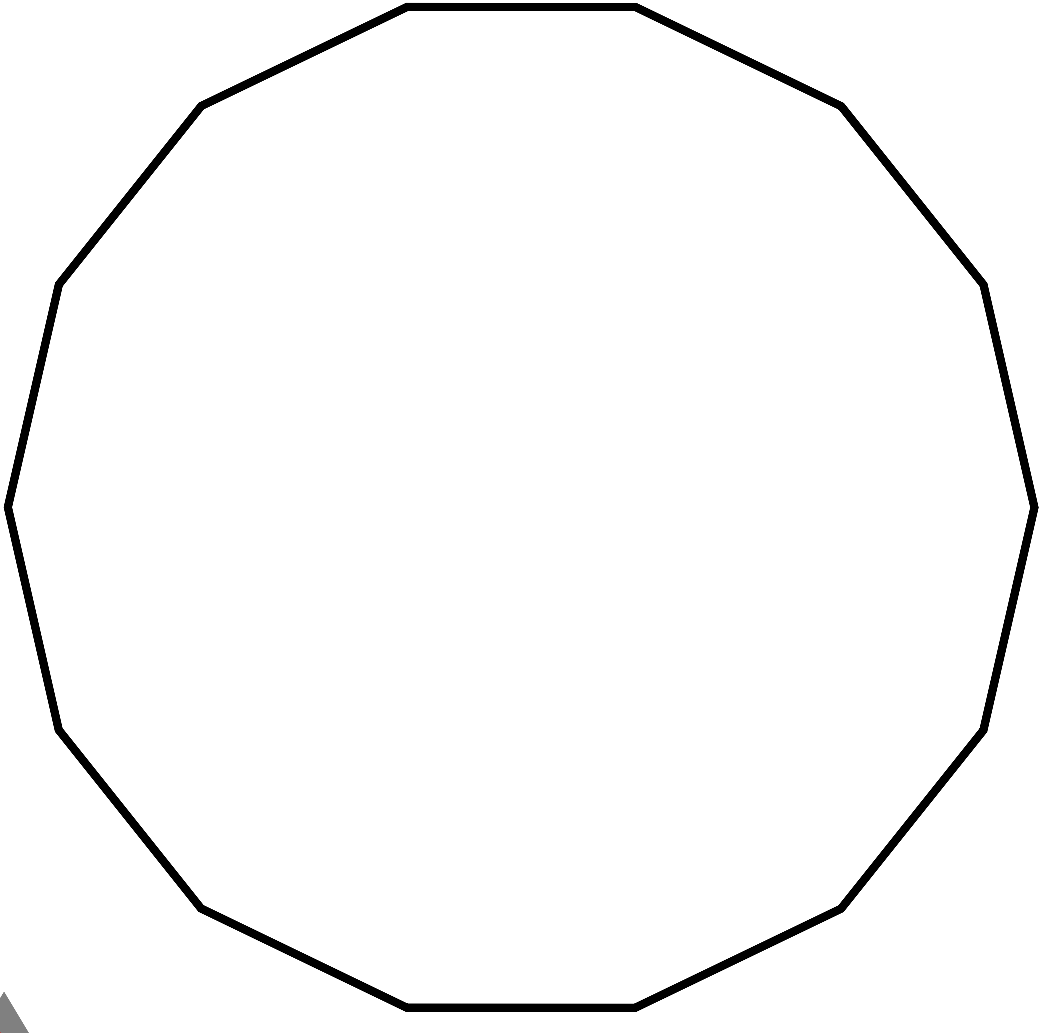
$$\mathrm{A}^{14} = \mathbb{1} \; , \;\; \mathrm{B}^2 = \mathbb{1} \; , \;\; \mathrm{A\,B\,A} = \mathrm{B}$$

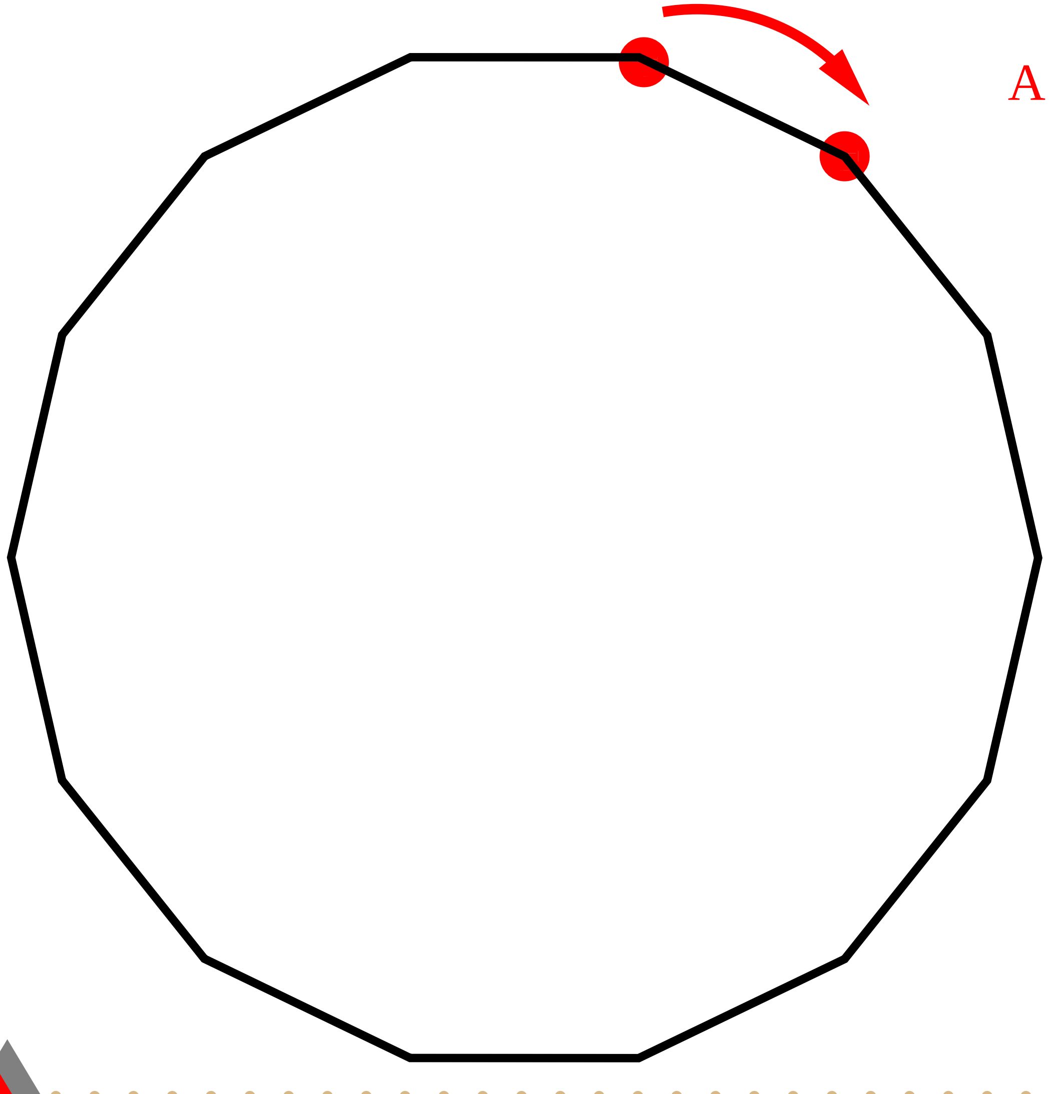
- Generators

$$\mathrm{A} = \left(\begin{array}{cc} \mathrm{e}^{\left(\frac{\pi i}{7}\right)\mathrm{j}} & 0 \\ 0 & \mathrm{e}^{-\left(\frac{\pi i}{7}\right)\mathrm{j}} \end{array} \right) \; , \; \mathrm{B} = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \; , \;\; \mathrm{j} = 1, ..., 6$$

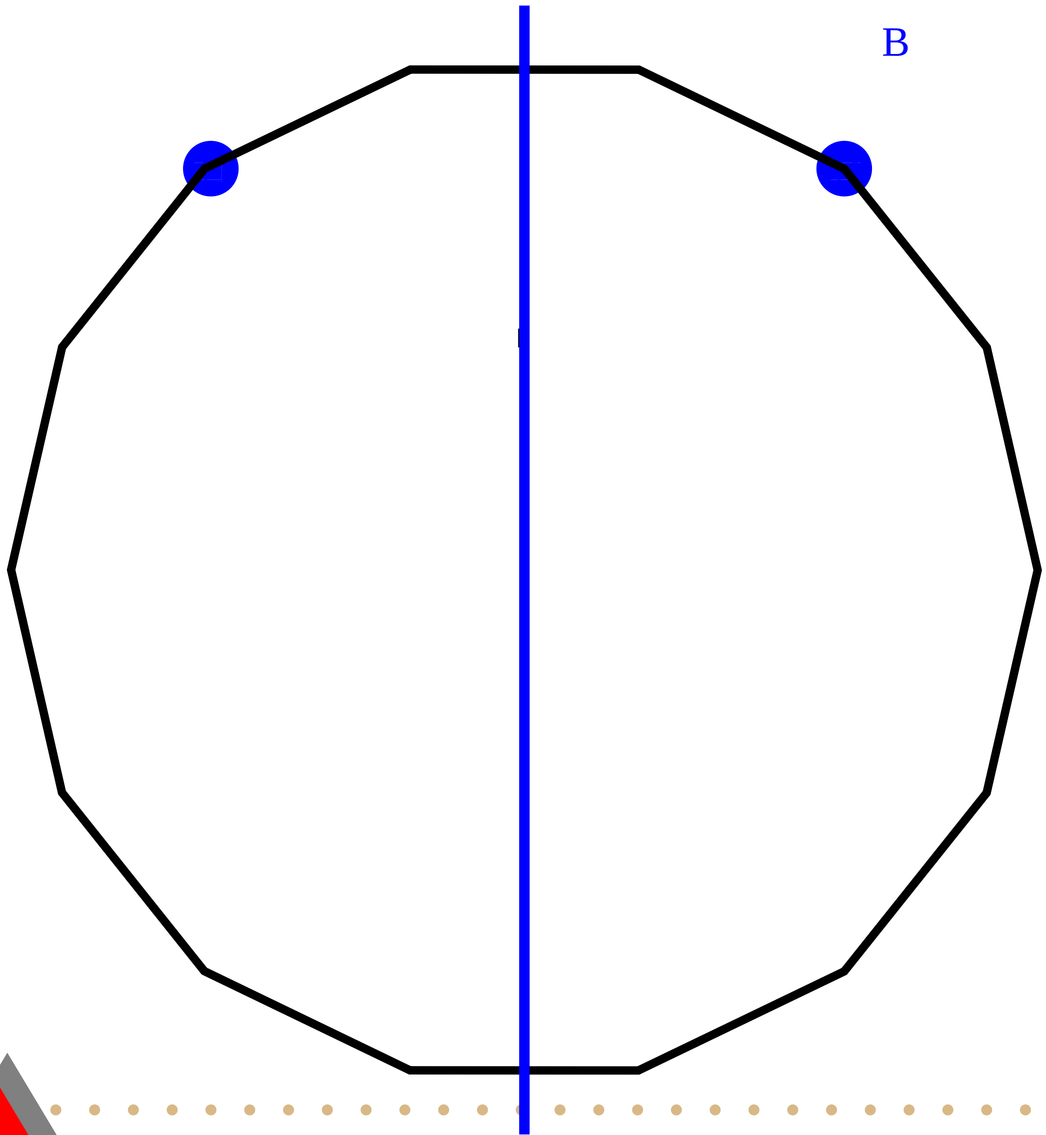
Visualization of D_{14}



Visualization of D_{14}



Visualization of D_{14}



Z_2 Subgroups of D_{14}

One type of Z_2 subgroup is generated by the element $\mathrm{B\,A}^k$ with $k = 0, ..., 13$

$\underline{\mathbf{1}}_{\mathbf{1}}$	:	$\mathrm{B\,A}^k = 1$	$\surd$
$\underline{\mathbf{1}}_{\mathbf{2}}$	:	$\mathrm{B\,A}^k = -1$	$-$
$\underline{\mathbf{1}}_{\mathbf{3}}$	:	$\mathrm{B\,A}^k = (-1)^k$	$\surd$ for k even
$\underline{\mathbf{1}}_{\mathbf{4}}$	:	$\mathrm{B\,A}^k = (-1)^{k+1}$	$\surd$ for k odd

Scalars $\varphi_{1,2}$ forming a $\underline{\mathbf{2}}_{\mathbf{j}}$ under D_{14} leave a Z_2 invariant, if

$$\begin{pmatrix} \langle \varphi_1 \rangle \\ \langle \varphi_2 \rangle \end{pmatrix} \propto \begin{pmatrix} \mathrm{e}^{-\frac{\pi\, i\, \mathrm{j}\, k}{7}} \\ 1 \end{pmatrix} \Rightarrow$$

Z_2 group in up quark sector with $k_1 = 0$
and in down quark sector with $k_2 \neq 0$ odd

D_{14} Quark Model

(Blum/H ('09))

- Flavor group $G_F = D_{14} \times Z_3 \times U(1)_{FN}$
- Framework: MSSM
- Quarks transform non-trivially under G_F
- MSSM Higgs doublets $h_{u,d}$ are singlets under G_F
- Necessity of gauge singlets (flavons) transforming under G_F

$$\{\psi^u_{1,2}, \chi^u_{1,2}, \xi^u_{1,2}, \eta^u\} \quad \text{and} \quad \{\psi^d_{1,2}, \chi^d_{1,2}, \xi^d_{1,2}, \eta^d, \sigma\}$$

- FN field θ is only charged under $U(1)_{FN}$
- Structure of Yukawa couplings

$$\frac{\theta^2}{\Lambda^4}(Q_D u^c \chi^u \xi^u) h_u \quad \text{and} \quad \frac{1}{\Lambda} Q_3 (b^c \eta^d) h_d$$

Setup of Quark Model

Field	Q_D	Q_3	u^c	c^c	t^c	d^c	s^c	b^c	$h_{u,d}$
D_{14}	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$
Z_3	1	1	1	1	1	ω^2	ω^2	ω^2	1
$U(1)_{FN}$	0	0	2	0	0	1	1	0	0

Field	$\psi^u_{1,2}$	$\chi^u_{1,2}$	$\xi^u_{1,2}$	η^u	$\psi^d_{1,2}$	$\chi^d_{1,2}$	$\xi^d_{1,2}$	η^d	σ	θ
D_{14}	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{1}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{4}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$
Z_3	1	1	1	1	ω	ω	ω	ω	ω	1
$U(1)_{FN}$	0	0	0	0	0	0	0	0	0	-1

Leading Order in Up Quark Sector

no ϕ

(33) $\quad Q_3\, t^c\, h_u$

$1\,\phi$

(13), (23) $\quad \frac{1}{\Lambda}(Q_D \psi^u) t^c h_u$

(32) $\quad \frac{1}{\Lambda} Q_3 (c^c \eta^u) h_u$

$2\,\phi$

(11), (21) $\quad \frac{\theta^2}{\Lambda^4}(Q_D u^c \chi^u \xi^u) h_u + \frac{\theta^2}{\Lambda^4}\left(Q_D u^c (\xi^u)^2\right) h_u + \frac{\theta^2}{\Lambda^4}(Q_D \psi^u \eta^u u^c) h_u$

(12), (22) $\quad \frac{1}{\Lambda^2}(Q_D c^c \chi^u \xi^u) h_u + \frac{1}{\Lambda^2}\left(Q_D c^c (\xi^u)^2\right) h_u + \frac{1}{\Lambda^2}(Q_D \psi^u)(\eta^u c^c) h_u$

Leading Order in Down Quark Sector

$$\boxed{1\,\phi}$$

$$(33) \qquad \frac{1}{\Lambda} Q_3 (b^c \eta^d) h_d$$

$$(32) \qquad \frac{\theta}{\Lambda^2}\, Q_3\, s^c \sigma h_d$$

$$(12),\ (22) \qquad \frac{\theta}{\Lambda^2}\, (Q_D \psi^d)\, s^c h_d$$

Vacuum Structure

Up quark sector $\langle \eta^u \rangle \neq 0$

$$\begin{pmatrix} \langle \psi_1^u \rangle \\ \langle \psi_2^u \rangle \end{pmatrix} = v^u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \chi_1^u \rangle \\ \langle \chi_2^u \rangle \end{pmatrix} = w^u \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \xi_1^u \rangle \\ \langle \xi_2^u \rangle \end{pmatrix} = z^u \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Down quark sector $\langle \eta^d \rangle \neq 0$ and $\langle \sigma \rangle \neq 0$

$$\begin{pmatrix} \langle \psi_1^d \rangle \\ \langle \psi_2^d \rangle \end{pmatrix} = v^d \begin{pmatrix} \mathrm{e}^{-\frac{\pi i k}{7}} \\ 1 \end{pmatrix}, \begin{pmatrix} \langle \chi_1^d \rangle \\ \langle \chi_2^d \rangle \end{pmatrix} = w^d \mathrm{e}^{\frac{\pi i k}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{2\pi i k}{7}} \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} \langle \xi_1^d \rangle \\ \langle \xi_2^d \rangle \end{pmatrix} = z^d \mathrm{e}^{\frac{2\pi i k}{7}} \begin{pmatrix} \mathrm{e}^{-\frac{4\pi i k}{7}} \\ 1 \end{pmatrix}$$

Results for Quarks at Leading Order

Assume $\frac{\langle \Phi^u \rangle}{\Lambda} \approx \epsilon$, $\frac{\langle \Phi^d \rangle}{\Lambda} \approx \epsilon$, $t = \frac{\langle \theta \rangle}{\Lambda} \approx \epsilon \approx \lambda^2 \approx 0.04$

then $\mathcal{M}_u$ and $\mathcal{M}_d$ read

$$\mathcal{M}_u = \begin{pmatrix} -\alpha_1^u \, t^2 \, \epsilon^2 & \alpha_2^u \, \epsilon^2 & \alpha_3^u \, \epsilon \\ \alpha_1^u \, t^2 \, \epsilon^2 & \alpha_2^u \, \epsilon^2 & \alpha_3^u \, \epsilon \\ 0 & \alpha_4^u \, \epsilon & y_t \end{pmatrix} \langle h_u \rangle$$

$$\mathcal{M}_d = \begin{pmatrix} 0 & \alpha_1^d \, t \, \epsilon & 0 \\ 0 & \alpha_1^d \, \mathrm{e}^{-\pi i k/7} \, t \, \epsilon & 0 \\ 0 & \alpha_2^d \, t \, \epsilon & y_b \, \epsilon \end{pmatrix} \langle h_d \rangle$$

Results for Quarks at Leading Order

Quark Masses

$$m_u^2 : m_c^2 : m_t^2 \sim \epsilon^8 : \epsilon^4 : 1 \,, \quad m_d^2 : m_s^2 : m_b^2 \sim 0 : \epsilon^2 : 1 \,,$$

$$m_b^2 : m_t^2 \sim \epsilon^2 : 1 \quad \text{for} \quad \text{small} \quad \tan\beta$$

CKM matrix

$$|V_{CKM}| = \begin{pmatrix} |\cos(\frac{k\,\pi}{14})| & |\sin(\frac{k\,\pi}{14})| & 0 \\ |\sin(\frac{k\,\pi}{14})| & |\cos(\frac{k\,\pi}{14})| & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \mathcal{O}(\epsilon^4) & \mathcal{O}(\epsilon^2) \\ \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon) \\ \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon^2) \end{pmatrix}$$

$$\Downarrow$$

$k = 1$ or $k = 13$ leads to $|V_{ud}| \approx 0.97493$

Experimental value: $|V_{ud}|_{\text{exp}} = 0.97419^{+0.00022}_{-0.00022}$

Completion: Add Leptons

(H/Meloni (to appear))

Goals :

- Add leptons in minimal way
- Predict $\mu\tau$ symmetry in the lepton sector
- Do not disturb quark sector

Completion: Add Leptons

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Goals :

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- Predict $\mu\tau$ symmetry in the lepton sector
- Do not disturb quark sector

Solution

Field	L_1	L_D	e^c	μ^c	τ^c	ν_1^c	ν_D^c	$\chi_{1,2}^e$
D_{14}	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{2}}_{\mathbf{3}}$	$\underline{\mathbf{2}}_{\mathbf{2}}$
Z_7	1	1	ω_7^5	ω_7^5	ω_7^5	ω_7^4	ω_7^4	ω_7^2

Neutrino Sector

Majorana mass matrix for right-handed neutrinos

$$\boxed{\;1\,\phi\;}$$

$$(11) \qquad \nu_1^c \nu_1^c \sigma$$

$$(23) \qquad (\nu_D^c \nu_D^c)\sigma$$

leads to

$$\mathcal{M}_R = \begin{pmatrix} \alpha_1^M & 0 & 0 \\ 0 & 0 & \alpha_2^M \\ 0 & \alpha_2^M & 0 \end{pmatrix} \epsilon\,\Lambda$$

Neutrino Sector

Dirac neutrino mass matrix

$$\boxed{1\,\phi}$$

$$(12), (13) \qquad \frac{1}{\Lambda}(L_1 \nu_D^c \xi^u) h_u$$

$$(21), (31) \qquad \frac{1}{\Lambda}(L_D \nu_1^c \chi^u) h_u$$

$$(23), (32) \qquad \frac{1}{\Lambda}(L_D \nu_D^c \psi^u) h_u$$

leads to

$$\mathcal{M}_\nu^D = \begin{pmatrix} 0 & \alpha_1^D & \alpha_1^D \\ \alpha_2^D & 0 & \alpha_3^D \\ -\alpha_2^D & \alpha_3^D & 0 \end{pmatrix} \epsilon \, \langle h_u \rangle$$

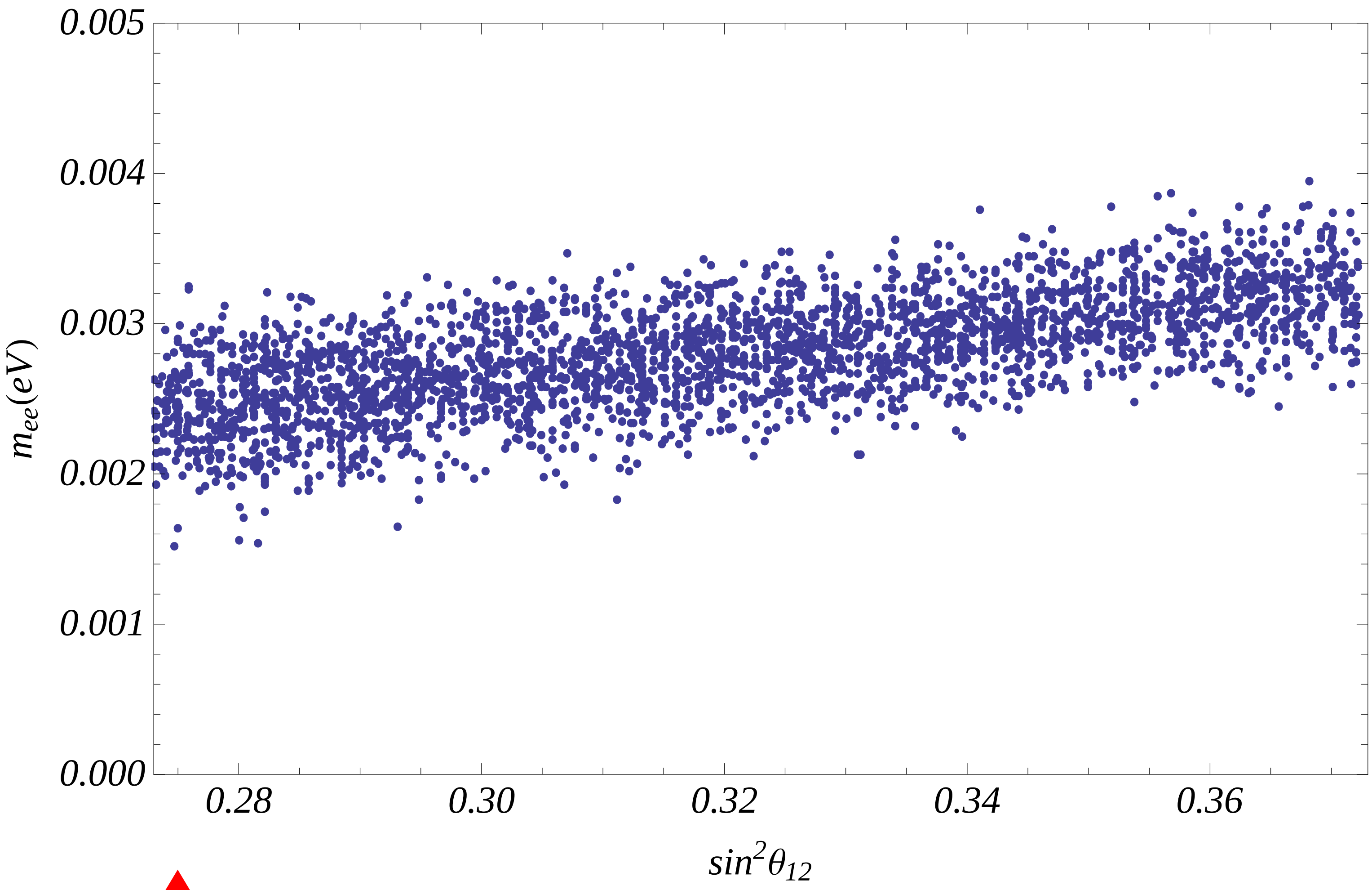
Neutrino Sector

Light neutrino mass matrix

$$\mathcal{M}_\nu = \begin{pmatrix} 2x^2/v & x & x \\ x & z & v-z \\ x & v-z & z \end{pmatrix} \epsilon \, \langle h_u \rangle^2 / \Lambda$$

- $\mu\tau$ symmetric neutrino mixing
- θ^ν_{12} is given by $$\tan(\theta^\nu_{12}) = \sqrt{2} \left| \frac{x}{v} \right|$$
- Normal ordering with $m_1 = 0$ is predicted and $$m_2^2 = \frac{(|v|^2 + 2|x|^2)^2}{|v|^2} \left(\frac{\epsilon \langle h_u \rangle^2}{\Lambda} \right)^2 \quad , \quad m_3^2 = |v - 2z|^2 \left(\frac{\epsilon \langle h_u \rangle^2}{\Lambda} \right)^2$$
- Additional relation $|m_{ee}| = m_2 \sin^2(\theta^\nu_{12}) = \sqrt{\delta m^2} \sin^2(\theta^\nu_{12})$

Neutrino Sector



Charged Lepton Sector

Alignment of new flavon $\chi^e_{1,2}$

$$\begin{pmatrix} \langle \chi^e_1 \rangle \\ \langle \chi^e_2 \rangle \end{pmatrix} = v^e \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

for which just one additional driving field σ^{0e} is needed

$$w_{f,l} = a_l \, \sigma^{0e} \, \chi^e_1 \, \chi^e_2$$

Nota bene: Also this alignment preserves a Z_2 subgroup of D_{14}, because $\underline{\mathbf{2}}_{\mathbf{2}}$ is unfaithful.

Yukawa Operators for Charged Leptons

$\boxed{1\,\phi}$

$$(3\alpha) \qquad \frac{1}{\Lambda}(L_D \chi^e)\,\alpha^c\,h_d$$

$\boxed{2\,\phi}$

$$(2\alpha) \qquad \frac{1}{\Lambda^2}(L_D \chi^e \xi^u)\,\alpha^c\,h_d$$

$\boxed{3\,\phi}$

$$(1\alpha) \qquad \frac{1}{\Lambda^3}(L_1 \chi^e \psi^u \xi^u)\alpha^c h_d$$

$$(1\alpha) \qquad \frac{1}{\Lambda^3}(L_1 \eta^u)(\chi^e \chi^u)\alpha^c h_d$$

Charged Lepton Mass Matrix

For $\frac{v^e}{\Lambda} \approx \epsilon \approx \lambda^2$ we get

$$\mathcal{M}_e = \begin{pmatrix} \alpha^e_1\,\epsilon^3 & \alpha^e_2\,\epsilon^3 & \alpha^e_3\,\epsilon^3 \\ \alpha^e_4\,\epsilon^2 & \alpha^e_5\,\epsilon^2 & \alpha^e_6\,\epsilon^2 \\ \alpha^e_7\,\epsilon & \alpha^e_8\,\epsilon & \alpha^e_9\,\epsilon \end{pmatrix} \langle h_d \rangle$$

- Charged lepton masses $m_e : m_\mu : m_\tau \sim \epsilon^2 : \epsilon : 1$
- Charged lepton mixing angles $\theta^e_{12} \sim \epsilon$, $\theta^e_{13} \sim \epsilon^2$, $\theta^e_{23} \sim \epsilon$

Lepton mixings are nearly $\mu\tau$ symmetric

$$\sin^2(\theta^l_{23}) = \frac{1}{2} + \mathcal{O}(\epsilon)\ ,\ \ \sin(\theta^l_{13}) = \mathcal{O}(\epsilon)\ ,\ \ \sin^2(\theta^l_{12}) = \mathcal{O}(1)$$

Conclusions

- There are many hints at a flavor symmetry: existence of three generations, fermion mass hierarchies, mixing patterns, no excessive flavor violation observed
- Discrete symmetries are very useful to understand fermion mixing, especially if non-trivially broken in different sectors of the theory
- Impact of T2K?! ... one might argue that many mixing patterns/models are now disfavored
  ... but: in all models corrections exist,
  T2K results need to be confirmed by others/more statistics
- Still we should search for new symmetries giving easily rise to $\theta_{13} \neq 0$ at LO, not giving up the non-trivial breaking pattern of the flavor group!

Thank you.