A Novel Method to Extract Dark Matter Parameters from Neutrino Telescope Data

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Dark matter

$$\rho_{DM}/(\rho_{DM}+\rho_{baryon}) = 82\ \%$$

What is DM composed of??

A popular class of candidates: WIMP

WIMP: Weakly Interacting Massive Particle

WIMP

- Lightest supersymmetric particle (LSP)
- Lightest KK mode in extra large dimension models
- .....
- AMEND and SLIM : Y.F, S. Pascoli and Schimdt, **JHEP 1010 (2010); Y.F., Phys.Rev. D80 (2009)**

WIMP

- Direct detection

- Indirect detection

Direct DM Detection

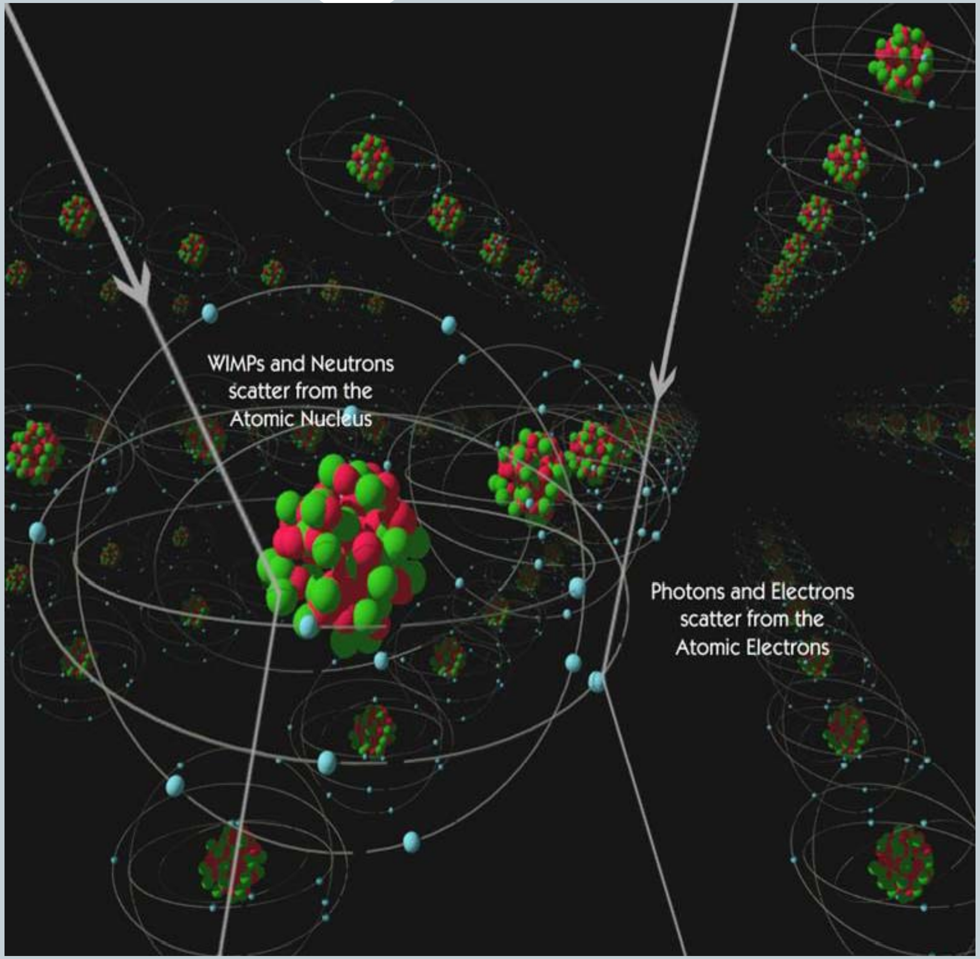
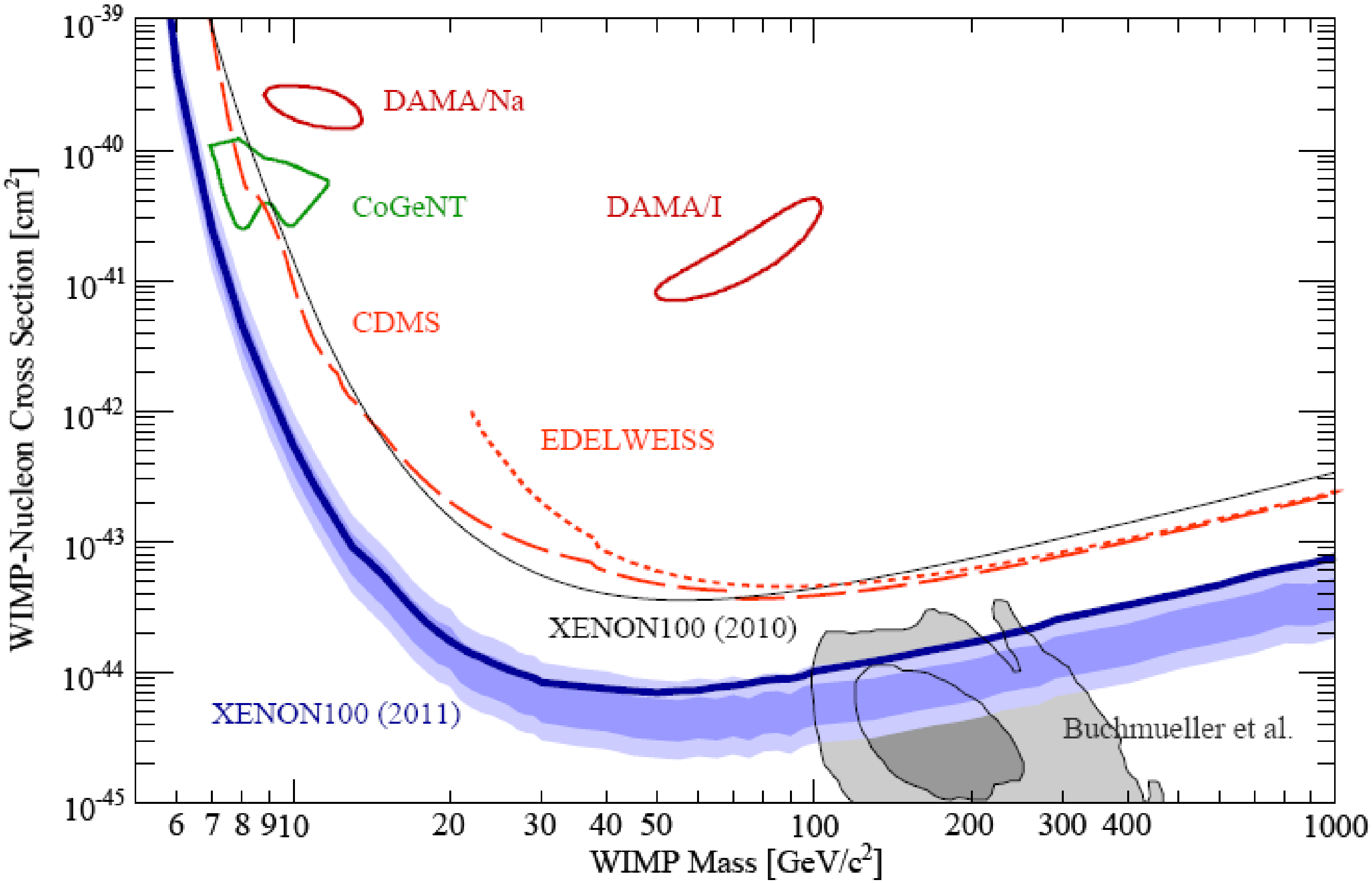


Image courtesy of:
http://cdms.berkeley.edu/Education/DMpages/science/directDetection.shtml

DAMA signal and Direct bounds



XENON100, arXiv:1104.2549

Indirect detection

- Detection of the products of DM pair annihilation in the DM halo, galaxy center, Sun, Earth ....

$$\mathrm{DM} + \mathrm{DM} \to e^- e^+$$

$$\mathrm{DM} + \mathrm{DM} \to \gamma\gamma$$

$$\text{Inverse Compton}: \quad e^{\pm} + \gamma \to e^{\pm} + \gamma$$

$$\text{pair annihilation}: \quad e^+ e^- \to \gamma\gamma$$

Signal from the Sun

$$\mathrm{DM}(v_i) + N \rightarrow \mathrm{DM}(v_f) + N$$

$$v_i \sim 200\ \mathrm{km/sec} \quad v_i > v_f$$

Trapped inside the gravitational well

n_{DM} Grows .

$$\Gamma(\mathrm{DM} + \mathrm{DM} \rightarrow \mathrm{anything}) \propto n_{DM}^2$$

Only ν and $\bar{\nu}$ come out of the Sun center and reach our detectors.

DM capture in the Sun

$$C \sim \frac{\rho_{DM}}{m_{DM} v_{DM}} \left(\frac{M_\odot}{m_p} \right) \sigma_{DM-nucleon} \langle v_{esc}^2 \rangle$$

$$\rho_{DM} = 0.39 \text{ GeV cm}^{-3}$$

$$v_{DM} \sim 270 \text{ km sec}^{-1}$$

The maximal possible capture rate is therefore $O[10^{24} \text{ sec}^{-1}]$.

$$E_{kinetic} = \frac{3kT}{2}$$

$$E_{Gravity} = G_N \left(\frac{4\pi}{3} \rho r_{DM}^3 \right) \frac{m_{DM}}{r_{DM}}$$

$$E_{kinetic} = E_{Gravity}$$

$$r_{DM} \approx \left(\frac{9T}{8\pi G_N \rho m_{DM}} \right)^{1/2}$$

$$r_{DM} = 0.003 R_\odot \left(\frac{100 \text{ GeV}}{m_{DM}} \right)^{1/2}$$

Some conventional neutrino production modes

$$\mathrm{DM} + \mathrm{DM} \to \nu\,\bar{\nu}$$

$$\frac{v}{c} \sim 10^{-4} \ll 1 \quad E_\nu \simeq m_{DM}$$

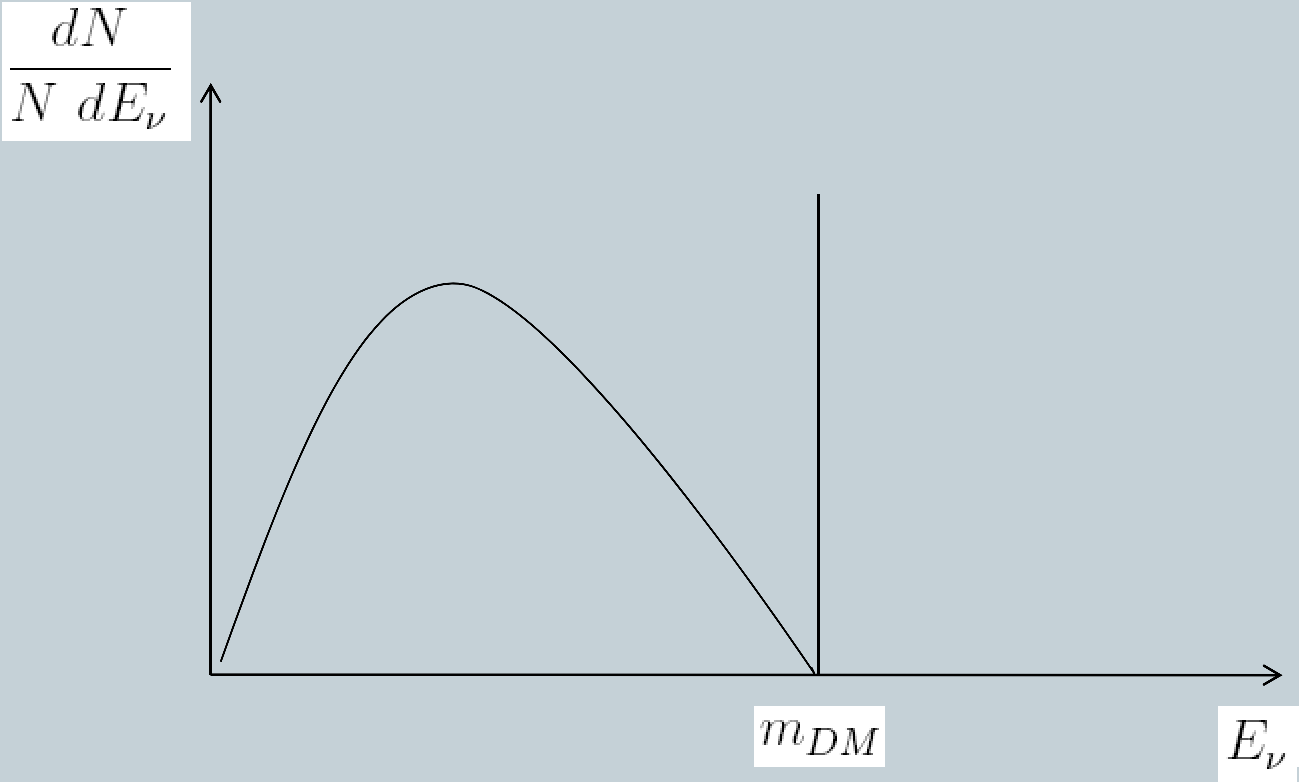
$$\mathrm{DM} + \mathrm{DM} \to ZZ, W^+W^-$$

$$W^- \to l_\alpha^- \bar{\nu}_\alpha \;, \quad Z \to \nu\bar{\nu}$$

$$\mathrm{DM} + \mathrm{DM} \to b\bar{b} \quad b \to c\bar{\nu}_\alpha l_\alpha$$

$$E_\nu < m_{DM}$$

Spectrum



models

- Lindner, Merle and Niro, **Enhancing Dark Matter Annihilation into Neutrinos, PRD82**

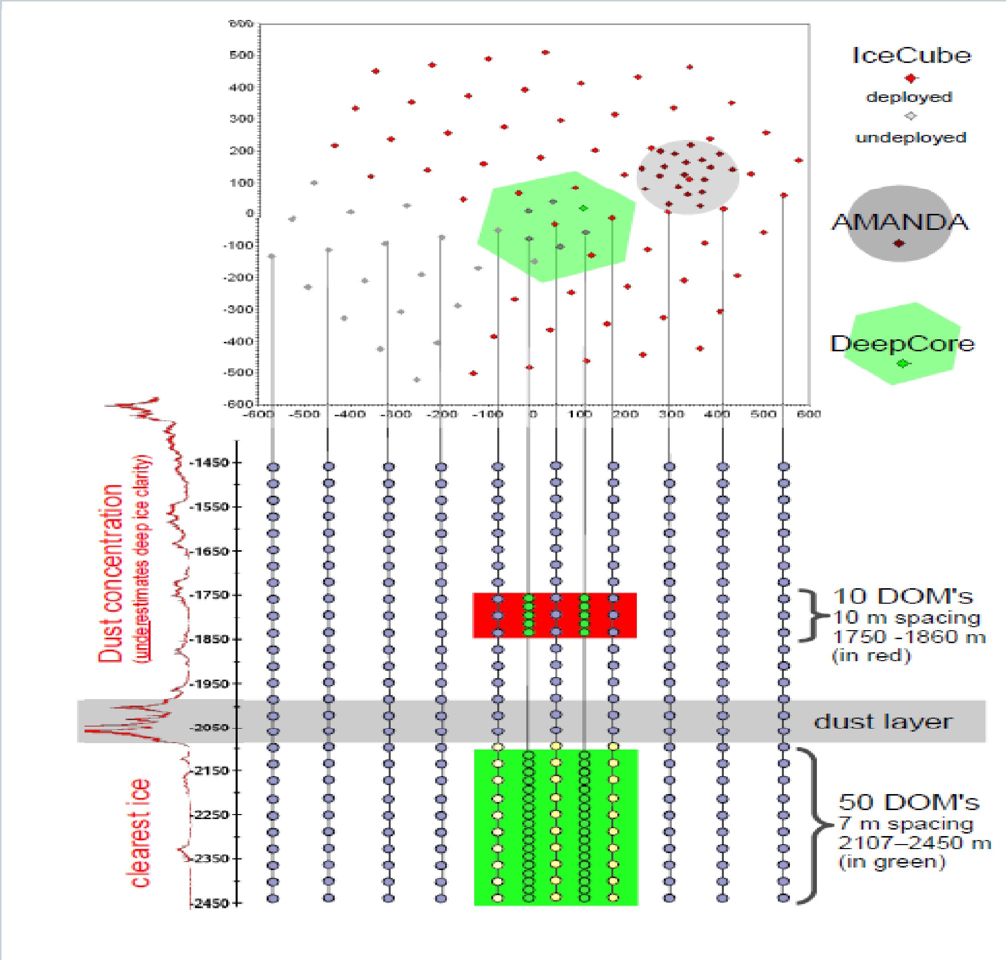
- Arman Esmaili and YF**, JCAP**

ICECUBE and DEEPCORE

Physics Capabilities of the IceCube DeepCore Detector.

, for the IceCube Collaboration.

arXiv:0907.2263



Shower and muon track

- CC: $\nu_\mu + N \rightarrow \mu + X$

Muon-track

Shower or cascade

- NC: $\nu + N \rightarrow \nu + X$
- CC: $\nu_e + N \rightarrow e + X$

$$\nu_\tau + N \rightarrow \tau + X \;,\; \tau \rightarrow \nu_\tau + X$$

Background

- Background from solar atmosphere<10 per year

Fogli et al, PRD74

Atmospheric muons: During summer and spring

The rest of ICECUBE acts as filter for DeepCore.

Atmospheric neutrino background

Atmospheric background

a cone with half angle 1° around the direction of Sun,

Whole ICECUBE: $\sim 6\ \mathrm{yr}^{-1}$

Deepcore: $\sim 2.5\ \mathrm{yr}^{-1}$

Fogli, Lisi, Kasahara PRD75

Scattering of neutrinos inside the Sun

- Scattered neutrinos: $$\nu N \to \nu X$$
- Absorbed neutrinos: $$\nu_\alpha N \to l_\alpha X$$
- Regenerated neutrinos: $$\nu_\tau N \to \tau X \ , \qquad \tau \to \nu_\tau \mu \bar{\nu}_\mu$$

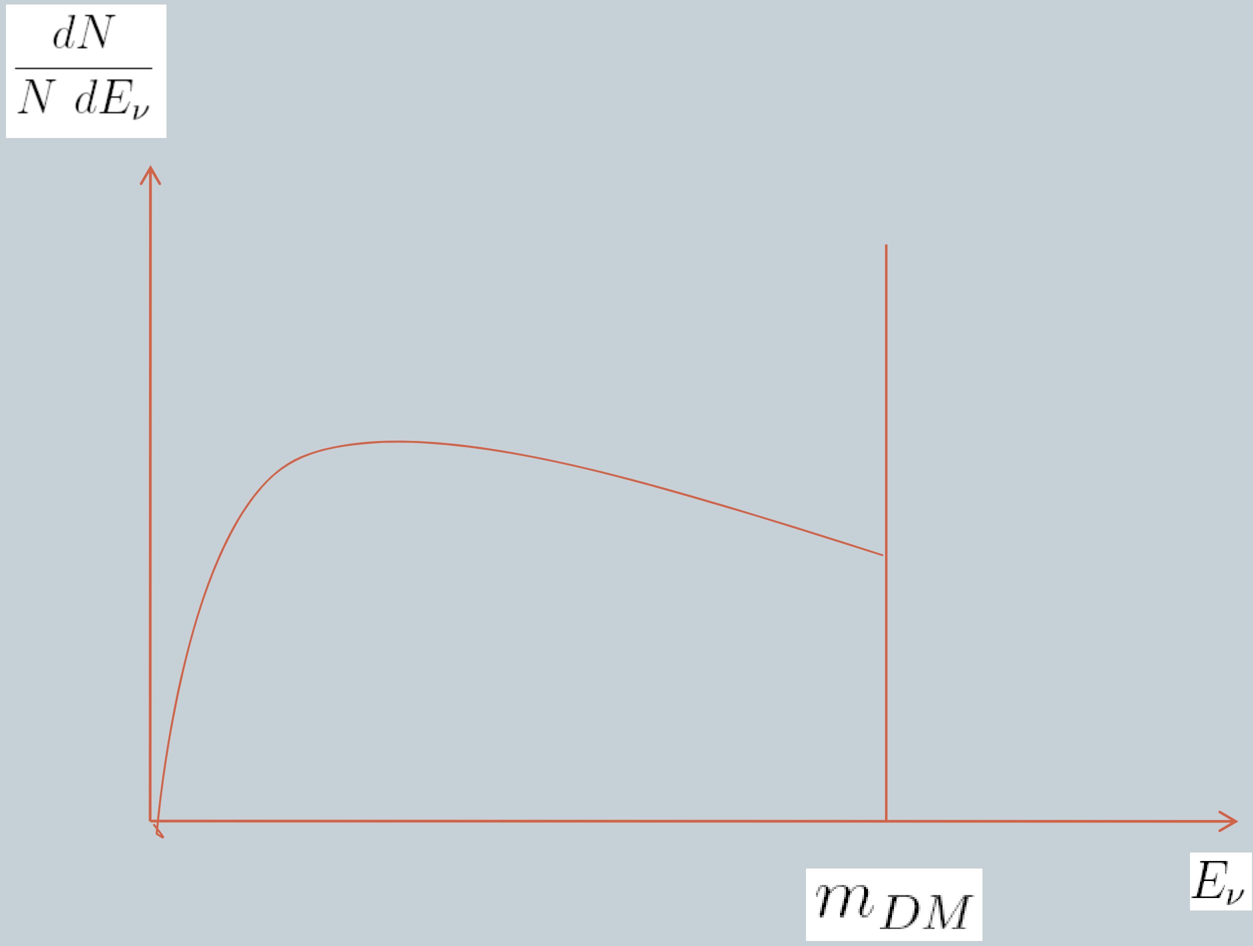
Sharp line remains sharp!

$$\frac{d\sigma[\nu(k)N \to \nu(k')X]}{dk'}$$

has no peak at

$$k' \to k$$

Spectrum after emerging from the Sun



Oscillation of neutrinos

- Oscillation inside the Sun
- Oscillation between Sun and Earth
- Oscillation in the Earth???

Oscillation in the Earth

$$\Delta m^2_{ij}/p \ll V_e$$

$$\theta_{eff} \to 0$$

Oscillation in matter

$$i\frac{d|\nu_\gamma\rangle}{dt} = [\frac{m_\nu^\dagger \cdot m_\nu}{2p} + \mathrm{diag}(V_e, 0, 0)]_{\gamma\sigma}|\nu_\sigma\rangle$$

$$i\frac{d|\bar{\nu}_\gamma\rangle}{dt} = [\frac{m_\nu^T \cdot m_\nu^*}{2p} - \mathrm{diag}(V_e, 0, 0)]_{\gamma\sigma}|\bar{\nu}_\sigma\rangle$$

$$V_e = \sqrt{2} G_F N_e .$$

$$N_e \begin{cases} \neq 0 & r < r_{sun} \\ = 0 & r > r_{sun} \end{cases}$$

Neutrino oscillation in matter

$$|\nu'_\alpha;\text{surface}\rangle = a_{\alpha 1}|1\rangle + a_{\alpha 2}|2\rangle + a_{\alpha 3}|3\rangle$$

$$|\nu'_\alpha;\text{detector}\rangle = a_{\alpha 1}|1\rangle + a_{\alpha 2}e^{i\Delta_{12}}|2\rangle + a_{\alpha 3}e^{i\Delta_{13}}|3\rangle$$

$$\Delta_{ij} \equiv \Delta m^2_{ij} L/(2E)$$

$$P(\nu_\alpha \to \nu_\mu) = \sum_i |a_{\alpha i}|^2 |U_{\mu i}|^2 +$$
$$2\Re[a^*_{\alpha 1} a_{\alpha 2} U_{\mu 1} U^*_{\mu 2} e^{i\Delta_{12}}] + 2\Re[a^*_{\alpha 1} a_{\alpha 3} U_{\mu 1} U^*_{\mu 3} e^{i\Delta_{13}}]$$
$$+ 2\Re[a^*_{\alpha 2} a_{\alpha 3} U_{\mu 2} U^*_{\mu 3} e^{i(\Delta_{13}-\Delta_{12})}]$$

Neutrino versus antineutrino

$$|\nu'_\alpha; \text{surface}\rangle = a_{\alpha 1}|1\rangle + a_{\alpha 2}|2\rangle + a_{\alpha 3}|3\rangle \; .$$

$$|\bar{\nu}'_\alpha; \text{surface}\rangle = \bar{a}_{\alpha 1}|\bar{1}\rangle + \bar{a}_{\alpha 2}|\bar{2}\rangle + \bar{a}_{\alpha 3}|\bar{3}\rangle .$$

Literature

- Lehner and Weiler, PRD77,Erkoca, Reno and Sarcevic PRD80, Cirelli, Fornengo, Montaruli, Sokalski, Strumia and Vissani, NPB727
- Barger ,Keung and Shaughnessy, PLB664
- Blennow, Melbeus and Ohlsson, JCAP 100
- Blennow, Edsjo and Ohlsson, JCAP 0801

WimpSIM

- **J. Edsjö, WimpSim Neutrino Monte Carlo, http://www.physto.se/~edsjo/wimpsim/**
- **M. Blennow, J. Edsjö and T. Ohlsson, [arXiv: 0709.3898]**

Oscillatory terms

$$|\nu'_\alpha; \text{detector}\rangle = a_{\alpha 1}|1\rangle + a_{\alpha 2}e^{i\Delta_{12}}|2\rangle + a_{\alpha 3}e^{i\Delta_{13}}|3\rangle$$

$$\Delta_{ij} \equiv \Delta m^2_{ij} L/(2E).$$

$$P(\nu_\alpha \to \nu_\mu) = \sum_i |a_{\alpha i}|^2 |U_{\mu i}|^2 +$$

$$2\Re[a^*_{\alpha 1} a_{\alpha 2} U_{\mu 1} U^*_{\mu 2} e^{i\Delta_{12}}] + 2\Re[a^*_{\alpha 1} a_{\alpha 3} U_{\mu 1} U^*_{\mu 3} e^{i\Delta_{13}}]$$

$$+ 2\Re[a^*_{\alpha 2} a_{\alpha 3} U_{\mu 2} U^*_{\mu 3} e^{i(\Delta_{13} - \Delta_{12})}]$$

Our work

- A. Esmaili and Y.F., "A **Novel Method to Extract Dark Matter Parameters from Neutrino Telescope Data,"** JCAP 1104 (2011) 007

A. Esmaili and Y.F., "**An Analysis of Cosmic Neutrinos: Flavor Composition at Source and Neutrino Mixing Parameters ,"** Nucl.Phys. B821 (2009) 197-214

Neutrino oscillation length

$$L_{osc} = \frac{4\pi E_\nu}{\Delta m^2_{12}} \sim 3 \times 10^{11}\ \text{cm}\ \left(\frac{E_\nu}{100\ \text{GeV}}\right)\left(\frac{8\times 10^{-5}\ \text{eV}^2}{\Delta m^2_{12}}\right)$$

Earth Sun distance: $L = 1.5 \times 10^{13}\ \text{cm}$

If the energy resolution $(\delta E/E)$ is worse than $1\ \%$ and the width of the spectrum is larger than $(\delta E/E)$, the oscillatory terms will average to zero.

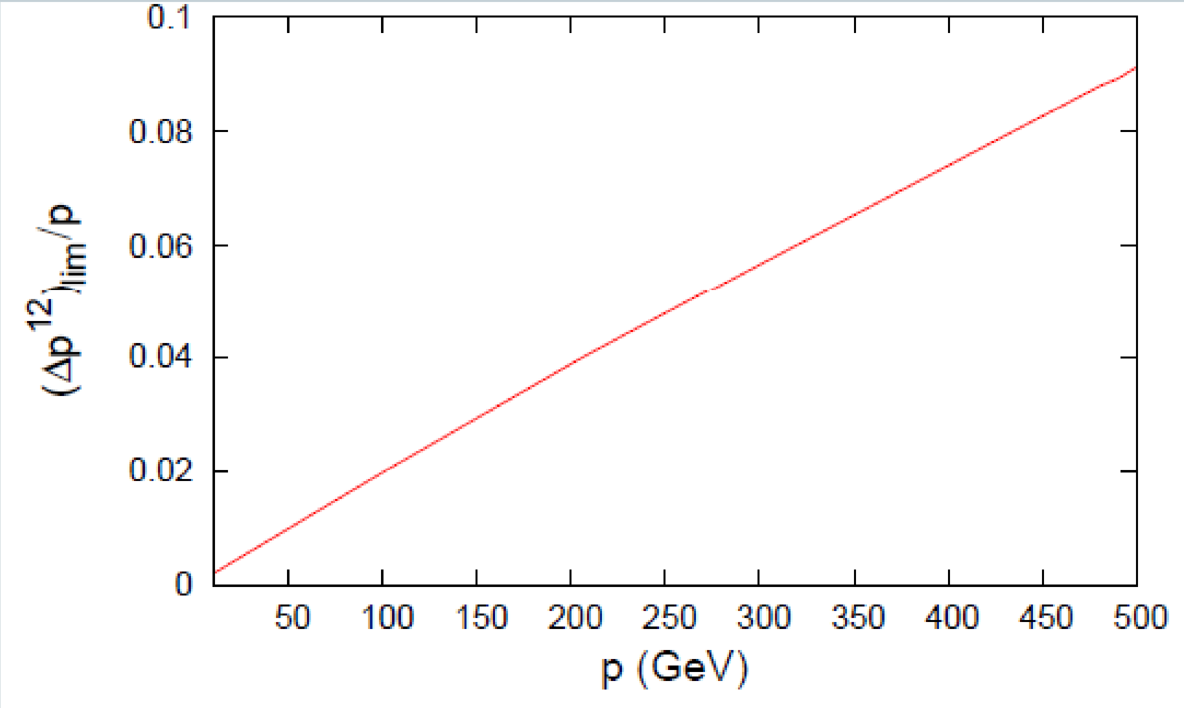
Averaging limit

$$|\nu_\alpha; p; L\rangle = a_{\alpha 1}(L)|1;p\rangle + a_{\alpha 2}(L)|2;p\rangle + a_{\alpha 3}(L)|3;p\rangle$$

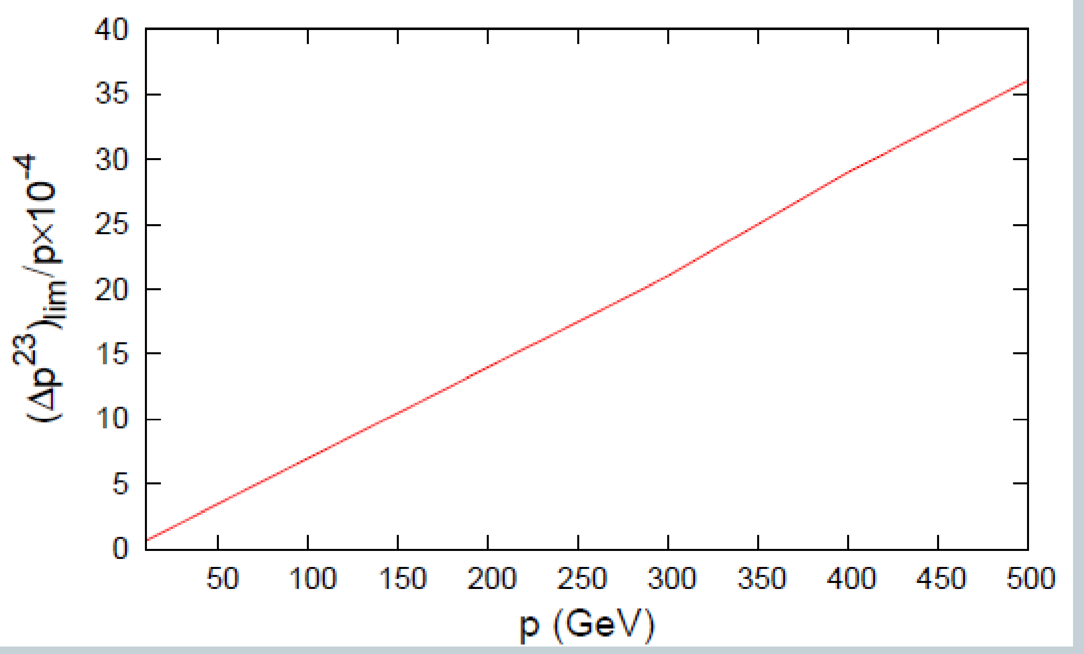
$$\arg\left[\frac{[a_{\beta i}(0)(a_{\beta j}(0))^*(a_{\alpha i}(L/c))^* a_{\alpha j}(L/c)]\,|_{p+(\Delta p^{ij})_{lim}}}{[a_{\beta i}(0)(a_{\beta j}(0))^*(a_{\alpha i}(L/c))^* a_{\alpha j}(L/c)]\,|_{p}}\right] = 2\pi$$

In vacuum: $\dfrac{(\Delta p^{ij})_{lim}}{p} = \dfrac{2\pi p}{L\Delta m_{ij}^2},$

Independent of α and β



Independent of θ_{13}



40
35
30
25
20
15
10
5
0
50
100
150
200
250
300
350
400
450
500
p (GeV)
(Δp²³)lim/p×10⁻⁴

Thermal widening

$$\bar{v} = (3T/m_{DM})^{1/2} \simeq 60\ \mathrm{km/sec}(100\ \mathrm{GeV}/m_{DM})^{1/2}.$$

$$\frac{\Delta E}{E} \sim \frac{\bar{v}}{c} \sim 10^{-4} \left(\frac{T}{1.3\ \mathrm{keV}}\right)^{1/2} \left(\frac{100\ \mathrm{GeV}}{m_{DM}}\right)^{1/2}.$$

Robust against solar models

Bottomline

THE THERMAL WIDENING OF THE MONOCHROMATIC LINE IS NOT ENOUGH FOR AVERAGING OUT THE OSCILLATORY TERMS.

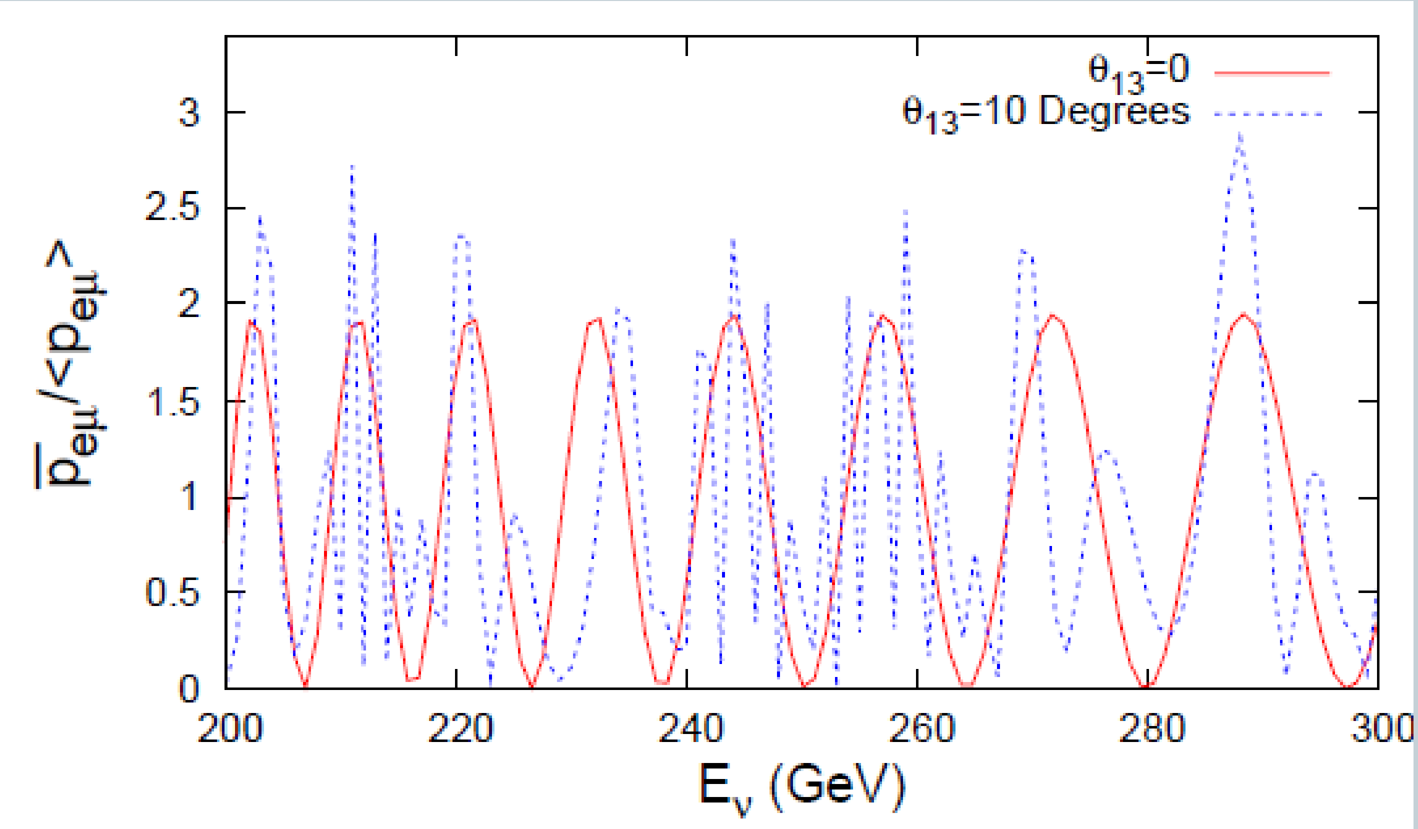
Averaging due to production point

$$r_{DM} \approx \left(\frac{9T}{8\pi G_N \rho m_{DM}}\right)^{1/2},$$

$$0.003 R_\odot (100\ \mathrm{GeV}/m_{DM})^{1/2}$$

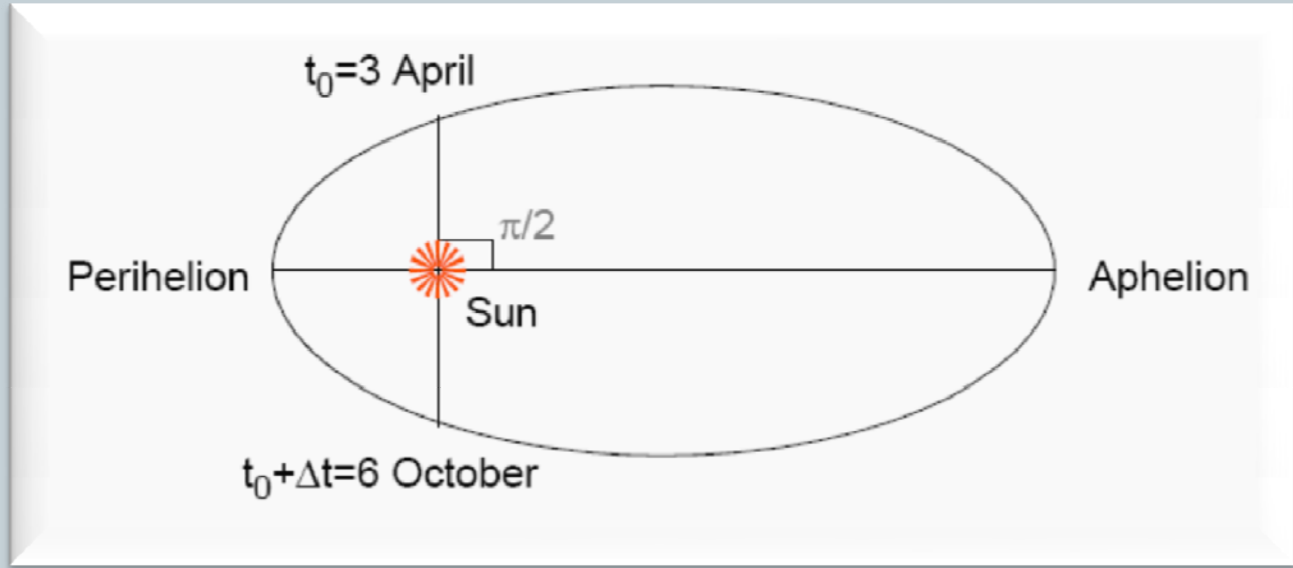
The average over production point: $\bar{P}_{\alpha\beta}$

Dropping the oscillatory terms: $\langle P_{\alpha\beta} \rangle$



θ13=0
θ13=10 Degrees
3
2.5
2
1.5
1
0.5
0
200
220
240
260
280
300
Eν (GeV)
p̄eμ/<peμ>

The earth orbit



Earth Sun distance variation is of order of

$$L_{osc} = \frac{4\pi E_\nu}{\Delta m^2_{12}} \sim 3 \times 10^{11}\ \text{cm} \left(\frac{E_\nu}{100\ \text{GeV}}\right)\left(\frac{8\times 10^{-5}\ \text{eV}^2}{\Delta m^2_{12}}\right)$$

$$P(\nu_\alpha \to \nu_\mu) = \sum_i |a_{\alpha i}|^2 |U_{\mu i}|^2 +$$

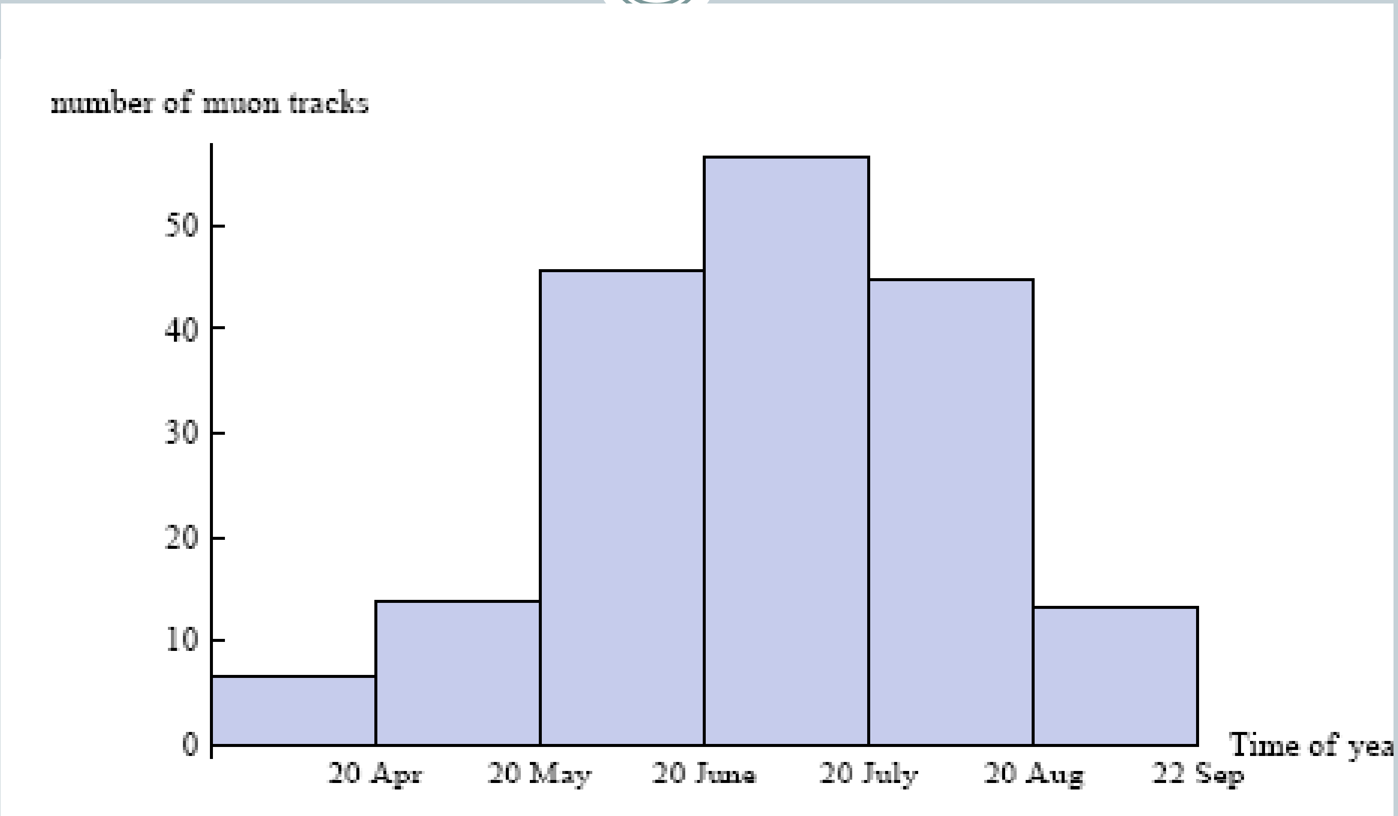
$$2\Re[a^*_{\alpha 1} a_{\alpha 2} U_{\mu 1} U^*_{\mu 2} e^{i\Delta_{12}}] + 2\Re[a^*_{\alpha 1} a_{\alpha 3} U_{\mu 1} U^*_{\mu 3} e^{i\Delta_{13}}] + 2\Re[a^*_{\alpha 2} a_{\alpha 3} U_{\mu 2} U^*_{\mu 3} e^{i(\Delta_{13}-\Delta_{12})}]$$

$$O_{12}(t, \Delta t) \equiv \frac{\int_t^{t+\Delta t} e^{i\Delta_{12}(t)} A_{eff}(t) L^{-2}(t) dt}{\int_t^{t+\Delta t} A_{eff}(t) L^{-2}(t) dt},$$

$$O_{13}(t, \Delta t) \equiv \frac{\int_t^{t+\Delta t} e^{i\Delta_{13}(t)} A_{eff}(t) L^{-2}(t) dt}{\int_t^{t+\Delta t} A_{eff}(t) L^{-2}(t) dt}.$$

For $E_\nu \sim 100$ GeV, $|O_{12}| \sim 1$ and $|O_{13}| \sim 0.1$

Variation of muon track



DM+DM→ $\nu_e\bar{\nu}_e$.

$m_{DM} = 270$ GeV

$C_\odot = 3.4 \times 10^{22}$ s^{-1}

Factoring out the distance factor

$$\tilde{N}(t_0, \Delta t) \equiv \frac{\int_{t_0}^{t_0+\Delta t} (dN_\mu/dt)\ dt}{\int_{t_0}^{t_0+\Delta t} A_{eff}(\theta[t])/[L(t)]^2\ dt}.$$

$$\Delta(t_0, \Delta t) \equiv \frac{\tilde{N}(t_0, \Delta t) - \tilde{N}(t_0 + \Delta t, 1\ \mathrm{year} - \Delta t)}{\tilde{N}(t_0, \Delta t) + \tilde{N}(t_0 + \Delta t, 1\ \mathrm{year} - \Delta t)}$$

E_ν (GeV)	Δ(20March, 186days)				Δ(3April, 186days)			
	$\theta_{13} = 0^\circ$		$\theta_{13} = 10^\circ$		$\theta_{13} = 0^\circ$		$\theta_{13} = 10^\circ$	
	NH	IH	NH	IH	NH	IH	NH	IH
100	18 %	18 %	9 %	11 %	12 %	12 %	6 %	7 %
300	57 %	57 %	37 %	42 %	60 %	60 %	39 %	43 %

$$\mathrm{DM} + \mathrm{DM} \rightarrow \nu_e + \nu_e$$

E_ν (GeV)	Δ(20March, 186days)				Δ(3April, 186days)			
	$\theta_{13} = 0^\circ$		$\theta_{13} = 10^\circ$		$\theta_{13} = 0^\circ$		$\theta_{13} = 10^\circ$	
	NH	IH	NH	IH	NH	IH	NH	IH
100	9 %	6 %	4 %	1 %	7 %	4 %	3 %	0.3 %
300	12 %	7 %	6 %	19 %	13 %	7 %	6 %	20 %

$$\mathrm{DM} + \mathrm{DM} \rightarrow \nu_\mu + \nu_\mu$$

A measure of oscillation

$$\Delta(t_1, \Delta t_1; t_2, \Delta t_2) \equiv \frac{\tilde{N}(t_1, \Delta t_1) - \tilde{N}(t_2, \Delta t_2)}{\tilde{N}(t_1, \Delta t_1) + \tilde{N}(t_2, \Delta t_2)}$$

$$\tilde{N}(t, \Delta t) \equiv \frac{\int_t^{t+\Delta t} (dN_\mu / dt) dt}{\int_t^{t+\Delta t} A_{eff}(\theta[t]) L^{-2}(t) dt}$$

Application

- Is there a sharp feature?
- Is production democratic?

$$n_0 \equiv n_{\nu_e} = n_{\nu_\mu} = n_{\nu_\tau}$$

$$\sum_\alpha P(\nu_\alpha \to \nu_\beta) = 1$$

$$\sum_\alpha n_{\nu_\alpha} P(\nu_\alpha \to \nu_e) = \sum_\alpha n_{\nu_\alpha} P(\nu_\alpha \to \nu_\mu) = \sum_\alpha n_{\nu_\alpha} P(\nu_\alpha \to \nu_\tau) = n_0$$

General case

$$\rho = \sum_\alpha n_\alpha |\nu'_\alpha\rangle\langle\nu'_\alpha|$$

$$|\nu'_\alpha; \text{surface}\rangle = a_{\alpha 1}|1\rangle + a_{\alpha 2}|2\rangle + a_{\alpha 3}|3\rangle$$

$$\bar{\rho} = \sum_\alpha n_\alpha |\bar{\nu}'_\alpha\rangle\langle\bar{\nu}'_\alpha|.$$

$$|\bar{\nu}'_\alpha; \text{surface}\rangle = \bar{a}_{\alpha 1}|\bar{1}\rangle + \bar{a}_{\alpha 2}|\bar{2}\rangle + \bar{a}_{\alpha 3}|\bar{3}\rangle$$

Definitions

$$O_{12}(t, \Delta t) \equiv \frac{\int_t^{t+\Delta t} e^{i\Delta_{12}(t)} A_{eff}(t) L^{-2}(t) dt}{\int_t^{t+\Delta t} A_{eff}(t) L^{-2}(t) dt},$$

$$O_{13}(t, \Delta t) \equiv \frac{\int_t^{t+\Delta t} e^{i\Delta_{13}(t)} A_{eff}(t) L^{-2}(t) dt}{\int_t^{t+\Delta t} A_{eff}(t) L^{-2}(t) dt}.$$

For $E_\nu \sim 100$ GeV, $|O_{12}| \sim 1$ and $|O_{13}| \sim 0.1$

Averaged oscillation probability

$$\langle P(\nu_\alpha \to \nu_\mu)\rangle|_t^{t+\Delta t} \equiv \frac{\int_t^{t+\Delta t} P(\nu_\alpha \to \nu_\mu) A_{eff} L^{-2}(t) dt}{\int_t^{t+\Delta t} A_{eff} L^{-2}(t)} =$$

$$\sum_i |a_{\alpha i}|^2 |U_{\mu i}|^2 + 2\Re[a^*_{\alpha 1} a_{\alpha 2} U_{\mu 1} U^*_{\mu 2} O_{12}] + \mathcal{O}(a^2_{\alpha i} U^2_{\mu i} O_{13}).$$

Muon tracks

- Muon track events from the sharp line are proportional to

$$K(t,\Delta t) \equiv \sum_{\alpha} n_{\alpha} \left(\langle P(\nu_{\alpha} \to \nu_{\mu}) \rangle |_{t}^{t+\Delta t} + r \frac{\sigma(\bar{\nu})}{\sigma(\nu)} \langle P(\bar{\nu}_{\alpha} \to \bar{\nu}_{\mu}) \rangle |_{t}^{t+\Delta t} \right)$$

Whole muon track events

$$\mathcal{A} + \mathcal{B} K(t,\Delta t),$$

What can be derived

$$\Delta m_{12}^2/m_{DM}$$

$$\mathcal{A}+\mathcal{B}\sum_{\alpha,i} n_\alpha \left(|a_{\alpha i}|^2|U_{\mu i}|^2 + r\frac{\sigma(\bar\nu)}{\sigma(\nu)}|\bar a_{\alpha i}|^2|U_{\mu i}|^2\right)$$

$$\mathcal{B}\left|\sum_{\alpha} n_\alpha \left(a^*_{\alpha 1}a_{\alpha 2}U_{\mu 1}U^*_{\mu 2} + r\frac{\sigma(\bar\nu)}{\sigma(\nu)}\bar a^*_{\alpha 1}\bar a_{\alpha 2}U^*_{\mu i}U_{\mu 2}\right)\right|$$

Valuable information but not enough to reconstruct the amplitude

- By dividing the time interval between spring equinox to autumn equinox to four periods, one can derive these combinations.

- If the present bounds are saturated, after 10 years of data taking, enough data can be collected to perform a ten per cent accuracy measurement.

The range for which the method is effective

$$m_{DM} < 500 \text{ GeV},$$

For larger masses, the sharp line will be reduced by scattering.

Conclusions

- For $100\ \mathrm{GeV} < m_{DM} < 500\ \mathrm{GeV}$, observing seasonal variation (i.e., nonzero Δ) means $\mathrm{DM} + \mathrm{DM} \to \nu\nu$ takes place dominantly and the flavor structure is not democratic.
- By studying the seasonal variation, the dark matter mass can be derived.
- Such a derivation (with 10 percent accuracy) requires more than 400 DM events.

Backup slides

Democratic production

$$n_0 \equiv n_{\nu_e} = n_{\nu_\mu} = n_{\nu_\tau}$$

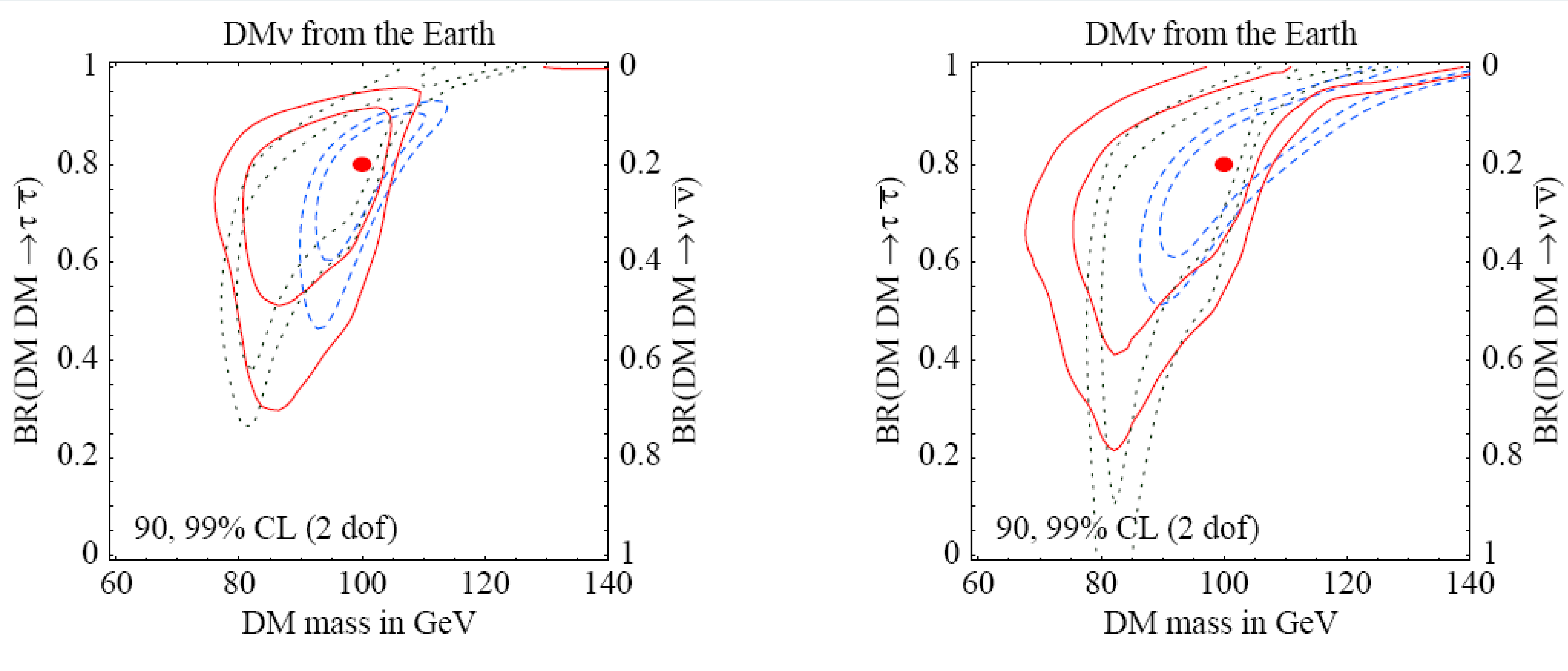
$$\sum_\alpha P(\nu_\alpha \to \nu_\beta) = 1$$

$$\sum_\alpha n_{\nu_\alpha} P(\nu_\alpha \to \nu_e) = \sum_\alpha n_{\nu_\alpha} P(\nu_\alpha \to \nu_\mu) = \sum_\alpha n_{\nu_\alpha} P(\nu_\alpha \to \nu_\tau) = n_0$$

Oscillatory terms do not show up.

In general $\mathrm{DM} + \mathrm{DM} \to \nu_\alpha + \nu_\beta$

Energy measurement



Cirelli, Fornengo, Montaruli, Sokalski, Strumia and Vissani, NPB727

Systematic analysis

$\mathcal{M}_{\alpha\beta}$ $\quad \mathrm{DM}+\mathrm{DM} \to \nu_\alpha + \overset{(-)}{\nu}_\beta$

$$|\psi\rangle = \sum_{\alpha\beta} \mathcal{M}_{\alpha\beta} |\nu_\alpha(\vec{p}_1) \overset{(-)}{\nu}_\beta(\vec{p}_2)\rangle$$

$$|\psi\rangle\langle\psi| = \sum_{\alpha\beta\gamma\sigma} \tilde{\rho}_{\alpha\beta,\gamma\sigma} |\nu_\alpha \overset{(-)}{\nu}_\beta\rangle\langle\nu_\gamma \overset{(-)}{\nu}_\sigma|$$

$$\tilde{\rho}_{\alpha\beta,\gamma\sigma} = \mathcal{M}_{\alpha\beta}\mathcal{M}^*_{\gamma\sigma} \qquad (\text{i.e., } \tilde{\rho}\log\tilde{\rho} = 0)$$

→ "reduced" matrix $\qquad (\text{i.e., } \rho\log\rho \neq 0)$

$$\rho_{\alpha\beta}|\nu_\alpha\rangle\langle\nu_\beta| \quad \text{with} \quad \rho_{\alpha\beta} = \sum_\gamma \tilde{\rho}_{\alpha\gamma,\beta\gamma} = (\mathcal{M}\mathcal{M}^\dagger)_{\alpha\beta}$$

Can we reconstruct $\mathcal{M}_{\alpha\beta}$?

$$\rho = V \rho_{diag} V^\dagger$$

$$\mathrm{DM} + \mathrm{DM} \to \nu_\alpha + \nu_\beta, \bar{\nu}_\alpha + \bar{\nu}_\beta$$

$$\mathcal{M} = V (\rho_{diag})^{1/2} V^T$$

$$\mathrm{DM} + \mathrm{DM} \to \nu_\alpha + \bar{\nu}_\beta$$

$$\mathcal{M} = V (\rho_{diag})^{1/2} V^\dagger$$